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Dynamics in Hybrid Cosmology within $f(R,\mathcal{G})$ Assisted by the Scalar Field ϕ

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Abstract

In this paper, we investigate a cosmic model in transition explaining the late expansion of the universe in $f(R,\mathcal{G})$ gravity where R and $\mathcal{G}$ are the Ricci scalar and the Gauss Bonnet invariant, respectively. The form of the scale factor used for this goal is $a(t)=t^{\lambda}e^{\sigma t}$ where λ and σ are free parameters but already constrained in order to meet the present value of Hubble constant. We deal with dynamical evolution of the universe through our model by describing the energy density, cosmic energy pressure and equation of state parameter of the scalar field ϕ considered as the only universe content. As results, our considered theoretical $f(R)$ model assisted by the scalar field ϕ indicates an phantom-like early evolution followed by accelerated expansion of universe leading to a conclusion that the investigated $f(R)$ model mimics the ΛCDM model in cosmological hybrid scale. Pacs numbers: 04.50.Kd, 95.36.+x, 98.80.-k.

Keywords

$f(R,\mathcal{G})$ Theory, FRW Space-Time and Observational Constraints and Sclar Field ϕ.

1. Introduction

Several cosmological theories suggest that we are living in the scenario of an accelerated cosmic expansion of the universe. Various observation results confirm this one the most treated enigma in cosmology. Among others, we can mention: the cosmic microwave background (*CMB*) [1], the type Ia supernova (*SNIa*) [2], the Weak lens [3], Baryon Acoustic Oscillations (*BAO*) [4], and redshift investigations such as 2*dF*, Galaxy Redshift Survey (2*dFGRS*) [5]. The study of redshift has shown that our Universe is almost spatially flat, homogeneous, isotropic on large scales [6] [7], Large-scale structures (*LSS*) [8]-[13]. These influences are caused by an unknown component called dark energy that appears to be two-thirds stored of the considerable energy density of the universe. Dark energy is considered as mysterious type of energy that possesses a massive amount of negative pressure responsable for the late expression of the cosmos at a higher rate. The simple and effective description of the accelerated expansion of the universe is based on a cosmological constant that is considered as the simplest candidate for dark energy. Other cosmological models are developed in attempt to deal with the current expansion of the universe as the cosmological parameter Λ [13]-[16].

The ΛCDM model shows good agreement with recent cosmological observations. Thus, this model is confronted to the question of coincidence and the problem of development on the theoretical level [17] [18]. To address these questions and describe their origin and the nature of dark energy, several dynamical models of dark energy, including modified gravity models that are based on extra dimension and scalar field while preserving the rules of General Relativity. Several interesting investigation like the those in [19]-[24] tell us more about this.

The scalar field of quintessence is one of the most recognized forms to explain dark energy with $w > -1$. It is kind of variable density scalar that serves as a dynamic quantity in space-time [25] based on the proportion of its potential energy (*PE*). Cosmological observations show that when the scalar field is repulsive, the potential energy is less than the kinetic energy and there is an accelerated expansion of the universe, while when the value of the potential energy is greater than the kinetic energy, the scalar field attracts and the universe is expanding at a slow pace. In addition, Chaplygin gas [26], K-petrol [27], Quintessence [28], Quintom [29] and tachyons [30] are some other dynamic dark energy models on offer. The model involving the ϕ scalar field as dark energy, generates a high negative pressure with a slow reduction potential. In the same context from the scalar field, several theories have been proposed to explain the evolutionary dynamics of the universe [31]-[33]. The idea of the tracker was first suggested by Johri [34], in which the evolution of the universe can be described by a single tracer with potential. Thus, many of the results from this theory strongly justify this. Our motivation for the studies of the dynamical properties of scalar fields in cosmology is the presence of the scalar field in many works published in renowned journals. In literature, a number of cosmic models have been discussed within the distinct framework of scalar field theories [35]-[39].

Referring to one of the work of Odintsov *et al.*, where they studied the inflationary phenomenon of the universe using a scalar field, we have defined in this work an action that depends on a function $f(R)$ where R is the Ricci scalar and the scalar field ϕ modifying the gravitational interaction. A system of equations governing the dynamics of the scalar and gravitational fields characterize the theory. These equations can be constructed, by adjusting the action using the scalar field (ϕ) and the metric tensor $g^{\mu\nu}$. Although the resulting equations are very complex and difficult to solve analytically, but in some cases, the resulting equations can be facilitated by the choice of a particular form for the function $f(R)$. Other key arguments in favour of his consideration is that gravity $f(R)$ can explain the easy accelerated expansion of the cosmos without the need for dark energy. In addition, the hypothesis could lead to alterations in gravity on the galaxy and provide a different explanation for the presence of dark matter. Recently, Moraes *et al.* [38], explore the cosmic model of the $f(R)$ theory describing different dynamic eras of the universe as the inflationary era, the radiation-dominated era, the matter-dominated era, and the current accelerated expansion age of the cosmos.

2. The Metric and $f(R)$ Gravity

The starting point of our work is the gravitational action, which for the $f(R)$ Einstein modified gravity has the following form

$$S=\int_{\Omega}\left(\frac{f(R)}{2\kappa^2}-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\xi(\phi)\mathcal{G}\right)\sqrt{-g}\,\mathrm{d}x^4 \tag{1}$$

A homogeneous and isotropic universe is modeled by the well known *FLRW* metric given by

$$\mathrm{d}s^2=-\mathrm{d}t^2+a(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{2}$$

where $a(t)$ denotes the scale factor. The Ricci scalar and the Gauss Bonnet invariant corresponding to the Friedmann metric are $R=6\left(2H^2+\dot{H}\right)$ and $\mathcal{G}=24H^2\left(H^2+\dot{H}\right)$, where $H=\frac{\ddot{a}}{a}$ denotes the Hubble rate and the dot "." represents the differentiation with respect to the cosmic time t. Furthermore, in order to simplify our work, we shall make the reasonable assumption that the scalar field is homogenous and thus it depends solely one the cosmic time.

Implementing the variation principle with respect to the metric tensor $g^{\mu\nu}$ and the scalar field ϕ, and by making using the Friedmann equation, we obtain the field equations for gravitational sector and the scalar field equation, which are,

$$\frac{3f_RH^2}{\kappa^2}=\frac{1}{2}\dot{\phi}^2+V+\frac{Rf_R-f}{2\kappa^2}-\frac{3H\dot{f}_R}{\kappa^2}+24\dot{\xi}H^3 \tag{3}$$

$$-\frac{2f_R\dot{H}}{\kappa^2}=\dot{\phi}^2+\frac{\ddot{f}_R-H\dot{f}_R}{\kappa^2}-16\dot{\xi}H\dot{H} \tag{4}$$

According to [39], we define $f(R)$ as

$$f(R)=\alpha R+\beta R^2, \tag{5}$$

where $f_R = \frac{\partial f}{\partial R} = \alpha + 2\beta R$, $\dot{f}_R = 12\beta\left(4H\dot{H} + \ddot{H}\right)$ and $\ddot{f}_R = 12\beta\left(4H\ddot{H} + 4\dot{H}^2 + \dddot{H}\right)$ Using the equations Equation (3)-(5), the expression for scalar field $\dot{\phi}^2$ and potential $V(\phi)$ after simplification are

$$\begin{aligned}\dot{\phi}^2 &= 12\beta H\left(H\dot{H} + \ddot{H}\right) + 16\dot{\xi}H\dot{H} - 2\dot{H}\left[\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right] \\ &\quad - 12\beta\left(4H\ddot{H} + 4\dot{H}^2 + \dddot{H}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}V(\phi) &= \left(3H^2 - 2\dot{H}\right)\left[\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right] + 36\beta\left(4H\dot{H} + \ddot{H}\right) \\ &\quad - 36\beta\left(2H^2 + \dot{H}\right)^2 - 24\dot{\xi}H^3 - 6\beta H\left(H\dot{H} + \ddot{H}\right) - 8\dot{\xi}H\dot{H} \\ &\quad + 6\beta\left(4\ddot{H}H + 4\dot{H}^2 + \dddot{H}\right)\end{aligned} \tag{7}$$

For a scalar field model to mimic a dark energy dynamic in the $FLRW$ universe, the energy density ρ_ϕ and cosmic pressure P_ϕ as the functions of scalar field ϕ are proposed as follows:

$$P_\phi = \frac{1}{2}\dot{\phi}^2\epsilon - V(\phi), \tag{8}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2\epsilon + V(\phi) \tag{9}$$

The induced relations from (6) and (7) are given by

$$\begin{aligned}P_\phi &= (1+\epsilon)6\beta H\left(H\dot{H} + \ddot{H}\right) + (1+\epsilon)8\dot{\xi}H\dot{H} + (2-\epsilon)\dot{H}\left(\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right) \\ &\quad + 24\dot{\xi}H^3 - (1+\epsilon)6\beta\left(4\ddot{H}H + 4\dot{H}^2 + \dddot{H}\right) - 3H^2\left(\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right) \\ &\quad - 36\beta\left(4H\dot{H} + \ddot{H}\right) + 36\beta\left(2H^2 + \dot{H}\right)^2\end{aligned} \tag{10}$$

$$\begin{aligned}\rho_\phi &= (\epsilon-1)6\beta H\left(H\dot{H} + \ddot{H}\right) + (\epsilon-1)8\dot{\xi}H\dot{H} - (\epsilon+2)\dot{H}\left(\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right) \\ &\quad + 24\dot{\xi}H^3 - (\epsilon-1)\beta\left(4H\ddot{H} + 4\dot{H}^2 + \dddot{H}\right) + 3H^2\left(\alpha + 12\beta\left(2H^2 + \dot{H}\right)\right) \\ &\quad + 36\beta\left(4H\dot{H} + \ddot{H}\right) - 36\beta\left(2H^2 + \dot{H}\right)^2\end{aligned} \tag{11}$$

The equation of state parameter is determined by

$$w_\phi = \frac{P_\phi}{\rho_\phi} \tag{12}$$

3. Cosmological Parameters in Hybrid Cosmology

The cosmological parameters from our model are a function of the Hubble parameter H as mentioned in the Equation (8), Equation (9). In the presence analysis, we investigate with some well known expressions of the scale factor. First, it is very important to recall here that scale factor plays a crucial role in dynamic survey of the universe. For example, when the scalar factor is a function of the cosmological constant $a(t) = \exp(\Lambda t)$, Λ is positive; the De Sitter universe is realized. Another inflationary scenario or dynamical evolution of the universe is supported by the so-called power-law scalar factor $a(t) = t^{2/3}$ in flat FRW

space-time. In the sequence, the scale factor in hibrid form effectively explains the necessary flipping behavior of the universe with the current accelerated expansion of the cosmos. Thus, to obtain an obvious transition model of the universe, we consider a hybrid scaling factor of the type $a(t)=t^{\lambda}\exp(\sigma t)$ where λ and σ are arbitrary constant. In several cosmological fields, several authors have used this hybrid form to explain the evolutionary behavior of the universe in transition [40]-[45]. The scale factor of such a hybrid provides a cosmic time dependent deceleration parameter defined as $q=\frac{\lambda}{(\lambda+\sigma t)^2}-1$ and the Hubble parameter is expressed as $H(t)=\sigma+\frac{\lambda}{t}$.

The hybrid scalar factor suggested above represents an expansion deceleration for $q>0$ of the cosmos when $t\frac{\sqrt{\lambda}-\lambda}{\sigma}$, while for $t\frac{\sqrt{\lambda}-\lambda}{\sigma}$, this explains the acceleration of the expansion era of universe ($q<0$). Thus, for the derived model, $V(\phi)$ and $\dot{\phi}^2$ for $\xi=\varrho a(t)=\varrho t^{\lambda}\mathrm{e}^{\sigma t}$ are recast as:

$$\begin{aligned}\dot{\phi}^2=&-\frac{12\beta}{t^4}(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)-\frac{16\lambda\varrho t^{\lambda+1}}{t^4}(\sigma t+\lambda)\\&+\frac{2\lambda}{t^4}\left[\left(24\sigma^2\beta+\alpha\right)t^2+2\lambda\varrho t+\lambda^2-\lambda\right]-\frac{24\lambda\beta}{t^4}(4\sigma t+4\lambda-3)\end{aligned}\tag{13}$$

$$\begin{aligned}V(\phi)=&\frac{3\sigma^2t^2+6\sigma\lambda t+3\lambda^2+2\lambda}{t^4}\left(\alpha t^2+24\beta\sigma^2t^2+48\beta\sigma\lambda t+24\beta\lambda^2-12\beta\lambda\right)\\&-\frac{72\beta\lambda t}{t_4}(2\sigma t+2\lambda-1)-\frac{36\beta}{t^4}\left(2\sigma^2t^2+4\sigma\lambda t+2\lambda^2-\lambda\right)^2\\&-\frac{24t^{\lambda+1}}{t^4}(\sigma t+\lambda)^3+\frac{6\beta}{t^4}(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)\\&+\frac{8\lambda t^{\lambda+1}\varrho}{t^4}(\sigma t+\lambda)\mathrm{e}^{\sigma t}+\frac{12\lambda\beta}{t^4}(4\sigma t+4\lambda-3)\end{aligned}\tag{14}$$

The pressure and the energy density previously defined in Equation (10) and Equation (11), respectively, are expressed according to the proposed model in the following form

$$\begin{aligned}\rho(\phi)=&-6\beta\frac{(\epsilon-1)}{t^4}(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)-(\epsilon-1)\frac{8\lambda t^{\lambda+1}\varrho}{t^4}(\sigma t+\lambda)\\&+\frac{\lambda}{t^4}(\epsilon+2)\left[\left(24\sigma^2\beta+\alpha\right)t^2+2\lambda\sigma t+\lambda^2-\lambda\right]+\frac{24t^{\lambda+1}\mathrm{e}^{\sigma t}}{t^4}(\sigma t+\lambda)^3-\mp_2\end{aligned}\tag{15}$$

and

$$\begin{aligned}P(\phi)=&-\frac{6\beta}{t^4}(\epsilon+1)(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)-(1+\epsilon)\frac{8\lambda\varrho t^{\lambda+1}\mathrm{e}^{\sigma t}}{t^4}(\sigma t+\lambda)\\&-\frac{\lambda(2-\epsilon)}{t^4}\left[\left(24\sigma^2\beta+\alpha\right)t^2+2\lambda\sigma t+\lambda^2-\lambda\right]+\frac{24t^{\lambda+1}}{t^4}(\sigma t+\lambda)^3\mathrm{e}^{\sigma t}-\mp_1\end{aligned}\tag{16}$$

The expression of the stade parameter is determined by

$$w(\phi)=\frac{-6\beta(\epsilon+1)(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)-(1+\epsilon)8\lambda\varrho\mathrm{e}^{\sigma t}(\sigma t+\lambda)t^{\lambda+1}-\dagger-\mp_1}{-6\beta(\epsilon-1)(\sigma t+\lambda)\left(\sigma\lambda t+\lambda^2-2\lambda\right)-(\epsilon-1)8\lambda t^{\lambda+1}\varrho(\sigma t+\lambda)+\ddagger-\mp_2}\tag{17}$$

where

$$\dagger = \frac{\lambda(2-\epsilon)}{t^4}\left[\left(24\sigma^2\beta+\alpha\right)t^2+2\lambda\sigma t+\lambda^2-\lambda\right]+\frac{24t^{\lambda+1}}{t^4}\left(\sigma t+\lambda\right)^3 e^{\sigma t} \tag{18}$$

$$\begin{aligned}\mp_1 &= \left(1+\epsilon\right)\frac{12\beta\lambda}{t^4}\left(4\sigma t+4\lambda-3\right)\\ &-\frac{3\left(\sigma t+\lambda\right)^2}{t^4}\left[\left(24\beta\sigma^2+\alpha\right)t^2+48\beta\sigma\lambda t+2\lambda^2-\lambda\right]\\ &+\frac{72\beta\lambda t}{t^4}\left(2\sigma t+2\lambda-1\right)+\frac{36\beta}{t^4}\left(2\sigma^2t^2+4\sigma\lambda t+2\lambda^2-\lambda\right)^2\end{aligned} \tag{19}$$

$$\ddagger = -\frac{\lambda}{t^4}\left(\epsilon+2\right)\left[\left(24\sigma^2\beta+\alpha\right)t^2+2\lambda\sigma t+\lambda^2-\lambda\right]+\frac{24t^{\lambda+1}e^{\sigma t}}{t^4}\left(\sigma t+\lambda\right)^3 \tag{20}$$

$$\begin{aligned}\mp_2 &= \left(\epsilon-1\right)\frac{12\beta\lambda}{t^4}\left(4\sigma t+4\lambda-3\right)\\ &+\frac{3\left(\sigma t+\lambda\right)^2}{t^4}\left[\left(24\beta\sigma^2+\alpha\right)t^2+48\beta\sigma\lambda t+2\lambda^2-\lambda\right]\\ &-\frac{72\beta\lambda t}{t^4}\left(2\sigma t+2\lambda-1\right)-\frac{36\beta}{t^4}\left(2\sigma^2t^2+4\sigma\lambda t+2\lambda^2-\lambda\right)^2\end{aligned} \tag{21}$$

4. Dynamics Study of the Universe through the Hybrid Cosmology

4.1. Digital Analysis

To study the viability of our model, we present in this section the numerical results of several cosmological parameters by comparing the results of the theory with the observational data.

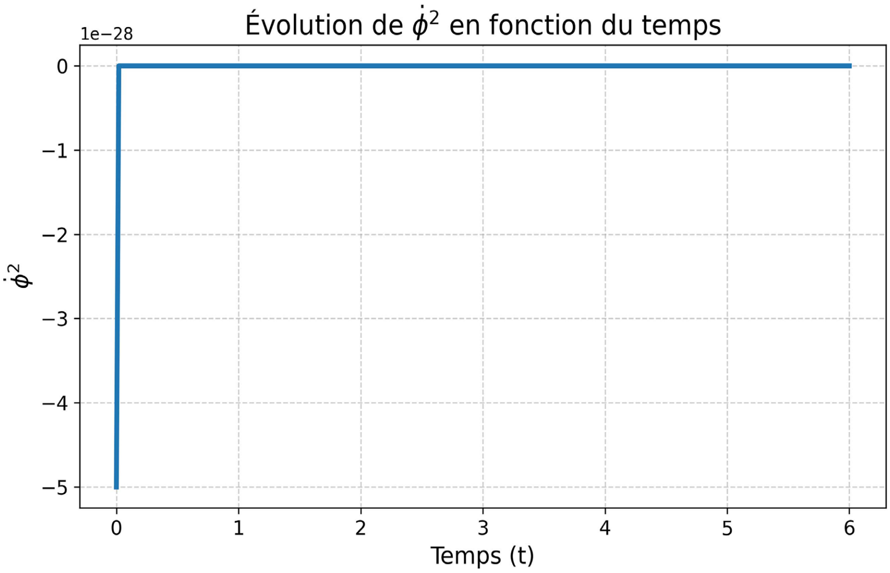

Figure 1. The graph presents the evolution of the scalar field parameter $\dot{\phi}^2$ over cosmic time t. The graph is plotted for $\lambda=1$, $\beta=0.96$, $\alpha=0.5186$, $\rho=10000$, $\kappa=1$, $\rho=1$ and $\epsilon=-1$.

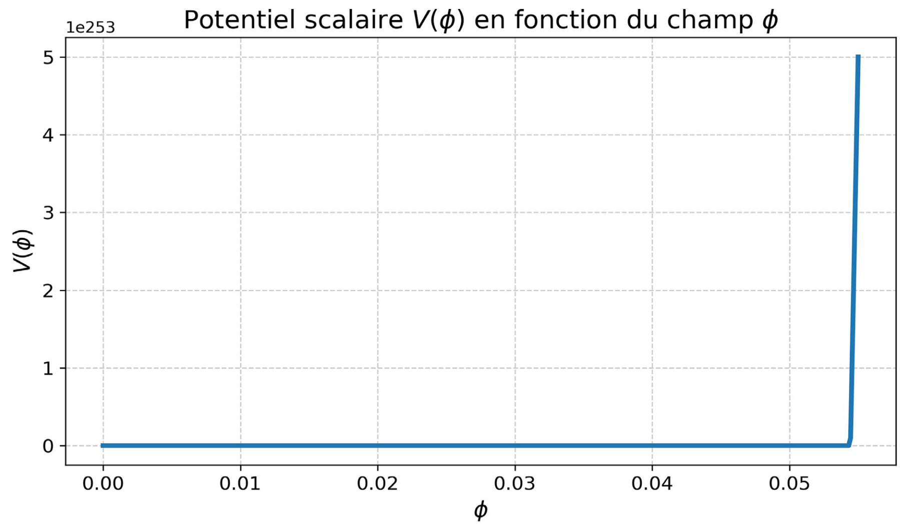

Figure 2. The graph shows the evolution of the potential of the scalar field $\dot{\phi}^2$ with respect to cosmic time $V(\phi)$ par rapport au temps cosmique t. The graph is plotted for $\lambda = 1$, $\beta = 0.96$, $\alpha = 0.5186$, $\rho = 10000$, $\kappa = 1$, $\rho = 1$ and $\epsilon = -1$.

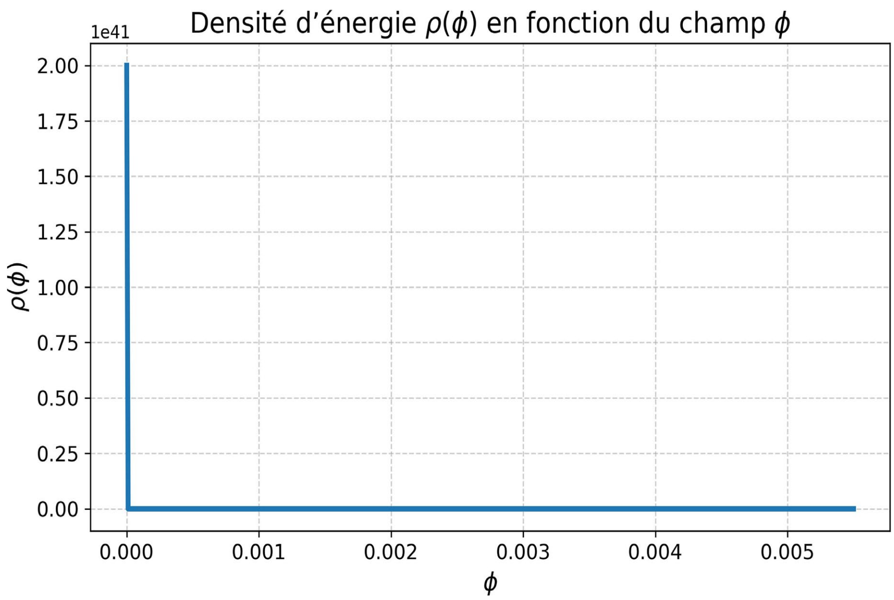

Figure 3. The graph shows the evolution of the energy density parameter $\rho(\phi)$ with respect to cosmic time t. The graph is plotted for $\lambda = 1$, $\beta = 0.96$, $\alpha = 0.5186$, $\rho = 10000$, $\kappa = 1$, $\rho = 1$ and $\epsilon = -1$.

4.2. Discussion

We provide numerically the behaviors of the scalar field in Figure 1, the potential of the scalar field in Figure 2, the energy density of the scalar field in Figure 3, the pressure density of the scalar field in Figure 4 and the equation of state parameter of the scalar field in Figure 5. The curve that describes the behavior of the scalar field density shows a decreasing rate with respect to cosmic time but its remains positive during the evolution of the universe. Moreover, the pressure density has rather remained negative, which explains the accelerated expansion of the Universe.

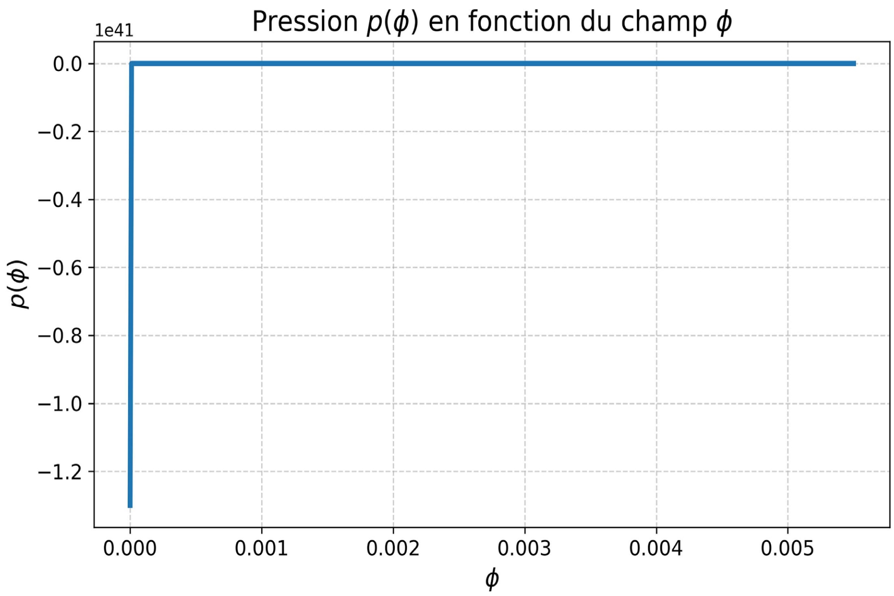

Figure 4. The graph shows the evolution of the parameter of the energy pressure $P(\phi)$ with respect to cosmic time t. The graph is plotted for $\lambda = 1$, $\beta = 0.96$, $\alpha = 0.5186$, $\rho = 10000$, $\kappa = 1$, $\rho = 1$ and $\epsilon = -1$.

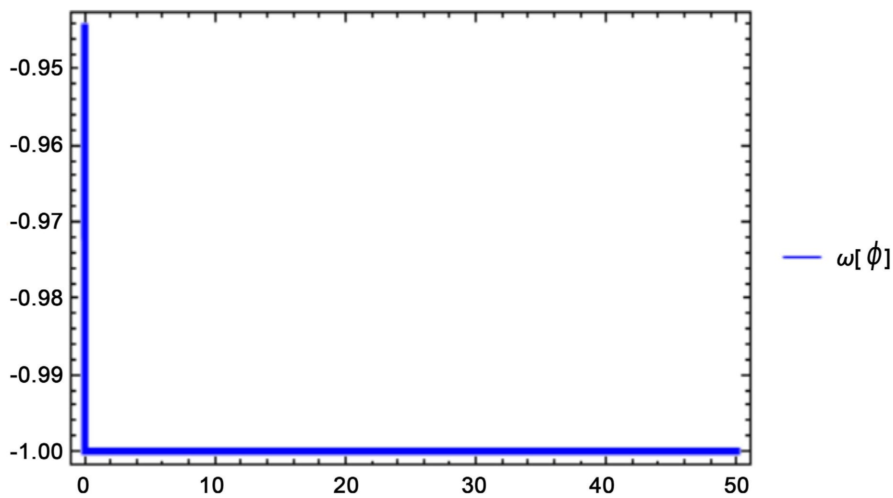

Figure 5. The graph shows the evolution of the state parameter $\omega(\phi)$ with respect to cosmic time t. The graph is plotted for $\lambda = 7$, $\beta = 2$, $\alpha = 5$, $\sigma = 20000$, $\kappa = 1$, $\rho = 3$ and $\epsilon = 1$.

When we take the better-fitting values of λ and σ, we notice that the plot of $\omega(\phi)$ as a function of cosmic time begins in the region of the quintessence ($-1 < \omega(\phi) < 0$) and tends ΛCDM ($\omega(\phi) \sim -1$)). The model shows short-term fluctuations during the first epoch but quickly demonstrates the essential characteristics of the model and is close to the ΛCDM. This scalar field hypothesis is confirmed by a number of cosmological measurements, such as ΛCDM radiation, *LSS* and observations of supernovae *Ia* .In short, our $f(R)$ model is a strong candidate for the to deal with the inflation era of the universe and the current acceleration of its expansion.

It is important to note that literature [46] [47], informs us that the astronomy of gravitational waves, which began 10 years ago with the famous LIGO detections, could, in principle, be fundamental for testing the actual viability of extended theories of gravity, including f(R) theories. The key point is that some differences between general relativity and modified gravity theories can be observed in linearized gravity. Indeed, modified gravity theories generally allow for more gravitational wave polarizations and, consequently, different interferometric response functions compared to standard general relativity.

In short, our model is a better candidate for explaining the universe's inflation

5. Conclusions

In this work, we explore a model of a universe in transition, describing the late accelerated expansion in the theory of gravity $f(R)$ where the model of $f(R)$ used is defined as $f(R)=\alpha R+\beta R^2$. We relied on one of the work of Odinsotsov *et al.* to be able to define our action S which is a function of the scalar field ϕ .

We proposed an explicit solution to the derived model using a scaling factor of the hybrid form $a(t)=t^{\lambda}\exp(\sigma t)$, and discussed the behavior of some cosmological parameters such as the kinetic form of scalar field, the scalar field potential, the energy density of the scalar field, the energy pressure of the scalar, and the state parameter as a function of cosmic time. The obtained results in the present investigation meet our goal and several results provided in literature.

The energy density $\rho(\phi)$ evolves positively and the scalar field pressure $P(\phi)$ for the derived model is negative throughout evolution. For the best- estimated values of α and β , the parameter *EoS* $\omega(\phi)$ starts in the region of the quintessence($-1<\omega(\phi)<0$) and approaches ΛCDM ($\omega(\phi)=-1$) at the end of the time.

To succeed in describing the inflation of the universe, there must be a scalar field with a potential that mocks the gravitational field. The late expansion of the cosmos in the phantom scenario is caused by a potential associated with a scalar field having negative kinetic energy. The behavior of the model in the shadow scenario can be seen in **Figure 1**.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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