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Some Consideration for a Quantum Theory of Gravitational Waves

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Abstract

In the present paper, a connection between gravity, space and quantum mechanics is discussed. In particular, the problem of directly **quantizing** space is by-passed by the suggestion, starting from a formal analogy with electrodynamic, to quantize the space metric. In this way, a possible path for the quantization of gravitational waves is proposed.

Keywords

Gravitational Waves, Quantum Theory, Graviton

1. Introduction

We are in debt to Albert Einstein for the great revolution which unified space and time and assigned to matter/energy the role of shaping the space. This great revolution had a mathematical description thank to the work of Minkowski and Riemann as extended by Ricci and Levi-Civita and allowed Einstein not only to give a better understanding of our Universe but also to give a key to make predictions on its possible evolution. The connection between the geometry of space and matter was accomplished in the theory of general relativity.

As it is well known Einstein [1] exposed his special theory of relativity in 1905 but soon after a few years in 1907 started to think to general relativity and worked on it until the theory had the first complete formulation in November 1915.

The leading idea in the development of special relativity was the constancy of the velocity of light in vacuum. The basis of the general relativity was the observation that the gravitational mass and the inertial mass are equal.

Relativity theory and quantum mechanics originated and developed in very different ways. The theory of relativity was a theoretical speculation of Einstein who made a severe critic of the usual ideas of space and time and started from the constancy of the light velocity, independent from the state of motion of emitter and absorber.

Later came the experimental confirmations of many of its predictions.

Quantum theory on the contrary originated by the necessity to explain the black body radiation, that classical laws were unable to explain and later from the study of the radiation emitted by atoms and molecules. Experimental facts played a rather subordinate role in the development of relativity while were fundamental in the development of quantum theory. The photoelectric effect, the Compton scattering, which proved the existence of the photon, the quantum of energy devised by Einstein on the base of a genial intuition entered as basic elements in the construction of quantum mechanics.

Einstein started to concentrate seriously on general relativity in 1911 after he arrived in Prague. During the summer 1912, when he returned to Zurich, he discovered that the space is not flat and the world geometry is not Euclidean but it is Riemannian.

Together with his friend the mathematician Marcel Grossmann he established the first connections between geometry and gravity. After months of strenuous work, he then presented the final version of the theory. To speak roughly, the general relativity geometrizes gravitation.

During his researches, Einstein was the first to formulate a theory of gravitational waves. The term *gravitational waves* appeared for the first time in 1905 in a paper by H Poincaré [2]. Eleven years later, in June 1916, Einstein [3] gave a first explicit formula to Poincaré's qualitative ideas.

Einstein was also the founder of the general relativistic cosmology, the modern theory of the universe on large scale, discussing how the University could be.

In his last years he tried to develop a unitary theory. The first idea of a unitary theory of fields starts in 1918 and his first proposal for a theory of this kind is in 1925. Einstein wanted to formulate a unitary theory of fields that not only unified gravitation with the electromagnetic forces but also gave the basis for a new interpretation of quantum phenomena that Einstein always considered not complete. To clarify this last point, may be useful to consider this letter in which he exposes his view [4].

The quantum mechanics is very imposing. But an inner voice tells me that it is still not Jacob. The theory yields much, but it hardly brings us nearer to the secret of the Old one. In any case I am convinced that he does not throw dices… I am toiling as deriving the equations of motion of material particles regarded as singularities from the differential equations of general relativity [5].

His view of a causal deterministic world was clearly exposed many years later in a paper with Natan Rose and Boris Podolsky [6] which originated the notion of entanglement.

The unitary theory of fields looked for by Einstein had to speak of the interaction between relativity and quantum theory as a unitary theory of fields. He was convinced that it must be possible to think of the external world as existing independently of the observing subject (the moon exists even if we do not look at it) and that the laws governing the objective world are strictly causal as exposed in his famous work with Natan Rosen and Boris Podolsky we quoted above. The world should be deterministic. He published several versions of this position but never succeeded in the task of making a unitary theory as he wanted.

In the general relativity theory geometry becomes an essential aid to describe the effect of gravity.

Notwithstanding the great advances in the theory, general relativity and quantum mechanics do not speak friendly each other and stand as two separate theories that are able to describe only a part of our world. All the attempts to construct a unified theory which encompasses all the laws of physics from a single law have failed.

Space in quantum mechanics is considered to be a continuous manifold, while theories of quantum gravity propose it is ***quantized*** with a discrete structure. An example is loop quantum gravity that suggest that space is made of fundamental units called spins.

In this theory, spins follow the laws of quantum mechanics. Other theories exist like string theory but no one has gained a general consensus.

To explain gravitation in Einstein's papers the Euclidean geometry needs to be replaced by Riemann geometry. Physics has to be described using a non-Euclidean geometry at 4 dimensions; its properties depend from ten coefficients of a quadratic form.

The gravitational field is characterized by these ten coefficients that are functions of the coordinates (t, x, y, z).

If the gravitational field is weak the special relativity is valid as a first approximation, only the general relativity is the sole able to explain the deviation of the light in a gravitational field, the exact value of the Mercury precession and the shift towards the red of light spectral lines coming from stars.

As far as the present author knows there is no fully accepted quantum theory for space or gravitation. Space seems to be a continuum manifold and no experimental evidence has been found at present of a fine structure. Gravitation has been linked to space by the general theory of relativity of Einstein and so it looks difficult to quantize it either. The present work tries to give a suggestion for a path to attain it, proposing to look in a slightly different direction and suggesting to quantize the space metric, rather than space directly.

2. Space Metric Quantization

The metric of a space may be defined considering the element of length that in an Euclidean space of three dimensions, referred to cartesian coordinates, is given by

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{1}$$

Riemann generalized this definition to n-dimensions defining

$$\mathrm{d}s^2 = g_{ij}\mathrm{d}x_i\mathrm{d}x_j \tag{2}$$

where the g's are functions of the x's. Relation (2) may be taken as a definition of space metric.

At this point, we may use of the Einstein's hypothesis of weak field [7] dividing the space metric $g_{\mu\nu}$ into two parts: the Minkowski [8] flat metrics $\eta_{\mu\nu}$ that is given by

$$\eta_{\mu\nu} = \begin{pmatrix} -\frac{1}{c^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{c^2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

by plus a small perturbative term $h_{\mu\nu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{4}$$

In this approximation $\left\| h_{\mu\nu} \right\| \ll 1$. If the squares and products of $h_{\mu\nu}$ can be neglected, one obtains the linear approximation of the field equations. This condition establishes that the gravitation field is weak and that the coordinate system is approximately cartesian. The quantities $h_{\mu\nu}$ bring to gravitational waves which propagate with the velocity of light. Gravitational waves may be considered small perturbations of space-time produced by movements of matter propagating with the light velocity.

In the approximation of weak field, it is possible to see not only the existence of gravitational waves but also to find that only two of the ten $h_{\mu\nu}$'s have a physical meaning. Accordingly, we may simplify the notation calling the two surviving values h_1 and h_2.

After 1918, Einstein mentioned anew gravitational waves when in 1937, he studied together with N. Rosen the exact solutions in the form of cylindrical waves of the gravitational wave equation [9]. Gravitational waves were discussed later by J. Weber [10]. They may be produced by the fusion between a black hole and a neutron star. This event has been effectively observed on June 28, 2021 by the collaboration Virgo, Ligo and Kagra [11].

Renormalizing $h_{\mu\nu}$ as

$$h_{\mu\nu} = h_{\mu\nu} - (1/2)\eta_{\mu\nu} h \tag{5}$$

where $h = \eta_{\mu\nu} h_{\mu\nu}$ is the trace of the perturbation, in Lorentz gauge, and inserting (4) in the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R \tag{6}$$

where $R_{\mu\nu}$ is the Riemann [12] tensor and

$$R = g_{il} R_{jl} \tag{7}$$

is the scalar curvature of space, it is possible to write the linearized Einstein equation in vacuum as [13]

$$\nabla^2 h_n + \frac{1}{c^2}\frac{\partial^2 h_n}{\partial t^2} = 0 \tag{8}$$

where from now on we consider only the two surviving components of $h_{\mu\nu}$ using a single subscript $n = 1, 2$.

Equation (8) is an equation similar of that for the electromagnetic vector potential in the electromagnetic theory. According we may quantise h_n by proceeding in the same way as in the electromagnetic theory, expanding h_n in a Fourier series

$$h_n = \sum\left[A_n(k,t)\exp(ikr) + A_n^*(k,t)\exp(-ikr)\right] \tag{9}$$

where the coefficients $A_n(k,t)$ and $A_n^*(k,t)$ should satisfy the equation

$$\nabla^2 A_n + \frac{1}{c^2}\frac{\partial^2 A_n}{\partial t^2} = 0 \tag{10}$$

or

$$k^2 A_n + \frac{1}{c^2}\frac{\partial^2 A_n}{\partial t^2} = 0 \tag{11}$$

with analogous equations for $A_n^*(k,t)$. We may write the Equation (11) also as

$$\frac{\partial^2 A_n(k,t)}{\partial t^2} + \omega_k^2 A_n(k,t) = 0 \tag{12}$$

where

$$\omega_k = ck \tag{13}$$

Now we replace A_n with an operator and observe that Equation (12) is the equation of a harmonic oscillator. The solution can be taken as

$$A_n(k,t) = A_n(k)\exp(-i\omega t) \tag{14}$$

and the complete h_n becomes

$$h_n = \sum\left[A_n(k,t)\exp(-i\omega t + ikr) + A_n^*(k,t)\exp(+i\omega t - ikr)\right] \tag{15}$$

The mode variables A_n and A_n^* can be replaced by generalized mode position coordinate q_n and mode momentum p_n in accordance with the transformation

$$A_n = B(\omega_n q_n + ip_n) \tag{16a}$$

$$A_n^* = B(\omega_n q_n - ip_n) \tag{16b}$$

where B is a suitable constant.

A single mode quantity that is a substitute of the "energy" can be defined as

$$E_n = \frac{1}{2}\left(p_n^2 + \omega_n^2 q_n^2\right) \tag{17}$$

This is precisely the usual form of the energy of a classical harmonic oscillator. The problem of the A_n associated with a mode has thus been made formally equivalent to the problem of a classical harmonic oscillator problem. The complete classical Hamiltonian H_0 is formed taking a sum over n of the single-mode expression (17).

$$H_0 = \frac{1}{2}\sum_n \left(p_n^2(t) + \omega_n^2 q_n^2(t) \right) \tag{18}$$

where

$$p_n(t) = \frac{\mathrm{d}q_n}{\mathrm{d}t}. \tag{19}$$

In this way, h_n is described in terms of a set of independent couples of conjugated variables, q_n and p_n, relative to a set of independent harmonic oscillators. The quantization is now achieved by regarding q_n and p_n as Hermitian operators obeying the commutation relations

$$[p_l, p_m] = [q_l, q_m] = 0 \quad [q_l, p_m] = i\hbar\delta_{lm} \tag{20}$$

according to the basic postulate of quantum mechanics. We may now use the standard procedure of quantisation of the harmonic oscillator with the introduction of a pair of non-Hermitian operators $\tilde{a}_n^+$ and $\tilde{a}_n$ by means of the equations

$$q_n = (\hbar/2\omega)^{1/2}\left[\tilde{a}_n^+ + \tilde{a}_n\right] \tag{21a}$$

$$p_n = (\hbar/2\omega)^{1/2}\left[\tilde{a}_n^+ - \tilde{a}_n\right] \tag{21b}$$

from which

$$\tilde{a}_n = (2\hbar\omega)^{-1/2}\left(\omega_n q_n + ip_n\right) \tag{22a}$$

$$\tilde{a}_n^+ = (2\hbar\omega)^{-1/2}\left(\omega_n q_n - ip_n\right) \tag{22b}$$

It can easily be seen from Equations (21a) and (21b) that $\tilde{a}_n$ and $\tilde{a}_n^+$ are Hermitian conjugate operators, while equation (20) shows that they obey the commutation relations

$$\left[\tilde{a}_l, \tilde{a}_m^+\right] = \delta_{lm} \quad \left[\tilde{a}_l, \tilde{a}_m\right] = \left[\tilde{a}_l^+, \tilde{a}_m^+\right] = 0 \tag{23}$$

The Hamiltonian of the system immediately follows as

$$H_0 = \sum_n H_n = \sum_n \hbar\omega_n \left(\tilde{a}_n^+, \tilde{a}_n + \tilde{a}_n, \tilde{a}_n^+\right). \tag{24}$$

If we now choose to use the Heisenberg picture, the time evolution of $\tilde{a}_n^+\tilde{a}_n$ and $\tilde{a}_n^+(t)$ is determined by the Heisenberg equations of motion

$$i\hbar\frac{\mathrm{d}\tilde{a}_n(t)}{\mathrm{d}t} = \left[\tilde{a}_n(t), H_0\right] = \hbar\omega_n\tilde{a}_n(t) \tag{25a}$$

$$i\hbar\frac{\mathrm{d}\tilde{a}_n^+(t)}{\mathrm{d}t} = \left[\tilde{a}_n^+(t), H_0\right] = -\hbar\omega_n\tilde{a}_n^+(t), \tag{25b}$$

having taken into account Equations (22a) and (22b). One has, according to Equations (25a) and (25b)

$$\tilde{a}_n(t) = \tilde{a}_n \exp(-i\omega_n t) \quad \tilde{a}_n^+(t) = \tilde{a}_n \exp(i\omega_n t) \tag{26}$$

where the quantities at zero time $\tilde{a}_n = \tilde{a}_n(0)$ and $\tilde{a}_n^+ = \tilde{a}_n^+(0)$ are, from now on, these operators in the Schrodinger picture.

The eigenvalues n_n of the operator associated with the eigenvalue equation

$$\tilde{a}_n^+ \tilde{a}_n |n_n\rangle = n_n |n_n\rangle \tag{27}$$

are furnished by all non-negative integers, so that n_n can be interpreted as the number of energy quanta (we may call them *gravitons*) in the mode n. The graviton is therefore a quantum of energy of the gravitational mode. The amplitude of the wave is proportional to the number of energy quanta or gravitons.

The eigenvalues of the graviton number operator n are unbounded, so that an arbitrary large number of gravitons may be found in the same quantum state, which means that gravitons are bosons and obey Bose-Einstein statistics. Equations (27) allows us to give $\tilde{a}_n^+ \tilde{a}_n$ the meaning of a numerical operator.

Furthermore, $\tilde{a}_n^+(t)$ and $\tilde{a}_n$ are the usual "creation" and "annihilation" operators, since they can be shown, respectively, to increase and lower by one the number of quanta, when they are operating on the eigenstates of $\tilde{a}_n^+ \tilde{a}_n$, according to

$$\tilde{a}_n^+ |n_n\rangle = (n_n + 1)^{1/2} |n_n + 1\rangle \tag{28a}$$

$$\tilde{a}_n |n_n\rangle = (n_n)^{1/2} |n_n - 1\rangle \tag{28b}$$

The quantity A_n can be written

$$A_n(r,t) = \sum_n (\hbar/2\varepsilon_0 V\omega_n)^{1/2} \left[\tilde{a}_n \exp(-i\omega_n t + ikr) + \tilde{a}_n^+ \exp(i\omega_n t - ikr)\right] \tag{29}$$

and consequently

$$\begin{aligned} h_n &= \sum_n \left[A_n(k,t)\exp(ikr) + A_n^*(k,t)\exp(-ikr)\right] \\ &= \sum_n (\hbar/2\varepsilon_0 V\omega_n)^{1/2} \left[\tilde{a}_n \exp(-i\omega_n t + ikr) + \tilde{a}_n^+ \exp(i\omega_n t - ikr)\right]. \end{aligned} \tag{30}$$

3. Conclusions

Equation (30) expresses the quantization of h_n. It means that the metric is fluctuating with a small granular structure with amplitude h_n.

Because the metric of space is linked to the amount of matter/energy present in it, this allows speculations about the granular structure of matter (may be quark at the final stage) and energy (it seems a bit more difficult perhaps to quantise the energy content in the Universe).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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