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Planck's Constant—A Bridge to Charge and Entanglement

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Abstract

Planck's constant was introduced phenomenologically to fit blackbody radiation, yet its physical origin has remained unclear. In this work, an elementary fermion is modeled as two coupled real strings. The coupling generates tension within the strings, and this tension is shown to be proportional to the coupling strength through Planck's constant. Equation (25) provides a direct interpretation: hhh emerges as the universal unit of action that characterizes the intrinsic response of coupled fields, independent of particle mass or size. This mechanical derivation grounds h in first principles, rather than treating it as an empirical input. While later work extends this framework to electric charge and entanglement through Noether symmetry and internal phase correlations, the present paper is restricted to establishing the physical basis of Planck's constant.

Keywords

Planck Constant, Strings, Coupling Interaction, Electric Charge, Entanglement

1. Introduction

In 1900, Max Planck introduced the constant h while analyzing blackbody radiation. His goal was to reconcile experimental spectra with thermodynamic laws.

He **postulated** that energy exchange between matter and radiation occurs in discrete quanta: $E = nh$, with $n \in \mathbb{Z}$.

At the time, this was a **mathematical trick** to fit the observed spectrum, not derived from deeper physics.

Planck himself was uneasy: he considered it a "purely formal assumption" without a clear physical mechanism.

Thus, h entered physics **phenomenologically**—a constant needed to explain data, not yet grounded in microscopic dynamics.

In our coupled-strings framework: We start with **two real strings** (germions) coupled by a constant κ.

Their **oscillatory exchange of energy and phase** naturally produces a fundamental unit of action.

The combination of **tension × oscillation period × phase increment** yields a universal constant with the same dimensions as Planck's h.

Unlike Planck's ad hoc introduction, here h is **derived** from a **deterministic, physical mechanism**: the dynamics of coupled fields in real spacetime.

Planck's constant is thought to be a fundamental physical constant defined in the realm of quantum theory. However, thus far, physicists do not have a convincing explanation for why action in the microcosmos is quantized or why h has a specific quantitative constant value.

Classical quantum theory is the basis for our concept of modern physics elementary particles theory. Ever since its introduction in the early years of the 20th century.

The birth of quantum mechanics is commonly attributed to the discovery of the Planck relation. In order to explain black-body radiation, Planck postulated that the radiation energy is transmitted in packages ("energy quanta"). Einstein later has found that light is absorbed by an electron in small "packets", which, like Planck's "energy quanta", is proportional to the light frequency ν. This relation is now called the Planck relation or Planck–Einstein relation: $E = h\nu$, where the constant "h" is "Planck's constant". Its value is [1] $6.662607015 \times 10^{-34}$ JSec and it usually appears as $\frac{h}{2\pi} = \hbar = 1.054571817 \times 10^{-34}$.

It has become one of the most important universal constants in physics. Yet, the exact physical meaning of Planck's constant is unknown; it has not been derived based on first principles.

Planck Constant also plays an important role in the creation of cosmological units such as the Planck length, Planck's time Planck's mass, etc. They all connect G-the gravitational constant and c-the speed of light.

Planck's constant links cosmological constants (G, c) with the quantum domain, bridging cosmological and microscopic phenomena (see for instance Wesson [2] and Kwiat [3]).

Several approaches have been described recently (e.g., Lipovka [4], Bruchholz [5] and Chang [6]), trying to derive h from basic principles.

Lipovka [4] suggested that the Planck constant is actually the adiabatic invariant of the electromagnetic field, characterized by scalar curvature of space of the Riemann-Cartan geometry. The main result of his work was to obtain the ratio between Riemannian scalar curvature of the Universe R, the Cosmological constant Λ and Planck's constant h.

Bruchholz [5] claims that since a photon must have a geometric boundary (which is why it behaves like a particle), the integration of its energy density (based on

Maxwell equations) over a bounded volume must have $E = h\nu$.

Chang [6], by using the Maxwell theory, have, in a similar manner to Bruchholz [5], assumed a finite size photon. Thus, a relationship is established between the total electromagnetic energy of a single photon, its frequency, its width (Q factor) and the dielectric qualities of the vacuum. This provides a similar relation $E = h\nu$.

In the current work, a different approach to quantum mechanics was used. Referring to wave functions as a combination of real fields and observing of the differential equations as representing geometrical qualities of coupled classical strings. Assume the coupled string-like real wave functions, undergo a mutual exchange interaction. This leads us to the understanding that Planck constant h is the result of exchange interactions between two coupled strings.

Though this work uses classical strings, it may be just as well extended to the concept of strings as the basic structure units of elementary particles. (Mukhi [7] and Dine [8]).

While this framework also offers insights into charge and entanglement, this work is deliberately restricted to the emergence of Planck's constant.

2. A Real Presentation of Schrödinger Equation

The basic equation of quantum mechanics is the one particle time-dependent Schrödinger equation:

$$-i\hbar \frac{\partial}{\partial t}\psi(x,t) = \mathcal{H}\psi(x,t) \tag{1}$$

where $\hbar$ is the reduced Planck constant which is $h/2\pi$ $\psi(x,t)$ is the complex wave function of the quantum system, x is the position in a one-dimensional coordinate system, and t the time. $\mathcal{H}$ is the Hermitian Hamiltonian operator (which characterizes the total energy of the system under consideration).

By decomposing the complex wave function $\psi(x,t)$ into real and imaginary components

$$\psi(x,t) = \Psi = \varphi_1 + i\varphi_2 \tag{2}$$

the Schrödinger equation may be written:

$$-i\hbar \frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi = (\mathcal{H}_r + i\mathcal{H}_i)(\varphi_1 + i\varphi_2) \tag{3}$$

$$+\hbar \frac{\partial}{\partial t}\varphi_2 = \mathcal{H}_r\varphi_1 - \mathcal{H}_i\varphi_2 \tag{4}$$

$$-\hbar \frac{\partial}{\partial t}\varphi_1 = \mathcal{H}_i\varphi_1 + \mathcal{H}_r\varphi_2 \tag{5}$$

In other words, the traditional Schrödinger equation is in fact two coupled equations of real wave functions, with real operators on a real 3-dimensional space [9].

For a time-independent classical Hamiltonian of a free particle, with mass m:

$$\mathcal{H} = \frac{p^2}{2m}$$

$$\mathcal{H}_r = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \quad \mathcal{H}_i = 0$$

When separated into real and imaginary components, these are equivalent to:

$$\mathcal{H}_r\varphi_1 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_1 = +\hbar\frac{\partial}{\partial t}\varphi_2 \tag{6}$$

$$\mathcal{H}_r\varphi_2 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_2 = -\hbar\frac{\partial}{\partial t}\varphi_1 \tag{7}$$

This provides two coupled equations of the two real wave functions:

$$\frac{\partial\varphi_1}{\partial t} = +\frac{\hbar}{2m}\frac{\partial^2\varphi_2}{\partial x^2} \tag{8}$$

$$\frac{\partial\varphi_2}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\varphi_1}{\partial x^2} \tag{9}$$

It will be assumed herewith, that the quantum description and characteristics of a single particle are the result of a coupling interaction between two components (fields) which composes the single "particle".

Based on this assumption, it will be described in the following, how can this real interpretation suggest an explanation to the non-relativistic Schrödinger equation through an interacting coupled two-strings classical model.

3. Tension in a Classical String

Let us start with a description of the forces in a classical one-dimensional, time independent, string (see **Figure 1**).

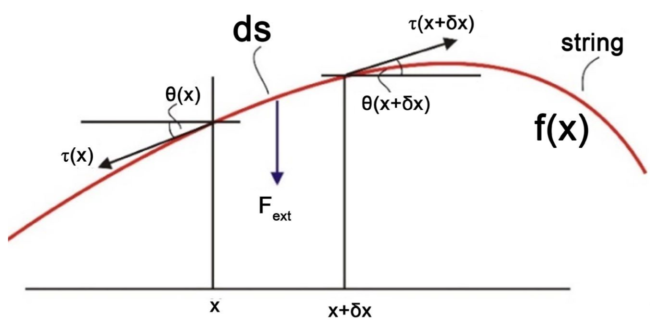

Figure 1. Tension forces on an infinitesimal string element.

The diagram illustrates the internal tension acting along both ends of a string segment and the external vertical force applied to it. The horizontal components follow the string axis, while the vertical component arises from external interaction. Together they demonstrate how classical string tension is distributed, providing the basis for extending the analysis to coupled strings and ultimately to the derivation of Planck's constant. Let the spatial distribution of a 1*-dimensional string of mass density* ρ *be described by the function* $f(x)$, f *being the amplitude.*

Internal tension forces on the string are at two opposite directions. We will assume that the magnitude of the tension $\tau(x)$ is the same along the string.

Additionally, there is an external force $F_{ext,y}$ acting vertically on the infinitesimal element d*s*. This external force is due to some external interaction.

The total horizontal component $F_{tot,x}$ of the force on the elemental d*s* is given by

$$F_{tot,x} = \tau(x+\delta x)\cos\theta(x+\delta x) - \tau(x)\cos\theta(x) \tag{10}$$

While the total vertical component $F_{tot,y}$ of the force on the elemental d*s* is given by

$$F_{tot,y} = \tau(x+\delta x)\sin\theta(x+\delta x) - \tau(x)\sin\theta(x) + F_{ext,y} \tag{11}$$

For infinitesimal small element d*s*, one may replace $\sin\theta \approx \tan\theta$ $\left(=\frac{\partial f(x)}{\partial x}\right)$.

Hence

$$F_{tot,y} \approx \tau(x+\delta x)\frac{\partial f(x+\delta x)}{\partial x} - \tau(x)\frac{\partial f(x)}{\partial x} + F_{ext,y} \tag{12}$$

$$F_{tot,x} \approx \tau(x+\delta x) - \tau(x) = \frac{\partial \tau(x)}{\partial x}\delta x \tag{13}$$

Thus

$$F_{tot,y} \approx \tau(x)\left[\frac{\partial f(x+\delta x)}{\partial x} - \frac{\partial f(x)}{\partial x}\right] + F_{ext,y} \tag{14}$$

d so

$$F_{tot,y} \approx \tau(x)\frac{\partial^2 f(x)}{\partial x^2}\delta x + F_{ext,y} \tag{15}$$

4. Interacting Strings

Consider next two strings φ_1 and φ_2. Let $\varphi_1(x,t)$ represent the amplitude of string 1 at time *t* and at position *x*. Let τ_s be some tension force in the string. As shown above, the net force exerted by this tension, on a small string element d*s* (see **Figure 2**) is connected to the amplitude change along the *x* axis and is described by:

$$F_s = \tau_s \frac{\partial^2 \varphi_1(x,t)}{\partial x^2} \tag{16}$$

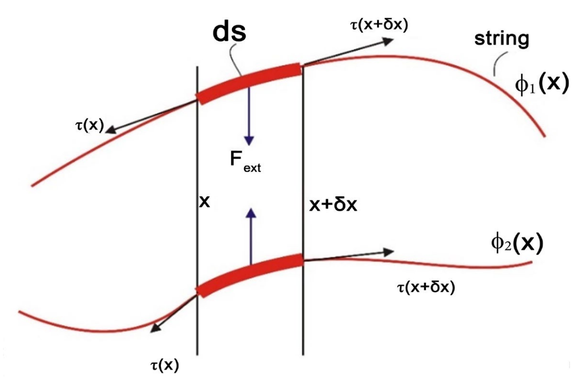

Figure 2. Tension and mutual forces between two interacting strings.

Each infinitesimal element of one string experiences internal tension along its axis and an additional coupling force exerted by the neighboring string. The coupling force, proportional to the displacement of the second string, either attracts or repels the first string in the opposite direction. This schematic illustrates the mechanism of mutual perturbation that unifies the two real strings into a single complex representation, forming the physical basis of the Schrödinger equation in this model.

Assume next, a second string is near the first one and is interacting with it by means of some coupling force, which couples the two strings together. Suppose now the second string, described by $\varphi_2(x,t)$, undergoes some small temporal perturbation

$$\Delta\varphi_2 \approx -\frac{\partial\varphi_2}{\partial t}\Delta t \tag{17}$$

This perturbation induces a change in the coupling force F_{21}, exerted by string 2 on string 1. This force is proportional to $\Delta\varphi_2$ and attracts or repels string 1, in the opposite direction of $\Delta\varphi_2$.

We denote this proportionality coupling constant by k_s.

We will also assume, without loss of generality, that the coupling between the two strings is proportional to the mass of d*s*. This is a reasonable assumption as we may think that the more mass, the stronger the coupling.

All in all, the assumptions made are the following:

Assumption 1 (Hook's Law): The coupling force is proportional to displacement $\Delta\varphi_2$ of string 2. We will denote this proportionality coupling constant by k_s.

Assumption 2 (mass law): The coupling between the two strings is proportional to the mass of the elemental d*s*.

The disturbance in the force is described by:

$$\Delta F_{ext} = -\rho \mathrm{d}s\left(k_s\Delta\varphi_2\right) = -\rho \mathrm{d}s k_s \Delta t\frac{\partial\varphi_2}{\partial t} \tag{18}$$

$$\Delta F_{ext} = -\rho \mathrm{d}s k_s \Delta\varphi_2 \tag{19}$$

And from the projection of d*s* on *x*:

$$\Delta F_{ext,y} = -k_s\frac{\partial\varphi_2}{\partial t}\Delta t\rho \mathrm{d}s\cos\theta = -k_s\frac{\partial\varphi_2}{\partial t}\Delta t\rho\delta x \tag{20}$$

k_s is the proportionality factor, which depends on the strength of the coupling. Therefore, by Equation (15):

$$\Delta F_{tot,y} \approx \left(\frac{\partial\tau(x)}{\partial t}\frac{\partial^2\varphi_1(x)}{\partial x^2} - k_s\rho\frac{\partial\varphi_2(x)}{\partial t}\right)\Delta t\delta x \tag{21}$$

At equilibrium $\Delta F_{tot,y} = 0$, and so:

$$\frac{\partial\varphi_2(x)}{\partial t} = \frac{1}{\rho k_s}\frac{\partial\tau}{\partial t}\frac{\partial^2\varphi_1}{\partial x^2} \tag{22}$$

By symmetry reason, the action of disturbance string 1 on tension in string 2 will be described by (force in the opposite direction)

$$\frac{\partial \varphi_1(x)}{\partial t} = -\frac{1}{\rho k_s}\frac{\partial \tau}{\partial t}\frac{\partial^2 \varphi_2}{\partial x^2} \tag{23}$$

Equations (22) and (23) represent a coupling between two real strings.

In fact, Equations (22) and (23) can be combined into a single equation. This is done by introducing a complex string $\psi = \varphi_1 + i\varphi_2$, so the two equations are unified to read

$$i\hbar\frac{\partial \psi(x)}{\partial t} = -\frac{\hbar^2}{2\rho}\frac{\partial^2 \psi(x)}{\partial x^2} \tag{24}$$

where $\frac{1}{\rho k_s}\frac{\partial \tau}{\partial t}$ was replaced by $\frac{\hbar}{2\rho}$.

Equation (24) is Schrodinger equation for a complex string ψ.

Are we allowed to assume $\frac{1}{\rho k_s}\frac{\partial \tau}{\partial t} = \frac{\hbar}{2\rho}$?

Looking at the term $\frac{1}{k_s}\frac{\partial \tau}{\partial t}$, we see that it has units of angular momentum. We will thus assume:

$$\boxed{\frac{1}{k_s}\frac{\partial \tau}{\partial t} = \frac{1}{2}\hbar} \tag{25}$$

Equation (25) assigns a clear physical interpretation to Planck's constant.

It demonstrates that h is not determined by the mass or size of a particle but instead arises from the intrinsic dynamics of the coupled string fields. In this framework, hhh quantifies the response of the string tension to perturbations generated by exchange interactions. The proportionality expresses how the internal restoring forces of the coupled strings universally react to coupling, thereby defining a fundamental quantum of action that is independent of any material parameters.

Unlike classical mechanics, where tension is an externally applied force, here the tension τ\tauτ is an inherent structural property of the fields themselves. When combined with the exchange interaction (with dimensions of 1/s^2; see Appendix B), the resulting proportionality naturally acquires the dimensions of angular momentum, making Planck's constant emerge as a direct consequence of field dynamics rather than as an empirical constant. The left hand side of Equation (25) is a constant. Therefore, one must have k_s as a time-dependent variable (or else, both τ_s and k_s are constants).

The above coupled Equations (22) and (23) now read

$$\frac{\partial \varphi_1}{\partial t} = +\frac{\hbar}{2\rho}\frac{\partial^2 \varphi_2}{\partial x^2} \tag{26}$$

$$\frac{\partial \varphi_2}{\partial t} = -\frac{\hbar}{2\rho}\frac{\partial^2 \varphi_1}{\partial x^2} \tag{27}$$

These equations are a coupled real presentation similar to Schrödinger equation.

This leads to the conclusion:

$$\tau_s(t) = \hbar \int k_s(t)\,\mathrm{d}t \tag{28}$$

So, the tension in the strings is proportional to Planck constant $\hbar$, and to the coupling between the two strings.

The tension τ has the physical dimension of force. In SI units this is expressed as [N] = [kg·m·s^{-2}]. When distributed along the string, $\tau(x)$ is often taken as a constant tension per unit cross-section, with dimensions [Force]. In the context of coupled strings, τ is not merely an external applied force but the internal restoring force per element of string. Thus, when expressed per unit length, its effective units become [N] = [kg·m·s^{-2}], consistent with standard string dynamics. This dimensional characterization is important for later steps, since combining τ with the exchange constant (with units of 1/s^2) yields a proportionality factor with the dimensions of angular momentum, naturally associated with Planck's constant ħ

5. Exchange Interaction

From its defining equation $\Delta F_{ext} = -m(k_s \Delta\varphi_2)$, the units of k_s are:

$$[k_s] = [\mathrm{N}/(\mathrm{kg}\cdot\mathrm{m})] = [1/\sec^2]$$

The fact that $[k_s(t)] = 1/\sec^2$ is indicative of the interaction type: the shorter the exchange, the stronger is the interaction.

This is characteristic of an exchange mechanism between the two strings. The higher the rate of exchange (particles/sec), the stronger the interaction.

Indeed, if the exchange rate is designated by R [particles/sec], then the constant $k_s(t)$ should be proportional with R^2 (two strings interacting with each other).

Therefore, $k_s(t)$ must have the units of 1/sec^2.

So, the tension in the strings is proportional to the Planck constant $\hbar$, and to the coupling between the two strings (**Figure 3**).

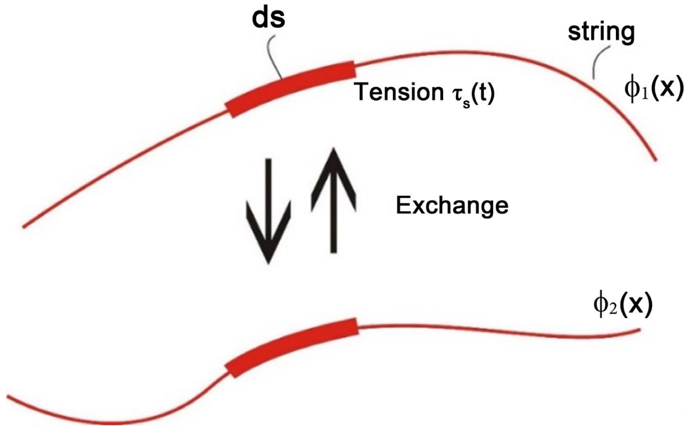

Figure 3. Exchange interaction between two adjacent strings.

The diagram represents how the coupling between neighboring strings is mediated by an exchange process. A higher exchange rate between the strings pro-

duces stronger interaction, which manifests as increased tension along each string. The proportionality between the exchange-driven force and the resulting string tension defines Planck's constant in this model, linking microscopic exchange dynamics to the universal quantum scale. The interaction caused be some sort of exchange mechanism between the two strings, results in tension in the strings, given by $\tau_s(t)=\hbar\int k_s(t)\,\mathrm{d}t$ *. The proportionality between the exchange force and the tension is the Planck constant* $\hbar$ *.*

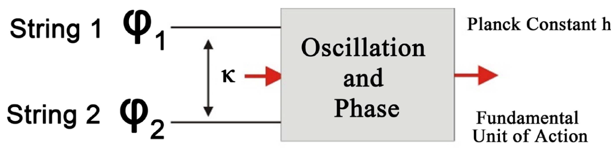

Figure 4. Anchor schematic for Planck's constant. Two coupled real strings with phases ϕ_1 and ϕ_2 interact through a coupling constant κ. Their oscillatory and phase-coupling dynamics define the Planck constant h as the universal quantum of action. The flow is parsed explicitly: **String coupling → Oscillation/Phase → Planck constant** h.

The schematic in Figure 4 summarizes the central result of this work: while Planck introduced h as a phenomenological constant, here it emerges from the deterministic dynamics of two coupled strings. The logical bridge from coupling → oscillation/phase → h is shown graphically.

This exchange mechanism is summarized in Figure 3. The universal constant that results from this process is illustrated in the anchor schematic, Figure 4.

Figure 4 captures the central outcome: the emergence of Planck's constant h from coupled string dynamics.

6. Conclusions

Based on the following assumptions:

1) A Classical Fermion is made up of two interacting string-like entities.

2) Tension in the strings is proportional to the coupling between the two strings.

3) The coupling between the two strings is proportional to the amount of time the exchange lasts.

One is lead to conclude, that Planck's constant $\hbar$, is the proportionality constant, between the total exchange (of some sort) between the two strings, and the tension in these strings.

7. Bridge to Charge and Entanglement

Interpreting the complex field ψ as an ordered pair of real, coupled string-fields (φ_1, φ_2) means that global rotations in the (φ_1, φ_2) plane are physical internal symmetries. By Noether's theorem, this continuous symmetry carries a conserved current J^μ and an associated scalar charge $Q=\int J^0\mathrm{d}^3x$. In subsequent work, this conserved quantity is identified with electric charge [10]: the unit normalization is fixed by the same coupling–tension response that here defines Planck's constant

$\hbar$. Thus, the Planck scale that emerges from two-string exchange also sets the natural scale for charge.

Furthermore, the complex representation endows the two-string system with a physically meaningful phase. When pairs of fermions are created, their internal two-string phases are correlated by the same coupling that determines $\hbar$. Those shared phases persist and yield the familiar quantum $E(\alpha,\beta)$ correlations without invoking nonlocal dynamics—entanglement appears as a conservation of internal orientation established at creation [11].

The associated conserved Noether current takes the familiar form (see **Appendix A**):

$$J^{\mu} = i\left(\psi^{*}\partial^{\mu}\psi - \psi\partial^{\mu}\psi^{*}\right) \tag{29}$$

This makes explicit how the internal U(1) symmetry of the two-string system yields a conserved quantity, identified in subsequent work with electric charge [10].

8. From $\hbar$ to Q and Correlations

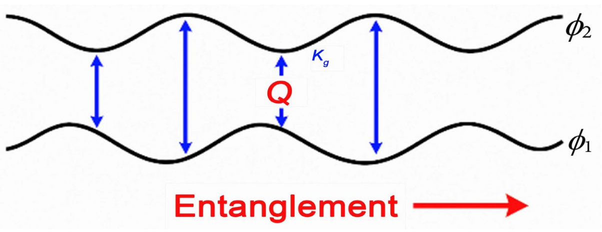

Figure 5. Coupled-strings schematic. Two real strings ("germions") oscillate with phases ϕ_1 and ϕ_2, linked by a coupling constant κ. Their interaction gives rise to emergent physical properties such as electric charge Q and entanglement correlations. The bidirectional arrows indicate the local coupling of the string displacements, forming the physical basis of the coupled-fields model.

The present analysis identifies $\hbar$ as the proportionality between a two-string exchange interaction and the induced tension. This places $\hbar$ as the primary scale in the double-string dynamics. In a Lagrangian framing of the same two real fields, global rotations of (φ_1, φ_2) yield a Noether current J^{μ} and its conserved scalar Q behaves as electric charge once the coupling is matched to data. In parallel, the complex phase that packages the two real fields encodes a physical internal orientation; shared orientation at pair creation—set by the same coupling that fixes $\hbar$—produces the observed entanglement correlations in local measurements (see **figure 5**).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: Compact Lagrangian and Noether Current

To make explicit the link from the two-string framework to electric charge, we sketch a compact Lagrangian and the resulting Noether current.

Consider two real string-fields φ_1 and φ_2, which we combine into a complex field $\psi = \varphi_1 + i\varphi_2$. A minimal Lagrangian density for these fields, with a symmetric internal rotation, is:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi^1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^2\right)^2 - V\left(\phi^1, \phi^2\right)$$

The Lagrangian is invariant under a global U(1) rotation $\psi \to e^{i\theta}\psi$. By Noether's theorem, the conserved current is:

$$J_\mu = \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \tag{30}$$

and the associated conserved quantity is the charge:

$$Q = \int J^0 \mathrm{d}^3 x \tag{31}$$

In the two-string interpretation, this current corresponds to the conserved internal rotation of (φ_1, φ_2). Identifying the normalization of this current with the coupling-tension scale introduced earlier links the conserved quantity Q to electric charge. Thus, the same mechanism that produces Planck's constant $\hbar$ from string coupling also fixes the natural scale for electric charge.

Furthermore, the phase of ψ is now understood as a physical orientation of the two real fields. When two fermions are created, conservation of internal orientation leads directly to correlated measurement outcomes—*i.e.*, entanglement correlations—without requiring nonlocality.

Appendix B: Dimensional Analysis of the Coupling Constant

This appendix verifies the dimensional consistency of the coupling constant κ as used in the derivation of Planck's constant within the coupled-strings framework. Equation (25) in the main text establishes that Planck's constant h arises from a proportionality between the string tension τ and the coupling constant κ:

$$h \propto \tau / \kappa$$

1) Dimensions of the String Tension (τ)

In classical mechanics, tension represents a force along a string. Its dimensions are:

$$[\tau] = [F] = \left[\mathrm{M\ L\ T^{-2}}\right] = \left[\mathrm{kg \cdot m \cdot s^{-2}}\right]$$

In the coupled-strings model, τ is an internal restoring force per unit element of string. When considered per unit length, it remains dimensionally equivalent to a force.

2) Dimensions of the Coupling Constant (κ)

From the exchange-interaction section, the coupling constant κ is defined as the proportionality factor relating the displacement of one string to the restoring force acting on the other. It represents an effective rate of exchange between the two strings. The model identifies κ with an inverse time-squared dependence:

$$[\kappa]=\left[\mathrm{T}^{-2}\right]=\left[\mathrm{s}^{-2}\right]$$

This corresponds physically to a restoring-force constant per unit mass, analogous to the square of an angular frequency in a harmonic oscillator.

3) Combined Dimensional Relation

Equation (25) implies that Planck's constant has the dimensions of angular momentum:

$$[h]=\left[\mathrm{M}\,\mathrm{L}^{2}\,\mathrm{T}^{-1}\right]=[\mathrm{J}\cdot\mathrm{s}]$$

Therefore, the product τ/κ has dimensions of action per unit length:

$$[\tau/\kappa]=\left[\mathrm{M}\,\mathrm{L}\,\mathrm{T}^{-1}\right]=\left[\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}\right]$$

Multiplying by a characteristic string length L yield:

$$[\tau L/\kappa]=\left[\mathrm{M}\,\mathrm{L}^{-2}\,\mathrm{T}^{-1}\right]=[\mathrm{J}\cdot\mathrm{s}]=[\mathrm{h}]$$

4) Summary Table

Quantity	Symbol	Dimensional formula	SI units
Tension	τ	$\mathrm{M\,L\,T^{-2}}$	$\mathrm{N = kg{\cdot}m{\cdot}s^{-2}}$
Coupling constant	κ	$\mathrm{T^{-2}}$	$\mathrm{s^{-2}}$
τ/κ		$\mathrm{M\,L\,T^{-1}}$	$\mathrm{kg{\cdot}m{\cdot}s^{-1}}$
$(\tau L)/\kappa$		$\mathrm{M\,L^{2}\,T^{-1}}$	$\mathrm{J{\cdot}s}$

The dimensional analysis confirms that the coupling constant κ has units of s^{-2}. When combined with the internal string tension τ, the proportionality $\tau L/\kappa$ yields a quantity with the same dimensions as Planck's constant h, demonstrating the consistency of the derivation.