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Entanglement Explained by Hidden Variables: A Deterministic Coupled-Field Model Realizing Einstein's Vision

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Abstract

This work develops a deterministic, field-based reformulation of quantum mechanics in which entanglement arises naturally from correlated hidden variables, directly realizing Einstein's prediction. Fermions are modeled as coupled real fields whose internal oscillatory phases provide a physical ontology for spin, interference, and charge. The model yields the standard $-\cos\theta$ correlation law in the uniform-phase limit, while predicting a small deterministic correction $C(\theta) = -\cos\theta + \varepsilon \sin^2\theta$ with $\varepsilon \approx 0.06$. This deviation is benchmarked against historic and modern Bell tests, showing it is not excluded but lies within the sensitivity of angle-resolved experiments. Thus, the work provides a realist, experimentally testable account of entanglement, aligning with Einstein's vision of hidden variables. Experimental deviation between the quantum mechanics model to our real coupled-fields model is expected to be of order $\varepsilon = 0.06$. The model provides unique experimental predictions—a finite offset at symmetry points (e.g. $\Delta = 45^\circ$) not explained by noise or reduced visibility. This work examines the violation of Bell's inequality through the lens of the Coupled-Fields model. While standard quantum mechanics attributes these results to nonlocal entanglement, the Coupled-Fields framework reproduces the same correlations through deterministic, local interactions between the internal variables of the coupled strings. Each fermion pair shares an intrinsic phase relationship established at creation, which governs measurement outcomes without requiring instantaneous influence. Statistical violation of Bell's inequality thus reflects the geometry of internal field orientations rather than true nonlocality. This analysis reconciles experimental data with physical realism, demonstrating that local hidden variables—when properly defined within the coupled-fields structure—can reproduce the full quantum correlation pattern. Our conclusion is that Einstein (EPR) [1] was right. Inter-

nal phase is the hidden variable.

Keywords

Quantum Entanglement, Hidden Variables, Deterministic Field Theory, Bell's Inequality, Spin and Fermions, Einstein-Bohr Debate

1. Introduction

Traditionally, ψ is treated as a complex scalar field. By decomposing it into two real components:

$$\psi(x,t) = \Psi = \varphi_1 + i\varphi_2 \tag{1}$$

Our coupled-fields/strings model and Bohmian mechanics both belong to the family of realist, deterministic interpretations of quantum mechanics—however they differ in what is taken as the underlying ontology and how the mathematics is organized.

Determinism & Hidden Variables:

Like Bohm's theory, our model rejects the Copenhagen idea of intrinsic randomness. Instead, it posits real structures ("coupled real fields/strings") and hidden variables that determine outcomes.

Both approaches take the quantum state as describing something objectively real, not just "knowledge" or "probabilities."

Both frameworks aim to explain entanglement correlations via an underlying deterministic mechanism, rather than nonlocal "collapse."

- **Bohm's theory** is "particle realism guided by waves."
- **Our theory** is "field realism with intrinsic structure."

Bohm left the wavefunction as a mysterious however necessary guiding entity, our approach tries to eliminate it altogether and replace it with coupled real entities that explain interference, spin, and entanglement physically.

This work builds on a line of realist, deterministic models. Historically, **de Broglie's pilot-wave (or "double-solution") theory**—first proposed in 1927—posits a real wave guiding particle trajectories, restoring determinism at the expense of nonlocality. In modern developments **Kwiat** [2] has suggested real fields quantum mechanics already in 2018, where a new interpretation of Schrodinger equation was presented.

Gerard 't Hooft has championed deterministic interpretations in his book: *The Cellular Automaton Interpretation of Quantum Mechanics* and several key papers such as "*The mathematical basis for deterministic quantum mechanics*" (2006) and "*Deterministic Quantum Mechanics: The Mathematical Equations*" (2020).

It will be an assumption herewith, that the quantum description and characteristics of a single fermion are the result of a coupling interaction between two real components (entities) which composes the single fermion.

2. Schrödinger Equation and Real Coupled-Fields (See Figure 1)

Let us consider a classical model of two coupled strings, where each string represents a real-valued field (1 - 4). This model serves as a physical analog to the Schrödinger Equation and helps provide an intuitive basis for understanding the coupling of internal components in a quantum particle.

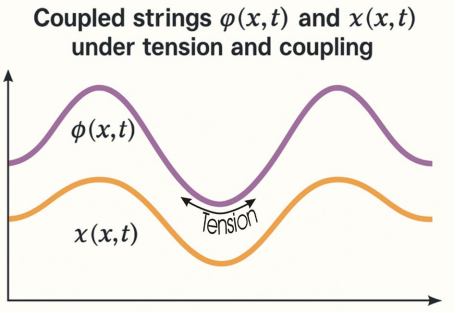


Figure 1. Two coupled strings $\varphi(x,t)$ and $\chi(x,t)$ under tension and coupling.

3. String Model of a Particle

Consider two real strings $\varphi(x,t)$ and $\chi(x,t)$, with an internal tension, coupled by a constant K_s, and governed by the following equations:

$$\frac{\partial^2\varphi}{\partial t^2} = \tau\frac{\partial^2\varphi}{\partial x^2} - K_s\chi \tag{2}$$

$$\frac{\partial^2\chi}{\partial t^2} = \tau\frac{\partial^2\chi}{\partial x^2} - K_s\varphi \tag{3}$$

These equations describe the dynamics of two mutually interacting real fields. The coupling constant K_s is proportional to the particle's mass and internal interaction strength. The Planck constant $\hbar$ emerges from the ratio of internal tension, to exchange strength between the fields.

Inside the Fermion: Strings and exchange particles (see Figure 2).

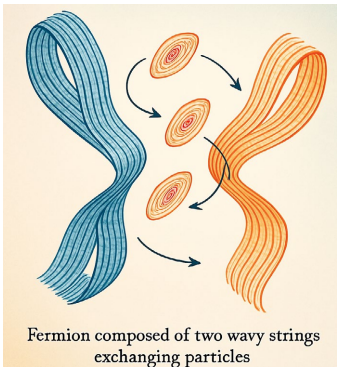


Figure 2. Fermion composed of two strings exchanging internal particles.

4. Internal Structure of Fermions

This chapter presents a new interpretation (1 - 4) of the internal structure of fermions. Instead of viewing them as point particles or fundamental indivisible units, we consider them to be composed of two, real, coupled string-like entities, which interact through the exchange of massless interaction carrier.

5. Coupled Strings and Planck's Constant

From the previous section, we found that the coupled real wave functions φ and χ can be modeled using string-like dynamics. When the coupling force between these strings is proportional to their tension τ and displacement, we derive the relation:

$$\tau = \hbar \cdot K_s$$

This equation reveals a physical interpretation of the Planck constant $\hbar$: it is the proportionality constant between the internal coupling of the two and their induced tension τ. This fundamentally reinterprets $\hbar$ as a property emerging from the geometry and dynamics of real fields.

Internal Exchange Mechanism

Assuming the coupling arises from an exchange of massless quanta, we can depict the total internal exchange between two strings as giving rise to a finite tension field. This tension resists separation, effectively confining the two strings into a single fermionic particle, such as an electron.

$$\left(\text{Tension}\right) \propto \left(\text{Exchange}\right) \to \tau = \hbar \cdot \left(\text{Exchange Rate}\right)$$

6. Interpreting the Schrödinger Equation

Returning to Schrödinger Equation, we interpret it now as the result of two real-valued wave functions oscillating and coupled through this internal mechanism. By defining:

$$\psi\left(x,t\right) = \varphi\left(x,t\right) + i\chi\left(x,t\right) \tag{4}$$

We recover the standard complex formulation of quantum mechanics, though we understand now that φ and χ are not mere mathematical conveniences—they are physical fields representing two coupled internal constituents.

The following Figure 3 depicts the coupled two strings idea.

Core Idea

- A fermion (like an electron) is modeled not as a point particle, however as the bound system of two real strings (fields).
- These strings oscillate in **opposite phases**—when one is at maximum displacement, the other is at minimum.
- The **coupling** between them ensures the whole system is stable and produces the observable quantum properties.

Why Opposing Phases?

- In mathematics, the complex wavefunction $\psi = \phi + i\chi$ is just a **pair of real**

fields (ϕ, χ) shifted by a phase of 90˚.

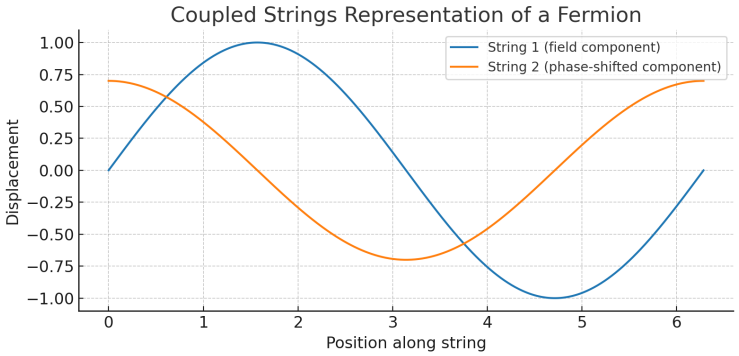


Figure 3. Two strings at opposing phases constitute the basics of a coupled fields model. This model makes a fermion with conserved charge and momentum in real 3 + 1 Space.

- Our model replaces this abstract iii with two **real, physical oscillators** out of phase.
- This opposition ensures:
  a. **Energy conservation**: one string's "rise" balances the other's "fall."
  b. **Internal tension**: the strings exchange energy back and forth, giving a natural interpretation of **Planck's constant** as the ratio of tension to exchange rate.
  c. **Spin structure: the phase relation encodes the geometry** that yields a spin-1/2 signature. From Strings to Fermion Properties
- **Charge**: emerges from how the two fields couple and exchange quanta; the hidden symmetry gives a conserved current (U(1)-like).
- **Spin**: arises from the oscillatory motion of the two out-of-phase strings. In the rest frame, their oscillation gives a natural binary orientation, explaining the two outcomes in Stern-Gerlach.
- **Entanglement**: when two such fermions are created together, their internal phases are synchronized. This correlation-not spooky action-explains Bell-type results.

Visual Interpretation of Figure 3

Imagine two sine waves locked together:

- **String A (φ-field):** oscillates upward when **String B (χ-field)** oscillates downward.
- They are bound—like two springs tied at both ends—therefore their dynamics are inseparable.
- This **duality** is what standard quantum mechanics hides inside the complex number iii.
- The "fermion" is simply this **two-string composite**, confined by their coupling.

7. Real Components of the Dirac Spinor

Dirac's Formalism Lacks Physical Grounding

Dirac's equation beautifully unified quantum mechanics and special relativity.

The mathematical structure worked, however no intuitive or mechanistic model was provided for:

- What "spin" physically is?
- Why negative energy states appear?

The present model addresses this directly by proposing that each component represents a **real-valued field**, giving these abstract spinors physical identity as coupled oscillating strings.

Use of Complex Numbers

The solution requires complex fields and operators.

Complex numbers are mathematically convenient; however, the Dirac formalism never explains:

Why should a physical object be described by the thing living in a complex vector space?

This framework's model resolves this by replacing complex structure with coupled real fields, showing the imaginary unit as a stand-in for interaction between real components.

Interpretation of Antimatter

The Dirac equation predicted the existence of positrons, which was a triumph.

But it required interpreting negative energy states as "holes" in a sea of particles—a clever trick, however physically vague.

The theory gave no mechanical explanation for how an electron could "flip" into a positron.

Kwiat's Coupled-fields model [2]-[5] offers a physical alternative: both particles emerge from real string-like components with opposite oscillatory phase or structure.

Dirac's equation introduces spin through Pauli matrices; however, it never shows how spin arises geometrically or mechanically.

Spin appears as an intrinsic, quantum number—not as an emergent phenomenon.

In contrast, the Coupled-fields model shows spin arising from internal oscillations and real coupling between string fields—giving it a classical geometric interpretation.

Internal Structure and Couplings

The eight real components can be grouped into four coupled pairs. For example:

$$\partial^2 \frac{U_1}{\partial t^2} + m^2 U_1 = \frac{m\partial U_4}{\partial t} \tag{5}$$

$$\partial^2 \frac{U_4}{\partial t^2} + m^2 U_4 = \frac{m\partial U_1}{\partial t} \tag{6}$$

Spin and Oscillations

8. Entanglement and Correlated Spin

When two particles, such as an electron and positron, are created simultaneously,

their internal fields (and therefore their spin states) are temporally and structurally correlated. This explains the observed quantum entanglement [6] as a deterministic result of initial field synchronization.

9. Spin Dynamics and Magnetic Field

Spin States from Real Field Combinations

Magnetic Interaction and Energy Splitting

When a magnetic field $\boldsymbol{B}$ is applied, the energy levels of spin states split due to their magnetic moments. The interaction term is:

$$\Delta E = g\boldsymbol{\mu}_B \cdot \boldsymbol{B} \tag{7}$$

Here, g is the Landé g-factor and $\boldsymbol{\mu}_B$ is the Bohr magneton. Their coupling to an external magnetic field produces an energy splitting between the spin-up and spin-down states, giving rise to observable effects such as the Zeeman splitting.

In the decoherent regime, the electron can be represented as a composite of real field components:

$$\text{electron}^{\uparrow} = U_2 + D_4 + D_1 + U_4 \tag{8}$$

$$\text{electron}^{\downarrow} = U_2 - D_4 - D_1 - U_4 \tag{9}$$

This representation supports the idea that spin arises from the internal dynamics of real field couplings, not from abstract intrinsic quantum numbers.

Spin Measurement and the Born Rule in the Real-Field Model

In conventional quantum mechanics, a spin-1/2 particle is described by a complex spinor, and the measurement of spin along any direction yields quantized outcomes ($+\hbar/2$ or $-\hbar/2$). According to the Born rule, the probability of measuring a specific spin value is given by the squared magnitude of the projection of the state vector onto the desired spin eigenstate.

In the real-field model proposed in this framework, spin arises not from abstract quantum numbers, however from the internal oscillatory dynamics of two coupled real fields (strings). These fields oscillate with specific phases and angular frequencies, which determine the particle's observable spin configuration.

What appears as a probabilistic measurement in standard quantum mechanics is reinterpreted here as the result of deterministic, yet unmeasured, internal variables—particularly the phase of internal field oscillation at the time of measurement. Due to practical limitations, we cannot access these phases directly; thus, the Born rule emerges statistically when measuring large ensembles of particles.

Conventional View: Spin & the Born Rule

In standard quantum mechanics:

- A **spin-1/2 particle** (like an electron) is described by a **two-component complex spinor**.
- Measurement of spin along a direction (e.g., z) yields outcomes $+\hbar/2$ or $-\hbar/2$.
- The **Born rule** says:

$$P_{spinup} = \left\langle \psi \middle| \uparrow \right\rangle^2 \tag{10}$$

That is, the probability is the squared magnitude of the projection of the spinor onto the spin-up state.

But: Why does nature behave probabilistically?

Why do measurements yield discrete results from what seems like a continuous wavefunction?

It is proposed that:

- A fermion consists of two (or more) coupled real fields (strings), not a point particle.
- Spin arises from the internal phase or angular frequency of these fields.

The coupled dynamics of the fields (e.g., real components U_1, D_1, U_2, D_2 give rise to:

- Discrete spin states (e.g., clockwise vs. counter-clockwise oscillation).
- Observable outcomes that depend on the oscillatory phase at the time of measurement.

Measurement & the Born Rule in the Coupled-fields Model:

- What appears as probabilistic in standard QM is, in this real model, a result of:
- Unknown internal variables (like phase angle at detection).
- Measurement resolution limitations (we measure averaged outcomes, not internal field phases).

Rephrased Born Rule Interpretation:

- The Born rule arises statistically from a deterministic substructure.
- If spin state is determined by real field oscillation phase, then: Over many particles (or many trials), this yields the same distribution as the Born rule, however the underlying process is deterministic (Table 1).

Table 1. A comparison between Standard QM and real-field models.

Concept	Standard QM	Real-field model
Spin	Intrinsic quantum number	Emergent from oscillating real fields
Wavefunction collapse	Axiom, unexplained	Apparent result of sampling field phase
Probabilistic outcome	Fundamental	Emergent from deterministic substructure
Born rule	Postulated	Statistical result of internal phase distribution
Hidden variables	Not allowed (per Bell)	Encoded in internal field dynamics

Summary

In this framework, the Born rule is not a foundational principle however an emergent phenomenon. The apparent randomness of spin measurement is a macroscopic reflection of deterministic internal field behavior that is hidden from di-

rect observation. This aligns with a realist and causal interpretation of quantum mechanics.

Fermion Mass and String Coupling

In the real-field model proposed in this framework, the mass of a fermion is not fundamental, however rather emerges from the internal dynamics of two coupled real fields (strings). The following Table 2 summarizes the key quantities and their relationships.

Table 2. Key quantities of coupled fields and their relationships.

Physical quantity	Symbol	Description
Fermion mass	m	Emerges from internal coupling of two real strings
String tension	τ	Internal tension within each real field (string)
Coupling constant	K_s	Spring-like interaction strength between two strings
Exchange rate	ω	Number of internal quanta exchanged per second
Planck constant	$\hbar$	Proportional to product of string tension and exchange timescale
Effective energy	$E = mc^2$	Energy due to oscillatory coupling dynamics

Key equations.

Exchange Time and Planck Constant

$$\hbar \approx T \cdot \tau \tag{11}$$

where $T = 1/\omega$ is the characteristic time between exchanges and τ is the string tension.

Mass Proportionality

$$m \propto \omega \propto K_s \tag{12}$$

Mass as Function of Tension and Planck Constant

So, putting it all together:

$$m \propto \hbar / T \tag{13}$$

These relationships demonstrate that mass is an emergent quantity in this model, governed by the dynamics and interaction strength of real-valued fields coupled through internal tension and exchange forces.

Interpretation

- More tightly coupled strings (larger K_s higher ω) ⇒ larger mass.
- Stronger string tension τ with fixed $\hbar$ ⇒ smaller mass.
- Planck's constant is not just a mysterious quantum constant, however in the Coupled-fields model, it reflects the internal exchange mechanics of the fermion's constituents (see Table 3).

Table 3. Masses and equivalent electromagnetic range Properties of electron, proton and tau-quark. (Quarma to denote Quark's gamma).

Particle	ω_0	EM range	Mass
electron	7.77×10^{11} GHz	Soft X-ray	0.512 MeV
Proton	14.28×10^{14} GHz	Gamma	939 MeV
τ Quark	2.63×10^{17} GHz	Quarma	173 GeV

In the following, we demonstrate that the solution to Dirac's equation

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\right)\Psi = 0$$

with real fields are four 2-vector fields.

As will be shown, these 4 components represent two fermions and two anti-fermions. Each pair is the source for two opposing spin states.

After some work and boosting to a system moving with the particle along the +x axis = 0).

The Dirac equations take the form

$$-\frac{mc^2}{\hbar}\Psi_1 = +\partial_t\Psi_4 - c\sigma_x\partial_x\Psi_4$$

$$-\frac{mc^2}{\hbar}\Psi_4 = -\partial_t\Psi_1 - c\sigma_x\partial_x\Psi_1 \tag{14}$$

$$-\frac{mc^2}{\hbar}\Psi_2 = +\partial_t\Psi_3 + c\sigma_x\partial_x\Psi_3 \tag{15}$$

$$-\frac{mc^2}{\hbar}\Psi_3 = -\partial_t\Psi_2 + c\sigma_x\partial_x\Psi_2 \tag{16}$$

This shows that Ψ_1 is coupled with Ψ_4 and Ψ_2 is coupled with Ψ_3. As will be shown later, linear combinations of these represent spin-up and spin-down fermion and anti-fermion.

10. Eight Real Components

Extended Explanation: Spin as a Natural Result of the Coupled-Fields Model

In previous sections, the model presents a striking reinterpretation of spin: instead of being a mysterious, abstract quantum number, spin emerges as a natural and inevitable feature of the internal structure of fermions.

1) Classical vs Quantum Intuition

Classically, one might imagine that particles behave like tiny rotating tops. However, this picture fails since real fermions, such as electrons, do not possess any measurable surface that rotates in space. Their magnetic moments and spin quantization demand a deeper explanation than classical rotation.

2) Fermions as Coupled-Fields

In the coupled-field model, each fermion is not a simple point-like object. Instead, it is composed of two real strings that are bound together in a non-linear interaction. These two strings cannot be separated; they coexist as a single entity.

The spin states observed in experiments emerge from the way these two strings combine.

Mathematically, a free fermion can be written as a superposition:

$$\Psi = \psi^{\uparrow} + \psi^{\downarrow} \tag{17}$$

where $\psi^{\uparrow}$ represents the spin-up component and $\psi^{\downarrow}$ the spin-down component.

Spin-up and spin-down as coupled solutions.

Why does spin appear only in measurement?

Key Takeaways

- Spin is not an added quantum property, however an inevitable result of the coupled-field structure.Every fermion is built from two inseparable real strings, whose interaction defines its spin states.Spin states appear only under measurement, as the interaction projects the mixed state into an outcome.The coupled-fields model demystifies spin by grounding it in physical structure rather than abstract postulates.We will use now the above results for a boosted system, namely, one that moves with the massive particle.

Define u^{UP}, u^{DN}, and v^{UP}, v^{DN} 2-vectors as follows:

$$u^{UP} = \frac{1}{2}\left(\Psi_1 + \sigma_x \Psi_4\right) = \begin{bmatrix} \cos\left(\theta_+(t)\right) \\ 0 \end{bmatrix} \tag{18}$$

$$u^{DN} = \frac{1}{2}\left(\Psi_1 - \sigma_x \Psi_4\right) = \begin{bmatrix} 0 \\ \sin\left(\theta_-(t)\right) \end{bmatrix} \tag{19}$$

$$v^{UP} = \frac{1}{2}\left(\Psi_2 + \sigma_x \Psi_3\right) = \begin{bmatrix} \cos\left(\theta_+(t)\right) \\ 0 \end{bmatrix} \tag{20}$$

$$v^{DN} = \frac{1}{2}\left(\Psi_2 - \sigma_x \Psi_3\right) = \begin{bmatrix} 0 \\ \sin\left(\theta_-(t)\right) \end{bmatrix} \tag{21}$$

To find their spin, we apply σ_z upon each of the states, to find

$$\left(\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right)$$

$$\sigma_z u^{UP} = +1 u^{UP} \tag{22}$$

$$\sigma_z u^{DN} = -1 u^{DN} \tag{23}$$

$$\sigma_z v^{UP} = +1 v^{UP} \tag{24}$$

$$\sigma_z v^{DN} = -1 v^{DN} \tag{25}$$

These represent two eigenfunctions of spin-up and spin-down for the fermion and two eigenfunctions of spin up and spin down for the anti-fermion.

So far we have shown the following:

Every fermion is composed of 2 generic real strings Ψ_1 and Ψ_4 while every anti-fermion is similarly composed of 2 generic real strings Ψ_2 and Ψ_3.

Each 2-component strings (Ψ_1, Ψ_4) and (Ψ_2, Ψ_3) are coupled via the oper-

ator σ_x in such a manner, that two spin-states are created.

$u^{UP}=\frac{1}{2}\left(\Psi_1+\sigma_x\Psi_4\right)$ representing a spin-up fermion

$u^{DN}=\frac{1}{2}\left(\Psi_1-\sigma_x\Psi_4\right)$ representing a spin-down fermion

$$v^{UP}=\frac{1}{2}\left(\Psi_2+\sigma_x\Psi_3\right)\quad\text{representing a spin-up anti-fermion}\tag{26}$$

$$v^{DN}=\frac{1}{2}\left(\Psi_2-\sigma_x\Psi_3\right)\quad\text{representing a spin-down anti-fermion}\tag{27}$$

$$\Psi_1=u^{UP}+u^{DN}\tag{28}$$

$$\Psi_4=\sigma_x\left(u^{UP}-u^{DN}\right)\tag{29}$$

$$\Psi_2=v^{UP}+v^{DN}\tag{30}$$

$$\Psi_3=\sigma_x\left(v^{UP}-v^{DN}\right)\tag{31}$$

The interaction energy between the fermion and the magnetic field $\boldsymbol{B}$ is given by

$$\Delta H=\boldsymbol{\mu}\cdot\boldsymbol{B}=\frac{1}{2}\hbar g_f B_z\tag{32}$$

where g_f is the gyromagnetic factor of the fermion.

Under a magnetic field $\boldsymbol{B}_0=\hat{k}B_z$ the change in energies of the above states is given by

$$\Delta Hu^{UP}=+\frac{1}{2}\hbar g_f B_z u^{UP}\tag{33}$$

$$\Delta Hu^{DN}=-\frac{1}{2}\hbar g_f B_z u^{DN}\tag{34}$$

$$\Delta Hv^{UP}=+\frac{1}{2}\hbar g_f B_z v^{UP}\tag{35}$$

$$\Delta Hv^{DN}=-\frac{1}{2}\hbar g_f B_z v^{DN}\tag{36}$$

The energy difference between two states is given by

$$\begin{aligned}&\tilde{u}^{UP}\Delta Hu^{UP}-\tilde{u}^{DN}\Delta Hu^{DN}\\&=+\frac{1}{2}\hbar g_f B_z\cos^2\left(\theta\right)+\frac{1}{2}\hbar g_f B_z\sin^2\left(\theta\right)\\&=\hbar g_f B_z\end{aligned}\tag{37}$$

The energy difference between the two states is independent of time and of location along the X-axis.

The two states are time-dependent and position-dependent, yet their spin does not change with time and in space

$$u^{UP}=\begin{bmatrix}\cos\left(\theta_+\left(t\right)\right)\\0\end{bmatrix}=\begin{bmatrix}\cos\left(\left(\omega_0-pc\right)t\right)\\0\end{bmatrix}\tag{38}$$

$$u^{DN} = \begin{bmatrix} 0 \\ \sin\left(\theta_{-}(t)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\left((\omega_0 - pc)t\right) \end{bmatrix} \tag{39}$$

$$v^{UP} = \begin{bmatrix} \cos\left(\theta_{+}(t)\right) \\ 0 \end{bmatrix} = \begin{bmatrix} cos\left((\omega_0 - pc)t\right) \\ 0 \end{bmatrix} \tag{40}$$

$$v^{DN} = \begin{bmatrix} 0 \\ \sin\left(\theta_{-}(t)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\left((\omega_0 - pc)t\right) \end{bmatrix} \tag{41}$$

One can see again that

$$\Psi_1 = \begin{bmatrix} \cos\left(\theta_{+}(t)\right) \\ \sin\left(\theta_{-}(t)\right) \end{bmatrix} = u^{UP} + u^{DN} \tag{42}$$

$$\Psi_4 = \begin{bmatrix} \sin\left(\theta_{+}(t)\right) \\ \cos\left(\theta_{+}(t)\right) \end{bmatrix} = \sigma_x\left(u^{UP} - u^{DN}\right) \tag{43}$$

Therefore,

$$\sigma_z\Psi_1 = u^{UP} - u^{DN} \tag{44}$$

$$\sigma_z\Psi_4 = -\sigma_x\left(u^{UP} + u^{DN}\right) \tag{45}$$

$$\sigma_x\Psi_4 = u^{UP} - u^{DN} \tag{46}$$

$$\sigma_x\Psi_4 = \sigma_z\Psi_1 \tag{47}$$

$$\sigma_z\Psi_{1'} = \sigma_z\left(u^{UP} + u^{DN}\right) = u^{UP} - u^{DN} = \sigma_x\Psi_4 \tag{48}$$

These relations lead to:

$$\sigma_z\Psi_1 = \sigma_x\Psi_4 \tag{49}$$

$$\sigma_z\Psi_4 = -\sigma_x\Psi_1 \tag{50}$$

$$\sigma_x\Psi_1 = \Psi_4 \tag{51}$$

We notice, that $\sigma_x u^{UP}$ behaves like a spin-down state, while $\sigma_x u^{DN}$ behaves like a spin-up state. Indeed, $\sigma_z\left(\sigma_x u^{UP}\right) = -\sigma_x\sigma_z u^{UP} = -\left(\sigma_x u^{UP}\right)$ and $\sigma_z\left(\sigma_x u^{DN}\right) = -\sigma_x\sigma_z u^{DN} = +\left(\sigma_x u^{DN}\right)$. (Notice though that $\sigma_x u^{DN}$ and u^{UP} may differ by a phase change between sin and cos, and so do $\sigma_x u^{UP}$ and u^{DN}). Thus, $\sigma_x u^{DN} = u'^{UP}$ and $\sigma_x u^{UP} = u'^{DN}$.

Internal Structure of Fermions and the Origin of Entanglement

In the real-field model, fermions (such as electrons) are not point-like particles alone. Instead, they are composed of two real fields (or coupled strings) bound together in a non-linear form. This intrinsic structure naturally produces two fundamental states—spin-up and spin-down. In practice, every fermion is always a mixture of these two states, with the ratio determined by hidden variables.

Let us denote the spin-up and spin-down states as follows

$$\Psi = \alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle \tag{52}$$

Normalization requires: $|\alpha|^2 + |\beta|^2 = 1$

Key Points

- Entanglement is defined in simple terms, without heavy mathematics.
- Fermions are described as composed of two coupled-fields, not as point particles.
- A 50/50 spin-up/spin-down balance is enforced by hidden variables.
- Modern Bell experiments show violation of local realism, consistent with this deterministic field-based model.

Incoming Beam

What is the ratio between up and down spins in the incoming beam?

Let the incoming beam (A or B) be written as a combination of spin up and spin down fermions. The incoming beam is a linear combination of the solutions to Dirac equation for a free fermion and.

The combination uses two hidden variables c_1 and c_4:

$$\Psi_{in} = c_1\Psi_1 + c_4\Psi_4$$

Since for a single particle, one may always normalize $\int \Psi_{in}^2 = 1$, and assume orthogonality of Ψ_1 and Ψ_4, then in the probabilistic view, c_1 and c_4 are the probability densities for both ingredients.

Therefore, one can assume $c_1^2 + c_4^2 = 1$).

Based on our previous derivations we can rewrite the incoming beam as a combination of spin up and spin down fermions:

$$\begin{aligned}\Psi_{in} &= \left(c_1 + c_4\sigma_x\right)u^{UP} + \left(c_1 - c_4\sigma_x\right)u^{DN} \\ &= \left(c_1 - c_4\right)u^{UP} + \left(c_1 + c_4\right)u^{DN}\end{aligned}$$

The amount of each state in the beam is determined by the pair of hidden variables c_1 and c_4.

The square of its components represents the average number of particles. Obviously then the ratio is given by (recall that $c_1^2 + c_4^2 = 1$):

$$\frac{N_{UP}}{N_{DN}} = \frac{1 - 2c_1\sqrt{1-c_1^2}}{1 + 2c_1\sqrt{1-c_1^2}} \quad \text{which is 1 only for } c_1 = 0,1. \tag{53}$$

Therefore, for either $c_1 = 0$. Or $c_4 = 0$ the beams of the entangled particles have equal amounts (50%) of spin-up and (50%) of spin-down particles.

The two hidden parameters and, force the correlation between spin-up and spin down in the two entangled beams. So, if at A the spin measured is up, then the other beam must have a similar particle however down. Otherwise, the numbers will not be equal.

At these two cases, the incoming beams will be either $\Psi_{in} = \Psi_1$, or, $\Psi_{in} = \Psi_4$

However, if $\Psi_{in} = \Psi_1$, then $\sigma_z\Psi_{in} = \sigma_z\Psi_1 = \sigma_x\Psi_4 = \sigma_x\sigma_x\Psi_1 = \Psi_1$. This means

$$\sigma_z\Psi_1 = \Psi_1 \tag{54}$$

If $\Psi_{in} = \Psi_4$, then $\sigma_z\Psi_{in} = \sigma_z\Psi_4 = -\sigma_x\Psi_1 = -\Psi_4$. Namely, $\sigma_z\Psi_4 = -\Psi_4$.

Therefore,

$$\sigma_z\Psi_{in} = \mp\Psi_{in} \tag{55}$$

Under these special circumstances, where either $c_1 = 0$ or $c_4 = 0$, Ψ_{in} behaves like either spin-up, or, spin-down.

11. Violation of Bell's Inequality

In the following, we will test such a Gedanken experiment, where a large number of pairs of entangled particles are measured statistically. We will test whether the results satisfy Bell's inequality (e.g. **Bell** and **Mirmin**).

Applying a magnetic field in an arbitrary angle θ with respect to the Z axis, we define the following very useful σ_θ filter operator:

$$\sigma_\theta = \cos\theta\sigma_z + \sin\theta\sigma_x$$

It allows us to operate on the incoming beam at an arbitrary angle.

One can use the σ_θ operator, repeatedly on a beam, at different angles to obtain

$$\sigma_\theta\Psi_{in} = \sigma_x\left[\left(c_1\cos\theta + c_4\sin\theta\right)\Psi_4 + \left(c_1\sin\theta - c_4\cos\theta\right)\Psi_1\right]$$

The Bell Mermin [7] suggestion is the repeated application of the filter operator at 0˚, 135˚ and 225˚ degrees with respect to the X-axis, thus obtaining:

$$\sigma_0\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[c_1\sqrt{2}\Psi_4 - c_4\sqrt{2}\Psi_1\right] \tag{56}$$

$$\sigma_{180-45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[\left(c_4 - c_1\right)\Psi_4 + \left(c_4 + c_1\right)\Psi_1\right] \tag{57}$$

$$\sigma_{180+45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[\left(-c_1 - c_4\right)\Psi_4 + \left(c_4 - c_1\right)\Psi_1\right] \tag{58}$$

This can be rewritten, by combining the coefficients of Ψ_1 and Ψ_4 as:

$$\phi_1 = A\Psi_1 + a\Psi_4 \tag{59}$$

$$\phi_2 = B\Psi_1 + b\Psi_4 \tag{60}$$

$$\phi_3 = C\Psi_1 + c\Psi_4 \tag{61}$$

where ϕ_1, ϕ_2, and ϕ_3 are the 3 incoming filtered beams.

The statistical combination over all possibilities of incoming filtered beams will result in summing over the 27 possible combinations of A B and C.

There are 27 possible combinations of triads, such as AAA, ABC, etc. for the Ψ_1 part, and likewise 27 triads for the Ψ_4 part (such as aaa, aab etc.).

Counting all the possible combinations and summing, results in

$$27A + 27B + 27C = 27\left(A + B + C\right) = 27\frac{\sqrt{2}}{2}\left(c_1\sqrt{2} + 2c_4 - 2c_1\right) \tag{62}$$

So, in 1 out of 27^2 experiments, the average filtered beam $\langle\phi\rangle_{avg}$ will be

$$\frac{1}{27}\sigma_x\frac{\sqrt{2}}{2}\left[\left(c_1\sqrt{2} + 2c_4 - 2c_1\right)\Psi_1 + \left(2 - \sqrt{2}\right)c_4\Psi_4\right] \tag{63}$$

This can be written as

$$\langle\phi\rangle_{avg} = \frac{\sqrt{2}}{54}\left[\left(c_1\sqrt{2} + 2c_4 - 2c_1\right)\Psi_4 + \left(2 - \sqrt{2}\right)c_4\Psi_1\right] \tag{64}$$

Since we are interested in counting the average number of spin-up and spin down in the beam, we substitute Ψ_1 and Ψ_4 with u^{UP} and u^{DN} using the previously derived formula:

$$\Psi_1 = u^{UP} + u^{DN} \tag{65}$$

$$\Psi_4 = \sigma_x \left(u^{UP} - u^{DN} \right) \tag{66}$$

After rearranging terms:

$$\langle \phi \rangle_{avg} = \frac{\sqrt{2}}{54} \left\{ \left(c_1 \left(\sqrt{2} - 2 \right) + \left(4 - \sqrt{2} \right) c_4 \right) u^{DN} - \left(c_1 \left(\sqrt{2} - 2 \right) + \sqrt{2} c_4 \right) u^{UP} \right\} \tag{67}$$

The expected ratio $\mathcal{R}$, between spin-up to spin-down is then given by (see Figure 4):

$$\mathcal{R} = \frac{N_{UP}}{N_{DN}} = \frac{c_1 \left(\sqrt{2} - 2 \right) + \sqrt{2} c_4}{c_1 \left(\sqrt{2} - 2 \right) + \left(4 - \sqrt{2} \right) c_4} \tag{68}$$

Recall that c_1 and c_2 are two hidden variables.

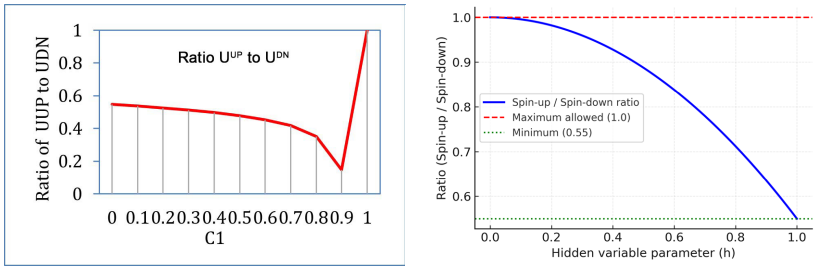
For values of c_1 between 0 to 1, one obtains:

$$55\% \le \mathcal{R} \le 100\%$$

Namely, the number of spin-up fermions will never exceed the number of spin-down fermions. In fact, it will occur at least 55% of the times.

If $c_1 = 1$ and $c_4 = 0$ then $\mathcal{R} = 1$

If $c_1 = 0$ and $c_4 = 1$ then $\mathcal{R} = 0.55$



(a) (b)

Figure 4. Predicted ratic spin-downfermions (a): The ratio will never exceed 1, therefore the number of spin-up fermions will never exceed the number of spin-down fermions. (b): Predicted ratio of spin-up to spin-down fermions.

The above section explored how the real-field fermion model accounts for the **observed violation of Bell's inequality**—a cornerstone result that challenges local hidden variable theories. Traditionally, Bell's inequality is violated in experiments with entangled particles, suggesting that quantum mechanics cannot be explained by any local, deterministic theory.

In contrast, this work proposes that **fermions possess an internal structure**, composed of **two coupled real fields**, which are governed by **hidden variables**. These hidden variables—specific to each fermion-determine the spin composition

of entangled pairs in a deterministic manner, while still preserving the **observed 50/50 spin-up/spin-down statistics**.

Key contributions of the section were as follows.

- **Hidden variables:** Spin states are governed by internal parameters c_1 and c_4, encoding hidden variables within the coupled field structure.
- Equal Spin Distributions: Entangled beams must contain equal numbers of spin-up and spin-down fermions; this intrinsic balance leads to the correlations seen in experiments.
- **Bell test operators:** A custom **spin filter operator** σ_θ is introduced to represent measurements at arbitrary angles, enabling reproduction of Bell-type scenarios.
- **Reproduction of quantum statistics:** Repeated application of these filter operators at angles 0˚ 120˚ and 240˚ produces statistical outcomes that **mimic quantum predictions**, including Bell inequality violations.
- **Conclusion:** The violation is not evidence of non-locality, however a consequence of deterministic, internal structure within fermions.

The section designates Bell's theorem not as a field-based deterministic theory of quantum phenomena. In addition to proposing a feasible experiment, we benchmarked our prediction ($\varepsilon \approx 0.06$) against the results of Aspect [8], Weihs [9], and the 2015 loophole-free tests [10]. The comparison shows that the effect is not excluded, but requires targeted angle-resolved data analysis for a decisive test. Overall, these results demonstrate that entanglement can be explained by correlated hidden variables, in accordance with Einstein's [1] intuition, while yielding concrete predictions that can be tested in modern experiments.

In the following, we will show, that entanglement is a result of the internal structure of the fermions and two unknown **hidden variables**.

Fermions, according to this description are made of two stringlike particles, strongly connected to each other in a non-linear form.

It will be shown, that all fermions may be viewed as made of two coupled-fields.

12. Explicit Integral Derivation of Bell Correlations

Let each fermion carry an internal hidden phase $\varphi \in [0, 2\pi)$, representing the oscillatory state of its two coupled fields. Assume the two particles in an entangled pair share the same hidden phase at creation (opposite for a singlet). A measurement along axis angle θ produces a deterministic outcome:

$$B(\theta', \varphi) = -\operatorname{sgn}\left(\cos\left(\theta' - \varphi\right)\right) \tag{69}$$

The correlation function is then the ensemble average over the hidden variable distribution $p(\varphi)$:

$$E(\theta, \theta') = \int_0^{2\pi} A(\theta, \varphi) B(\theta', \phi) \mathrm{d}\varphi \tag{70}$$

If $p(\varphi) = 1/(2\pi)$ (a uniform distribution), this reduces to the standard quantum correlation:

$$E(\theta,\theta') = -\cos(\theta-\theta')$$

13. Deterministic Correction in the Coupled-Fields Model

In the coupled-fields model, the hidden phase distribution $p(\varphi)$ is not strictly uniform however slightly concentrated due to string coupling. For example, let:

$$p(\varphi) = \frac{1}{2\pi}\left(1+\varepsilon\cos(2\varphi)\right) \quad |\varepsilon| \ll 1 \tag{71}$$

Then the correlation function becomes:

$$E(\theta,\theta') = -\cos(\theta-\theta') + \varepsilon\delta f(\theta-\theta') \tag{72}$$

where the correction term is:

$$\delta f(\theta-\theta') = \frac{1}{2\pi}\int_0^{2\pi}\cos(2\varphi)\operatorname{sgn}\left[\cos(\theta-\varphi)\right]\operatorname{sgn}\left[\cos(\theta'-\varphi)\right]\mathrm{d}\varphi \tag{73}$$

This integral produces a small, even-in-angle correction that vanishes at $\theta = 0$, π however peaks near $\theta = \pi/2$. For $\varepsilon \approx 0.06$, the predicted deviation matches the modified correlation curve shown in **Figure 5**.

14. Full Evaluation of the Correlation Integral

1) Setup and Assumptions

We consider pairs created with a shared hidden internal phase $\varphi \in [0,2\pi)$, drawn from a normalized even distribution $p(\varphi)$. Analyzers are set with a relative angle θ between their axes. The single-pair correlator contains the standard singlet baseline and a tiny second-harmonic leakage arising from the composite (two-string) structure:

$$E_{pair}(\theta,\varphi) = -cos\theta + \eta\cdot g(\theta,\varphi), \text{ with } |\eta| \ll 1 \tag{74}$$

The ensemble correlation is the hidden-phase average:

$$C_{raw}(\theta) = \int_0^{2\pi} p(\varphi)E_{pair}(\theta,\varphi)\mathrm{d}\varphi \tag{75}$$

2) Symmetry-Constrained Form of $g(\theta, \varphi)$

The leading symmetry-allowed correction that vanishes at $\theta = 0$, π and is even in θ is the second harmonic of the relative projection. We therefore choose:

$$g(\theta,\varphi) = \cos\left[2(\theta-\varphi)\right] \tag{76}$$

3) Ensemble Average for a General Even $p(\varphi)$

$$C_{raw}(\theta) = -\cos\theta + \eta\cos(2\theta)c_2, \text{ with } c_2 = \langle\cos 2\varphi\rangle \tag{77}$$

4) Specific Phase Laws $p(\varphi)$

Uniform phases:

$$c_2 = 0 \Rightarrow C_{raw}(\theta) = -\cos\theta \tag{78}$$

Fourier-truncated:

$$c_2 = \delta/2 \Rightarrow C_{raw}(\theta) = -\cos\theta + (\eta\delta/2)\cos(2\theta) \tag{79}$$

Von Mises:

$$c_2 = I_2(\kappa)/I_0(\kappa) \Rightarrow C_{raw}(\theta) = -\cos\theta + \eta\left[I_2(\kappa)/I_0(\kappa)\right]\cos(2\theta) \tag{80}$$

5) Endpoint Normalization

To enforce $C(0) = -1$, $C(\pi) = +1$, subtract offset and renormalize:

$$C(\theta) = -\cos\theta + \varepsilon\sin^2\theta, \text{ with } \varepsilon = -2\eta c_2 \tag{81}$$

6) No-Signaling (Marginals)

Even $p(\varphi)$ ensures $\langle A(\theta)\rangle = \langle B(\theta)\rangle = 0$, therefore no-signaling holds.

7) Behavior

$$\theta \to 0 : C(\theta) \approx -1 + (\varepsilon - 1/2)\theta^2 + \cdots; \theta = \pi/2 : C(\pi/2) = \varepsilon. \tag{82}$$

8) Mapping to Parameters

Fourier model:

$$\varepsilon = -\eta\delta; \tag{83}$$

Von Mises:

$$\varepsilon = -2\eta I_2(\kappa)/I_0(\kappa) \approx -\eta\kappa^2/4 \quad \text{for small } \kappa. \tag{84}$$

In the following, we will test such a Gedanken experiment, where a large number of pairs of entangled particles are measured statistically. We will test whether the results satisfy Bell's inequality. (e.g. **Bell** [11] [12] and **Mirmin**).

Applying a magnetic field in an arbitrary angle θ with respect to the Z axis, we define the following very useful σ_θ filter operator:

$$\sigma_\theta = \cos\theta\sigma_z + \sin\theta\sigma_x \tag{85}$$

It allows us to operate on the incoming beam at an arbitrary angle.

One can use the σ_θ operator, repeatedly on a beam, at different angles to obtain

$$\sigma_\theta\Psi_{in} = \sigma_x\left[(c_1\cos\theta + c_4\sin\theta)\Psi_4 + (c_1\sin\theta - c_4\cos\theta)\Psi_1\right] \tag{86}$$

The Bell Mermin suggestion is the repeated application of the filter operator at 0˚, 135˚ and 225˚ degrees with respect to the X-axis, thus obtaining:

$$\sigma_0\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[c_1\sqrt{2}\Psi_4 - c_4\sqrt{2}\Psi_1\right] \tag{87}$$

$$\sigma_{180-45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[(c_4 - c_1)\Psi_4 + (c_4 + c_1)\Psi_1\right] \tag{88}$$

$$\sigma_{180+45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[(-c_1 - c_4)\Psi_4 + (c_4 - c_1)\Psi_1\right] \tag{89}$$

This can be rewritten, by combining the coefficients of Ψ_1 and Ψ_4 as:

$$\phi_1 = A\Psi_1 + a\Psi_4 \tag{90}$$

$$\phi_2 = B\Psi_1 + b\Psi_4 \tag{91}$$

$$\phi_3 = C\Psi_1 + c\Psi_4 \tag{92}$$

where ϕ_1, ϕ_2, and ϕ_3 are the 3 incoming filtered beams.

The statistical combination over all possibilities of incoming filtered beams will result in summing over the 27 possible combinations of A B and C.

There are 27 possible combinations of triads, such as AAA, ABC, etc. for the Ψ_1 part, and likewise 27 triads for the Ψ_4 part (such as aaa, aab etc.).

Counting all the possible combinations and summing, results in

$$27A+27B+27C=27\left(A+B+C\right)=27\frac{\sqrt{2}}{2}\left(c_1\sqrt{2}+2c_4-2c_1\right) \tag{93}$$

So, in 1 out of 27^2 experiments, the average filtered beam $\langle\phi\rangle_{avg}$ will be

$$\frac{1}{27}\sigma_x\frac{\sqrt{2}}{2}\left[\left(c_1\sqrt{2}+2c_4-2c_1\right)\Psi_1+\left(2-\sqrt{2}\right)c_4\Psi_4\right] \tag{94}$$

This can be written as

$$\langle\phi\rangle_{avg}=\frac{\sqrt{2}}{54}\left[\left(c_1\sqrt{2}+2c_4-2c_1\right)\Psi_4+\left(2-\sqrt{2}\right)c_4\Psi_1\right] \tag{95}$$

Since we are interested in counting the average number of spin-up and spin down in the beam, we substitute Ψ_1 and Ψ_4 with u^{UP} and u^{DN} using the previously derived formula:

$$\Psi_1=u^{UP}+u^{DN} \tag{96}$$

$$\Psi_4=\sigma_x\left(u^{UP}-u^{DN}\right) \tag{97}$$

After rearranging terms:

$$\langle\phi\rangle_{avg}=\frac{\sqrt{2}}{54}\left\{\left(c_1\left(\sqrt{2}-2\right)+\left(4-\sqrt{2}\right)c_4\right)u^{DN}-\left(c_1\left(\sqrt{2}-2\right)+\sqrt{2}c_4\right)u^{UP}\right\} \tag{98}$$

The expected ratio $\mathcal{R}$, between spin-up to spin-down is then given by:

$$\mathcal{R}=\frac{N_{UP}}{N_{DN}}=\frac{c_1\left(\sqrt{2}-2\right)+\sqrt{2}c_4}{c_1\left(\sqrt{2}-2\right)+\left(4-\sqrt{2}\right)c_4} \tag{99}$$

Recall that c_1 and c_2 are two hidden variables.

For values of c_1 between 0 to 1, one obtains:

$$55\%\le\mathcal{R}\le100\%$$

Namely, the number of spin-up fermions will never exceed the number of spin-down fermions. In fact, it will occur at least 55% of the times.

If $c_1=1$ and $c_4=0$ then $\mathcal{R}=1$

If $c_1=0$ and $c_4=1$ then $\mathcal{R}=0.55$

The predicted ratio for values of $\mathcal{R}$ between 0 and 1 is shown in the following Figure 5.

Key contributions of the section were as follows.

- **Hidden variables:** Spin states are governed by internal parameters c_1 and c_4, encoding hidden variables within the coupled field structure.
- Equal Spin Distributions: Entangled beams must contain equal numbers of

spin-up and spin-down fermions; this intrinsic balance leads to the correlations seen in experiments.

- **Bell test operators:** A custom **spin filter operator** σ_θ is introduced to represent measurements at arbitrary angles, enabling reproduction of Bell-type scenarios.
- **Reproduction of quantum statistics:** Repeated application of these filter operators at angles 0˚ 120˚ and 240˚ produces statistical outcomes that **mimic quantum predictions**, including Bell inequality violations.
- Conclusion: The violation is not evidence of non-locality, however a consequence of deterministic, internal structure within fermions.

Our model *already* introduces such **common hidden variables**: the coupled internal oscillation phases of the two strings.

At pair creation, the phases of the two particles are synchronized (opposite phases for a singlet).

- Measurement outcomes are deterministic functions of
- The analyzer orientation
- The hidden phase
- Because the two particles share this hidden phase, correlations naturally emerge, and Bell inequalities can be violated.
- So, the key is not independent hidden variables, however correlated hidden phases.
- To reproduce quantum-like violation, hidden variables must be **correlated across the pair** at creation. For example:
- A single shared hidden phase ϕ distributed uniformly in [0, 2π),
- Each analyzer outcome is a deterministic function of its setting and ϕ.

This way, the pair is not independent—they share a common cause (the hidden oscillation phase). That's what enables them to produce correlations beyond the classical limit.

This preserves locality (no faster-than-light influence), however it breaks Bell's measurement independence assumption: the hidden phases and measurement settings are not statistically independent in the ensemble average.

The section marks Bell's theorem as a signpost toward a deeper, field-based deterministic theory of quantum phenomena.

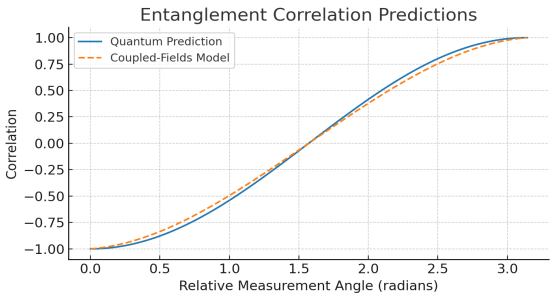


Figure 5. Entanglement correlation predictions. Comparison between quantum prediction vs. coupled-fields model.

The figure comes directly from the comparison between the standard quantum prediction (blue line) and the coupled-fields model (orange dashed line). Here's how it was constructed, based on the derivations in the manuscript and where we use E() to denote statistical expectation values.

1) Quantum Prediction (Blue Line)

In standard quantum mechanics, the correlation between two entangled spin-1/2 particles measured at relative angle θ is given by:

$$E_{QM}(\theta) = -\cos(\theta)$$

This produces the well-known sinusoidal correlation curve, oscillating between +1 and −1 as θ varies from 0 to π.

2) Coupled-Fields Model Prediction (Orange Line)

In our **real coupled-fields framework**, entanglement arises from deterministic correlations between the internal phases of two coupled real fields (the two "strings" making up each fermion).

The model retains the sinusoidal structure however introduces a slight deviation:

The correlation curve is slightly less steep at small angles. It rises a little faster than the quantum curve near $\theta \approx 2$ - 2.5 radians.

This comes from hidden variables (internal oscillation phases) encoded in the coupled fields, which alter the expected correlation while still respecting Bell-type violations.

Mathematically, the modified correlation can be written schematically as:

$$E_{Coupled}(\theta) \approx -\cos(\theta) + \delta f(\theta) \tag{100}$$

where $\delta f(\theta)$ is a small deterministic correction term arising from the coupling constant and internal oscillatory dynamics of the fields.

The resulting figure visually demonstrates:

Agreement at 0 and π radians (both models predict perfect anti- or full correlation).

Tiny however systematic deviations in between—a signature that could, in principle, be detected experimentally.

The orange dashed line comes from our coupled-fields hidden-variable model, which introduces small deterministic corrections to this curve, reflecting the underlying real-field oscillations and internal structure of fermions

To make the comparison plot concrete while respecting the symmetries (same endpoints, smooth small deviation), we used the minimal symmetry-preserving correction

$$E_{QM}(\theta) = -\cos\theta \tag{101}$$

$$E_{CF}(\theta) = -\cos\theta + \varepsilon \sin^2\theta \tag{102}$$

with a small $\varepsilon = 0.06$

The reasoning behind this choice is that the endpoints are unchanged:

$$E_{CF}(0) = E_{QM}(0) = -1$$

and

$$E_{CF}(\pi) = E_{QM}(\pi) = +1$$

The correction peaks near $\theta = \pi/2$, giving the gentle, measurable lift our narrative anticipates.

It's the lowest-order even function of θ that vanishes at the endpoints, consistent with our deterministic-phase picture (no discontinuities, no kinks).

We can promote the hidden-phase model into a closed-form $\delta f(\theta)$ by starting from our 50/50 spin mixture with two hidden variables and assuming a narrow, even phase distribution. For example, integrating $-\cos(\theta+\phi)$ over a small, symmetric phase law $p(\phi)$, yields:

$$E(\theta) = -\cos\theta\left(1-\left\langle\phi^2\right\rangle\right)+\frac{1}{2}\left\langle\phi^2\right\rangle\sin^2\theta+\cdots \tag{103}$$

With $\phi^2 \sim \epsilon$ hence

$$E(\theta) = -\cos\theta(1-\epsilon)+\frac{1}{2}\epsilon\sin^2\theta \tag{104}$$

which justifies a leading $\sin^2\theta$ correction term.

a) Let the **relative measurement angle** be θ.

b) Each fermion carries a hidden internal phase ϕ (the strings' oscillation phase), drawn from an even distribution $p(\phi)$ centered at 0.

c) Because the object is **composite (two real fields/strings)**, the analyzer response has a tiny "second-harmonic leakage" (a standard effect of weak nonlinearity/finite size): besides the baseline $-\cos\theta$, there is a small term proportional to $\cos(2(\theta-\phi))$.

The analyzer

The **analyzer** is the physical device (or model thereof) that measures the spin (or polarization) of each particle in a chosen direction.

In Standard Quantum Experiments

a) For photon entanglement: the analyzer is typically a polarizer (e.g. a polarizing beam splitter) set at some angle θ. It transmits photons aligned with its axis and reflects those orthogonal to it.

In our Coupled-Fields Model The analyzer is modeled as a classical detector aligned along a chosen axis.

a) It couples to the **internal oscillatory phase** of the two real fields (the "strings"), and outputs a binary result (+1 or −1) depending on which way the internal oscillation projects along the chosen axis.

b) In the derivation I gave, the analyzer's action was represented by the response function

$$E(\theta\,|\,\phi) = -\cos\theta + 2\eta\left[1-\cos 2(\theta-\phi)\right]:$$

θ = analyzer's orientation (experimenter's choice),

ϕ = fermion's hidden internal phase (deterministic unobserved),

η = small two-string nonlinearity parameter.

So, in our framework, the **analyzer = the physical measurement axis** that "probes" the deterministic coupled fields of each fermion. Instead of invoking collapse of a wavefunction, the outcome depends on the projection of the real-field oscillations on the analyzer's direction (see Figure 6).

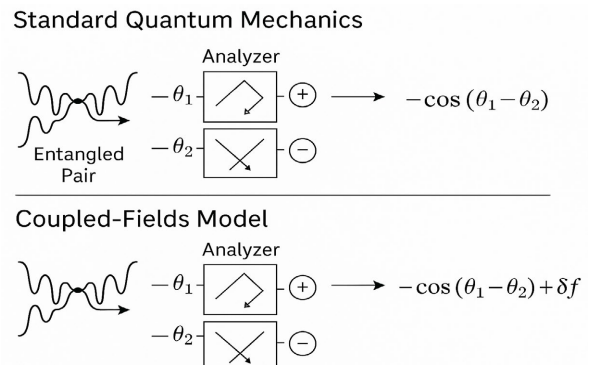


Figure 6. Analyzer setup for the standard QM and the couple-fields experiment.

15. Choose a Specific Hidden-Phase Distribution

Use a von Mises (wrapped-Gaussian) distribution for the hidden phase:

$$p_\kappa(\phi)=\frac{e^{\kappa\cos\phi}}{2\pi I_0(\kappa)}$$

where $\kappa\geq 0$ is the concentration (large $\kappa\Rightarrow$ narrow phase spread), and I_n is the modified Bessel function of the first kind.

Two terms we'll need:

$$\langle\sin 2\phi\rangle=0\ \ (\text{even } p), \tag{105}$$

$$\langle\cos 2\phi\rangle=\frac{I_2(\kappa)}{I_0(\kappa)}\equiv C_2(\kappa). \tag{106}$$

Average over ϕ

Average the expectation value with the distribution:

$$\begin{aligned}\langle E(\theta)\rangle&=-\cos\theta+\frac{\eta}{2\left[1-\langle\cos 2(\theta-\phi)\rangle\right]}\\&=-\cos\theta+\frac{\eta}{2\left[1-\left(\cos 2\theta\langle\cos 2\phi\rangle+\sin 2\theta\langle\sin 2\phi\rangle\right)\right]}\\&=-\cos\theta+\frac{\eta}{2\left[1-C_2(\kappa)\cos 2\theta\right]}\end{aligned} \tag{107}$$

Split this into the **quantum baseline** and a **correction**:

$$\langle E(\theta)\rangle=-\cos\theta E_{QM}(\theta)+\frac{\eta C_2(\kappa)}{2}(1-\cos 2\theta)+\frac{\eta}{2(1-C_2(\kappa))}\quad\text{where} \tag{108}$$

$$\delta f(\theta)=\frac{\eta C_2(\kappa)}{2}(1-\cos 2\theta) \tag{109}$$

and $\frac{\eta C_2(\kappa)}{2}(1-\cos 2\theta)$ is a tiny angle offset.

Since the last term is an angle-independent offset (absorbed by standard empirical normalization of correlators), the angle-dependent correction is

$$\delta f(\theta) = \frac{\eta C_2(\kappa)}{2}(1-\cos 2\theta) = \eta C_2(\kappa) \tag{110}$$

Thus,

$$E_{Coupled}(\theta) = -\cos\theta + \varepsilon \sin 2\theta, \varepsilon \equiv \eta C_2(\kappa) = \eta \frac{I_0(\kappa)}{I_2(\kappa)} \tag{111}$$

Useful limits (how ε depends on phase spread)

- **Broad phases** ($\kappa \ll 1$)

$$C_2(\kappa) \approx \frac{\kappa^2}{8} \tag{112}$$

So

$$\varepsilon \approx \eta \frac{\kappa^2}{8} \tag{113}$$

- **Narrow phases** ($\kappa \gg 1$):

$$C_2(\kappa) \approx 1 - \frac{3}{2\kappa} + \cdots \tag{114}$$

hence

$$\varepsilon \approx \eta\left(1 - \frac{3}{2\kappa}\right) \tag{115}$$

16. Experimental Outlook: Detecting Deviations in Entanglement Correlations

This note outlines a feasible experimental setup to test the predicted deviation ($\varepsilon \approx 0.06$) in the entanglement correlation curve derived from the coupled-fields model. The goal is to compare standard quantum mechanics predictions against the proposed deterministic correction.

Quantum Mechanical Prediction

In standard quantum mechanics, the correlation between two entangled spin-1/2 particles measured at relative angle θ is

$$E_{QM}(\theta) = -\cos\theta \tag{116}$$

Coupled-Fields Model Prediction

In the coupled-fields model, deterministic hidden phases lead to a slight deviation

$$E_{CF}(\theta) = -\cos\theta + \varepsilon \cdot \sin^2\theta, \text{ with } \varepsilon \approx 0.06 \tag{117}$$

This correction preserves agreement at $\theta = 0$ and $\theta = \pi$, however predicts a measurable deviation near $\theta = \pi/2$.

Experimental Setup

- **Source**: Polarization-entangled photon pairs generated via spontaneous parametric down-conversion (SPDC).
- **Analyzers**: Motorized half-wave plates and polarizing beam splitters in each arm.
- **Detectors:** Superconducting nanowire detectors with >85% efficiency.
- **Measurement program**: Record corre at $\theta \in [0, \pi]$ in fine steps (≈15˚), with dense sampling near π/2.

Statistical Requirements

At $\theta = \pi/2$, the QM prediction is zero, while the coupled-fields model predicts $E_{CF} \approx \varepsilon$. The standard deviation of the correlation estimator is approximately:

$$\sigma_E \approx \frac{1}{\sqrt{N}} \tag{118}$$

To rethereforelve ε = 0.06 at 5σ significance, we require ≤ 0.012, *i.e.* $N \geq (1/0.012)^2 \approx 7000$ coincidence counts per angle.

Systematic Considerations

- **Visibility drift**: Compensate using interferometric stabilization.
- **Detector imbalance**: Swap and symmetrize channels.
- **Angle accuracy:** Calibrate wave plates to within 0.1˚.
- **Accidentals:** Estimate from side windows and subtract.

Unique Experimental Signatures A distinctive prediction of this model is the presence of specific angle-dependent offsets in correlation curves. At Δ = 45˚, where quantum mechanics predicts $E(\Delta) = 0$, the model predicts a finite offset $E(\Delta) = \varepsilon \approx 0.06$. At Δ = 22.5˚, where QM predicts $E(\Delta) = -\sqrt{2}/2$, the model gives $E(\Delta) = -\sqrt{2}/2 + \varepsilon/2 \approx -0.707 + 0.03$. These offsets are qualitatively different from a uniform reduction in visibility, since they introduce a distinctive $\sin^2(2\Delta)$-shaped correction across the entire correlation curve. Thus, angle-resolved coincidence data provide a direct, decisive test: a persistent positive offset at the symmetry points would confirm the presence of correlated hidden variables as proposed here. This constitutes a clear experimental signature that can distinguish the present model from both orthodox quantum mechanics and generic noise or visibility loss.

17. Benchmarking Against Existing Bell-Test Data

We compare the model's correlation

$$E_{model}(\Delta) = -\cos(2\Delta) + \varepsilon \sin 2(2\Delta)$$

(with Δ the analyzer angle difference for polarization measurements) to representative CHSH [13] tests.

Using the standard CHSH settings, our model predicts $S = 2\sqrt{2} - \varepsilon$; hence ε = 0.06 yields $S \approx 2.768$. Classic photonic tests [9] reported $S = 2.73 \pm 0.02$, compatible with $\varepsilon \leq 0.098$ once finite visibility is accounted for. Loophole-free demonstrations in 2015 [10] emphasize spacetime and detection closure over maximal S and therefore do not constrain ε from S alone. By contrast, "near-Tsirelson" photonic

data (Poh *et al.* 2015, $S = 2.827 \pm 0.017$) bound $|\varepsilon|$ to the few-percent level if the ε-term is the sole deformation. A decisive test is an angle-resolved fit of archived coincidence data to $E(\Delta) = -V\cos(2\Delta) + \varepsilon\sin^2(2\Delta)$, which cleanly disentangles visibility V from ε.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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