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Higgs Mechanism, Elementary Particles and Quantum Foundations: A Geometrical Connection

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Abstract

A specific structure of Standard Model (SM) particles is proposed and investigated. According to this proposal, the precursor of an elementary particle is a de Sitter space tangent to ordinary spacetime; the value of the gravitational constant within such a space does not necessarily have to coincide with that relating to ordinary spacetime, and it is chosen as a function of the Higgs vacuum. The curvature of this space is sized by the Higgs boson mass. This space is a solution of the corresponding Einstein gravitational equations, if the internal density is suitably chosen and the internal pressure is assumed to be negative. An elementary fermion of the SM derives from this precursor through a redefinition of the internal gravitational constant to ensure the proportionality between mass and coupling constant to the Higgs field as required by the SM. The de Sitter radius then turns out to be, at the same time, the classical radius of the fermion and the "gravitational" radius in the sense of the internal gravitational constant. The quantum version of the fermion is obtained by passing from the Einstein gravitational equations to the Wheeler-de Witt (WdW) equation. There are free solutions, both harmonic and exponential. The former correspond to de Broglie plane waves and can be superposed in order to provide the usual solutions of the relativistic wave equations. The latter describes quantum jumps in full compliance with Einstein locality. The reduction of the projection postulate to a dynamical consequence on the level of elementary particles implies the production of true decoherence induced by usual microscopic interactions, without any tracing out of environmental degrees of freedom, and the calculation of the decoherence time is illustrated in a simple case. The interaction of SM gauge bosons with elementary fermions (with and without production of quantum jumps) is modeled according to the same scheme. A possible interpretation of the fine structure constant and a formula for calculating the coefficients of the CKM, PMNS matrices are derived in this context. Other consequences of potential theoretical interest are reviewed. The existence of particle de Sitter spaces can be tested in gravitational scattering processes between particles.

Keywords

Wheeler de Witt Equation, Decoherence, Standard Model, Higgs Mechanism, Flavour Mixing

1. Introduction

The Higgs mechanism [1] [2] is one of the essential ingredients of the Standard Model (SM), which in turn constitutes the current overall framework for the theoretical study of elementary particles. Initially proposed as a hypothesis aimed at the "soft" introduction (*i.e.* without loss of renormalizability) of the mass terms into the SM Lagrangian, it has been placed on a secure basis by subsequent experimental results, culminating in the confirmation of the existence of the Higgs boson H_0 in the past decade [3]. Here we propose a geometric reinterpretation of it, which we use as a basis for a more general geometric interpretation of the elementary fermions of the SM: quarks and leptons. As we will try to illustrate, this approach admits implications that connect particle physics and quantum foundations; furthermore, its premises are—at least in principle—experimentally testable.

In order to clarify the idea behind the construction, let us first illustrate it in a classical (*i.e.* pre-quantum) context, postponing the quantization to a later step. We propose the origination of an SM elementary fermion as conceptually (not physically) divisible into two consecutive steps. In the first step, a de Sitter space tangent to the spacetime V is created in a five-dimensional Euclidean space E_5 (V is thought to be immersed in E_5). The reasons for the choice of this particular space will be clarified later. It is assumed that distinct elements of matter belonging to this space influence each other through the "gravitational" interaction internal to it. It should be noted that this interaction is not the gravitational interaction acting on spacetime and well known to us; in particular, the internal "gravitational" constant of the de Sitter space considered, which regulates the interaction between the elements of matter contained in it, *is not* Newton's gravitational constant. Matter is assumed to be distributed, within the single de Sitter space, according to a homogeneous and constant density sized both by the vacuum expectation value (VEV) of the Higgs field and by the mass of the Higgs boson [4]. By imposing that the density is a source of negative pressure, the de Sitter space can be obtained as a solution to Einstein's gravitational equations according to an appropriate choice of the internal "gravitational" constant. The second step consists of a redefinition of the internal "gravitational" constant in accordance with a total mass content proportional to the VEV, as prescribed by the SM for elementary fermions. The related space is a rescaled version of the previous space associiated with the VEV, and is solution of the same gravitational equations with a different value of the gravitational constant. It can be considered as a version of the de Sitter space associated with the VEV but with a different energy content. It then implements a quantum entity that is an *elementary particle* (elementary fermion) of the SM [5]. No mutual "gravitational" interaction exists between mass elements of distinct spaces, even in the case of contact or superposition of these spaces in E_5. It is in principle possible to avoid contacts and overlaps between distinct de Sitter (dS) spaces by assuming a five-dimensional space $V \times R_i$ depending on the particle index i, with R_i, R_j orthogonal if i, j are distinct; in the following, however, we will not use this stronger condition. The two-step process thus described is a geometric equivalent of the Higgs mechanism, with respect to the generation of massive fermionic fields.

The transition to the quantum description can, in turn, be divided into two distinct phases, concerning respectively the free elementary fermions and their interactions mediated by gauge bosons. The first phase is achieved by moving from Einstein's gravitational equations to their quantum counterpart, consisting of a suitably chosen Wheeler-de Witt (WdW) equation. Among the asymptotic solutions of this equation, the harmonic and the decreasing exponential ones are of particular interest. The harmonic solutions depend on a time parameter defined by the distance scale, which can be connected to the internal global time of the de Sitter space. We identify this variable with the particle's proper time, measured by an external observer "at rest" with respect to it. It can be related to the internal closed slicing time. The application of the relativistic covariance rules transforms the harmonic solution into a de Broglie plane phase wave, delocalized over the entire space of contemporaneity of the particle in spacetime. From the linearity of the evolution equation it follows that the most general free solution is given by the generic superposition of plane waves that satisfies the dispersion relation. We then obtain the Klein-Gordon equation. In summary, the Higgs mechanism defines the mass of the SM particles, and therefore also their proper time, conjugated to the mass. The fact that the evolution of the particles is defined primarily by the proper time is the origin of the spatial delocalization of these quantum entities [6]. Exponential solutions do not support the relation between internal time of the dS space and external time measurable by an observer; they are therefore related to physical situations in which the evolution of the particle in external time stops. Their further characteristics are evanescence and violation of unitarity. These solutions therefore describe instantaneous processes of disappearance of the particle state from spacetime or of appearance of the particle state in spacetime, or both. We propose to use them to model the discontinuous variation of the particle state, *i.e.* quantum jumps, thus providing a dynamical basis for the projection postulate [7] [8].

The possibility of describing in dynamical terms the state reduction processes as effects of the ordinary interactions between elementary particles described by SM solves an old conceptual problem of quantum theories [9]. It also allows the construction of classicalization models that are not affected by the traditional problems of the absence of selection and justification of the preferred basis [10]. The decoherence involved in these models implies an actual violation of unitarity at the microscopic level, and not a mere concealment of coherence through the trace operation on the environmental degrees of freedom; the latter, in fact, are not relevant in this description. The proposed mechanism is therefore also operational for single quantum systems in total isolation (for example, decay of excited atomic states in interstellar gas).

The redefinition of elementary particles as quantum de Sitter spaces tangent to spacetime (with a quantum delocalized tangency point) implies the need to consequently redefine the interaction of SM elementary fermions with gauge bosons (γ, $W^{\pm}$, Z_0, H_0, gluons, graviton); this brings us to the second, previously anticipated, phase of the quantization procedure. The discontinuous nature of a quantum micro-interaction does not allow its modeling in classical terms of continuous phenomena. Therefore, neither mass exchanges between the dS spaces of the different elementary fermions convergent in an interaction vertex, nor direct couplings between their matter elements are admissible. The only way to describe the variation of the fermionic state in the coupling with a gauge boson is therefore to assume that the boson couples the entire dS space of the incoming fermion with the entire dS space of the outgoing fermion. As we will show, this choice entails the usually accepted form of the vertex amplitude; it also justifies the impossibility of extending to bosons the geometric mechanism postulated for fermions. The boson coupling mechanism is studied in some detail, and it is shown that only in weak interactions mediated by charged current does flavor mixing occur. A calculation of the amplitude of the interaction vertex is proposed and it is shown that it is diagonalized, on the basis of flavors, by the experimental mixing matrices (Cabibbo-Kobayashi-Maskawa or CKM for quarks, Pontecorvo-Maki-Nakagawa-Sakata or PMNS for leptons). An other, speculative, application concerns the photon, and consists in an attempt to justify the value of its coupling constant (fine structure constant).

The finite value of the de Sitter radius of elementary particles implies a finite value of the energy associated by the uncertainty principle to this radius. As we will show, it is possible, on this basis, to reinterpret the cosmological constant as a structural constraint connected to the potential processes of localization of matter in spacetime, without relation to the vacuum energy. This result constitutes a possible way out of the gigantic discrepancy between vacuum energy and the cosmological constant.

According to the proposed description, the elementary fermions of the SM should be point-like in impact processes mediated by interactions between their electric, weak or strong charges, in accordance with the available experimental data. However, they should behave as extended objects (on a scale $\leq 10^{-13}$ cm) in gravitational scattering processes. Their inertia would in fact be a property distributed in a finite region sized by the de Sitter radius. Some processes in which effects of this type could be observed (whose detection is beyond the possibilities of current technology, but could prove feasible in the near future) are briefly mentioned in the text of the paper and in the conclusions.

The plan of the presentation is as follows. The basic principles and assumptions are stated in Section 2. Subsections 2.1, 2.2 present the two-step geometric representation of the SM elementary fermions, in essentially semiclassical terms. The transition to quantization, via the WdW equation, is described in Subsection 2.3. A suitable physical interpretation of the harmonic solutions of the WdW allows the introduction of elementary particles as spatially delocalized entities described by quantum-relativistic wave equations, an aspect that is discussed in Subsection 2.4. The exponential solutions of the WdW are interpreted as a formal description of quantum jumps in Subsection 2.5.

In Subsection 2.6 this dynamic description of the quantum discontinuity, which completes the usual Von Neumann projection postulate [7], is applied to the problem of decoherence and classicalization, with the discussion of a specific example. Subsection 2.7 is a short note illustrating the entropic meaning of the fine structure constant in the context of the present approach. In Subsection 2.8 a conjecture is presented which appears necessary (given the scarcity of data currently available) for the application of the approach to neutrinos. Subsection 2.9 is devoted to an interpretation of the cosmological constant in the terms already exposed, with some numerical evaluations. Subsection 2.10 is a marginal note referring to previous works and discusses the possible extension of the line of reasoning of the paper to the hadronic case. In particular, the possibility of defining, within the proposed scheme, an *ab initio* confinement condition both in geometric and color terms is recalled.

Section 3 is devoted to the SM gauge bosons. Subsection 3.1 introduces the concept of interaction boson and shows its congruence with the usual expression of the vertex amplitude in the SM. Subsection 3.2 is an attempt to justify the value of the fine structure constant, inspired by the entropic meaning of this constant, illustrated in Subsection 2.7. Subsection 3.3 is devoted to the computation of the mixing parameters of the charged current-mediated weak interaction. Conclusions are given in Section 4.

2. Basic Principles

2.1. Pre-Quantum Description

Let us consider Einstein's gravitational equations with cosmological term:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda_0 g_{\mu\nu} = \chi T_{\mu\nu} \tag{1}$$

with the usual meaning of the terms. Let us consider the case in which matter can be schematized as an ideal fluid with mass density η, which we will assume to be homogeneous and constant, and four-velocity u_μ:

$$T_{\mu\nu} = \left(\eta + \frac{p}{c^2}\right) u_\mu u_\nu + p g_{\mu\nu} \tag{2}$$

For a negative pressure p with $p + \eta c^2 = 0$, Equation (1) becomes:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \left(\Lambda_0 + \chi\eta c^2\right) g_{\mu\nu} = 0 \tag{3}$$

which is the equation of a de Sitter space with an "effective" cosmological constant $\Lambda = \Lambda_0 + \chi\eta c^2$. If $\Lambda_0 = 0$, we have $\Lambda = \chi\eta c^2 = R$. We note that it is possible to arrive at the same result if we start from the full contraction of Equation (1) without a cosmological term:

$$2R = \frac{8\pi G}{c^4} T \tag{4}$$

(where we placed $\chi = 8\pi G/c^4$), remarking that for a negative pressure $p = -\eta c^2$ we have:

$$T_{\mu\nu} = \left(\eta + \frac{p}{c^2}\right) u_\mu u_\nu + p g_{\mu\nu} = p g_{\mu\nu} \Rightarrow T = g^{\mu\nu} T_{\mu\nu} = -2p = 2\eta c^2 \tag{5}$$

and therefore:

$$R = \frac{8\pi G}{c^4} \eta c^2 = \frac{3}{r_{dS}^2} \tag{6}$$

where r_{dS} is the de Sitter radius of the space described by Equation (6).

Let us now consider the Higgs field, which is introduced in the usual way, as an iso-doublet Ω of scalar fields defined on the space SU(2) of the weak isospin. These fields will depend on the spacetime position. We denote by ε (having the dimensions of an energy) the expectation value of the vacuum (VEV), defined in the usual way. Let's consider the energy e defined by the relation:

$$|\Omega|^2 = \Omega^+ \Omega = \varepsilon e \tag{7}$$

The symmetry breaking, that is the passage from $e = 0$ (the "false vacuum") to $e = \varepsilon$ (the "true vacuum"), corresponds in the SU(2) space to the choice of a specific direction of the spinor Ω of norm ε as the new vacuum state. $|\Omega| = \varepsilon$ must be a stable point of the potential of the free Higgs field. Therefore, this potential must be even in $\varepsilon(e - \varepsilon)$ and then be expressed by a sum of even powers of $(|\Omega|^2 - \varepsilon^2)$. The only possibility compatible with renormalization is that of a completely harmonic dynamics of the variable $\varepsilon(e - \varepsilon)$, in accordance with the usual choice of a potential proportional to $\varepsilon^2 (e - \varepsilon)^2$, that is $(|\Omega|^2 - \varepsilon^2)^2$.

After these premises, let us now come to the idea underlying our proposal. It consists in assuming that spacetime is immersed in an E_5 space, and that each point-event of spacetime is the potential point of tangency, on spacetime, of a de Sitter space in E_5. We assume that these spaces are non-interacting in the sense that, although spaces tangent to the same point-event or to nearby point-events can overlap or intersect in E_5, the matter contained in the regions of overlap interacts only with the matter located in the same space. In other words, the matter of each space does not "see" the matter of other spaces (a more complicated, but equivalent construction is mentioned in the Introduction). As regards the interaction of distinct portions of matter of the same space, we assume that it is of a single type, that is, it is a gravitational interaction describable through Equation (1). A crucial observation at this point is the following: there is no reason why the gravitational constant G internal to these spaces should coincide with Newton's gravitational constant operating on spacetime. In fact, these are entirely different spaces between which there exists a mere geometric relation of tangency. There is no relationship between the internal gravitation of these spaces and the "external" gravitation acting on spacetime. It is then possible to consider the situation in which the tangent spaces are solutions of Equation (6) with the conditions:

$$G = \frac{\hbar c}{\left(\frac{\varepsilon}{c^2}\right)^2} = G_0 \tag{8}$$

and:

$$r_{dS} = \frac{\hbar}{M_{H_0} c} = r_{dS,0} \tag{9}$$

where M_{H0} is the mass of the Higgs boson H_0. As can be seen from Equation (6), the solution exists if:

$$\eta = \eta_0 = \frac{M_{H_0}^2 c^4}{\hbar^2} \left(\frac{\varepsilon}{c^2}\right)^2 \frac{3}{8\pi\hbar c} \tag{10}$$

that is, if the density is chosen as a suitable function of the constants M_{H0} and ε. Denoting with M the total mass content of space:

$$\eta = \frac{M}{\frac{8\pi}{3}\left(\frac{k\hbar}{Mc}\right)^3} = \frac{3}{8\pi} \frac{M^4 c^3}{\hbar^3 k^3} \tag{11}$$

(we will see the meaning of the factor k in a subsequent subsection), where:

$$r_{dS} = \frac{k\hbar}{Mc} \tag{12}$$

then η_0 turns out to be the geometric mean of the following densities computed with $k = 1$:

$$\eta_{H_0} = \frac{3}{8\pi} \frac{M_{H_0}^4 c^3}{\hbar^3} \tag{13}$$

$$\eta_\varepsilon = \frac{3}{8\pi} \frac{\left(\frac{\varepsilon}{c^2}\right)^4 c^3}{\hbar^3} \tag{14}$$

that is:

$$\eta_0 = \sqrt{\eta_{H_0} \eta_\varepsilon} \tag{15}$$

The mass content of space is then:

$$\frac{8\pi}{3}\eta_0 \left(\frac{\hbar}{M_{H_0} c}\right)^3 = M_{H_0}^{-1} \left(\frac{\varepsilon}{c^2}\right)^2 \tag{16}$$

as one might expect from (15). The physical idea is that the VEV of the Higgs field is nothing more than a measure of the gravitational constant (8), while the mass of the Higgs boson is a measure of the de Sitter radius (9) or the curvature $-R_0 = 3/r_{dS,0}^2$. Another way to define the same physical situation [4] is to write the self-interaction potential of the field in the form:

$$V(\Omega) = \frac{\sqrt{Det(g_{\mu\nu})}}{4\pi}\left[\frac{R_0}{6}\Omega^+\Omega - \frac{\chi_0 R_0}{36}(\Omega^+\Omega)^2\right] \tag{17}$$

where $\chi_0 = 8\pi G_0/c^4$. Posing:

$$R_0 = 6\mu^2 \tag{18}$$

and assuming the conventional relation between the VEV and the mass M_W of the W boson:

$$\varepsilon = \frac{\sqrt{2}c^2}{g}M_W \tag{19}$$

(g represents the electroweak coupling constant), from the expansion around ε in the usual way we then have:

$$\varepsilon = c^2\left(\frac{3}{\chi_0}\right)^{1/2} \Rightarrow \chi_0 = \frac{3}{2}\frac{g^2}{M_W^2} \tag{20}$$

$$M_{H_0} = \frac{\hbar}{c}\left(-\frac{R_0}{3}\right)^{1/2} \Rightarrow R_0 = -\frac{3M_{H_0}^2c^2}{\hbar^2} \tag{21}$$

$$V(\Omega) = \mu^2\Omega^+\Omega + \lambda(\Omega^+\Omega)^2 \tag{22}$$

where $\lambda = -\chi_0 R_0/36$ and:

$$\mu^2 = -\frac{1}{2}\frac{M_{H0}^2c^2}{\hbar^2} \tag{23}$$

We can now note that *in general*, substituting (11), (12) in (6) we have:

$$G = \frac{k\hbar c}{M^2} \tag{24}$$

This result can be obtained by replacing, in (8), ε/c^2 with $M/k^{1/2}$. This allows us to extend the representation to the elementary fermions of the SM. According to the SM, the mass M of the fermion is proportional to the VEV, the constant of proportionality being the coupling constant of the fermion to the Higgs field. We therefore have $(M/k^{1/2}) = f(\varepsilon/c^2)$ and therefore, from (8) and (24), $G = G_0/f^2$. In other words, the genesis of the mass M of the fermion (more correctly, of the ratio $M/k^{1/2}$) derives from a rescaling of the internal gravitational constant, which in turn induces a new de Sitter radius expressed no longer by (9), but by (12).

Thus, the classical equivalent of the positional eigenstate of a SM elementary fermion will in fact be associated not to the point-event O, but to the fermionic de Sitter space tangent to O, solution of (6) with gravitational constant (24), de Sitter radius (12) and density (11). The mass content of this space is:

$$\frac{8\pi}{3}\eta\left(\frac{\hbar}{Mc}\right)^3 = M \tag{25}$$

as can be easily verified by direct substitution of (11). For the particles we therefore have the following scaling relations:

$$\frac{r_{dS}^2}{r_0^2} = \frac{M_0^2}{M^2} = \frac{G}{G_0} = \frac{1}{f^2} \tag{26}$$

where:

$$r_0 = \sqrt{k}\,\frac{\hbar c}{\varepsilon}; \qquad M_0 = \sqrt{k}\,\frac{\varepsilon}{c^2} \tag{27}$$

The replacement of the VEV of the Higgs field, ε, with the rest energy of the fermion, $Mc^2 \propto \varepsilon$, can be seen as the passage from the Higgs field to a scalar field ξ, internal to the fermionic de Sitter space, whose VEV reproduces $\xi_0 = Mc^2/k^{1/2}$. We then obtain $U(\xi) = [\xi^2 - (Mc^2)^2/k]^2$. Therefore, $\xi_0 \big/ \left(8\pi r_{dS}^3/3\right) = p\big/k^{1/2}$, where p is the pressure modulus. It is possible to introduce the dimensionless self-potential:

$$V(\xi) = \frac{U(\xi)}{\left(Mc^2\right)^2} = \left[\left(\frac{\xi}{Mc^2}\right)^2 - \left(\frac{1}{k^{1/2}}\right)^2\right]^2 = \left(\xi^2 - \xi_0^2\right)^2 \tag{28}$$

where ξ, ξ_0 are now dimensionless and $\xi_0 = 1/k^{1/2}$. For continuity with previous works [11], we will also express, without any loss of generality, the dimensionless quantity ξ as $\xi = r/r_{dS}$, where r is a length of positive, zero or negative value.

2.2. Digression on the *k* Factor

In Equation (12) we have inserted a factor k that converts the Compton wavelength of the particle into the de Sitter radius of the space associated with it. We now want to clarify the physical meaning of this factor. Consider two interacting elementary fermions, 1 and 2, and let:

$$E_{int} = \frac{\hbar c}{L} \tag{29}$$

be their positional interaction energy in the center-of-mass frame. Let d be their spatial distance in that frame and:

$$E_{sol} = \frac{\hbar c}{d} \tag{30}$$

the energy required by the uncertainty principle to resolve this distance. Given the symmetry of the system, it is possible to set:

$$\frac{E_{int}}{E_{sol}} = \frac{d}{L} = \frac{q_1 q_2}{\hbar c} \tag{31}$$

where q_1, q_2 are attributes of the interacting fermions 1 and 2 respectively. This relation can also be written in the more familiar Coulomb form:

$$E_{int} = \frac{q_1 q_2}{d} \tag{32}$$

from which it follows that q_1, q_2 are (the moduli of) the interaction charges of the two fermions. As can be seen from (30), the singularity of Coulomb's law at $d = 0$ arises from the singularity of E_{sol} in accordance with the uncertainty principle. In the case of two fermions of the same type (for example, two electrons) we have $|q_1| = |q_2| = q$ and:

$$\frac{d}{L} = \frac{q^2}{\hbar c} \tag{33}$$

The threshold for the creation of a fermion corresponds to a value of L defined by the relation:

$$E_{int} = \frac{\hbar c}{L} = Mc^2 \Rightarrow L = \frac{\hbar}{Mc} \tag{34}$$

where M is the mass of the single fermion. The corresponding value of d is:

$$d = \frac{q^2}{\hbar c} L = \frac{q^2}{Mc^2} \tag{35}$$

The interaction energy $E_{int} = Mc^2$, the threshold for the production of the fermion, must correspond to a resolution energy E_{sol} of the interaction distance d equal to that required by the uncertainty principle to distinguish the de Sitter radius r_{dS} of the generated fermion. As can be seen from (35), this implies that d equals r_{dS}. Equation (35) therefore becomes:

$$r_{dS} = \frac{q^2}{Mc^2} \tag{36}$$

The de Sitter radius of an elementary fermion therefore coincides with its classical radius. Comparing (36) and (12) we have:

$$k = \frac{q^2}{\hbar c} \tag{37}$$

This is then the expression of the factor k. It can be noted that the charge comes to play a dual role: that of coupling constant between fermions in Coulomb law (32) and that of coupling constant between mass and curvature of the single fermionic space in (36). In relation to this aspect it must be noted that the energetic singularity expressed by (32) does not correspond, in this description, to any "charged material point" located in the tangency point O of the fermionic de Sitter space on the spacetime. The charge, as can be seen from (36), is a global property of this space and is not "located in O". Inserting (37) into (24) we obtain the relation:

$$q^2 = GM^2 \tag{38}$$

From (38) and (36) we obtain:

$$r_{dS} = \frac{GM}{c^2} \tag{39}$$

In other words, the de Sitter radius is at the same time the classical radius of the fermion and its "gravitational" radius in the sense of the internal "gravitational" interaction. It can be noted that by setting $r_{dS} = c/H_0$, where H_0 is the Hubble constant of the fermionic de Sitter space, from (39), (11), (12) it follows that:

$$\eta = \frac{3H_0^2}{8\pi G} \tag{40}$$

That is, the density of matter inside such a space is always the critical density. In relations (36), (37), (38) the squared charge q^2 must be understood as the sum of the squared charges associated with the different interactions to which the fermion contributes:

$$q^2 = q_{electromagnetic}^2 + q_{weak}^2 + q_{strong}^2 \tag{41}$$

For charged leptons the last term is zero, while the first term dominates the second; we can therefore assume $q \approx q_{electromagnetic}$. In the case of neutrinos the first and third terms are zero, so we have $q = q_{weak}$. For quarks the situation is analogous to that of charged leptons, with the difference that the third term is not zero. We will return to the correct definition of q_{weak} in later Sections.

From this semiclassical treatment it follows that the mass and charge of an elementary fermion are not associated with a point-like entity, but with an extended dS space. However, the magnitude of the interaction diverges as d approaches zero (Equation (32)), in agreement with the behavior observed in the high-energy scattering of these particles, on the basis of which a point-like structure is usually attributed to them. An important point is that while no notion of continuous charge distribution on the dS space is introduced here, a notion of continuous mass distribution on this space is introduced instead through (Equation (11)). This difference entails a prediction that is absent in the conventional treatment, namely that the gravitational interaction of SM elementary fermions will deviate from Newton behavior for energies larger than $\hbar c/r_{dS}$. Indeed, if the equivalence principle is to be applied to the mass elements of a fermionic dS space, each element will contribute both to the inertia of the particle (inertial mass), as prescribed by Equation (25), and to its coupling with an external gravitational field (gravitational mass). The gravitational charge of the particle is therefore the only interaction charge to have a continuous distribution on the dS space even if, as specified in the Introduction, it is always the entire fermionic dS space that interacts with each single graviton by emitting or absorbing it. Gravitational scattering experiments of elementary fermions (for example, neutrinos) would therefore reveal the finite size of their dS spaces, allowing on the one hand the verification of the present hypotheses, on the other the measurement of r_{dS}. As regards the gravitational interaction between fermions in the static limit we note that if, in accordance with the equivalence principle, the inertial mass density η is to be taken as the source of the gravitational field, d must be averaged over the finite extension of the mass distribution of the two fermions in E_5. This results in a maximum value of (32) ≈ $G_N M_1 M_2/(\alpha\hbar/(M_1 M_2)^{1/2} c)$, where M_1 and M_2 are the masses of the interacting fermions and G_N is the Newtonian gravitational constant. Even in the extreme case $M_1 = M_2 = M_{top} = 170\ \text{GeV}/c^2$, the ratio between this expression and $G_N M_{Pl}^2 / L_{Pl}$ (where M_{Pl} and L_{Pl} are the Planck mass and length respectively) amounts to 10^{-51}.

In general, since the entire mass distribution on the fermionic dS space is involved in the exchange of each single graviton, the latter must have a maximum energy ~ $\hbar c\gamma/r_{dS}$ and it must be exchanged in a minimum time interval ~ $r_{dS}\gamma/c$, where γ is the relativistic contraction factor. The energy exchanged in the unit of time is therefore smaller than $\sim \hbar c^2/r_{dS}^2$; in other words, the maximum intensity of the interaction is independent on γ. Since the total masses involved in the interaction, and therefore the number of gravitons emitted by them in the unit of time, are proportional to γ, it follows that the coupling constant G_N must be $\propto \gamma^{-2}$ on spatial scales smaller than r_{dS}.

2.3. Quantization

The conveyance of the energy Mc^2 and the charge q in the formation of an elementary fermion of renormalized charge q and renormalized mass M can therefore be conceptually seen as a two-stage process (in effect simultaneous and instantaneous). In the first stage a de Sitter space is formed whose internal gravitational constant G_0 and radius r_0 are dimensioned by the properties of the Higgs field (Equations (8), (9)). In the second stage, these quantities undergo a rescaling $G_0 \to G$, $r_0 \to r$ so that M and q are encoded in the geometry of the "final" de Sitter space according to the relations $q = rc^2/G^{1/2}$, $M = rc^2/G$ (Equations (36), (38)). This space has a definite point of tangency on the ordinary spacetime V.

However, in order to exist over a finite interval of duration, the particle must possess a momentum vector along the external temporal direction in V. This requirement obviously conflicts with the uncertainty principle, according to which spatial position and momentum must be conjugate quantities with an uncertainty product in the order of h. In the transition from the "classical" description adopted up to now to the quantum one, the point of tangency will therefore become generally undefinite.

The spacetime propagation of the particle is described by a quantum-mechanical equation of motion. In order to derive it, it is preliminarily necessary to move from the description of the particle de Sitter space based on Einstein's gravitational equations, developed in the previous subsections, to the corresponding quantum description based on the quantum version of those equations; that is, on the Wheeler-De Witt (WdW) equation [12] [13]. We therefore consider a Friedmann-Robertson-Lemaitre-Walker space with closed spatial sections, filled with a homogeneous and isotropic fluid whose pressure is p and whose density is η, these quantities being connected by the equation of state $p = w\eta$ (we use here units $c = 1$). In addition is assumed the presence of a massless scalar field ξ. The relevant Wheeler-De Witt equation can be written in the form [14]:

$$\left(a^2\frac{\partial^2}{\partial a^2} + a\frac{\partial}{\partial a} + \frac{\omega_0^2 a^{2q} - a^4}{\hbar^2} - \frac{\partial^2}{\partial \xi^2}\right)\Psi(a,\xi) = 0 \tag{42}$$

where a is the scale factor and ω_0 is an integration constant dependent on the energy density on a defined hyper-surface Σ. The parameter $q = 3(1 - w)/2$ defines the matter scheme: radiation for $w = 1/3$, dust for $w = 0$ and pure cosmological constant for $w = -1$. Our case is the latter; we have $q = 3$ and $\omega_0^2 = \Lambda$. Posing $\zeta = \ln(a/a_0)$, where a_0 is the reference scale factor, Equation (42) can be rewritten as:

$$\left(\frac{\partial^2}{\partial\zeta^2} - \frac{\partial^2}{\partial\xi^2} + \frac{\Lambda a_0^6 e^{6\zeta} - a_0^4 e^{4\zeta}}{\hbar^2}\right)\Psi(\zeta,\xi) = 0 \tag{43}$$

The Wheeler-De Witt equation does not contain time. This fact constitutes the origin of the well-known "problem of time" [15] [16], which we do not delve into here. But what if the quantity ζ, which appears in Equation (43) as a delocalized quantum variable, is taken as a time label? In this case it becomes an external parameter which, being dimensionless, can be written in the form $\zeta = T/\theta$, where the variable T and the constant θ are time intervals. The scale factor then takes the form $a = a_0\exp(T/\theta)$. This is in fact the expression of the distance scale in a de Sitter space with cosmic time T and a cosmological constant $\Lambda = 3/\theta^2$, solution of the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \tag{44}$$

in which the point denotes the derivation with respect to T. Limiting ourselves to considering only negative values of T (*i.e.* values prior to the instant at which $a = a_0$), for $|T| \to \infty$ the non-differential term of Equation (43) vanishes and it becomes a D'Alembert equation:

$$\left(\frac{\partial^2}{\partial\zeta^2} - \frac{\partial^2}{\partial\xi^2}\right)\Psi(\zeta,\xi) = 0 \tag{45}$$

In fact, (43) is well approximated by (45) for $|T| \gg \theta$. As can be seen from (44), θ is the de Sitter time $\theta = r_{ds}/c$. We now look for separable solutions of (45), of the form $\Psi(\zeta, \xi) = \Phi(\zeta)\Gamma(\xi)$. We first have exponential solutions of the type:

$$\Phi(\zeta) = e^{C\zeta} = e^{-\frac{C|T|}{\theta}} \tag{46}$$

$$\Gamma(\xi) = e^{-C\xi} \tag{47}$$

where C is an arbitrary constant. Then there are harmonic solutions of the type:

$$\Phi(\zeta) = e^{\pm ik\zeta} = e^{\pm\frac{ikT}{\theta}} = e^{\pm\frac{iMc^2T}{\hbar}} \tag{48}$$

$$\Gamma(\xi) = e^{\pm ik\xi} \tag{49}$$

In the last step of (48) the constant k, in principle arbitrary, has been assigned the value defined by (37). This choice allows, thanks to (36), the derivation of the de Broglie phase factor [17] if the variable $\pm T$ is identified with the proper time t of the particle. Naturally this identification, which we will discuss later, is only possible for harmonic solutions. To underline this difference we will indicate T, in exponential solutions, with the symbol τ. Setting, in Equation (45), $\xi = r/r_{ds}$, $\zeta = T/\theta$ it becomes:

$$\left(\frac{\partial^2}{c^2\partial T^2}-\frac{\partial^2}{\partial r^2}\right)\Psi(T,r)=0 \tag{50}$$

This is the form in which it has been discussed in a previous work [11].

We draw attention to the fact that the wave function solution of the equation WdW (Equations (43), (45), (50)) is associated with the whole space, not with material components (such as fields, particles, etc.) located in that space. In other words, the "particle" here is the whole space. This idea, which we will take to its extreme consequences in the next Sections, is a mirror image of that of treating quantum microuniverses, in quantum cosmology, as particle-like structures within a third quantization formalism (see [18] for an introduction to this topic). The formalism is in fact the same, albeit with a different physical interpretation.

2.4. Quantum Delocalization

As we saw in the previous subsection, for $|T| \gg \theta$ the WdW equation is approximated by a D'Alembert equation, whose harmonic solutions can be identified with the de Broglie phase factor of the particle. This identification consists in assuming that $t = \pm T$ is the proper time of the particle. Writing the scale factor in the form:

$$\frac{a_0}{a} = e^{-\frac{T}{\theta}} = \cosh\left(\frac{t_g}{\theta}\right) \tag{51}$$

where t_g is the closed slicing time, connected to the global time X_0 by the relation [19]:

$$\frac{X_0}{\theta} = \sinh\left(\frac{t_g}{\theta}\right) \tag{52}$$

then for positive values of t_g much greater than θ, we have:

$$\frac{a_0}{a} \approx e^{\frac{t_g}{\theta}+\ln 2} \quad \Rightarrow \quad T \approx -t_g - \theta\ln 2 \tag{53}$$

That is, the "external" time T equals the opposite of the internal closed slicing time, up to a translation. This means that if the point of tangency of the particle de Sitter space is slid, on this space, along a time axis without varying its spatial coordinates, the variation of the "internal" time t_g is opposite in sign to the variation of the "external" time T. This variation corresponds, in E_5, to a variation of X_0 according to (52). From now on we will use the proper time variable $t = \pm T$ instead of the variable T, with respect to the harmonic solutions.

According to this position, the particle of mass M corresponds (Equation (48)) to a phase wave $\exp(\pm iMc^2 t/\hbar)$. Under a Lorentz transformation $(ict, 0, 0, 0) \rightarrow (x_0, x_1, x_2, x_3)$, this phase wave becomes an ordinary plane wave in Minkowski spacetime:

$$e^{\pm i\frac{Mc^2t}{\hbar}} \rightarrow e^{\pm i\frac{Et'-p\bullet x}{\hbar}} \tag{54}$$

with:

$$E^2 - p^2c^2 = M^2c^4 \tag{55}$$

In (54), t' is the new time in the frame in which the particle is in motion with momentum $\boldsymbol{p}$ and energy E. It is immediately seen that the plane wave (54) is a solution of the Klein Gordon equation:

$$\hbar^2c^2\left(\frac{\partial^2}{c^2\partial t'^2} - \nabla\right)\Psi + M^2c^4\Psi = 0 \tag{56}$$

Since (56) is linear, a linear combination of plane waves of the type (54) satisfying (55) is still a solution of (56). This remains true even if the momenta $\boldsymbol{p}$ of the different superposed plane waves are different, provided that the superposition coefficients are functions of these momenta only. The general solution of (56) is, in fact, the most general superposition of plane waves satisfying constraint (55) [20]:

$$\Psi(\vec{x},t) = \int d^3(p)A(p)e^{i\frac{p\bullet x - Et}{\hbar}} \tag{57}$$

of course under the general conditions of existence of such a solution (in (57) we have included only the positive frequencies and removed the apex in t). We can therefore introduce wave packets into spacetime and study their evolution starting from given initial conditions.

The introduction of spin does not seem to present any particular difficulties. For example, the free motion of an electron will be described by a plane wave of the type (54) multiplied by a column vector whose four elements are independent of the coordinates. It is easily demonstrated (ref. [20], page 399 and following) that this product is a solution of the Dirac equation if the condition (55) is satisfied.

There are two points that appear relevant. The first is that, although each de Broglie factor (54) carries information related to the particle/de Sitter space, the insertion of more factors in a wave packet comes to represent additional information that is absent at the level of de Sitter space. This information is related to the statistical distribution of the four-momentum of the particle. This distribution is a characteristic of the relationship that is created between de Sitter space—which here constitutes the "particle"—and spacetime, and therefore it cannot be defined by the pure internal dynamics of de Sitter space. The same considerations apply to the position of the point of tangency on spacetime, and to its statistical distribution within the packet. We also remark that one can select n dS spaces associated with n distinct particles; the set of spaces thus selected (each with its own distribution of four-momentum and spatial position) corresponds to the tensorial product of n states of these particles. This product "lives" on the configurational space of the n particles.

This brings us to the second point. The fact that four-momentum and position are not encoded in the internal dynamics of the particle/de Sitter space, but emerge from its relation with "external" spacetime, leads to the consequence that these quantities are, in general, indefinite. In (54), the relation of the particle/space to spacetime is mediated by t; the spatial position appears secondly after application of the relativistic transformation that leaves the phase unchanged. There is not, at the "native" or "primordial" level, something that defines the position (*i.e.* the point of tangency). From the application of the usual interpretation of quantum theory, the meaning of (54) is that if an interaction localized the particle at a specific point, this would happen with the same probability at every point. Equation (57), or its equivalent relation according to a given wave equation, allows the calculation of that probability in the general case. Similarly, $\boldsymbol{p}$ appears in (54) after application of the relativistic transformation. The Fourier transform of the initial condition on the packet provides the function $A(\boldsymbol{p})$ which expresses the delocalization of $\boldsymbol{p}$ at the initial instant. From Equations (56), (57) it is then possible to obtain the statistical distribution of the momentum at each subsequent instant.

We can therefore conclude by stating that in the proposed scheme the particle/space appears, from the spacetime perspective emerging under the condition $|t_g| \gg \theta = r_{ds}/c$, as a delocalized entity according to the usual quantum theory. The localization of a particle in space occurs through its *local* interactions with other particles. Such interactions are in fact expressed by operators that are diagonal in the representation of the spatial coordinates. To give an example, consider a particle described by the wave function $\psi(x,t)$, which at time t interacts with a second particle described by the interaction operator $O(x', t)$ and let $\phi(y, t)$ be the wave function of the particle emerging from this interaction. Posing $\psi(x,t) = \langle x|\psi\rangle$, $\phi(y, t) = \langle y|\phi\rangle$, $O(x', t) = \Sigma_{x'}|x'\rangle\langle x'|O|x'\rangle\langle x'|$, the interaction amplitude is expressed as:

$$\begin{aligned}\langle \phi|O|\psi\rangle &= \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x' \langle \phi|y\rangle\langle y|\big[|x'\rangle\langle x'|O|x'\rangle\langle x'|\big]|x\rangle\langle x|\psi\rangle \\ &= \int \mathrm{d}x\,\phi^*(x,t)O(x,t)\psi(x,t)\end{aligned} \tag{58}$$

which is a superposition of terms in the usual three-dimensional space, no longer labeled by the particle index. The diagonality of the interaction operator in fact induces a trace with respect to the coordinates of the three particles involved, and the result is an integral over a function of the point in a three-dimensional space independent of the particle. This space is the space of our usual perception, generated by the local nature of the interactions. Spacetime, in other words, is not the set of possible positions of a specific particle (which instead constitutes its configurational space), but the geometric locus of possible interactions *between* particles. This geometric locus is independent of the specific particles participating in the interactions.

Thus, while the coordinates of the configurational space of a particle are the Fourier conjugate variables of its momentum and therefore a property of the particle, spacetime is a space-trace of interactions between particles completely independent of the individual particles [21]. The emergence of spacetime is a reflection of the locality of interactions.

Before closing this Section, it is worth dwelling on two important details. First of all, Equations (51)-(53) determine a relation between the internal time of the particle de Sitter space and the external "proper" time of the particle. Therefore, the space of contemporaneity of the de Sitter space that contains the point of tangency (or on which such point is delocalized) varies continuously with the passage of the external proper time. This condition, which appears essential to define a common notion of existence of the particle space both in external time and for "internal observers", presupposes that the causal structures of the particle space and of the spacetime V are the same. This condition is satisfied for a de Sitter space, but not for an anti-de Sitter space [22], since the latter has a different signature than that of V. In an anti-de Sitter space the time domain is periodic, so an infinite time line on V could not be mapped onto an infinite time line in such a domain. This is the reason why we have chosen, from the beginning, the de Sitter space as the particle space. An elementary particle is by definition devoid of an internal structure that can be modeled with an inhomogeneous and/or variable density of matter: we therefore expect it to correspond to a maximally symmetric solution of the gravitational equations. The only possible solutions with finite curvature are the de Sitter and anti-de Sitter spaces. Excluding the latter for the reasons mentioned, the de Sitter space remains.

We would then like to draw attention to the fact that the correspondence expressed by Equations (51)-(53) is valid in the approximation $|T| \gg \theta$, with $\theta = q^2/Mc^3 \leq h/Mc^2$. In fact, even in the extreme case of strong interactions at about 100 MeV, $q^2/\hbar c = \alpha_{strong} \approx 0.1$. The condition $|T| \gg \theta$ is therefore verified on intervals of external proper time $\gg h/Mc^2$, *i.e.* on a scale larger than the Compton scale. This is compatible with a particle propagation on V described by wave equations such as (56). These equations (and more generally the concept of wave function of a particle) in fact lose meaning on intervals smaller than the Compton scale, as a consequence of the virtual dissociation of the particle into pairs. On this scale, as is known, this description should be replaced by another based on second quantization procedures. In particular, the loss of validity of the relation between T and the proper time t (Equation (53)) on the sub-Compton scale is consistent with the well-known loss of t-orientation of the particle dynamics on this scale, implied by the superposition of terms of both signs of the energy in the solution of (56). The non-differential term that we left out in (43) under the condition $|T| \gg \theta$ therefore corresponds to the sub-Compton regime and is related to the internal "gravitational" interaction of the spaces associated with single elements of the fermion-antifermion pairs existing on this scale. Given the introductory nature of this paper, this interesting detail will not be explored here.

2.5. Quantum Discontinuity

Let us now return to the exponential solutions (46), (47). The solution $\exp(-C|\tau|/\theta)$, as we have chosen to denote it, can exist for $\tau \in (-\infty, 0]$ or $\tau \in [0, +\infty)$. We assume a physical interpretation for these solutions, which also allows us to fix the otherwise arbitrary parameter C.

The position eigenstate of a particle is relative to the point of tangency O of the particle dS space on spacetime. The dS space is closed because, in any internal spatial direction, its maximum extension is $2\pi r_{dS} = 2\pi c\theta$, where $\theta = r_{dS}/c$ is the de Sitter time of the particle. It follows that the energy required to create a positional eigenstate of the particle in spacetime is $hc/(2\pi c\theta) = \hbar/\theta$. This should not be confused with the energy required to locate the eigenstate exactly (which is obviously infinite), nor with the energy Mc^2 required to create the particle. When the particle is removed from spacetime, its de Broglie phase factor stops. The de Sitter space tangent to spacetime at the current point-event O, and associated with the position eigenstate $|O\rangle$, therefore vanishes as a transient state of duration T. This duration, of course, is to be understood in the domain of the internal variable $\tau \in [0, +\infty)$ and not in terms of external time measurable by an observer. In fact, the de Broglie phase factor has been locked at a certain instant of external time. The duration T can be estimated from the uncertainty principle $(\hbar/\theta)T \approx \hbar$ and we can set $T = \theta$.

We assume that the transient associated with the removal of the particle from spacetime is expressed by the product of the wave function at the instant of arrest by the amplitude (46). Since, by the uncertainty principle, the time constant of (46) must equal $T = \theta$, we have $C = 1$. The removal process is then described by the transformation $|x\rangle \rightarrow |x\rangle \exp(-|\tau|/\theta)$ applied at each point-event (x, t), where t is the instant of arrest. Since the particle state $|\psi\rangle$ is decomposed according to the relation $|\psi\rangle = \Sigma_x \langle x|\psi\rangle |x\rangle = \Sigma_x \psi(x, t)|x\rangle$, it then undergoes the transformation $\exp(-|\tau|/\theta)|\psi\rangle = \exp(-|\tau|/\theta)\Sigma_x \psi(x, t)|x\rangle$ or, what is the same:

$$\psi(x,t) \rightarrow \psi(x,t)e^{-\frac{|\tau|}{\theta}} \tag{59}$$

As can be seen from (59), the instant $\tau = 0$ corresponds to the instant t of the stop in external time. The evolution $\tau \rightarrow +\infty$ extinguishes the wave function $\psi(x, t)$. By reversing the arrow, (59) becomes the description of the appearance, in spacetime, of the function $\psi(x, t)$ at the instant $\tau = 0$, with activation of the phase factor at the instant t of external time. This occurs as the final act of an evolution started at $\tau = -\infty$. The extinction and build-up processes described by (59) clearly involve the whole space. However, they are not associated with signalling processes between spatially separated portions of the wave function, in accordance with Einstein principle of locality.

The extinction of a wave function can be contextual to the build up of another wave function; this requires that the instant of arrest of the wave function that is extinguished and the instant of activation of the new wave function coincide in the external time t. Furthermore, the process must respect the principles of conservation. In this case, there is a discontinuous variation of the quantum state of a system of one or more particles. This variation can be identified in the phenomenon of the *quantum jump* (QJ), which breaks the unitarity of the temporal evolution (in the sense of the observer's time) of the quantum state. We note that the QJ is associated with the interaction with other particles and represents the nonunitary aspect of such interaction [11].

To understand how quantum delocalization and quantum discontinuity define the spacetime behavior of the particle, let us consider the well-known double-slit experiment. The wave $|\psi\rangle$ incident on the screen interacts with it, giving it momentum and energy (and then an impact occurs), or not. In both cases:

$$|\psi\rangle \rightarrow e^{-\frac{|\tau|}{\theta}} \sum_{x} \langle x|\psi\rangle |x\rangle \tag{60}$$

where x is the position of the current point on the spatial region occupied by the screen. For $\tau = 0$, Equation (60) returns the ordinary wave function incident on the screen (entry state). The interaction between particle and screen is described in the domain $\tau \in [0, +\infty)$, starting from $\tau = 0$ and evolving for $\tau \rightarrow +\infty$. This evolution starts, at each point x, from the local amplitude $\langle x|\psi\rangle$ for $\tau = 0$ and leads to the complete extinction of the quantum amplitude of the particle at that point. For the reasons previously explained, this evolution is not traceable from the perspective of external time. For the external observer, it is reduced to a single time instant.

We now come to the outgoing state $|\phi\rangle$. In the first case (the one in which an impact occurs), we will have:

$$e^{-\frac{|\tau|}{\theta}} |x_0\rangle \rightarrow |\phi\rangle \tag{61}$$

where x_0 is the impact point. The evolution in $\tau \in (-\infty, 0]$ starts from $\tau = -\infty$ and continues until $\tau = 0$, where Equation (61) returns the conventional value of $|\phi\rangle$, that of the positional eigenstate relating to the impact. Equation (61) expresses the transfer of physical quantities, released by the annihilation of $|\psi\rangle$, to the new state $|\phi\rangle$; a transfer that is completed at the instant $\tau = 0$. In this case the local amplitude $\langle x|x_0\rangle$ appears as the final condition on the process. Even this process is, for the external observer, "instantaneous"; he/she perceives a discontinuous variation $|\psi\rangle \rightarrow |\phi\rangle$.

In the second case (absence of impacts) we will instead have:

$$e^{-\frac{|\tau|}{\theta}} \sum_{x \in H_1, H_2} \langle x|\phi\rangle |x\rangle \rightarrow |\phi\rangle \tag{62}$$

where H_1, H_2 are the spatial regions of the two slits. The evolution in the domain $\tau \in (-\infty, 0]$ starts from $\tau = -\infty$ and continues up to $\tau = 0$ with the reproduction of the final amplitude $\langle x|\phi\rangle$. In this case, which is that of the so-called "negative" or "zero" interaction, the physical quantities released by the annihilation of $|\psi\rangle$ are not transferred to the screen; they are instead transferred to the final state $|\phi\rangle$. What changes is therefore only the statistical distribution of the particle (*i.e.* the spatial distribution of potential future impacts with release of these quantities to other particles), according to the renormalization:

$$\langle x|\phi\rangle = \frac{\langle x|\psi\rangle}{|\langle \phi|\psi\rangle|} \tag{63}$$

which preserves the phase relation between the incoming state and the outgoing state. We note that at the connection point $\tau = 0$, Equation (62) can be rewritten as:

$$|\varphi\rangle = \sum_{x \in H_1} \langle x | \phi \rangle | x \rangle + \sum_{x \in H_2} \langle x | \phi \rangle | x \rangle \quad (64)$$

and therefore the distribution of subsequent impacts (on the rear screen, for example) is subject to interference effects.

From the perspective of particle dS space, the QJ can therefore be seen as the passage from a Lorentzian internal time to an Euclidean one, or vice versa (it is also possible to speculate on a toroidal geometry of the τ-domain, with coincident points $\tau = \pm\infty$). There is therefore a relationship of alternation between harmonic and exponential solutions of the WdW. Let us reconsider Equation (42):

$$\left(a^2 \frac{\partial^2}{\partial a^2} + a \frac{\partial}{\partial a} + \frac{\omega_0^2 a^{2q} - a^4}{\hbar^2} - \frac{\partial^2}{\partial \xi^2} \right) \Psi(a,\xi) = 0 \quad (65)$$

Under our specific assumptions, is possible rewrite it as [23]:

$$\left[\hbar^2 \frac{\partial^2}{\partial a^2} + \frac{\hbar^2}{a} \frac{\partial}{\partial a} - \frac{\hbar^2}{a^2} \frac{\partial^2}{\partial \xi^2} + \left(\Lambda a^4 - a^2 \right) \right] \Psi(a,\xi) = 0 \quad (66)$$

Promoting the wave function Ψ to operator, we have its decomposition in normal modes:

$$\hat{\Psi}(a,\xi) = \int \mathrm{d}k \left[e^{ik\xi} A_k(a) \hat{c}_k^{+} + e^{-ik\xi} A_k^{*}(a) \hat{c}_k \right] \quad (67)$$

The amplitudes A_k then satisfy the equation of the damped harmonic oscillator:

$$\left(\hbar^2 \frac{\partial^2}{\partial a^2} + \frac{\hbar^2}{a} \frac{\partial}{\partial a} + \omega_k^2(a) \right) A_k(a) = 0 \quad (68)$$

where:

$$\omega_k(a) = \sqrt{\Lambda a^4 - a^2 + \frac{\hbar^2 k^2}{a^2}} \quad (69)$$

The creation and annihilation operators of mode k of the de Sitter space are expressed by:

$$\hat{c}^{+} = \sqrt{\frac{\omega_{0k}}{2\hbar}} \left(\hat{\Psi} - \frac{i}{\omega_{0k}} \hat{p}_{\Psi} \right); \quad \hat{c} = \sqrt{\frac{\omega_{0k}}{2\hbar}} \left(\hat{\Psi} + \frac{i}{\omega_{0k}} \hat{p}_{\Psi} \right) \quad (70)$$

where ω_{0k} is the value derived from (69) on Σ. It should be noted that the values of $\omega_k(a)$ given by (69) can be as well as real than complex. The transition from the real to the complex domain represents the transition from the Lorentzian to the Euclidean time region. For example, in the case $k = 0$ one has a de Sitter space for $a > \Lambda^{-1/2}$, while for $a < \Lambda^{-1/2}$ one has a de Sitter instanton collapsing on Euclidean time.

In our case a corresponds to $a'/(a/a_0)$, where a' is a constant with the dimensions of a length while the variable $(a/a_0) \in [0,1]$ is dimensionless. The radicand of (69), for $k = 0$, is therefore positive for $a'/(a/a_0) > \Lambda^{-1/2}$, that is for $(a/a_0) < a'\Lambda^{1/2}$, and negative for $(a/a_0) > a'\Lambda^{1/2}$. Choosing $a' = 1/\Lambda^{1/2}$, these two regions become respectively $(a/a_0) < 1$, which is always true, and $(a/a_0) > 1$. We therefore have a harmonic solution, or in general a superposition of harmonic solutions, for $(a/a_0) < 1$; each harmonic component represents an evolution in time T, as illustrated in Section 2.3.

For $(a/a_0) > 1$, a region outside the definition interval of the variable (a/a_0) adopted by us, there would be the asymptotically exponential collapse of a Euclidean instanton. This instanton can be put in correspondence with the exponential solutions illustrated in Section 2.3, the evolution of which is labeled by the time variable τ. The point of separation $(a/a_0) = 1$ between the two regions is then a quantum jump (QJ).

2.6. Decoherence and Classicalization

The double-slit example already contains, *in nuce*, a decoherence mechanism of the wave function mediated by the common SM interactions (this aspect is further detailed in the following Section 3). If we indicate the final wavefunctions (61), (62) with A and B respectively, the described process involves the diagonalization of the statistical operator:

$$\begin{pmatrix} AA^* & AB^* \\ A^*B & BB^* \end{pmatrix} \to \begin{pmatrix} AA^* & 0 \\ 0 & BB^* \end{pmatrix} \tag{71}$$

in dramatic violation of unitarity. This decoherence mechanism has no relation to statistical averaging operations and does not depend on the existence of environmental degrees of freedom: it is in fact operational at the level of micro-interactions between elementary particles. The proposed mechanism is not affected by the selection problem [10], because it does not limit itself to diagonalizing the density matrix according to (71): in each single repetition of the experiment, as we have seen, it selects A or B as the outgoing state. There does not even seem to be a specific preferred basis problem [10]. In fact, the fixed scheme is that of the alternative between positive interaction and negative (null) interaction; therefore the preferred basis is the one that separates these two cases as distinct terms in a superposition, and it is ultimately determined by the structure of the interaction.

One may wonder whether, in a quantum system subject to many quantum jumps, the decoherence mechanism discussed here can lead to the emergence of "locked" classical states. This seems to be an important step towards the emergence of classical systems from the underlying quantum level (so-called classicalization problem [24]). One can reconsider in this perspective the model system originally proposed by Simonius [25], showing that one can arrive at the same relaxation time without assuming any coupling with external probes and without tracking the degrees of freedom of such probes, but only by admitting the decoherence produced by *internal* QJs. The essential difference is that in the first case the overall system (probes included) remains coherent and the tracking on the degrees of freedom of the probes produces an appearance of internal decoherence. In the second case, instead, the coherence is effectively destroyed at the level of the elementary interactions. The model system is a two-state system:

$$\Psi = \left(\frac{a}{2}e^{\frac{iAt}{\hbar}} + \frac{b}{2}e^{-\frac{iAt}{\hbar}}\right)\left|1\right\rangle + \left(\frac{a}{2}e^{\frac{iAt}{\hbar}} - \frac{b}{2}e^{-\frac{iAt}{\hbar}}\right)\left|2\right\rangle \tag{72}$$

where $A/\hbar = \omega$. We also assume:

$$\Psi(t=0) = \alpha\left|1\right\rangle + \beta\left|2\right\rangle; \qquad \alpha\alpha^* + \beta\beta^* = 1 \tag{73}$$

From which:

$$\alpha = \frac{a+b}{2}; \quad \beta = \frac{a-b}{2}; \quad a = \alpha+\beta; \quad b = \alpha-\beta. \tag{74}$$

We will assume that the QJs occur at regular intervals of duration τ (of course this parameter should be understood as an *average* recurrence time of the QJs), and that therefore the evolution of the density matrix begins as follows:

$$\begin{pmatrix} \alpha\alpha^* & \alpha\beta^* \\ \alpha^*\beta & \beta\beta^* \end{pmatrix}_{t=0} \to \begin{pmatrix} \alpha'\alpha'^* & \alpha'\beta'^* \\ \alpha'^*\beta' & \beta'\beta'^* \end{pmatrix}_{t=\tau^-} \to \begin{pmatrix} \alpha'\alpha'^* & 0 \\ 0 & \beta'\beta'^* \end{pmatrix}_{t=\tau^+} \tag{75}$$

We will also assume $\tau \ll 1/\omega$, and this allows us the following approximate expansions:

$$\begin{aligned}
\alpha'\alpha'^* &= \left(\frac{a}{2}e^{i\omega\tau} + \frac{b}{2}e^{-i\omega\tau}\right)\left(\frac{a^*}{2}e^{-i\omega\tau} + \frac{b^*}{2}e^{i\omega\tau}\right) \\
&= \left(\frac{aa^*}{4} + \frac{bb^*}{4}\right) + \frac{ab^*}{4}e^{2i\omega\tau} + \frac{a^*b}{4}e^{-2i\omega\tau} \\
&= \left(\frac{aa^*}{4} + \frac{bb^*}{4} + \frac{ab^*}{4} + \frac{a^*b}{4}\right) - \frac{ab^*}{4} + \frac{ab^*}{4}e^{2i\omega\tau} - \frac{a^*b}{4} + \frac{a^*b}{4}e^{-2i\omega\tau} \\
&= \alpha\alpha^* + \frac{ab^*}{4}\left(e^{2i\omega\tau} - 1\right) + \frac{a^*b}{4}\left(e^{-2i\omega\tau} - 1\right) \\
&\approx \alpha\alpha^* + \left(\frac{ab^*}{4} + \frac{a^*b}{4}\right)\left[\cos(2\omega\tau) - 1\right] \\
&= \alpha\alpha^* + \left(\frac{2\alpha\alpha^* - 2\beta\beta^*}{4}\right)\left[\cos(2\omega\tau) - 1\right] \\
&= \frac{\alpha\alpha^*}{2} + \frac{\beta\beta^*}{2} + \left(\frac{\alpha\alpha^* - \beta\beta^*}{2}\right)\cos(2\omega\tau) \\
&= \frac{1}{2} + \left(\frac{\alpha\alpha^* - \beta\beta^*}{2}\right)\cos(2\omega\tau) \\
&\approx \frac{1}{2} + \left(\frac{\alpha\alpha^* - \beta\beta^*}{2}\right)\left(1 - 2\omega^2\tau^2\right)
\end{aligned} \tag{76}$$

It has been considered that $ab^* = (\alpha + \beta)(\alpha^* - \beta^*) = \alpha\alpha^* - \alpha\beta^* + \alpha^*\beta - \beta\beta^*$ and then $a^*b = \alpha\alpha^* - \alpha^*\beta + \alpha\beta^* - \beta\beta^*$. Analogously:

$$\begin{aligned}
\beta'\beta'^{*} &= \left(\frac{a}{2}e^{i\omega\tau} - \frac{b}{2}e^{-i\omega\tau}\right)\left(\frac{a^{*}}{2}e^{-i\omega\tau} - \frac{b^{*}}{2}e^{i\omega\tau}\right) \\
&= \left(\frac{aa^{*}}{4}+\frac{bb^{*}}{4}\right) - \frac{ab^{*}}{4}e^{2i\omega\tau} - \frac{a^{*}b}{4}e^{-2i\omega\tau} \\
&= \left(\frac{aa^{*}}{4}+\frac{bb^{*}}{4}+\frac{ab^{*}}{4}+\frac{a^{*}b}{4}\right) - \frac{ab^{*}}{4} - \frac{ab^{*}}{4}e^{2i\omega\tau} - \frac{a^{*}b}{4} - \frac{a^{*}b}{4}e^{-2i\omega\tau} \\
&= \alpha\alpha^{*} - \frac{ab^{*}}{4}\left(e^{2i\omega\tau}+1\right) - \frac{a^{*}b}{4}\left(e^{-2i\omega\tau}+1\right) \\
&\approx \alpha\alpha^{*} - \left(\frac{ab^{*}}{4}+\frac{a^{*}b}{4}\right)\left[\cos(2\omega\tau)+1\right] \\
&= \alpha\alpha^{*} - \left(\frac{2\alpha\alpha^{*}-2\beta\beta^{*}}{4}\right)\left[\cos(2\omega\tau)+1\right] \\
&= \frac{\alpha\alpha^{*}}{2} + \frac{\beta\beta^{*}}{2} - \left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\cos(2\omega\tau) \\
&= \frac{1}{2} - \left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\cos(2\omega\tau) \\
&\approx \frac{1}{2} - \left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)
\end{aligned} \tag{77}$$

Equations (76), (77) are the expressions of the diagonal terms that survive in (75) for $t = \tau^{+}$, that is, after the first QJ. We note that:

$$\begin{aligned}
&\alpha'\alpha'^{*} - \beta'\beta'^{*} \\
&\approx \left[\frac{1}{2}+\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)\right]-\left[\frac{1}{2}-\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)\right] \\
&= \left(\alpha\alpha^{*}-\beta\beta^{*}\right)\left(1-2\omega^{2}\tau^{2}\right)
\end{aligned} \tag{78}$$

Then, the next QJ at $t = 2\tau$ will produce the diagonal elements:

$$\alpha''\alpha''^{*} \approx \frac{1}{2}+\left(\frac{\alpha'\alpha'^{*}-\beta'\beta'^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right) = \frac{1}{2}+\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)^{2} \tag{79}$$

$$\beta''\beta''^{*} \approx \frac{1}{2}-\left(\frac{\alpha'\alpha'^{*}-\beta'\beta'^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right) = \frac{1}{2}-\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)^{2} \tag{80}$$

Therefore, after a succession of n quantum jumps spaced by τ the surviving elements of the density matrix (*i.e.* the diagonal ones) will be:

$$\alpha_n\alpha_n^{*} \approx \frac{1}{2}+\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)^{n} \tag{81}$$

$$\beta_n\beta_n^{*} \approx \frac{1}{2}-\left(\frac{\alpha\alpha^{*}-\beta\beta^{*}}{2}\right)\left(1-2\omega^{2}\tau^{2}\right)^{n} \tag{82}$$

Since $\omega\tau \ll 1$, the approximation $1 - 2\omega^2\tau^2 \approx \exp(-2\omega^2\tau^2)$ is permitted so that $(1 - 2\omega^2\tau^2)^n \approx \exp(-2\omega^2\tau^2 n) = \exp(-2\omega^2\tau^2 t/\tau) = \exp(-2\omega^2\tau t) = \exp(-t/\lambda)$, where the relaxation time λ is expressed by:

$$\lambda = \frac{1}{2\omega^2\tau} \tag{83}$$

It is $\lambda \gg 1/2\omega$, because $\omega\tau \ll 1$. Denoting with δ the semi-difference of the diagonal elements at $t = 0$:

$$\delta = \frac{\alpha\alpha^* - \beta\beta^*}{2} \tag{84}$$

The density matrix becomes:

$$\begin{pmatrix} \frac{1}{2} + \delta e^{-t/\lambda} & 0 \\ 0 & \frac{1}{2} - \delta e^{-t/\lambda} \end{pmatrix} \tag{85}$$

and it relaxes to the chaotic matrix:

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{86}$$

on times $\gg\lambda$, with total loss of memory of the initial condition. The fact that in the regime $\omega\tau \ll 1$ the memory is lost on a time scale (83) inversely proportional to the interval τ between the jumps is not surprising, because too small an interval favors a Zeno effect.

2.7. A Brief Note on the Entropy-Action Relation

The Bekenstein entropic constraint [26] for a particle de Sitter space is given by the expression (in units of the Boltzmann constant):

$$S_{Bekenstein} = \frac{rE}{\hbar c} \tag{87}$$

where $r = c\theta = q^2/Mc^2$ is the de Sitter radius and $E = Mc^2$ is the energy content of space (in an external frame of reference where it is at rest); M and q are the mass and charge of the particle, respectively. We therefore have:

$$S_{Bekenstein} = \frac{q^2}{\hbar c} = \alpha \tag{88}$$

where α is the fine structure constant (multiplied by q^2/e^2). The maximum rate of entropy production in the rest frame of the particle is therefore α/θ. In a proper time interval of the particle of duration $t \geq \theta$ there is therefore a maximum entropy production $\alpha t/\theta = \alpha(mc^2t)/(mc^2\theta) = \alpha(mc^2t)/(q^2/c) = (mc^2t)/\hbar$. This is nothing but the variation of action in the interval, expressed in units $\hbar$. This result is an expression of the well-known de Broglie thermomechanical relation between entropy and action [27]. It shows that the thermomechanics of the free particle, originally developed by de Broglie, can be connected to the present description, if the entropy produced is interpreted in terms of the entropic constraint on the particle dS space.

2.8. Digression on Neutrinos

In applying (36) to the elementary charged fermions of the SM, it is necessary to keep in mind the three different situations represented by the charged leptons, the quarks and the neutrinos. The application to the charged leptons does not present any difficulties; the charge q essentially coincides with the electric charge and the mass M is that measured starting from the inertial properties of the particle. The de Sitter radius therefore coincides, for these particles, with the classical radius and the largest value is for the electron, for which $r_{dS} = r_0 = c\theta_0 = 2.81 \times 10^{-13}$ cm. The application of (36) to quarks requires additional precautions, given the permanent confinement of these particles within the hadron to which they belong; the only exception is represented by the top quark which however, due to its fast decay, remains essentially confined to the interaction region in which it is generated in pairs. The charge q here essentially coincides with the strong Coulomb charge of the Cornell potential, while the mass M is the current mass deduced a posteriori from the chromodynamic modeling of the hadrons to which the quark belongs.

The most problematic case is that of neutrinos. Since (36) contains a definite value of mass, it must apply to the mass eigenstates rather than to the superpositions (electron, muon, tau neutrino) generated by weak interactions. The mass eigenvalues are currently unknown. In addition to this, the neutrino mass generation mechanism is still debated [28]. In this work we will rely on the correspondence that seems to experimentally exist between charged leptons and neutrinos and reformulate it as follows: the i-th generation charged lepton and the i-th neutrino mass eigenstate have dS spaces with identical cosmological constant, but different "gravitational" constant. This assumption allows us to extend to neutrinos the geometrized Higgs mechanism exposed in the previous Sections; the price to pay, however, is that the charge q appearing in (36), while retaining the role of coupling constant between the curvature of space dS and mass precisely expressed by this equation, loses the meaning of coupling constant of the particle in weak interactions.

More precisely, the "gravitational" constant is rescaled according to the relation $G_i = G_{i,charged} \times M_{i,charged}/M_i$, where G_i,M_i are respectively the "gravitational" constant and the mass of the i-th neutrino mass eigenstate and $G_{i,charged}$,$M_{i,charged}$ are respectively the internal "gravitational" constant and the mass of the i-th generation charged lepton. From this relation and Equation (39) follows that the de Sitter radii of the two particles are equal. According to the relations already seen: $q = rc^2/G^{1/2}$, $M = rc^2/G$ (Equations (36), (38)), it then turns out that the charge q of the neutrino is much smaller than the charge of the corresponding charged lepton, if the mass of the latter is much larger than the mass of the neutrino. It can be assumed that the weak charge q of the neutrino equals that q_{weak} of the corresponding charged lepton. Note that, starting from the expression of Fermi's theory of beta decay $[2^{1/2}g^2/8(M_W c^2)^2] = 1.166 \times 10^{-5}$ GeV^{-2}, we have $g^2\hbar c \approx 0.42\hbar c$. The quantity $(g^2\hbar c)/(M_W)^2$ has the dimensions of a gravitational constant. Assuming that the $(q_{weak})^2$ of the electron is the product of this "gravitational" constant by $(m_e)^2$, where m_e is the mass of the electron, we have $(q_{weak})^2 \approx 1.7 \times 10^{-11}\ \hbar c$. From the assumed identity of the q_{weak} of the electron and the lightest mass eigenstate of the neutrino, ν_1, and from the assumed identity of their de Sitter radii we have $M_{\nu 1}c^2 = (q_{weak})^2/r_0 = 0.23 \times 10^{-8}\ m_e c^2 = 1.5$ meV. This value appears reasonable; the other mass eigenvalues can be estimated under the same assumptions. As is clear, the purpose of these conjectures is only to support the "reasonableness" of parameters such as q_{weak}, not to define their value.

Of course, the correctness of this description should be verified by directly measuring the de Sitter radii of the considered particles, through the deviation of their behavior from the Newton trend in gravitational scattering. Knowing the de Sitter radii and the masses, the values of the charges can be deduced from (36). Assuming the present description to be true, the largest value of the de Sitter radius for a free (*i.e.* unconfined) elementary fermion of the SM turns out to be the classical radius of the electron. It is, moreover, experimentally proven that this radius coincides, in order of magnitude, with the effective radius of action of the strong interactions. In this sense, therefore, the classical radius of the electron fixes the scale of the internal geometry of the elementary particles.

2.9. Cosmological Constant

The positional eigenstate of an elementary fermion of mass M and charge q is associated with a space dS of radius q^2/Mc^2. Now, the uncertainty principle associates the energy $\alpha^{-1}Mc^2$, where $\alpha = q^2/\hbar c$, with a positional indeterminacy:

$$\frac{\hbar c}{\alpha^{-1}Mc^2} = \frac{q^2}{Mc^2} = r \tag{89}$$

This energy $\alpha^{-1}Mc^2$, concentrated in a region of extension $\sim r$, can interact electromagnetically with a charge q. The interaction energy is:

$$\approx \frac{q^2}{\hbar c}\ \alpha^{-1}Mc^2 = Mc^2 \tag{90}$$

The uncertainty principle associates this interaction energy with an uncertainty in position:

$$\approx \frac{\hbar c}{Mc^2} = \frac{\hbar}{Mc} \tag{91}$$

which is nothing other than the Compton wavelength of the particle. It measures the extension of the spatial region in which the electromagnetic *interaction vertex* is delocalized. We note that this reasoning presupposes that $q^2/\hbar c$ is a real coupling constant, and therefore does not apply to neutrinos for the reasons already seen. The fact that the interaction vertex involving the fermion is delocalized over a region of extension equal to (91), determined only by the mass of the fermion, excludes the case of permanently confined elementary fermions, *i.e.* quarks. The present reasoning is therefore applicable only to charged leptons.

Now, regardless of the actual occurrence of the interaction (90), the energy concentration $\alpha^{-1}Mc^2$ in a region of extension ~ r self-interacts gravitationally. The self-interaction energy is:

$$\approx \frac{\left(\sqrt{G_N}\alpha^{-1}M\right)\left(\sqrt{G_N}\alpha^{-1}M\right)}{r} = \frac{G_N M^2\alpha^{-2}}{r} \tag{92}$$

where G_N is the Newtonian gravitational constant. According to the uncertainty principle, this self-interaction energy corresponds to a positional uncertainty of the (gravitational) interaction vertex, which can be evaluated as:

$$\approx \frac{\hbar c}{\frac{G_N M^2\alpha^{-2}}{r}} = \frac{\hbar c}{G_N M^2\alpha^{-2}} r = \frac{q^2\alpha r}{GM^2} \propto \frac{q^6}{M^3} \tag{93}$$

Since we have restricted our attention to the charged leptons only, we must keep in mind that for them $q = e$, elementary charge, and therefore $\alpha = e^2/\hbar c$. The maximum value of (93) on the set of charged leptons occurs for $M = m_e$, the mass of the electron. It corresponds to:

$$L_0 = \frac{\alpha e^2 r_0}{G_N m_e^2} \tag{94}$$

where $r_0 = c\theta_0$ is the classical radius of the electron. We interpret the length $L_0 = ct_0$ as the radius of the cosmological horizon of a generic interaction vertex, *i.e.* of a generic event. The value of the cosmological constant connected to the horizon is $\Lambda_0 = 3/(ct_0)^2$. Substituting the nominal values of the quantities on the right-hand side of (94) we obtain $\Lambda_0 = 4.1 \times 10^{-56}$ cm^{-2}. Cosmological observations provide a value around 1.10×10^{-56} cm^{-2} [29], so the result is correct in order of magnitude. We note that, introducing the Planck time:

$$t_P = \sqrt{\frac{\hbar G_N}{c^5}} \tag{95}$$

Equation (94) can be rewritten as:

$$\left(\frac{\theta_0}{t_P}\right)^2 = \frac{t_0}{\theta_0} \tag{96}$$

One has $(t_0/t_P)^2 \sim 10^{121}$, a value very close to the one considered by Penrose [30].

This reasoning brings the value of the cosmological constant Λ_0 back to the process of localization of elementary particles, as a sort of ineluctable infrared limit involved in this process, freeing it from the energy density of the vacuum. The difficulty originating from the observed disagreement between Λ_0 and the value of this density is therefore avoided. Support for this interpretation is given by the fact that if we set the mass content of all elementary fermions to zero, their de Sitter radii will diverge as a consequence of (36). At the same time, the radius of the cosmological horizon will diverge as a consequence of (94), with the result that empty fermionic dS spaces will collapse onto an empty Minkowskian spacetime. In the latter there is no cosmological constant.

2.10. A Side Note on Hadrons

Although the topic of the present paper is the elementary SM fermions and the gauge bosons mediating their interactions, it can be considered that the symmetry breaking mechanism hypothesized in hadronic formation by several phenomenological models, such as the sigma models [31] [32], is substantially analogous. The role of the Higgs field is covered in these models by one or more mesonic scalar fields, combined in such a way as to generate a self-interaction potential of the same shape as the Higgs one. The VEV of the mesonic field is in the order of $M_\pi c^2 \approx 140$ MeV, where M_π is the mass of the pion (the lightest hadronic state). In principle it seems possible to reproduce the general lines of the reasoning developed for the elementary fermions, and thus introduce hadronic de Sitter spaces [11] [33]-[38].

Quarks and gluons will then be confined within such spaces. It is possible to formulate a simple confinement condition *ab initio* if we admit that the points of tangency of the de Sitter spaces of quarks, belonging to a given hadron, must be contained within the vertical projection of the de Sitter space of the hadron on spacetime. The vertical raised in E_5 from the point of tangency of a quark then intersects the hadronic space in two points, and it can be conjectured that the strong charge of the particle is localized in one of these two points. From the application of Gauss theorem, taking into account that the de Sitter space is closed, we then find that the total color charge on this space must be zero. In other words, the hadron must be colorless [36] [38]. The hypothesis of hadronic de Sitter spaces, besides offering the possibility of introducing confinement in a simple way, also seems to be in accordance with the statistics of the production of hadronic states in interactions between particles mediated by the strong interaction [39].

3. Gauge Bosons

3.1. Coupling between Elementary Fermions and Gauge Bosons

So far we have considered the elementary SM fermions in isolation. We now want to focus on their interactions. As we pointed out in the Introduction, there are no direct couplings between the volume elements of dS spaces associated with distinct particles. The coupling between two fermions involves their two dS spaces each considered as a whole, and it is mediated by a gauge boson. This leads to the usual concept of interaction vertex, where a boson B couples to two elementary fermions (a, b). We can consider one of the two fermions, say a, as "afferent" to the vertex, and the other—in our case b—as "effluent" from the vertex. These labels are conventional, and the opposite choice is equally valid. In particular, the notions of "afference" and "effluence" are here understood in a sense that is completely independent on the directions of the fermionic lines a, b with respect to external time. For example, if the quadrimpulses of the fermions a, b and of the boson B are time-like, three possible situations can arise:

1) a and b are in the past of the vertex, while B is in the future of the vertex; this is a pair annihilation;

2) a and b are in the future of the vertex, while B is in the past of the vertex; this is a pair creation;

3) a (b) is in the past of the vertex, while b (a) is in the future of the vertex; in this case, if the energy of a (b) exceeds the energy of b (a) we have the emission of B and B is in the future of the vertex; if, instead, the opposite condition is valid we have the absorption of B and B is in the past of the vertex.

The notions of affererence and effluence allow us to formalize the concepts of vertex and boson. This is possible if we associate the afferent fermion with the product of its second quantization amplitude and a "down" spinor:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_a(x) \tag{97}$$

Similarly, the effluent fermion is associated with the (conjugate of) the product of its second quantization amplitude and an "up" spinor:

$$\bar{\psi}_b(x)\begin{pmatrix} 1 & 0 \end{pmatrix} \tag{98}$$

Spinors are introduced here to represent the concept of affluence and effluence and have no relation to physical quantities such as spin or isospin. The symbol x represents the current point in spacetime. In this context, the B boson is represented by the product operator of the four-potential of the field associated with it (which is an operator of Dirac algebra in itself) and the matrix σ_1:

$$\gamma^{\mu} A_{\mu}(x)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{99}$$

where $\mu = 0, 1, 2, 3$. In the case of the Higgs boson, the four-potential is replaced by the scalar doublet associated with this boson. In general, then, the gauge boson B is a field of operators that couples the afferent fermion with the effluent fermion:

$$\begin{aligned} & g\,\bar{\psi}_b(x)\begin{pmatrix} 1 & 0 \end{pmatrix}\gamma^{\mu} A_{\mu}(x)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\psi_a(x) \\ &= g\,\bar{\psi}_b(x)\begin{pmatrix} 1 & 0 \end{pmatrix}\gamma^{\mu} A_{\mu}(x)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\psi_a(x) \\ &= g\,\bar{\psi}_b(x)\,\gamma^{\mu} A_{\mu}(x)\,\psi_a(x) \end{aligned} \tag{100}$$

In (100), g is the coupling constant and the expression obtained is the usual one for a SM interaction vertex. The fermionic amplitudes and the bosonic four-potential will have a dynamics derived from the ordinary SM field equations. The relevant point here is that the minimal notion of boson represented by (99) does not imply any notion of dS space associated with the boson. In particular, there is no notion of position (although delocalized) attributable to the boson, since no point of tangency of any dS space on spacetime is defined. For the same reason, it is not possible to apply to the Higgs mechanism for gauge bosons the same geometric interpretation applied here to elementary fermions.

What changes with respect to the ordinary interpretation of the SM vertex is that now the bosonic field (99) connects fermionic dS spaces. If the lines connected to the fermions (a, b) and to the B boson are on shell, this connection defines an extreme (initial or final) to their existence; that is, there is a redefinition of the initial conditions on the propagation of the fields, described by the ordinary equations of quantum field theory. In the vertex, the conserved physical quantities are transferred from one fermionic line to the other and this involves two transients of the type (59): one in which the dS space that gives up the packet is extinguished in the interval [$\tau \in (0, +\infty)$]; the other in which the dS space that acquires the packet is created starting from a fluctuation in the interval [$\tau \in (-\infty, 0)$]. The involvement of transients, which connect the fermionic states to the vacuum, is possible because the mass of the fermions is well definite, as required by equations (59). Each transient breaks unitarity, as the fermionic amplitude vanishes or emerges from nothing. The complex of transients concurrent at the vertex constitutes what is usually called a quantum jump (QJ).

If the lines connected to the fermions (a, b) and to the boson B are off shell, the intervention of transients of the type (59) is inhibited by the fact that the fermionic mass is not definite. In this case, quantum jumps cannot occur. The connection of the lines a, b consists simply in a modification of the external dynamical state of the dS space (*i.e.*, relative to spacetime, and qualified by its quadrimpulse, spin, etc.) to which can be added, in the case of flavor changes, also a modification of the internal dynamical state (*i.e.*, of the internal scalar field that defines the mass). Since quantum jumps do not occur, there is no breaking of unitarity. Vertices and lines of this type normally appear as internal to interaction diagrams, not associated with asymptotic states, and are representative of virtual processes. Virtual processes emerge as terms of the development of the time evolution operator, which is unitary; they therefore do not include any breaking of unitarity. A fermionic line connecting several intermediate virtual vertices represents the propagation (usually without a definite direction in external time) of a single dS space, without transients that break unitarity. Internal fermionic and bosonic lines eventually yield the conserved physical quantities transmitted to them by an external line to another external line and then to an outgoing asymptotic state.

It should be noted that the notion of "being or not on shell" depends on the choice of the frame of reference, if global transformations of coordinates including accelerations are admitted. In fact, acceleration modifies the norm of the quadrimpulse. For example, consider a free particle of four-momentum p_μ and on-shell mass M, within a frame of reference in which it undergoes a constant four-acceleration a_μ and is $p_\mu = 0$ for $t = 0$. We have $p^\mu p_\mu = m^2(t)c^4 = m^2(0)a^\mu a_\mu t^2$ for $t > 0$; let us consider, in this reference, the amplitude $\exp[(ip^\mu x_\mu - \delta(\mu)Mc^2|\tau|)/\hbar]$, being $m^2 - M^2 = \mu^2$. There can therefore exist an instant t' such that $m(t') = M$; at that instant the amplitude becomes $\exp[(ip^\mu x_\mu - Mc^2|\tau|)/\hbar]$, with $p^\mu p_\mu = M^2c^4$, which represents a QJ during which the particle is created (if $p_0(t') > 0$) or annihilated (if $p_0(t') < 0$). For $t \neq t'$ the amplitude instead becomes $\exp[(ip^\mu x_\mu|)/\hbar]$, with $p^\mu p_\mu =$

m^2c^4, $m \neq M$, which is relative to a virtual propagation. Consequently, the absence of quantum jumps in a frame of reference may correspond to the presence of quantum jumps, associated with the same physical processes, in a frame accelerated with respect to it. In other words, the notion of "quantum jump" is not absolute. An example is offered (in the classical context) by the uniform rectilinear motion of a charged material point that does not generate radiation, which is instead present in a system accelerated with respect to the charge.

3.2. A Hypothesis on the Meaning of the De Vries Formula

We now want to show how a purely empirical formula proposed many years ago to express the renormalized fine structure constant can be interpreted in the present conceptual framework. The conjecture presented here is based on the entropic interpretation of this constant (Equation (88)), and on the additional assumption that the interaction vertex is a fluctuation of this entropy. In this treatment we will not consider any restriction on the amplitude of this fluctuation: we will therefore not refer either to the mass of the boson B, or to particular scales of confinement. It is therefore intended to be applied to a free-scale situation in which the mass of B is zero and there is no confinement; these requirements limit this application to the case of the photon γ only. For other gauge bosons, the value of the interaction constant will be modified by factors that express a symmetry breaking by selecting a scale (think of the relation between the electromagnetic and weak fine structure constants mediated by the Weinberg angle, and therefore by the mass of the intermediate vector bosons), or there will be a value running divergent at the confinement scale (gluon interaction).

Let us begin by observing that, in the single coupling vertex with the photon, an elementary fermion charged with charge q exchanges an action ~ Q^2/c, with q^2 ~ $Q^2/2$. The factor 2 is because the photon couples with fermions of the same flavor (electron-electron, electron-positron, muon-muon, etc.); the distribution of the exchanged action between the two fermions can therefore be assumed as symmetric on the average. We can then define an angle φ through the proportion:

$$\frac{Q}{\sqrt{c}} : \pi = \frac{x}{\sqrt{c}} : \varphi \Rightarrow \varphi = \pi \frac{x}{Q} \tag{101}$$

where the charge x, exchanged by the single fermion, is a random variable with mean zero and variance $(Q)^2/2$. It follows that:

$$\langle \varphi \rangle = 0; \quad \langle \varphi^2 \rangle = \frac{\pi^2}{2} \tag{102}$$

Let us remark that $\varphi^2 \propto x^2$, which is itself an entropy by virtue of (88). We then interpret φ^2 as the amplitude of an entropy fluctuation corresponding to the vertex, measured in suitable units. The extra-vertex situation should then be considered as an equilibrium state whose (maximum) entropy is expressed by (88); under these hypotheses the probability of the fluctuation is estimated as:

$$e^{-\langle \varphi^2 \rangle} = e^{-\frac{\pi^2}{2}} \tag{103}$$

and this value constitutes a first, rough estimate of the fine structure constant. Its reciprocal is about 139.

Since the charges of the two fermions *a*, *b* are equal in modulus and opposite in sign, the two exchange processes will be symmetric. Since Q^2 is the total squared charge exchanged, each of the two fermions will exchange a squared charge ~ $Q^2/2$. The exchange of action implied in each of these two charge exchanges will be $Q^2/2c$. On the other hand, the action exchanged in the *whole* (renormalized) process of interaction between photon and fermionic field (not in the single vertex) is ~ h. For the same symmetry reasons, the amount of this action exchanged with the single afferent or effluent fermionic line is ~ $h/2$. The coupling constant of the single element of the pair (*a* or *b*) is therefore:

$$\frac{Q^2}{2c} : \frac{h}{2} = \frac{Q^2}{\hbar c} \cdot \frac{1}{2\pi} \tag{104}$$

This is, actually, the action exchanged between one of the two fermionic lines (*a* or *b*) and the electromagnetic field. If we indicate the direction of this exchange with $1 \to 2$ (where 1 denotes the fermionic line and 2 the electromagnetic field or vice versa), we can qualify Q^2 as $(x_{1\to 2})^2$. The actor 2 can then return to the actor 1 the squared charge $(x_{1\to 2})^2$, scaled by the factor $1/2\pi$ according to (104), which does not depend on the direction of the exchange. We therefore have $(x_{2\to 1})^2 = (x_{1\to 2})^2/2\pi$. The process is clearly iterative and we can set:

$$x_{i+1}^2 = \frac{x_i^2}{2\pi} \tag{105}$$

where i is the iteration index. Equation (105) can be written in the form:

$$x_i^2 = \frac{x_0^2}{(2\pi)^i} \tag{106}$$

This means that at the i-th iteration the coupling constant is $\alpha/(2\pi)^i$. The contribution of the i-th iteration to the overall coupling constant of the whole sequence of iterations is given by the product of $\alpha/(2\pi)^i$ by the total contribution of the previous iterations. That is: $\alpha(i + 1) = \alpha(i)\alpha/(2\pi)^i$, where $i = 0, 1, 2, 3\ldots$ and $\alpha(0) = 1$. We then have $\alpha(1) = \alpha(0)\alpha/(2\pi)^0 = \alpha$, as is natural; $\alpha(2) = \alpha(1)\alpha/(2\pi)^1 = \alpha^2/2\pi$; $\alpha(3) = \alpha(2)\alpha/(2\pi)^2 = \alpha^3/(2\pi)^3$; $\alpha(4) = \alpha(3)\alpha/(2\pi)^3 = \alpha^4/(2\pi)^6$, and so on. For each of the two sequences of iterations relating respectively to the exchanges involving *a* or *b* and the electromagnetic field, we therefore have a total coupling constant defined by the series:

$$\Gamma(\alpha) = 1 + \frac{\alpha}{(2\pi)^0}\left\{1 + \frac{\alpha}{(2\pi)^1}\left[1 + \frac{\alpha}{(2\pi)^2}\left(1 + \frac{\alpha}{(2\pi)^3}\cdots\right)\right]\right\} \tag{107}$$

The first term is related to the absence of exchanges and it represents the total action h not yet exchanged as reservoir of squared charge $\hbar c$, but available for exchange as Q^2.

The overall coupling constant is therefore the product of the two functions $\Gamma(\alpha)$, associated respectively with the two sequences related to a, b, and the factor $\exp(-\pi^2/2)$, that is $\alpha' = [\Gamma(\alpha)]^2\exp(-\pi^2/2)$. This must therefore be the coupling constant of the photon to the charged fermion. But this constant must equate α. We therefore have $\alpha' = \alpha$ and hence:

$$\alpha = \left[\Gamma(\alpha)\right]^2 e^{-\frac{\pi^2}{2}} \tag{108}$$

This relation defines the value of α. It was proposed, without any physical justification, by Hans De Vries more than twenty years ago [40]. We note that if z is the charge q_f of the fermion with which the photon couples, expressed in units e, *i.e.* $z = q_f/e$, Equation (108) actually does not give α but α/z^2. That is:

$$\frac{\alpha}{z^2} = \frac{1}{137,\ldots} \quad \Rightarrow \quad \alpha = \frac{z^2}{137,\ldots} \tag{109}$$

as it should be. However, in common usage α is defined as 1/137, ... and the coupling constant then becomes: $z^2\alpha = z^2/137, \ldots$

Equation (108) can be solved iteratively by assigning an initial value to α and inserting it in the right-hand side, then substituting the value thus obtained again in the right-hand side and so on. The obtained value and the experimental one are reported below [41]:

CODATA 2018 (source: NIST): $7.2973525693(11) \times 10^{-3}$

Hans de Vries: $7.2973525686 \times 10^{-3}$

We must however point out that the CODATA 2022 adjustment differs from the computed result by 4 standard deviations (new recommended figure: $7.297\,3525643(11) \times 10^{-3}$) [42]. At the moment it is impossible for us to say whether this discrepancy is a demonstration of the fallacy of the present argument or the simple consequence of having neglected, in the estimation of the constant, finer effects. Our intention here is not so much to solve an age-old mystery of theoretical physics (the origin of alpha), but to illustrate a possible connection between this problem and the issues discussed in this article.

3.3. Mixing Matrices

Equation (100) is inclusive of flavour mixing, if a suitable mixing factor is included in the coupling constant g. We now want to investigate this phenomenon in the context of the present model. Let us consider the dissociation $B \to q_1 + \overline{q}_2$ of a gauge boson B into two fermions of flavour a, b and charge q_1, $-q_2$ respectively. If B is self-conjugated, the self-conjugation property must extend to the pair $(q_1, \overline{q}_2)$; this is only possible if $a = b$ and $q_1 = q_2$. This is the situation for $B = H_0$, γ, Z_0, graviton; hence the interactions of these bosons do not exhibit flavour mixing. In the case of gluons, the exchange of flavours a, b in the quark-antiquark pair $(q_1, \overline{q}_2)$ is equivalent to a charge conjugation. But the exchange of flavors cannot induce a modification of the color charge, given the ontological independence of color from flavor. This contradiction can be resolved only by imposing the equality $a = b$ of flavors and therefore the absence of flavor mixing in interactions mediated by gluons. This leaves only the case of the W boson mediating weak interactions with charged currents. As in the gluonic case, the exchange of flavors a, b inverts the charge of the W. But, unlike the gluonic case, the electric charge is not ontologically independent of flavor; on the contrary, it is a function of it. Therefore there are no *a priori* reasons to assume the absence of flavor mixing in interactions mediated by the W. In this Section we postulate a specific mixing mechanism, building on the concepts introduced in the previous Sections. This mechanism gives rise to a quantitative expression for the amplitude of the interaction vertex between the W boson and two fermions (Equation (114)). Subsequently, we explore the compatibility of this expression with experimental mixing matrices. It hardly needs saying that the whole argument is conjectural; but, on the other hand, it seems to us that this is roughly the state of all current proposals concerning the physics of flavor.

Let us consider an interaction vertex in which a W boson connects two fermionic lines: one, afferent, of flavor a and generation i_a; the other, effluent, of flavor b and generation i_b. Let $n = |i_a - i_b|$. We will suppose that the transition from line a to line b occurs through a defined succession of steps, at the end of each of which a redefinition of the internal gravitational constant of the fermion *could occur*. The sequence is defined as follows: first we move from a to the conjugate fermion of a of the same generation (for example, if a is the u quark, this conjugate will be the d quark); then, starting from this second fermion, we move on to the subsequent fermions of the same weak isospin in increasing or decreasing order of generation, up to b (to continue the example, $d \to s \to \ldots$). The number of steps is clearly $n + 1$. The transition from line b to line a occurs with the same procedure. First, we move to the conjugate of b of the same generation (for example, if b is the s quark, this conjugate will be the c quark); then, starting from this second fermion, we move to the subsequent fermions of the same weak isospin in decreasing or increasing order of generation, up to a (to continue the example, $c \to u$). The number of steps is still $n + 1$.

The probability amplitude that a single step starts from a state in which the internal gravitational constant is not redefined with respect to the initial state of the sequence is assumed to be $p/(p + 1)$, where p is the number of previous steps. Therefore, the probability amplitude A of the sequence satisfies the rule $A(p + 1) = A(p)\, p/(p + 1)$. It follows that $A(p + 1)/A(p) = [1/(p + 1)]/(1/p)$, or $A(p) = 1/p$. For the entire sequence we have $p = n + 1$ and therefore an amplitude $A(n) = 1/(n + 1)$. Since the two sequences $a \to b$ and $b \to a$ are assumed to be independent, their total amplitude is the product of the two individual amplitudes, that is $1/(1 + n)^2 = 1/(1 + |i_a - i_b|)^2$. This will be the first factor in the expression of the vertex amplitude. The internal gravitational constant is redefined at the final endpoints a and b of the two sequences, and the sequences themselves together form a loop or cycle connecting these endpoints.

From a temporal perspective, the endpoints of the two fermionic lines a and b are positioned, in complex time (t, τ), at the same value of t, the external time of the laboratory; this value is the instant in which the event (real or virtual) constituted by the interaction vertex occurs. The endpoints of the two lines a and b are, consequently, the endpoints of an imaginary time segment. In external real time, the interaction vertex is delocalized in a time interval of extension $\approx h/M_W c^2$, where M_W is the mass of the W boson. This delocalization of the instant t, together with the difference $m_a - m_b$ between the masses m_a, m_b of the fermions a, b will give rise to a phase factor over the uncertainty interval:

$$e^{-i\frac{(m_a - m_b)c^2}{\hbar}\frac{h}{M_W c^2}} = e^{-2\pi i\frac{(m_a - m_b)}{M_W}} \tag{110}$$

which will be the second factor in the expression of the vertex amplitude. The action of the W boson can then be schematized as the propagation between a and b, along the imaginary time segment of extension $\Delta\tau \approx ih/M_W c^2$, of the (positive) energy $|m_a - m_b|c^2$ connected to the difference in mass between the two fermions and oriented from the heavier fermion to the lighter fermion. This propagation generates an amplitude scaling factor:

$$e^{-i\frac{|m_a - m_b|c^2}{h}\frac{ih}{M_W c^2}} = e^{\frac{|m_a - m_b|}{M_W}} \tag{111}$$

which is the dual of (110) in the imaginary time domain. We take (111) as the third factor of the vertex amplitude.

The further action of the W boson in imaginary time will be given by the trace operation on the scalar fields ξ_a, ξ_b inside the fermions a, b respectively. We will assume that the statistics of these fields in imaginary time is given by Equations (46), (47), where $C = 1$ according to what was established in Section 2.5., that is, from the product of functions $\exp(-\xi_a - \xi_b)$. In the propagation of the W boson between the ends a and b of the segment, these fields will be hypothetically correlated by the relation $\xi_{a,b} = r/r_{a,b} = c|\sigma|/r_{a,b}$, where the variable r has the dimensions of a length and therefore the variable σ has the dimensions of a time. r_a, r_b are the de Sitter radii of the fermions of flavor a, b; c is the limit speed. In essence, in the coupling with the W the two scalar fields are identified, while retaining the memory of their original de Sitter radii.

We will normalize the wave functions $y = A\exp(-c|\sigma|/r_{dS})$ according to the condition:

$$\int_0^\infty |y|^2 \, \mathrm{d}|\sigma| = 1 \quad \Rightarrow \quad A = \sqrt{\frac{2c}{r_{dS}}} \tag{112}$$

The trace operation is then expressed by the integration on the variable $|\sigma|$. We have:

$$S_{a,b} = \sqrt{\frac{2c}{r_a}}\sqrt{\frac{2c}{r_b}}\int_0^\infty e^{-\frac{c|\sigma|}{r_a} - \frac{c|\sigma|}{r_b}} \, \mathrm{d}|\sigma| = \frac{2}{\sqrt{r_a r_b}} \frac{r_a r_b}{r_a + r_b} \tag{113}$$

The integration is limited to positive values of the scalar field, which can be admitted if these variables are considered as an rms value of the field. The expression (113) will appear as the last factor in the expression of the vertex amplitude.

Putting all the factors together, we obtain the expression for the vertex amplitude:

$$f_{a,b} = \frac{S_{a,b}\ e^{\frac{|m_a - m_b|}{M_W}}}{\left(1 + |i_a - i_b|\right)^2} e^{-2\pi i \frac{m_a}{M_W} + 2\pi i \frac{m_b}{M_W}} \quad (114)$$

In the absence of mixing, we have $a \equiv b$ and therefore, as can be easily seen, $f_{ab} = \delta_{ab}$; the vertex amplitude is then reduced to the coupling constant. In the case of W, however, f is neither the unit matrix nor a diagonal matrix, and the problem therefore arises of how it can be diagonalized. As can be seen, the matrix f is Hermitian, that is $(f_{ab})^* = f_{ba}$. Let us consider the matrix V that diagonalizes the matrix f_{ab}, that is, such that VfV^{-1} = Diag is diagonal. We immediately have $(VfV^{-1})^+ = (V^{-1})^+ f^+ (V)^+ = (V^{-1})^+ f (V)^+ = (\text{Diag})^+ = \text{Diag} = VfV^{-1}$, from which it follows that V is unitary. In the basis where f is diagonal, the W dissociates into one of three fermion-antifermion pairs. In the case of quarks these three pairs will be (d', u), (s', c), (b', t) or (d, u'), (s, c'), (b, t'); in the following we will limit ourselves to the first case. The amplitudes d', s', b' are linear combinations of the amplitudes d, s, b and therefore there is flavor mixing. The matrix V is the mixing matrix. A similar reasoning applies to the dissociations of the W into leptonic a, b states (charged lepton and neutrino). The mixing matrix will be the CKM matrix for quarks, the PMNS matrix for leptons.

The relations presented in this section allow us to define the elements of the $f_{a,b}$ matrix for both quarks and leptons (we will assume that the de Sitter radius of a neutrino mass eigenstate is the same as that of the charged lepton of the same generation). The correctness of the reasoning is then verified if the mixing matrix constituted by the best fit of the experimental data diagonalizes the $f_{a,b}$ matrix for suitable choices of the fermion masses. Both the quark and neutrino masses are in fact known within experimental error bars, and the diagonalization will be better approximated by suitable choices of the values of these masses within the respective error bars. The situation is exposed in the next two subsections.

3.3.1. The CKM (Cabibbo-Kobayashi-Maskawa) Matrix

The calculation strategy is as follows. We start from the mixing matrix in the "standard" parametrization [43]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad (115)$$

where $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$, $(i, j) = 1, 2, 3$. The matrix V is therefore entirely determined by the angles θ_{ij} and the phase δ_{13}. For these parameters we assume the experimental best fit values [43]:

$\theta_{12} = 13.04^\circ \pm 0.05^\circ$ $\theta_{23} = 2.38^\circ \pm 0.06^\circ$ $\theta_{13} = 0.201^\circ \pm 0.011^\circ$

$\delta_{13} = 68.08^\circ \pm 4.5^\circ$

Running the quark masses on their experimental error bars [44]:

$m_u = 1.9 - 2.6$ MeV $m_d = 3.5 - 5.5$ MeV $m_s = 84 - 104$ MeV

m_c = 1.15 – 1.35 GeV m_b = 4.2 – 4.4 GeV m_t = 172 GeV (fixed)

We first compute the matrix f_{ab}, then the matrix VfV^{-1} = Diag, choosing the values of the masses that minimize the ratio between the sum of the moduli of the off-diagonal terms of Diag and the sum of the moduli of its diagonal terms. The lowest value of this ratio (0.0039) is obtained with the following masses:

m_u= 1.97 MeV m_d = 3.5 MeV m_s = 84 MeV

m_c = 1.15 GeV m_b = 4.2 GeV m_t = 172 GeV (fixed)

The list of elements of Diag, defined up to a common factor, is as follows (for each element the real part is reported first, then the imaginary part):

Diag (1, 1) = (9.907985479E−02, 4.445020854E−02)

Diag (1, 2) = (−2.097622026E−03, 1.963231713E−03)

Diag (1, 3) = (2.270226367E−03, −1.590122236E−03)

Diag (2, 1) = (8.128318004E−04, 2.398663666E−03)

Diag (2, 2) = (0.134869426, 4.873338714E−02)

Diag (2, 3) = (−9.121133015E−03, 1.912320405E−02)

Diag (3, 1) = (1.110927667E−03, −1.323248027E−03)

Diag (3, 2) = (2.465872327E−03, −8.105657063E−03)

Diag (3, 3) = (0.766050756, −9.318360686E−02)

These results are consistent with the proposal of this paper, that the mixing matrix V is the diagonalization matrix of the vertex amplitudes f_{ab}.

3.3.2. The PMNS (Pontecorvo-Maki-Nakagawa-Sakata) Matrix

The procedure is the same as in the case of quarks. The matrix is considered in the same standard parametrization seen in the previous subsection, with the following parameter values [45]:

θ_{12} = 33.41˚ + 0.75˚ − 0.72˚ θ_{23} = 49.1˚ + 1.0˚ − 1.3˚

θ_{13} = 8.54˚ +42˚ − 25˚ δ_{13} = 197˚ + 42˚ - 25˚

Precisely, the central value is taken. The variation of the parameters within their error bars is not considered, leaving this possibility to possible future investigations. The masses of the charged leptons are considered fixed, while those of the neutrino mass eigenstates are varied within reasonable error bars, according to the following scheme (compatible with PDG limits):

m_e = 0.511 MeV m_μ = 105.658 MeV m_τ = 1776 MeV

$m_{\nu 1}$ = 0.0001 − 0.0006 eV $m_{\nu 2}$ = 0.0001 − 0.001 eV $m_{\nu 3}$ = 0.0001 − 0.06 eV

The lowest value of the ratio of the sum of the moduli of the off-diagonal terms of Diag to the sum of the moduli of its diagonal terms (0.1257) is obtained with the following masses:

$m_{\nu 1}$ = 0.0006 eV $m_{\nu 2}$ = 0.001 eV $m_{\nu 3}$ = 0.06 eV

That is, practically the upper extremes of the adopted bars. The list of the corresponding Diag elements is the following:

Diag (1, 1) = (0.318358690, 4.858494736E−03)

Diag (1, 2) = (2.038531750E−02, 1.348413900E−02)

Diag (1, 3) = (1.689680666E−02, −1.703028567E−02)

Diag (2, 1) = (1.838763803E−02, 1.354285795E−02)

Diag (2, 2) = (0.313012689, 7.760077715E−04)

Diag (2, 3) = (−3.567337990E−04, 1.792673953E−02)

Diag (3, 1) = (1.815088838E−02, −1.137212384E−02)

Diag (3, 2) = (1.312024891E−03, 1.500429027E−02)

Diag (3, 3) = (0.368628621, −5.634509027E−03)

It can therefore be stated that the calculation methodology illustrated in this Section also works in the case of leptonic couplings of the W boson. The results seem consistent with a direct mass hierarchy (although this point should be explored with more extensive numerical studies). In the calculation, the identity of the de Sitter radii of the states ν_1, ν_2, ν_3, and of the charged states e, μ, τ, respectively, has been assumed.

4. Conclusions

In these conclusions, we would like to summarize in points, from a bird's eye view, the topics touched upon in the various sections of the article. The first hypothesis presented in this work is related to the vacuum of the Higgs field: it is associated with de Sitter (dS) spaces tangent to spacetime. The characteristics of these spaces (de Sitter radius, cosmological constant, gravitational constant, average density of matter-energy) are defined by the VEV and the mass of the Higgs boson. We then hypothesized that elementary particles are also dS spaces tangent on spacetime, whose properties are defined starting from the *renormalized* charge and mass of the specific particle.

The well-known Higgs mechanism, which provides mass to the elementary particles of the SM, can then be reinterpreted geometrically [4], as a rescaling of the internal gravitational constant of the dS space associated with the vacuum of the Higgs field, which produces the dS space associated with the particle. This gravitational constant has nothing to do with Newton's external gravitational constant, which governs the gravitational interactions *between* particles. In the context of this description, which is essentially pre-quantum (dS spaces are solutions of Einstein's gravitational equations in the *internal* cosmological constant), the spacetime coordinates are assimilable to degrees of freedom of the Higgs vacuum, inherited by the elementary particles in the rescaling process that constitutes the essence of the Higgs mechanism.

The quantum description is obtained by passing from Einstein's gravitational equations to their quantum counterpart, constituted by the WdW equation [12] [13] (again in the *internal* cosmological constant). This passage imposes, as a consequence of the more general "time problem" [15] [16] associated with this equation, the choice of an internal evolution variable of the dS spaces associated with a specific particle. It is possible to choose a variable that, in the Lorentzian regime, can be connected with the proper time of the particle. The harmonic quantum amplitudes in the proper time, solutions of the WdW equation, are then interpreted as the usual plane wave functions of the particle. They describe fields of dS spaces of definite mass and direction of the proper time axis (direction of the four-momentum), but completely indefinite position of the point of tangency on the spacetime. This results in the well-known quantum delocalization of elementary particles [6]. From the superposition of the plane wave functions thus constructed, with different time axes but the same mass (delocalization of the momentum), we obtain the general wave function of the particle, solution of the ordinary quantum-relativistic wave equations.

The Euclidean-time solutions of the WdW equation correspond, as usual in a third quantization scheme [46]-[50], to an evanescence of the field of dS spaces associated with a particle, or to their build up from the annihilation of previous states. The transition of a field of dS spaces from Lorentzian to Euclidean time, or vice versa, therefore models a quantum jump (QJ) [51] [52]. QJs violate the unitarity of the evolution of the wave function, effectively implementing its "collapse". The breaking of unitarity occurs at the level of micro-interactions between particles, without the participation of "environmental" degrees of freedom. This phenomenon is at the basis of *classicalization* [10] [53].

A very important role is played by the local nature of interactions, consisting in the diagonality of the interaction operators on the basis of position. In a real event, *i.e.* associated with a QJ, the interacting particles are localized in the *same* spatial position (even if this can be delocalized in an interaction region of finite volume). The spatial degrees of freedom of matter then become the coordinates that express the *position of real events* (interaction regions) in a three-dimensional space *independent of the particle indices*. This fact becomes the decisive aspect of classicalization: macroscopic objects, *which are not constituted by particles but by clusters of real interactions between particles*, become stable entities in a three-dimensional space-arena external to them.

The nature of three-dimensional space as the geometric locus of real events of particle interaction is reflected in the existence of a positive cosmological constant. In fact, the localization of a positional eigenstate of a particle requires a concentration of energy in that space, around the position represented by the eigenvalues of that eigenstate. This concentration of energy self-interacts gravitationally, and the minimum self-interaction energy corresponds to an infrared limit on the delocalization of the gravitational interaction vertex, associated with a cosmological constant. The cosmological constant is therefore implied by the very property of space of being the locus where particle interactions are localized; it is completely independent of the zero-point energy of empty space. This explanation of the "small" value of the cosmological constant seems to be simpler than others based on arguments of a much more technical nature [54]-[57].

Finally, we investigated the interactions between elementary fermions and gauge bosons, proposing that gauge bosons are operator fields that couple fermionic lines and, as such, are ontologically different from the elementary fermions of the SM. They do not appear to be associated to dS spaces; therefore, the geometric representation of the Higgs mechanism formulated for fermions cannot be extended to bosons. This notion of gauge boson, which is found to be fully compliant with the ordinary vertex amplitude of the SM, has been tested on two fronts. First, inspired by the entropic nature of the fine structure constant elucidated by the present approach, we have conjectured a possible justification for the formula proposed by De Vries for the determination of its value [40]. Second, it turns out that, in the case of an interaction vertex mediated by weak charged currents, the vertex amplitude comes to depend on the fermionic flavors. The unitary transformation of the fermionic states that diagonalizes the vertex amplitude then constitutes the mixing matrix. Preliminary numerical computations, reported in the paper, support this prediction for reasonable values of the fermionic masses. It should be noted that the research program here is only outlined, by indicating dependencies between the free parameters of the Standard Model (fermionic masses, interaction constants, mixing parameters), which points in the direction of reducing the logical arbitrariness of the values of these parameters, and thus of a future deeper theory of elementary particles.

Although the quantitative evaluations concerning the fine structure constant, the cosmological constant and the mixing parameters in the weak interaction paint a picture of substantial convergence with experimental and observational data, additional to the conventional SM narrative, they cannot be considered as evidence in support of the hypotheses presented in this paper. Since the central idea presented here is that of the association of dS spaces to the elementary SM fermions, the problem arises of finding suitable experimental tests of this association. We recall here that the material fields originating the fermion mass (*i.e.*, the rescaled Higgs fields) are distributed within the fermionic dS space. In classical terms, the inertia of the fermion is distributed continuously within the entire dS space. The principle of proportionality between inertial and gravitational mass then implies a continuous distribution, within the dS space, of the gravitational charge of the fermion. In other words, while the SM fermions appear point-like in non-gravitational interactions, they should instead appear as extended entities in gravitational interactions. This prediction of the deviation of elementary fermions from Newtonian behavior on scales $\leq 10^{-13}$ cm leads to two possible lines of experimental-observational verification, unfortunately not immediately implementable. The first consists in the study of hyperdense states of matter, generated by extreme gravitational collapse. The spatial superposition of different elementary particles in the same volume of radius $\sim 10^{-13}$ cm leads in fact to finite gravitational self-interaction energies, although relevant if the collapse is extensive. This prevents the formation of space-time singularities. The difficulty here is that the origination of event horizons around the collapse region prevents the direct observational study of the collapsed matter and therefore the verification of the prediction. A second possibility is represented by the study of gravitational interactions between elementary fermions in particle accelerators or terrestrial laboratories. The most relevant processes to study in this context are, in our opinion: 1) the gravitational scattering of the photon on the neutrino: $\nu + \gamma \rightarrow \nu + \gamma$, which should allow the measurement of the de Sitter radius of the neutrino and of the behavior of the gravitational constant at high energies for values of the impact parameter ≤ 10^{-13} cm (end of Section 2.2); 2) the gravitational radiation of a neutrino in free fall in a homogeneous and constant gravitational field: $\nu \to \nu +$ graviton. This second study presupposes the measurability of single gravitons, which is, for now, far beyond present possibilities. This would provide the value of the gravitational mass of the neutrino; from the mass and the de Sitter radius, it would then be possible to estimate the parameter q^2 attributed in the present description to the neutrino. Similar studies with charged leptons would be affected by the superposition of the electroweak interaction. Despite the evident technological difficulties, the experimental study of gravitational interactions between elementary particles could be the new frontier of this discipline, as suggested by recently published reports [58]-[60].

Perhaps the most important aspect of the hypotheses discussed here is the emergence of a three-dimensional space-arena in the context of the classicalization process, and the unexpected role that the Higgs field would play in this process. The peculiarity of the Higgs field would no longer be only that of "giving mass to particles", but also that of inducing their proper time (conjugated to the mass). The free evolution of massive particles is labelled by the proper time and, only to the accessory extent constituted by the delocalization of the momentum, also by the spatial position (conjugated to the momentum). QJs, and with them classicalization, would then be further aspects of this dynamical emergence of the spacetime ordering of real events. Removing the Higgs field from this process, we are left with a static space-arena, inhabited by matter alien to it in a sort of static, frozen, relation. This situation reproduces the background of the current theoretical representation. This paper can therefore be seen as a stimulus to overcome the traditional separation between spacetime and material fields; an overcoming somehow already announced by previous attempts, such as the Maldacena conjecture and holography [61] [62].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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