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The Hyperflower: Strange Attractors in Complex Hilbert Space Cosmology

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Abstract

This paper presents a novel extension of cosmological dynamics into a complex Hilbert space, where trajectories in spacetime are influenced by dual attractors—one causal, one anticausal—embedded in a complexified manifold. Motivated by recent proposals that interpret black holes as sources of tachyonic emissions flowing backward through time, we construct a mathematical formalism using complex-valued time coordinates, Lagrangian field theory, and chaotic attractor models generalized to infinite dimensional settings. This framework offers a unifying picture of entropy balance, and black hole information conservation, with testable implications for cosmic structure and future observables.

Keywords

Black Holes, White Holes, Big Bang, Hilbert Space, Chaos Theory, Strange Attractors, Lorenz Systems

1. Introduction

The search for a unified framework that reconciles the fundamental nature of quantum mechanics with the geometry of spacetime remains one of the most ambitious goals in modern physics. At the heart of this quest lies the persistent challenge of interpreting the role of causality, entropy, and information flow across phenomena ranging from black hole evaporation to cosmological expansion [1]. While general relativity provides a powerful geometric language for gravitation and spacetime curvature, and quantum mechanics governs microscopic phenomena with astonishing precision, the two theories remain fundamentally incompatible when it comes to the nature of time, measurement, and singularity.

In recent developments, strange attractors from nonlinear dynamical systems theory have emerged as fertile analogies for understanding complex state evolution, including quantum state reduction and classical chaos. This paper builds upon a radical hypothesis: that the global structure of the universe—including the emergence of matter, the arrow of time, black hole horizons, and entanglement—can be modeled as trajectories through a complexified Hilbert space, with strange attractors acting as fixed points governing the causal and anti-causal flows of energy and information.

This hypothesis finds its roots in the notion of Backflow Cosmology [2], wherein black holes are treated not as endpoints of information loss, but as dynamic portals that convert infalling matter into tachyonic fields propagating backward in time. This leads to a time-symmetric picture in which the Big Bang is interpreted not as a unique beginning, but as a white hole boundary condition through which the universe continually re-emerges from its own interior dynamics. The central insight of the present work is that such backflow dynamics can be formally encoded using the machinery of complex Hilbert spaces and attractor bifurcations. Sen's analysis of tachyon condensation in brane–antibrane systems offers a formal precedent for interpreting tachyons as indicators of vacuum instability, with transitions to new spacetime structures—a mechanism analogous in spirit to the cosmological phase shifts proposed in this work [3].

We propose that the apparent separation between quantum indeterminacy and cosmological determinism dissolves when one allows the evolution of the universe to be described by complex trajectories in Hilbert space, where imaginary time corresponds to anti-causal, tachyonic components of the field. In this framework, strange attractors act as global regulators of phase space flow—analogous to black holes, entangled states, and cosmological inflation—each representing a stable configuration that attracts trajectories based on their initial phase orientation.

This paper develops the mathematical scaffolding for this theory, integrating methods from dynamical systems, complex analysis, quantum field theory, and general relativity. In doing so, we aim to illuminate how complex attractor dynamics not only mirror known physical behavior but also predict new phenomena such as observable asymmetries in cosmic background radiation, deviations in gravitational lensing, and reinterpreted quantum nonlocality.

2. Mathematical Preliminaries

To formalize the hypothesis of complex Hilbert space cosmology, we begin by establishing the necessary mathematical background. This includes the structure of Hilbert spaces (both real and complex), the behavior of dynamical systems in infinite-dimensional spaces, and a brief review of strange attractors. We then introduce a complexified time coordinate and reinterpret the evolution of physical fields within this generalized framework.

2.1. Hilbert Spaces and Complex Extension

Let H be a real Hilbert space of square-integrable states, typically associated with the wavefunctions of quantum fields:

$$\mathcal{H} = L^2\left(\mathbb{R}^n, \mathbb{C}\right),$$

with the inner product

$$\langle \psi, \phi \rangle = \int_{\mathbb{R}^n} \psi^*(x)\phi(x)\,\mathrm{d}x.$$

We extend this structure to a complexified Hilbert space $\mathcal{H}_{\mathbb{C}}$, allowing not only complex-valued fields but also complex-valued spacetime parameters:

$$x^\mu \in \mathbb{C}^4, \quad t \to t + i\tau.$$

This permits the introduction of imaginary time components, which will play a key role in distinguishing causal from anti-causal evolution.

2.2. Dynamical Systems and Strange Attractors

A dynamical system is described by a flow

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = F(\Psi),$$

where $\Psi(t) \in \mathcal{H}_{\mathbb{C}}$. In classical settings, nonlinear F may give rise to chaotic dynamics and attractors in phase space. A *strange attractor* is a fractal-like structure in state space toward which trajectories converge, despite sensitivity to initial conditions. The Lorenz attractor is a classical example in three real dimensions:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z,$$

whose behavior exhibits bifurcation, lobe switching, and deterministic chaos. In this work, we propose a generalization of such systems into complex Hilbert space, modeling both quantum and cosmological flows.

2.3. Time Symmetry and Complex Time Coordinates

We adopt a dual-time coordinate system:

$$t \in \mathbb{R}\ (\text{causal time}), \quad \tau \in \mathbb{R}\ (\text{anti-causal or imaginary time}).$$

The full time coordinate is then complex:

$$T = t + i\tau,$$

such that the evolution operator becomes:

$$\Psi(T) = \mathrm{e}^{-iHT/\hbar}\Psi(0) = \mathrm{e}^{-iHt/\hbar}\mathrm{e}^{-H\tau/\hbar}\Psi(0),$$

where evolution in τ induces exponential decay or growth depending on the sign of the energy eigenvalues. This structure allows us to interpret τ as a flow toward or away from attractors in imaginary time—a mechanism we associate with tachyonic behavior and black hole-white hole phase transitions.

2.4. Tachyonic and Photonic Field Definitions

In this formalism, we define two classes of propagating modes:

- **Photonic fields:** $\psi(x,t)$ evolve along real time, with velocity $v=c$, and define forward-time causality.
- **Tachyonic fields:** $\psi_T(x,\tau)$ evolve along imaginary or negative time, with effective velocity $v_T=-c$ (or $v_T=ci$ under a complex formulation), defining anti-causal behavior.

These dual components coexist in the total Hilbert space evolution:

$$\Psi(x,T)=\psi(x,t)+\psi_T(x,\tau),$$

suggesting that all physical entities may contain both causal and anti-causal components, modulated by their projection onto real or imaginary time.

2.5. Notation

Throughout this paper, we will use:

- $\mathcal{H}_{\mathbb{C}}$: Complex Hilbert space.
- $T=t+i\tau$: Complex time coordinate.
- ψ: Real-time field component.
- ψ_T: Imaginary-time (tachyonic) component.
- Γ: Imaginary-time connection or arc in spacetime.

These foundations set the stage for interpreting cosmological dynamics as flows between strange attractors embedded in a complex phase space.

3. Phase Flow in Complex Hilbert Space

In classical dynamical systems, the evolution of a system is often described by the flow of a point in a finite-dimensional phase space governed by differential equations [4]. In our extended cosmological model, the phase space becomes infinite-dimensional and complex: a *complex Hilbert space* $\mathcal{H}_{\mathbb{C}}$, where each point corresponds to a possible configuration of the spacetime-field system, including both causal (real-time) and anti-causal (imaginary-time) components.

Let $\Psi(t)\in\mathcal{H}_{\mathbb{C}}$ represent the state of the universe at complex time $t=t_R+it_I$, evolving according to a generalized Hamiltonian flow:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t}=-i\hat{H}_{\mathbb{C}}\Psi,$$

where $\hat{H}_{\mathbb{C}}$ is a self-adjoint or pseudo-Hermitian operator acting on $\mathcal{H}_{\mathbb{C}}$. This generalization extends the usual Schrödinger equation into complex time, such that the imaginary component corresponds to anti-causal evolution associated with tachyonic or negative-time flow.

The system's phase trajectory is now a complex curve:

$$\Gamma:\mathbb{C}\to\mathcal{H}_{\mathbb{C}},\ \Gamma(t)=\Psi(t),$$

and can exhibit attractor behavior in both real and imaginary temporal directions. We define the *causal attractor* $\mathcal{A}_+\subset\mathcal{H}_{\mathbb{C}}$ as the set toward which all real-time trajectories asymptotically converge as $t_R\to\infty$, and the *anti-causal attractor* $\mathcal{A}_-$

as the set toward which they converge as $t_R \to -\infty$.

The phase flow can be characterized by a vector field $V : \mathcal{H}_\mathbb{C} \to \mathcal{H}_\mathbb{C}$, where:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = V(\Psi).$$

If V is analytic, the evolution preserves the holomorphic structure of the state manifold. In this case, Cauchy-Riemann conditions guarantee that real and imaginary components are not independent, linking causal and anti-causal flows.

3.1. Example: Linearized Flow

For illustrative purposes, consider a linear complex evolution governed by:

$$\hat{H}_\mathbb{C} = \alpha\mathbb{I} + i\beta\mathbb{I},$$

with constants $\alpha, \beta \in \mathbb{R}$. Then:

$$\Psi(t) = \Psi_0 \exp\left[-i(\alpha + i\beta)t\right] = \Psi_0 \exp(-\beta t)\exp(-i\alpha t),$$

which shows exponential decay or growth in the imaginary direction and oscillatory behavior in real time. This kind of structure naturally accommodates the asymmetric time evolution hypothesized for photon-tachyon dual fields.

3.2. Topology and Boundedness in Infinite Dimensions

In finite-dimensional dynamical systems, attractors are typically defined as compact, invariant subsets of the phase space toward which trajectories asymptotically converge. These sets—such as fixed points, limit cycles, or strange attractors—are compact in the topological sense: closed and bounded within a finite-dimensional normed space.

However, in our framework, the phase space $\mathcal{H}_\mathbb{C}$ is an infinite-dimensional complex Hilbert space. In such spaces, compact subsets are rare or trivial due to the Riesz theorem: the closed unit ball in an infinite-dimensional Hilbert space is not compact under the norm topology. This implies that traditional definitions of compact attractors must be reinterpreted.

Instead, we consider *bounded absorbing sets* and *weakly compact invariant sets*. These are subsets $B \subset \mathcal{H}_\mathbb{C}$ such that:

- Every trajectory $\Psi(t)$ eventually enters and remains in B as $t \to \pm\infty$, depending on the direction of flow.
- The dynamics restricted to B is topologically invariant and may exhibit chaotic or structured behavior.
- B may be compact under weaker topologies, such as the weak topology or compact convergence, even if not in the norm topology.

This generalization allows us to meaningfully define causal and anti-causal attractors $\mathcal{A}_+$, $\mathcal{A}_- \subset \mathcal{H}_\mathbb{C}$ as weakly compact invariant sets toward which trajectories converge in real or imaginary time. In cosmological terms, these attractors correspond to:

- $\mathcal{A}_+$: forward-time (causal) attractors such as black holes, which absorb infalling information.

- $\mathcal{A}_-$: backward-time (anti-causal) attractors associated with the Big Bang boundary or white hole-like past states that emit information via tachyons.

The physical significance of these abstract attractors is that they define the global flow of information in the universe. Despite the absence of norm-compact attractors, the existence of such absorbing sets ensures that the cosmological evolution remains dynamically bounded and that entropy-like invariants can be meaningfully analyzed in this setting.

4. Strange Attractors and Causal Duality

Having established that the phase space of our cosmological model is a complex Hilbert space $\mathcal{H}_{\mathbb{C}}$, we now explore the structure and dynamics of attractors within this space. In particular, we examine the emergence of *strange attractors* as geometric and dynamical signatures of causal and anti-causal flows.

4.1. Definition and Role of Strange Attractors

In classical dynamical systems, a strange attractor is a fractal-like, topologically intricate set toward which a trajectory converges over time [5]. It is typically associated with sensitive dependence on initial conditions—hallmarks of deterministic chaos.

In our complexified cosmological context, strange attractors arise as the asymptotic endpoints of the real-time (t_R) and imaginary-time (t_I) evolution of field states in $\mathcal{H}_{\mathbb{C}}$. These attractors may be "folded" structures, in the sense that they simultaneously span both the real and imaginary components of time. The causal trajectory of matter and radiation thus evolves toward a forward-time strange attractor (e.g., a black hole), while anti-causal field states such as tachyons follow a mirrored trajectory toward the Big Bang past-boundary attractor.

$$\lim_{t_R \to +\infty} \Psi(t) \in \mathcal{A}_+, \quad \lim_{t_R \to -\infty} \Psi(t) \in \mathcal{A}_-,$$

with $\mathcal{A}_+ \cap \mathcal{A}_- = \varnothing$ except at a possible entropic saddle or symmetry point, which may correspond to a unique feature of the initial singularity.

4.2. Causal vs. Anti-Causal Attractors

The dual attractors $\mathcal{A}_+$ and $\mathcal{A}_-$ define distinct dynamical roles:

- $\mathcal{A}_+$: absorbs causal mass-energy (e.g., black holes).
- $\mathcal{A}_-$: absorbs anti-causal fields, such as tachyons or backward-traveling antimatter, converging into the Big Bang boundary (a white hole).

This duality implements a time-symmetric flow structure across the universe and explains the emergence of *information conservation* through two mirrored attractors: one in the future, and one in the past.

4.3. Phase Space Folding and the Butterfly Geometry

The metaphor of the "butterfly" often used to visualize classical strange attractors becomes geometrically meaningful in this setting. Each "wing" of the butterfly corresponds to a distinct attractor—one in positive-time evolution, the other in negative-time evolution. The central fold represents the photon-tachyon symmetry boundary, where a phase transition in the geometry of time occurs:

$$t = 0 : \text{phase transition surface } (\text{e.g., photon} \leftrightarrow \text{tachyon conversion}).$$

We propose the existence of a higher-order attractor structure—possibly fractal—in which black holes function as localized forward-time attractors, while the Big Bang boundary behaves as a global attractor in negative time. This framework replaces the classical thermodynamic arrow with a *bifurcated attractor landscape* embedded in complexified time.

4.4. Dynamical Symmetry and Reversibility

The existence of mirrored attractors enforces a dynamical symmetry under time reversal:

$$T : t \to -t,\ \ \Psi(t) \to \Psi^*(-t).$$

This symmetry ensures that any entropy generated by matter falling into black holes is balanced by reverse-time flow of equivalent anti-information into the Big Bang white hole structure. In this model, entropy does not increase globally—it circulates across a time-symmetric attractor geometry.

This conceptual shift provides a new interpretation of the second law of thermodynamics: instead of entropy growing indefinitely, it is asymmetrically distributed across temporally distinct attractors. The universe is thus not "running down," but dynamically re-balancing information flows via a complex, attractor-governed architecture.

5. Global Causal Structure and Tachyonic Bifurcation

The global structure of spacetime in this framework is governed by a complex-valued manifold $\mathcal{M}$ embedded in a Hilbert space $\mathcal{H}_{\mathbb{C}}$. In this formulation, real-valued causal trajectories associated with photons and massive particles evolve forward in proper time, while their tachyonic and antimatter counterparts trace anti-causal trajectories backward in a complexified time coordinate τ. This structure naturally leads to a bifurcation of the causal cone, extending both into the future and the past in dual symmetry.

5.1. Forward and Backward Time Attractors

In the forward-time (causal) direction, gravitational collapse results in localized curvature singularities interpreted as black holes [6]. These act as strange attractors for matter-energy flow, concentrating entropy and reducing spatial degrees of freedom.

In the backward-time (anti-causal) direction, the proposed model treats the Big Bang not merely as an initial condition but as a *global white hole attractor* for all anti-causal flows.

This attractor draws in information via tachyons and time-reversed antimatter, leading to a low-entropy but high-information-density origin state. The bifurcation is illustrated by the duality of worldlines: a causal trajectory $x^\mu(\lambda)$ terminating in a black hole maps via complex conjugation to an anti-causal trajectory $x^\mu(\bar{\lambda})$ terminating in the Big Bang. These are connected by an imaginary-time geodesic segment $\Gamma(\tau)$, forming a continuous path in $\mathcal{H}_\mathbb{C}$.

5.2. The Curvature Equation and Modified Field Tensor

To encode this structure mathematically, we propose a modified Einstein field equation that distinguishes causal from anti-causal stress-energy via tensorial decomposition:

$$\Delta G_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu}^{\text{photon}} + T_{\mu\nu}^{\text{tachyon}}\right),$$

where $T_{\mu\nu}^{\text{tachyon}}$ is defined on the imaginary-time hypersurface and is formally related to the complex conjugate of the photon sector via:

$$T_{\mu\nu}^{\text{tachyon}} = \mathcal{I}\left[\psi_T^\dagger\left(i\gamma^\mu\nabla_\mu - m\right)\psi_T\right],$$

with ψ_T representing the tachyon field and $\mathcal{I}$ denoting projection onto the anti-causal subspace.

5.3. Boundary Conditions and the Role of the Big Bang

The Big Bang, in this model, serves as a boundary condition at $\tau = 0$ for all anti-causal flows. It is topologically distinct from black hole singularities in that it is not emergent from collapse, but rather serves as a globally defined attractor for backward trajectories. This implies a compression of the entire complexified past light cone into a singular point—not in the real-time manifold, but in the orthogonal imaginary component. This structure eliminates the need for a low-entropy "initial condition" by instead viewing the Big Bang as a thermodynamically maximal point of anti-causal convergence. The entropy gradient is thus symmetric across time, but directionally reversed.

5.4. Constraints on Multiple White Hole Attractors

While the existence of multiple black hole attractors is a natural consequence of localized collapse, the emergence of multiple white hole attractors in the negative-time direction is more tightly constrained. Our model assumes that the Big Bang is the *unique* global attractor in the $\tau < 0$ domain due to the manifold's topological and thermodynamic boundary conditions. However, we acknowledge that exotic topologies (e.g., non-simply connected or branched coverings) could admit additional attractor-like structures. These would represent future extensions of the theory.

5.5. Implications

Figure 1 illustrates the bifurcated structure of the causal manifold, showing

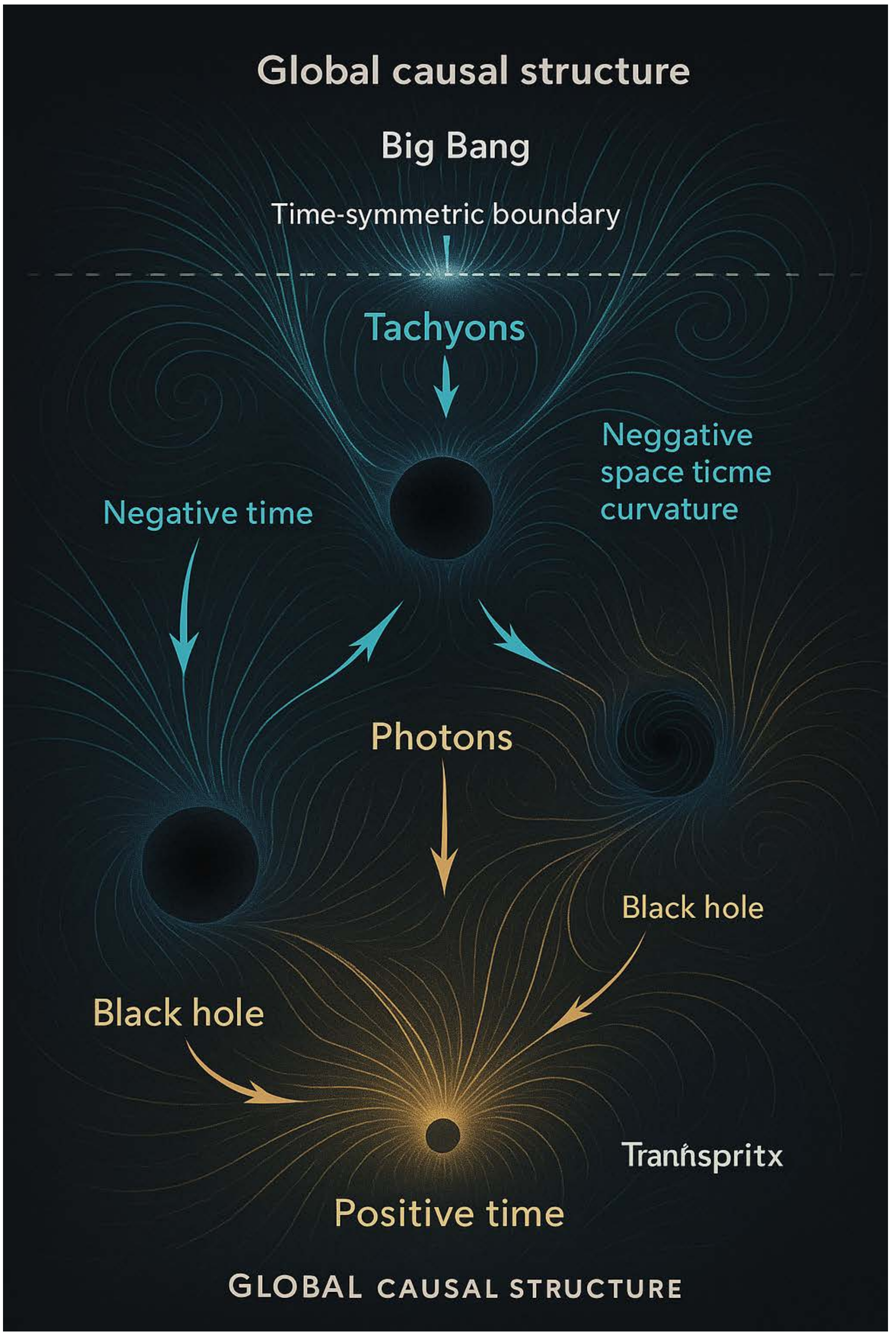

Figure 1. Bifurcation of causal and anti-causal trajectories in complex Hilbert spacetime. Tachyonic flows (blue) converge in $\tau < 0$ on the Big Bang attractor. Photon trajectories (red) terminate in distributed black hole attractors in $t > 0$.

forward-time trajectories terminating in distributed black hole attractors, and backward-time tachyonic flows converging on a singular Big Bang attractor via imaginary-time geodesics. The recognition of the Big Bang as a strange attractor in negative-time flow provides a compelling solution to the entropy problem and a mechanism for global time symmetry. It also explains the scarcity of antimatter in forward-time evolution and the gravitational influence of dark matter and dark energy as emergent effects of anti-causal information compression.

6. Entropic Balance and Information Conservation

The apparent arrow of time, characterized by the Second Law of Thermodynamics, has traditionally stood in tension with the time-reversal symmetry of the underlying laws of physics [7]. In the present framework, we propose that this asymmetry is only apparent within a local causal domain, while the global structure of spacetime enforces entropic balance through the inclusion of anti-causal (tachyonic) information flow.

6.1. Black Holes as Entropic Sinks

In the forward-time (causal) region of the universe, black holes act as entropy-accumulating endpoints of matter evolution. The entropy associated with a black hole of mass M is given by the Bekenstein-Hawking formula:

$$S_{\mathrm{BH}} = \frac{kc^3 A}{4G\hbar},$$

where A is the area of the event horizon. As photons and matter fall into black holes, their informational content appears to be lost to an external observer. However, under our model, this information is conserved via a transformation into tachyonic degrees of freedom that propagate anti-causally.

6.2. The Big Bang as an Entropic Convergent Point

Contrary to traditional models which assume the Big Bang as an improbable low-entropy initial condition, we reinterpret it as the global convergence point of all anti-causal information trajectories. Tachyons and reversed-time antimatter return information from the distributed future black holes toward this origin point. The Big Bang thus represents a coherent, information-dense attractor in negative time, ensuring that the net entropy of the entire complexified manifold remains conserved. We define the global entropy flux across the manifold $\mathcal{M} = \mathcal{M}_+ \cup \mathcal{M}_-$ as:

$$\int_{\mathcal{M}_+} \nabla_\mu J^\mu_{\text{causal}} \, \mathrm{d}^4 x + \int_{\mathcal{M}_-} \nabla_\mu J^\mu_{\text{anti-causal}} \, \mathrm{d}^4 x = 0,$$

where J^μ_{causal} and $J^\mu_{\text{anti-causal}}$ denote the entropy current vectors in the forward and backward-time regions, respectively.

6.3. Tachyonic Flow and Holographic Encoding

Tachyons, modeled as anti-causal spacelike fields ψ^T, serve as the conduits of information conservation. These fields arise from horizon-scale phase transitions (e.g., photon-to-tachyon conversion) and carry information retrocausally through the complexified spacetime manifold. The information encoded on the event horizon is thus not lost, but rather redirected to the Big Bang attractor through tachyonic propagation. This reinterpretation of black holes as phase-transition boundaries rather than true singularities aligns with the holographic principle. The encoding of three-dimensional information on two-dimensional surfaces (e.g., event horizons) is preserved even across the causal divide, reinforcing the idea of global coherence.

6.4. Predictions and Implications

While detailed observational consequences of our time-symmetric cosmological model are developed in our companion paper, the present attractor-based formalism offers a new framework for interpreting potential anomalies and information patterns in the early universe. In particular, the dynamics of strange attractors in complex Hilbert space introduces the possibility that Cosmic Microwave Background (CMB) anisotropies are not simply statistical relics of inflationary noise, but structured signatures of global phase flow stability.

Specifically, if black holes act as local attractors whose tachyonic outflows converge retrocausally into the Big Bang—as a global white hole—then the geometric configuration and bifurcation structure of these attractors may leave residual imprints in the CMB. Features such as large-scale alignment axes ("axis of evil"), hemispherical power asymmetry, or low-multipole suppression could, in this framework, arise from the nonlinear coupling of local spacetime flows to the global strange attractor geometry.

Moreover, entropic balance across event horizons suggests that the universe's information budget is not thermodynamically erased but dynamically encoded. The structure of the global attractor may constrain this encoding in a way that produces observable, testable effects—particularly in precision measurements of the CMB's angular power spectrum or its polarization anisotropies. Future work may extend this prediction framework using tools from ergodic theory, attractor stability, and bifurcation analysis, potentially allowing for a classification of observable patterns based on the topology of causal flow.

6.5. Clarifying the Horizon Encoding Hypothesis

The claim that information is "stored on the horizon" must be revisited in light of the present framework. In conventional holography, the black hole event horizon encodes information in its area-based entropy, though the mechanism of retrieval (via Hawking radiation) remains contested. In our model, this encoding is interpreted as a boundary condition for a phase transition: infalling photon states undergo conversion to anti-causal tachyonic modes, which transport the information retrocausally toward the Big Bang. The event horizon thus serves both as an informational boundary and a conduit, enforcing global unitarity without invoking speculative late-time radiation correlations or violating the equivalence principle. This recontextualizes the black hole information paradox not as a loss, but a redirection across the causal boundary of complexified spacetime.

7. Generalized Lorenz Systems in Complex Hilbert Space

Strange attractors have long served as archetypes of deterministic chaos in dynamical systems. Among these, the Lorenz system occupies a central role, revealing deep insights into sensitivity to initial conditions, bifurcation structures, and long-term unpredictability. In this section, we explore the extension of Lorenz-like dynamics into a complex Hilbert space, where time-symmetric causal duality and tachyonic bifurcation are encoded in the structure of the attractor landscape.

7.1. Forward and Backward Time Attractors

The classical Lorenz equations describe a simplified model of atmospheric convection:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z,$$

where $x, y, z \in \mathbb{R}$ are state variables and σ, ρ, β are real parameters. This system admits a strange attractor characterized by non-periodic, bounded trajectories in state space and sensitive dependence on initial conditions.

7.2. Generalization to Higher-Dimensional Lorenz Systems

Generalizations of the Lorenz system to higher-dimensional real spaces have been developed, for example:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - xy_1 - \beta z,$$
$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = xz - 2xz_1 - d_0 y_1,$$
$$\frac{\mathrm{d}z_1}{\mathrm{d}t} = 2xy_1 - 4\beta z_1,$$

where y_1 and z_1 capture nonlinear feedback from additional spatial or energetic dimensions. These higher-dimensional systems yield even richer attractor structures and bifurcation behaviors.

7.3. Complexification and Hilbert Space Embedding

To generalize Lorenz dynamics into the framework of complexified spacetime, we promote the state variables to elements in a separable complex Hilbert space $\mathcal{H}_{\mathbb{C}}$. Let the state vector $\vec{\Psi}(t) \in \mathcal{H}_{\mathbb{C}}$ be defined as:

$$\vec{\Psi}(t) = \begin{pmatrix} x(t) + ix'(t) \\ y(t) + iy'(t) \\ z(t) + iz'(t) \end{pmatrix},$$

with $x'(t), y'(t), z'(t)$ encoding imaginary-time or tachyonic contributions to the flow. The complex Lorenz dynamics are then governed by coupled nonlinear differential equations of the form:

$$\frac{\mathrm{d}\vec{\Psi}}{\mathrm{d}t} = \mathbf{F}\left(\vec{\Psi}, \bar{\vec{\Psi}}, \nabla\vec{\Psi}\right),$$

where $\bar{\vec{\Psi}}$ denotes the complex conjugate and $\mathbf{F}$ is a nonlinear vector functional encoding symmetry-breaking interactions and feedback terms from the anti-causal domain.

7.4. Strange Attractors and Dual Flow

The attractors of such a system now occupy a submanifold of the Hilbert space defined by both real and imaginary components. Forward-time trajectories are drawn toward classical Lorenz-like lobes (e.g., black hole sinks), while backward-time (anti-causal) branches emerge from the origin-like attractor (Big Bang), forming a mirrored structure. This naturally leads to the notion of a *causal bifurcation* surface, a hypersurface in $\mathcal{H}_{\mathbb{C}}$ where the real and imaginary components of the dynamical flow intersect orthogonally. Near this surface, the complexified Lorenz attractor mediates the conversion between real-time photon states and imaginary-time tachyonic states, effectively representing a dynamical phase transition across the causal boundary.

7.5. Implications for Cosmological Dynamics

This framework allows for the description of a time-symmetric universe with spatially localized sinks (black holes) and a single universal source (Big Bang). The Lorenz-like structure captures the chaotic yet bounded evolution of regions in the universe, with each attractor reflecting a quasi-stable configuration determined by local curvature and informational flow. Further work may extend this model by coupling the complex Lorenz system to Einstein field equations in a semiclassical regime, enabling simulations of phase-space geometry in the presence of spacetime curvature and entropy gradients.

8. Event Horizons as Separatrix Surfaces

In classical general relativity, the event horizon of a black hole is defined as the boundary beyond which causal signals cannot escape to future null infinity. However, when considered within the framework of complexified spacetime and dynamical systems theory, the event horizon can be reinterpreted as a *separatrix*—a critical boundary in phase space separating distinct dynamical regimes.

8.1. Forward and Backward Time Attractors

In nonlinear dynamics, a separatrix is a manifold in the system's phase space that divides trajectories exhibiting qualitatively different behavior. For example, in a double-well potential, the separatrix marks the boundary between basins of attraction for different fixed points. Similarly, in chaotic systems such as the Lorenz attractor, separatrices divide the flow between distinct lobes or attractor basins. In the context of cosmological attractors, we interpret the event horizon as a separatrix between real-time (causal) evolution and imaginary-time (anti-causal) flow. This follows naturally from our complexified dynamical framework, where tachyonic trajectories diverge from black hole interiors into the negative-time direction, sourcing the Big Bang as a global attractor.

8.2. Metric Behavior near the Separatrix

Let the complexified spacetime metric be expressed as

$$g_{\mu\nu} = g_{\mu\nu}^{(\mathrm{R})} + i g_{\mu\nu}^{(\mathrm{I})},$$

where $g_{\mu\nu}^{(\mathrm{R})}$ describes the real (causal) geometry, and $g_{\mu\nu}^{(\mathrm{I})}$ encodes imaginary-time (anti-causal) curvature effects. The condition defining the event horizon as a separatrix is that the tangent vector to a null geodesic satisfies

$$g_{\mu\nu} k^{\mu} k^{\nu} = 0 \text{ with } \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\arg\left(k^{\mu}\right)\right) \to \infty,$$

where $\arg\left(k^{\mu}\right)$ indicates the complex argument of the null vector, diverging as it transitions across the causal boundary.

8.3. Phase Transition across the Horizon

In this view, the event horizon becomes a phase transition hypersurface between:

- **Causal flow (Re[t] increasing)**: Matter and light evolve forward in real time, governed by standard general relativity and thermodynamic entropy increase.
- **Anti-causal flow (Im[t] dominant)**: Tachyons and possibly antimatter evolve "backward" in time, sourced from the black hole interior toward the Big Bang boundary.

This transition is analogous to a bifurcation in nonlinear systems, where infinitesimal perturbations near the separatrix grow exponentially, leading to divergent phase trajectories on opposite sides.

8.4. Entropic and Informational Constraints

From the perspective of holography and black hole thermodynamics, the separatrix/horizon carries encoded information about both forward-time infalling matter and backward-time tachyonic emissions. The generalized entropy functional $S\left[g_{\mu\nu}, \psi\right]$ must remain invariant under flow across the separatrix:

$$\delta S = 0 \text{ across } \Sigma_{\mathrm{EH}}.$$

This constraint enforces consistency between the information absorbed into the black hole and the information embedded in the tachyonic (imaginary) outflow. In effect, the event horizon acts as a conserving transducer: absorbing causal information and emitting its anti-causal dual, preserving unitarity and time-symmetric dynamics globally.

8.5. Cosmological Implications

If event horizons are indeed separatrix surfaces in complexified dynamical systems, this reframes our understanding of black holes and their interiors. Rather than singular endpoints of matter and information, they are local phase boundaries across which causality and entropy undergo inversion. This view also supports the idea of a single global attractor—the Big Bang—which lies at the origin of all backward-propagating (imaginary-time) trajectories. All black holes thus become local sinks for forward-time matter and sources for backward-time tachyons, dynamically linked via separatrix structures embedded in the complexified spacetime manifold.

9. Complex Field Dynamics and the Generalized Action Principle

To unify the causal and anti-causal dynamics in our cosmological framework, we construct a Lagrangian that extends Landau and Lifshitz [8] by incorporating both standard matter fields and their tachyonic counterparts in a complexified spacetime. This formulation is designed to preserve global information conservation, encode the transition between causal and anti-causal domains, and allow coupling to curvature.

9.1. Complex Scalar Field Lagrangian

We begin with a complex scalar field $\psi\left(x^{\mu}\right)\in\mathbb{C}$, where the field contains both a causal (real time) and anti-causal (imaginary time) component. The tachyonic field ψ_T is defined as the time-reversed dual of ψ:

$$\psi_T\left(x\right)=\psi^T\left(x\right)=\psi\left(x^0\to -x^0\right).$$

The free-field Lagrangian in flat complexified spacetime becomes:

$$\mathcal{L}_{\text{free}}=\partial_{\mu}\psi^*\partial^{\mu}\psi-m^2\psi^*\psi,$$

$$\mathcal{L}_{\text{tachyon}}=\partial_{\mu}\psi_T^*\partial^{\mu}\psi_T+m_T^2\psi_T^*\psi_T,$$

where the tachyon mass term appears with the opposite sign. This ensures that $m_T^2<0$, consistent with faster-than-light propagation and imaginary mass.

9.2. Complex Time Covariant Derivative

To generalize this to a curved spacetime with complexified time, we define the derivative operator as:

$$D_{\mu}=\frac{\partial}{\partial x^{\mu}}+i\delta_0^{\mu}\frac{\partial}{\partial\tau},$$

where τ is the imaginary time coordinate. This leads to a kinetic term involving the full complexified manifold:

$$\mathcal{L}_{\text{kinetic}}=g^{\mu\nu}D_{\mu}\psi^*D_{\nu}\psi.$$

9.3. Full Lagrangian with Coupling to Gravity

The total Lagrangian, including gravitational curvature R, standard matter ψ, and tachyonic field ψ_T, takes the form:

$$\mathcal{L} = \frac{1}{2\kappa} R - \partial_\mu \psi^* \partial^\mu \psi + m^2 \psi^* \psi + \partial_\mu \psi_T^* \partial^\mu \psi_T + \left|m_T\right|^2 \psi_T^* \psi_T + \lambda\left(\psi^* \psi_T + \psi_T^* \psi\right),$$

where $\kappa = \frac{8\pi G}{c^4}$, and λ is a coupling constant mediating the photon-tachyon phase transition. The cross terms act as a symmetry-breaking interaction facilitating transitions across the causal/anti-causal boundary.

9.4. Field Equations via Variation

Varying the action $S = \int \sqrt{-g}\,\mathcal{L}\,\mathrm{d}^4 x$ with respect to ψ^* and ψ_T^*, we obtain the coupled Klein-Gordon-type equations:

$$\Box \psi + m^2 \psi + \lambda \psi_T = 0,$$

$$\Box \psi_T - \left|m_T\right|^2 \psi_T + \lambda \psi = 0.$$

The system is symmetric under time reversal and supports oscillatory solutions for particular values of λ, potentially modeling the vacuum fluctuations observed near event horizons.

9.5. Stress-Energy Tensor and Modified Einstein Equations

The total stress-energy tensor includes both causal and anti-causal contributions:

$$T_{\mu\nu}^{\text{total}} = T_{\mu\nu}^{\psi} + T_{\mu\nu}^{\psi_T},$$

$$T_{\mu\nu}^{\psi} = \partial_\mu \psi^* \partial_\nu \psi + \partial_\nu \psi^* \partial_\mu \psi - g_{\mu\nu} \mathcal{L}_\psi,$$

$$T_{\mu\nu}^{\psi_T} = \partial_\mu \psi_T^* \partial_\nu \psi_T + \partial_\nu \psi_T^* \partial_\mu \psi_T - g_{\mu\nu} \mathcal{L}_{\psi_T}.$$

The Einstein field equations are accordingly modified:

$$\Delta G_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu}^{\psi} + T_{\mu\nu}^{\psi_T}\right),$$

where $\Delta G_{\mu\nu}$ includes corrections due to complexified curvature and anti-causal flow. The presence of imaginary mass terms and non-Hermitian interactions may at first seem to violate unitarity. However, when interpreted in the full complex Hilbert space with appropriate analytic continuation, the total action remains real, and the Hamiltonian generates a unitary evolution in the combined photon-tachyon system. This provides a promising avenue for modeling horizon entropy, vacuum transitions, and the preservation of information across event horizons.

10. Stability, Bifurcation, and Symmetry Breaking

In the framework of Complex Hilbert Space Cosmology, the emergence of structured spacetime regions, such as black holes and the Big Bang, can be viewed as bifurcations of the dynamical geometry itself. These bifurcations are governed by stability transitions in the attractor landscape of the system, leading to local and global symmetry breaking. In this section, we explore how these phenomena arise naturally from the coupled dynamics of causal and anti-causal fields in the complexified manifold.

10.1. Linear Stability and Complex Eigenmodes

To understand the local behavior near fixed points (e.g., the Big Bang or a black hole horizon), we linearize the phase flow equations:

$$\frac{\mathrm{d}\vec{X}}{\mathrm{d}t} = \vec{F}\left(\vec{X}\right),$$

around a critical point $\vec{X}_0$. In a complex Hilbert space, the Jacobian matrix $J = \partial\vec{F}/\partial\vec{X}$ has complex eigenvalues:

$$J\vec{v} = \lambda\vec{v},\ \ \lambda \in \mathbb{C}.$$

The real part of λ governs local stability (growth or decay), while the imaginary part governs oscillatory or anti-causal behavior (e.g., backward time evolution). Tachyonic modes correspond to eigenvalues with negative real components and large imaginary parts, driving rapid phase transitions across temporal boundaries.

10.2. Bifurcation of Attractors

In classical dynamical systems, bifurcations occur when a system parameter is tuned through a critical value, resulting in a qualitative change in long-term behavior [9]. In our cosmological model, bifurcations emerge when the curvature or entropy density crosses a critical threshold:

$$\left.\frac{\mathrm{d}^2\mathcal{S}}{\mathrm{d}t^2}\right|_{\text{crit}} = 0 \Rightarrow \text{onset of tachyonic bifurcation.}$$

This triggers a spontaneous "branching" of solution space into new attractor basins—e.g., a collapsing star transitions into a Kerr black hole, which simultaneously seeds a backward-time flow of tachyons toward the Big Bang.

10.3. Symmetry Breaking and Temporal Asymmetry

A core insight of this framework is that temporal symmetry [10] is spontaneously broken by the global structure of spacetime itself. Though the underlying equations are time-symmetric, boundary conditions at singular attractors (Big Bang and black holes) break this symmetry:

- Forward-time evolution is dominated by photon emission, expansion, and decoherence.
- Backward-time evolution is dominated by tachyon flow, entropy inversion, and convergence.

We can formalize this using a double-well potential in complex time:

$$V\left(\tau\right) = -\alpha\tau^2 + \beta\tau^4,$$

where the system initially rests at $\tau = 0$ (time-symmetric state), but bifurcates into one of the minima at $\tau = \pm\tau_0$, corresponding to forward or backward causal domains. This mirrors the Higgs mechanism in field theory, but applied to temporal geometry.

10.4. Application to Cosmic Phase Transitions

This symmetry-breaking mechanism naturally extends to inflationary dynamics, black hole entropy, and the emergence of matter-antimatter asymmetry. For example:

1) The inflationary epoch corresponds to a bifurcation from the imaginary-time-dominated vacuum (tachyonic) to a photon-dominated real-time expansion.

2) Black holes form when matter locally exceeds a gravitational entropy threshold, leading to re-entry into the anti-causal flow.

3) Entropy balance across these bifurcations ensures conservation of information across time-reversed trajectories.

10.5. Stability of the Global Attractor Network

The global attractor network is dynamically stabilized by the entropic flow across spacetime. It consists of the Big Bang as a single white-hole attractor in negative time, and a distribution of black hole attractors in positive time. Perturbations decay or converge toward these attractors, depending on the direction of time and the causal domain. This global structure ensures:

- The conservation of total mass-energy in both causal and anti-causal sectors.
- The matching of boundary conditions at spacetime infinity and at singularities.
- The emergence of a cosmic time arrow without fundamental asymmetry in the underlying equations.

10.6. Illustrative Example: Entropy-Curvature Bifurcation Diagram

To clarify the role of stability and bifurcation in complex Hilbert space cosmology, we construct a simplified dynamical model in which spacetime undergoes a phase transition governed by the interplay between curvature $\mathcal{R}$ and local entropy density $\mathcal{S}$.

Assume the system is described by an effective potential of the form:

$$V(\mathcal{R}, \mathcal{S}) = a\mathcal{R}^2 - b\mathcal{S}\mathcal{R} + c\mathcal{S}^2,$$

where $a, b, c > 0$ are constants that encode the coupling strength between entropy and curvature. The critical points of this potential correspond to the stable attractor states of the universe. Setting $\partial V / \partial \mathcal{R} = 0$, we obtain the critical curvature at which the bifurcation occurs:

$$\mathcal{R}_c = \frac{b}{2a}\mathcal{S}.$$

Substituting back, we derive the effective potential along the critical path:

$$V_{\text{crit}}(\mathcal{S}) = -\frac{b^2}{4a}\mathcal{S}^2 + c\mathcal{S}^2 = \left(c - \frac{b^2}{4a}\right)\mathcal{S}^2.$$

Interpretation:

- If $c > \frac{b^2}{4a}$, the potential is positive definite and the system remains in a stable, symmetric configuration—no tachyonic bifurcation occurs.
- If $c < \frac{b^2}{4a}$, the potential becomes negative, and the system bifurcates into two asymmetric states—one corresponding to forward-time causality (photonic), the other to backward-time anti-causality (tachyonic).

Diagram:

- The x-axis as entropy density $\mathcal{S}$.
- The y-axis as curvature $\mathcal{R}$.
- A bifurcation line $\mathcal{R} = \mathcal{R}_c(\mathcal{S})$ separating symmetric (single attractor) from broken-symmetry (dual-attractor) regimes.

The toy model shown in **Figure 2** captures the qualitative behavior expected near singularities and during early-universe inflation: when entropy density crosses a critical threshold relative to curvature, the system undergoes a topological shift that generates a bifurcated attractor structure—one flowing forward in real time and the other backward in imaginary time. The transition underlies the arrow of time and the causal separation between observable matter and its tachyonic complement. While mathematically grounded in the relationship between

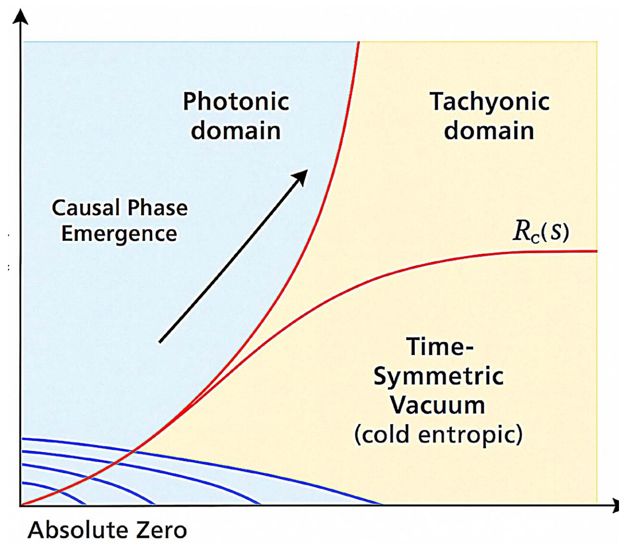

Figure 2. Entropy-Curvature Phase Diagram. This schematic illustrates phase behavior in spacetime dynamics as a function of entropy density S (horizontal axis) and spacetime curvature R (vertical axis). The bifurcation line $R = R_c(S)$ separates a time-symmetric vacuum phase (below the line) from a broken-symmetry phase (above the line), where distinct photonic and tachyonic attractors emerge. This framework draws analogy to thermodynamic phase transitions while remaining agnostic about specific boundary conditions at extreme regimes (e.g., cosmological origins).

entropy flow and spacetime curvature, deeper thermodynamic implications remain to be explored in future work.

11. Conclusions and Future Directions

In this work, we have explored a novel cosmological framework wherein the geometry of spacetime is extended into a complex Hilbert space. This complexification allows for a unified treatment of causality and anti-causality via the introduction of complex time and mass components. Within this framework, black holes are proposed to act as strange attractors that locally concentrate information, curvature, and entropy. The Big Bang, in contrast, serves as a unique global attractor in negative time that shapes the large-scale structure and temporal flow of the universe.

We have constructed this model by coupling general relativistic curvature to complex-valued fields, developing an extended Lagrangian formulation, and interpreting the resulting dynamics in terms of strange attractors and bifurcation phenomena. Notably, the appearance of tachyonic solutions is no longer an artifact but a necessary component of a fully time-symmetric cosmology. This offers a reinterpretation of dark matter and dark energy as manifestations of mass-energy propagating along the negative-time axis, preserving total information and entropic balance.

In extending this model to complex Hilbert spaces, we find that the evolution of the universe can be understood as a phase flow between attractors, with bifurcation events marking critical symmetry-breaking transitions between photonic and tachyonic domains. The analogy to generalized Lorenz systems underscores the dynamic complexity of cosmological evolution, suggesting chaotic but deterministic structures that are sensitive to initial conditions yet globally consistent.

Future Directions

Several important research avenues emerge from this work:

- **Formal Rigorous Embedding in Quantum Field Theory:** The extension of standard quantum field theory to complex Hilbert manifolds remains an open mathematical challenge. A rigorous axiomatization may yield testable constraints on tachyonic fields and entanglement geometry.
- **Numerical Simulations of Strange Attractors in Cosmological Spacetimes:** We aim to build dynamical simulations of tachyon-photon interactions in curved complex spacetimes using discretized Lorenz-like systems and field equations.
- **Experimental Predictions:** We predict subtle, time-evolving patterns in the Cosmic Microwave Background (CMB) that may correlate with the formation and growth of black holes—reflecting their role as backward-time information sources. Additionally, deviations from unitarity at black hole horizons could be probed via entanglement entropy measurements.
- **Gravitational Wave and Anti-Causal Echoes:** Future detectors could be tuned to search for exotic polarization patterns or reversed-time gravitational signa-

tures that arise from tachyonic outflows or anti-matter annihilation at causal boundaries.

- **Further Development of the "Hyperflower" Model:** The attractor structure of complex cosmological systems, from local black holes to the global Big Bang, can be visualized as an entangled multi-nodal structure. This structure resides in Hilbert space and reflects deep connections across vastly different scales. The "hyperflower" model offers a unifying topological language for space, time, and information flow.

In sum, this work initiates a new geometric paradigm for cosmology, in which causality, entropy, and curvature are unified through complex geometry and dynamical attractors. It bridges previously disjoint phenomena such as the arrow of time, dark matter, inflation, and entanglement. These elements are unified into a coherent, testable, and deeply beautiful mathematical structure. Much remains to be explored, but the conceptual tools developed here open a promising path forward in the search for a time-symmetric, information-conserving theory of the cosmos.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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