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A Dark Energy Hypothesis VIII

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Abstract

Hubble's Law leads to a cosmological second law of thermodynamics. A global entropy increase accompanies expansion. Topics explored are the second law's relationship to the cosmic microwave radiation, the early and late universes, and the interpretation of the expansion.

Keywords

Second Law of Thermodynamics, Entropy, Expansion

1. Introduction

Clausius' summary of the two cosmic laws of thermodynamics is that the energy of the universe is a constant, and the entropy of the universe tends to a maximum [1]. His universe is Newton's static universe of fixed stars. The notion of a maximum entropy implies the notorious concept of a cosmic heat death, which the discovery of the expanding universe obviates. The idea of the heat death, however, preceded Clausius.

Tolman [2] explores the relationship between entropy and expansion, beginning with the Clausius inequality.

$$\oint \frac{\mathrm{d}Q}{T} \le 0 \quad \text{or} \quad \delta S \ge \frac{\delta Q}{T}$$

The equality applies to reversible heat flow in which case T is the temperature of the system and the inequality to irreversible heat flow in which case T is that of the surroundings. In § 57 he shows how to write the inequality if change is governed by special relativity and in § 119 if general relativity governs. In § 167 the appliction of § 119 is to cosmology where a great simplification occurs:

$$\frac{\mathrm{d}\left[\sigma \delta V\right]}{\mathrm{d}t} \ge 0$$

The right-hand side means that the process is adiabatic. On the left-hand side

σ is the entropy density and δV an element of volume, so the argument is just δS, the entropy change in the volume element. He notes that since the universe at this scale of size is homogeneous, adiabaticity was to be expected. So the entropy either remains constant or increases, but since the former is for reversibility, which is an idealized limiting process, the inference is that entropy increases on the cosmological scale. He leaves the argument at this local stage, but let it be noted that by the Cosmological Principle the entropy density is the same everywhere and all volume elements of a given size are the same, so the global entropy change is $\Delta S > 0$. Hence, the DEH argument for a global entropy increase accompanying expansion is a complement of Tolman's.

An inspection of major works by Eddington [3], Lemaître [4], Schrödinger [5] and the historian J. D. North [6] shows little or no interest in the relationship of expansion and entropy. The discovery of the cosmic microwave background brings radiation entropy with it. Two contemporary writers treat the CMR as the only cosmic entropy of importance [7] [8].

2. DEH Handy Formulas

Papers DEH I and DEH II provide the basic principles of the Dark Energy Hypothesis formalism [9] [10]. They are sources for the following handy formulas that should be useful in the developments that follow.

All numerical work derives from an energy inventory for dark energy/dark matter/baryonic matter in the proportions 70/25/5 for the current epoch.

Instead of a cosmological constant Λ for dark energy, the DEH proposes a variable cosmological parameter, $\Lambda = 1/\eta^2 a^2$, where "a" is the scale factor and η is the conformal time: $a\mathrm{d}\eta = c\mathrm{d}t$.

The conformal time gives parametrically the time and scale factor at any epoch:

$$ct = \frac{\Gamma}{6}\left[\sinh(\eta) - \eta\right]$$

$$a = \frac{\Gamma}{6}\left[\cosh(\eta) - 1\right] = \lambda \eta^2 \Gamma$$

$$\lambda = \frac{\cosh(\eta) - 1}{6\eta^2}$$

In the DEH formalism, total energy is conserved, which is expressed in units of length by the parameter Γ. To convert into an energy, divide by the Einstein gravitational constant, κ:

$$\kappa = \frac{8\pi G}{c^4} = 2.076 \times 10^{-43}\ \mathrm{m} \cdot \mathrm{J}^{-1}$$

$\Gamma = 6.306 \times 10^{24}$ m. λ is the dimensionless dark energy parameter that depends only on the time and in itself defines a cosmological epoch: $\lambda = 7/10$ for the present epoch, from which it follows that $\eta = 5.571$.

A dimensionless formula for energy conservation is

$$\lambda + \chi(dm) + \chi(b) = 1$$

where $\chi(dm) = 1/4$ for the present epoch and $\chi(b) = 1/20$ for all epochs. Since the dark energy term increases with time, the dark matter term must decrease, that is, dark energy is the sink of dark matter.

3. The Hubble Law Approach to Cosmological Entropy

Introduction. In thermodynamics, the search for a parameter of spontaneous change is for a physical quantity that changes monotonically with time. In cosmology that's easy to find: the Hubble Law scale factor, which is easy to convert into entropy. This avoids the problem posed by the Kelvin and Clausius approaches where thermodynamic space divides into system and surroundings, but globally the universe can have no surroundings. The method employed here has close similarities to that of Carathéodory, which is briefly summarized in Appendix IV.

Cosmological second law. *The scale factor increases with time*. This is the case in a DEH with its hyperbolic space and in the standard ΛCDM theory whose space is Euclidean. The subsequent development compares the two cases.

Cosmological entropy in a DEH. Dark energy is proportional to the scale factor and increases monotonically with time:

$$U_\lambda = \frac{a}{\kappa\eta^2} = \frac{\lambda\Gamma}{\kappa} \tag{1}$$

Divide by a temperature to get a quantity with the units of entropy, that behaves like entropy, and can rightly be called dark entropy.

$$S_\lambda = \frac{U_\lambda}{T} \tag{2}$$

The entropy difference between two different epochs, $\lambda_2 > \lambda_1$ is

$$\Delta S = S_{\lambda 2} - S_{\lambda 1} = \frac{U_{\lambda 2}}{T_2} - \frac{U_{\lambda 1}}{T_1} > 0 \tag{3}$$

because $U_{\lambda 2} > U_{\lambda 1}$ and $T_2 < T_1$. There cannot be equilibrium states such that $S_{\lambda 2} = S_{\lambda 1}$, meaning that entropy increases irreversibly without limit. If the temperature is that of cosmic microwave radiation, then it obeys the conservation law

$$aT = a_0 T_0 = 3.73\times10^{26}\ \text{m}\cdot\text{K}$$

The numerical value has been computed from values for the present epoch in DEH II. Given this temperature dependence, it follows that $S_\lambda \propto (a/\eta)^2 \propto (\lambda\eta)^2$ so that dark entropy depends only on the time.

$$S_\lambda = C[\lambda\eta]^2 \tag{4}$$

where

$$C = \frac{\Gamma^2}{\kappa a_0 T_0} = 5.13\times10^{65}\ \text{J}\cdot\text{K}^{-1}$$

Dark entropy is the arrow of time and could appropriately be called the *Hubble entropy*.

Equation (3) is the entropy of expansion and Eq. (4) gives the absolute entropy at any time. For example, the conformal time for this epoch in a DEH is $\eta_0 = 5.571$, so the absolute entropy is

$$S_\lambda(\eta_0) = 7.80 \times 10^{66}\ \mathrm{J \cdot K^{-1}}$$

This is absolute in the sense that $S_\lambda(0) = 0$ by L'Hopital's Rule.

Appendix III gives a numerical illustration of what may not be apparent in the above argument: the entropy increase accompanying the conversion of dark matter into dark energy is the same as the entropy of expansion, which suggests three interpretations.

1) The notion of an entropy of expansion is redundant, is just another name for the entropy of conversion of dark matter into dark energy, adds nothing to our understanding, and can be discarded.

2) Rather than just a redundancy, it shows a correlation between the disappearance of dark matter and expansion. The increase in dark energy is just the energy of expansion.

3) Rather than just a correlation, it shows causation. The disappearance of dark matter drives expansion. It is the analog of an irreversible adiabatic change of state in an ideal gas, which features an entropy increase with a volume increase accompanied by a temperature drop because the internal energy of the gas must supply the energy of expansion.

The point of view taken here is that the choice is between 2) and 3), that they are consistent with Tolman's analysis, that there are other thermodynamic changes of state that connect an entropy increase to an increase in volume, and that Hubble's Law is the natural starting point for connecting cosmology and thermodynamics. Choosing between 2) and 3) is not straightforward: correlation is a necessary but not sufficient condition for causation. What would constitute sufficiency is not obvious.

Cosmological entropy in the Benchmark Model. The Benchmark Model [11] is an idealized version of the ΛCDM cosmology in which space is identically flat with a non-zero cosmological constant. The derivation in Appendix I gives the scale factor of the Benchmark Model as

$$a = \alpha^{1/3}\left[\sinh(3\gamma t/2)\right]^{2/3}$$

where

$$\alpha = \frac{\kappa M c^2}{\Lambda} \quad \text{and} \quad \gamma = c\sqrt{\frac{\Lambda}{3}}$$

The reference gives for the present epoch, $t_0 = 13.74$ Gyr and $\Lambda = 1.12 \times 10^{-52}$ $\mathrm{m^{-2}}$ = constant, making the hyperbolic sine term of order unity and thus the scale factor is $a \sim \alpha^{1/3}$. In the spirit of the Hubble Law approach to entropy, the entropy should be

$$S = \frac{a}{\kappa T}$$

Comparing entropies for the same value of the scale factor gives

$$S(\text{Benchmark}) = \eta^2 S(\text{DEH})$$

Two entropic invariants of the cosmic microwave radiation.

1) The CMR entropy

The entropy of thermal radiation is

$$S = \frac{4\sigma V T^3}{3}$$

where σ is the Stefan-Boltzmann constant. For the CMR this becomes

$$S = \frac{4\sigma (aT)^3}{3} \tag{5}$$

But this is invariant because (aT) is invariant in an expanding universe; the increase of entropy caused by increased volume is exactly off-set by the cooling that accompanies expansion. For the present epoch, $a = 1.37 \times 10^{26}$ m (DEH II) and $T = 2.7255$ K, giving $S = 5.25 \times 10^{64}$ J·K^{-1}.

Evidently, the calculations in this section challenge the idea that the CMR is the principal source of cosmological entropy.

2) **S**, the CMR entropy per baryon [12]

The definition is

$$\mathbf{S} = \frac{S}{k_B N_b} \tag{6}$$

The numerator is the CMR entropy from 1) and N_b is the number of baryons, which is conserved in the models under consideration. Dividing by the Boltzmann constant turns the ratio into a dimensionless invariant.

Source	**S**
Barrow & Tipler	$\sim 10^9$
Benchmark model	3.8×10^9
DEH	4.9×10^{11}

Appendix II shows how to derive the numbers in the bottom two rows. The flat space of the Benchmark model contains more baryons than that of the hyperbolic DEH; hence, the denominator is smaller in the latter case.

Barrow and Tipler make this interesting statement: "The value of **S** is responsible for the gross pattern of cosmic history". The following remarks illustrate that thought and invite readers to think about it critically.

In the standard ΛCDM cosmology, the times of matter/radiation equality and of recombination are roughly the same, 0.050 Myr and 0.25 Myr, respectively ([11]: p. 157). By contrast, in a DEH there is no matter/radiation equality; matter is always dominant [13].

Imagine a flowchart of ideas in mathematical cosmology. There is a central cluster from which ideas flow; the cluster includes the Friedmann-Lemaître equation and the three geometries, $k = 0, \pm 1$. Among the deductions are the invariance of **S** and the "coincidence" described above for $k = 0$. Invariants occupy a special place in physical theory. For Barrow and Tipler, **S** seems to be a secondary center from which new deductions radiate including the coincidence. Yet **S** and the coincidence come from the original central cluster, and it is by no means clear how to deduce the coincidence from **S**. In addition, **S** is an invariant in a DEH but there is no coincidence to deduce.

4. The Early Universe in a DEH

"Early universe" here means that conformal time is less than unity, $\eta < 1$. Overview: the early universe divides into causally disconnected cells; the disconnection increases the entropy from what it would be in a causally connected space.

Let D(PH) be the distance to the particle horizon at time η, which is the time that the observer receives a light-ray from the particle horizon. At the time of reception, the distance to the particle horizon is D(PH) = ηa. If $\eta < 1$, D (PH) < a, and the space defined by the scale factor divides into causally disconnected regions. Let $a_c = D$(PH) be the edge length of a cube whose volume is $a_c^3 = \eta^3 a^3$; the number of cubes is $N = 1/\eta^3$. The entropy of one cube in a DEH is

$$S_1 = \frac{a_c}{T\kappa\eta^2} = \frac{a}{T\kappa\eta}$$

The total entropy of N cells is

$$S_\lambda = NS_1 = \frac{1}{\eta^2}\left[\frac{a}{T\kappa\eta^2}\right]$$

The quantity in square brackets is the entropy for a causally connected space, so the causal disconnection enhances the entropy by a factor of $1/\eta^2$.

Equation (4) for the absolute entropy cannot be used here because the temperature of the early universe is not that of the cosmic microwave background.

The loss of causality introduces a disorder. The derivation is for N cells of equal volume, but there is no reason to suppose that other models of disorder cannot be considered. For example, there might be a distribution of cell sizes centered around the single cell size used above. The disorder can be likened to a kind of chaos or turbulence that passes away as time elapses, in rough analogy to water gushing from the base of a high pressure pipe that is turbulent at emergence but becomes laminar with the passage of time. In a DEH, the early universe is dominated by dark matter and radiation, so if the analogy is of any value there might be a viscosity associated with those quantities.

5. The Late Universe

Here's a reminder about the definition of the decleration parameter q:

$$q = -\frac{a\ddot{a}}{(\dot{a})^2} = 1 - \frac{aa''}{(a')^2}$$

where the primes mean differentiation with respect to conformal time.

DEH. When $\lambda = 19/20$, $\eta = 6.033$, the last dark matter changes into dark energy. At this point, the two parameters decouple, there is a second order phase transition, meaning that the slope of the $S(\eta)$ curve changes discontinuously, and henceforth the entropy increases quadratically and without limit with the conformal time. The acceleration parameter q tends towards zero asymptotically, while the expansion rate that has been greater than the speed of light, $\mathrm{d}a/\mathrm{d}t > c$, approaches that limit asymptotically.

$$S = (19/20)^2 C\eta^2 \quad \& \quad q = \frac{1}{1+\cosh(\eta)} \to 0$$

Benchmark model. For large values of the time, the scale factor becomes

$$a \to \left(\frac{\alpha}{4}\right)^{1/3} \exp\left[ct\sqrt{\frac{\Lambda}{3}}\right]$$

With a constant Λ, the expansion enters a so-called de Sitter state in which the scale factor and the entropy increase exponentially with an ever increasing acceleration, while the deceleration parameter $q \to -1$.

The latter behavior is well known and the former probably hardly at all. They offer two different viewpoints about the expansion of the universe.

One view is that the cosmological constant, Λ, which is dark energy, drives the expansion, most spectacularly in the late universe. On the other hand, the cosmological parameter of a DEH falls asymptotically to zero, $\Lambda \to 0$, which makes it ineffective in the late universe. An authoritative history of twentieth century cosmology notes that more than one cosmologist speculated that the zero limit seems plausible, even "natural" ([8]: pp. 56-62).

Another view is plausible although not necessary in a DEH, viz., that the "reason" for expansion is anthropic. The large-scale structure of the universe is intrinsically unstable—there can be no stable, static state in which $\Delta S = 0$; Einstein's static, metastable universe is the premier historical example. Hence, the universe must either expand or contract, and if the latter there could be no biological evolution of observers to take note of the fact. Entropy increase accompanies the expansion but does not drive it.

6. Conclusion

Hubble's law is the link between cosmology and the second law of thermodynamics. The scale factor can be converted into an absolute entropy of expansion. The magnitude of the entropy of expansion challenges that of the cosmic microwave radiation. Causal disconnection in the early universe enhances the entropy of expansion. The cosmological parameter of the DEH vis-à-vis the cosmological constant of the ΛCDM theory challenges the prevailing view of the nature of expansion.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix I

The Scale Factor in the Benchmark Model

The Friedmann-Lemaître equation is

$$\left(\frac{\dot{a}}{a}\right)^2+\frac{kc^2}{a^2}=\frac{8\pi G\rho}{3}+\frac{\Lambda c^2}{3}$$

In the Benchmark model, $k = 0$, Λ = constant, and $\rho = M/a^3$. The first task is to collect constants into two terms, separate variables, and integrate: the result is

$$\int \mathrm{d}a\sqrt{\frac{a}{\alpha+a^3}}=\gamma t+C$$

where

$$\alpha=\frac{\kappa \mathrm{M}\mathrm{c}^2}{\Lambda} \quad \& \quad \gamma=c\sqrt{\frac{\Lambda}{3}}$$

When $t = 0$, $a = 0$, so $C = 0$. Make the substitution $a^3 = \alpha\sinh^2(\xi)$, which leads to the rather astonishing simplification

$$\int \mathrm{d}a\sqrt{\frac{a}{\alpha+a^3}}=\frac{2}{3}\int \mathrm{d}\xi$$

From here, it's just rearranging to get

$$a=\alpha^{1/3}\left[\sinh\left(3\gamma t/2\right)\right]^{2/3}$$

Appendix II

On Calculating the Invariant **S**

The calculation requires the total number of baryons. Assume that the baryons consist of 90% H and 10% He by number, which leads to an average baryon mass of $m = 2.17 \times 10^{-27}$ kg.

DEH. The total mass of baryons is

$$M_b=\frac{0.05\Gamma}{\kappa c^2}=1.69\times10^{49}\ \mathrm{kg}$$

Then $N_b = M_b/m = 7.78 \times 10^{75}$.

Benchmark model. Since space is flat, let the mass density be the critical mass density:

$$\rho=\rho_{cr}=8.70\times10^{-27}\ \mathrm{kg}\cdot\mathrm{m}^{-3}$$

A reasonable choice for the scale factor is the Hubble length $a_H = 1.26 \times 10^{26}$ m, which is about the same as the scale factor in a DEH for this epoch. Then $M_b = 2.19 \times 10^{52}$ kg and $N_b = 1.01 \times 10^{79}$.

Appendix III

Conversion of dark matter into dark energy vis-à-vis expansion

The comparison must be between two epochs, the choice here being $\lambda_1 = 3/10$, $\eta_1 = 4.166$ and $\lambda_2 = 4/10$, $\eta_2 = 4.670$. The following numbers result using the handy formulas and, if necessary DEH II.

λ	U_λ (J)	$M(dm)c^2$ (J)	T(K)	S_λ (J·K⁻¹)
3/10	9.112×10^{66}	1.974×10^{67}	11.37	8.013×10^{65}
4/10	1.215×10^{67}	1.671×10^{67}	6.786	1.790×10^{66}

Entropies found by both U/T and Equation (4) are in good agreement.

$$\Delta U_\lambda = U_{\lambda 2} - U_{\lambda 1} = 3.04 \times 10^{66}\ \text{J} \quad \text{and} \quad \Delta\left[M(dm)c^2\right] = -3.03 \times 10^{66}\ \text{J}$$

as required.

$$\Delta S = S_{\lambda 2} - S_{\lambda 1} = 9.89 \times 10^{65}\ \text{J} \cdot \text{K}^{-1}$$

which by the table must be the entropy change accompanying the conversion of dark matter into dark energy, but it is also the entropy of expansion starting with Hubble's Law. Hence, the conversion and expansion are at least correlated.

Appendix IV

Short Notes on Carathéodory's Second Law Method [14]

Heat, δQ, is not a state function, but the goal is to convert it into one. To do so, imagine a small volume δV in thermodynamic state space. If in the vicinity of δV there exist states that are inaccessible by an adiabatic transition, then there exists an integrating denominator that will convert heat into a state function. In cosmology, all neighboring states will be adiabatic, but by the Hubble version of the second law there will be inaccessible states, namely those in which the scale factor stays the same or decreases. Hence an integrating denominator exists. Carathéodory shows that this must have the units of temperature, that is, it is a thermodynamic temperature, and then shows that it is identical to the ideal gas temperature. Hence, the argument in this paper is just replacing heat by dark energy, although the latter is already a state function:

$$\delta S = \frac{\delta Q_{rev}}{T} \Rightarrow S_\lambda = \frac{U_\lambda}{T}$$