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Force Acting on the Photon. Elementary Theory and Astrophysical Implications

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Abstract

The relativistic equation of Newton's second law of motion for massive photons is expressed in terms of frequency with the aid of the optical dispersion equation in vacuum—dependence of the speed of light on frequency of radiation in free space resulting from the Proca equation for massive vector bosons of spin 1. The force causing the acceleration of a massive photon is proportional to the first order of time derivative of its frequency. It turns out the expression of the force retains its physical meaning for a massless photon as well if the force and the light velocities are colinear vectors. But in this case, the force reveals itself not through the acceleration of the photon (which is impossible), but through the change in frequency over time. The effect of the massiveness in a wide range of astrophysical scenarios is extremely weak because the detection of the rest mass of the photon lies far below the threshold of experimental and observational possibilities. Therefore, when estimating the magnitude of the force acting on the photon, we can neglect that effect and consider only massless photon for simplicity. The magnitude of a force, acting upon the photon in the visible part of spectrum in different physical and astrophysical scenarios involving gravitational shift in frequency of radiation (such as Pound-Rebka experiment, light deflection by the Sun and clusters of galaxies) and the expanding accelerating Universe, was estimated to vary between ($\sim 10^{-45}$ - 10^{-31}) N which is many orders of magnitude falls below the magnitude of the weakest force ever recorded (4.2×10^{-23} N). The effect of massiveness of the photon on the change in frequency of galaxies turned out to be extremely small and virtually undetectable ($\sim 10^{-58}$).

Keywords

Massive Photon, Optical Dispersion in Vacuum, Redshift, Blueshift, Cosmological Expansion

1. Introduction

It is widely accepted in physics that the photon is a massless particle and as such, not accelerating because it always moves at the same speed of light. In courses of general physics, the lack of mass of the photon is usually explained by the gauge invariance of Maxwell equations, the principle applicable in many modern physical theories. Despite the claim "all fundamental physical interactions must be gauge invariant" is an important heuristic principle in physics, it is not the law of nature yet. As it is stated in [1], the gauge argument, which is pretending to be a "logic of nature", must "be taken with a grain of salt". Instead of saying that the massiveness of the photon is forbidden by the principle of gauge invariance, we can reverse the formulation and say that the lack of massiveness of the photon makes the Maxwell equations gauge invariant. But if the photon is endowed with the rest (invariant) mass, Maxwell equations can be replaced by the generalized Proca equation for vector bosons with the spin equal to one [2] [3], which is not gauge invariant. The possibility of finding a generalized version of gauge invariance in this case was discussed by Arbab [4]. In his earlier work [5], he showed "that a nonzero superconductivity of vacuum leads to nonzero mass for the photon". In [4], he introduced "extended gauge transformations" involving currents and field, which "leads to a massive boson field (photon) that is equivalent to Proca field". He pointed out that "a formal paradigm to generate mass term for interacting particles is the Higgs mechanism", which can be considered as superconductivity in the vacuum where electric currents are generated in the presence of a very strong magnetic field, reaching out to $\sim 10^{16}$ T [6] [7]. In [8], the authors have concluded that gauge invariance doesn't require the bare photon mass to be zero. As stated in [5], "existence of massive photons is a consequence of breaking the Lorentz gauge".

It follows from the Proca equation that the speed of motion of the massive photon in vacuum depends on its frequency. As a result, acceleration becomes possible under specific physical circumstances within an extremely small domain of velocities with magnitudes very close to the speed of light. In this paper, we discuss one aspect of the hypothesis of the massive (heavy) photon, related to the acceleration possibility or the possibility of being acted upon by an external force in free space. The theory of photon acceleration described by Mendonca [9] (see also [10]-[12]), unlike the situation considered in this paper, assumes that "photon acceleration can only be observed in a dense medium, with a large number of particles at the incident photon wavelength scale". In his case, the acceleration of the photon is induced by the ionization front triggered by a high-energy laser pulse in a gas. The photon is assigned an "effective mass" only in the medium, but not in vacuum (that effective mass depends on the frequency of plasma oscillations (see also [13])).

However, if the photon is endowed with the rest (invariant) mass, its motion could possibly be affected by an external force in the absence of a medium. Gravity (or dark energy, which is assumed to be responsible for the accelerated expansion of the Universe) is an example of such a force: it may cause additional deflection of light additional to what is known for massless photons [14] [15] traveling near a gravitational center or change in frequency of radiation passing through a gravitational field. The free fall of the photon in gravity is discussed by Pardy [16]. But in his work, the speed of light does not depend on frequency. Here, we consider one simple consequence resulting from the possible acceleration of the photon in a free flat spacetime in which the effect of the optical dispersion in vacuum is used. It helps to estimate the force acting on the photon passing near a gravitating mass in the weak-field approximation. It is interesting to note that in their publications about the light deflection observations (Dyson, Eddington, Davidson [17]) and the gravitational redshift measurements (Pound, Rebka [18]), the authors were talking about "heavy" or "weighted" light.

2. Newton's Second Law of Motion for the Massive Photon

In special relativity, the force $\boldsymbol{F}$ acting upon a moving particle in the direction of its velocity $\boldsymbol{v}$ is given by the formula for Newton's relativistic equation of motion [19]:

$$\boldsymbol{F} = \frac{m_\gamma}{\left(1-\frac{\boldsymbol{v}^2}{c^2}\right)^{3/2}}\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{1}$$

Here time is measured in the observer's reference frame, m_γ is the "rest" (invariant) mass of the photon, c—invariant speed of light used in Lorentz transformations. If $m_\gamma > 0$ the speed of the photon depends on its frequency ν. This effect is known as the "optical dispersion in vacuum" [2] and is linked to the relativistic dispersion and the Klein-Gordon equations [15], derivable from the Proca equation. It can be expressed in terms of the group velocity for the photon's wave packet which can be written as follows [2] [13] [20]:

$$v = c\sqrt{1-\nu_0^2/\nu^2} \tag{2}$$

$$F = \frac{m_\gamma c}{\nu_0\sqrt{1-\nu_0^2/\nu^2}}\frac{\mathrm{d}\nu}{\mathrm{d}t} \tag{3}$$

For a repulsive force we can conventionally assume $F < 0$ and so is $\mathrm{d}\nu/\mathrm{d}t < 0$ (redshift). If the force is attractive, $F > 0$ then $\mathrm{d}\nu/\mathrm{d}t > 0$ (blueshift). The force is undefined if $\nu = \nu_0$.

In fact, formula (3) represents Newton's second law of motion for the massive photon. It can be rewritten in a different form, if we recall that by the definition of ν_0, shown above, $m_\gamma/\nu_0 = h/c^2$. Thus, we have

$$F = \frac{h}{c\sqrt{1-\nu_0^2/\nu^2}}\frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{2.209\times10^{-42}}{\sqrt{1-\nu_0^2/\nu^2}}\frac{\mathrm{d}\nu}{\mathrm{d}t}\ \mathrm{N\cdot s^2} \tag{4}$$

after substituting numerical values of the physical constants $h = 6.626\times10^{-34}\ \mathrm{J\cdot s}$, $c = 3.0\times10^8\ \mathrm{m/s}$. For massless photons ($\nu_0 = 0$) $F \neq 0$, (which does not seem true from Equation (1)), and is equal to

$$F_1 = 2.209 \times 10^{-42} \frac{\mathrm{d}\nu}{\mathrm{d}t} \,\mathrm{N} \cdot \mathrm{s}^2 \tag{5}$$

(the derivative $\mathrm{d}\nu/\mathrm{d}t$ has a dimension s^{-2}). The effect of the massiveness of the photon is extremely small because, as a rule, $\nu_0 \ll \nu$ and the force in (4) is $F \approx F_1 \left(1 + \nu_e^2/2\nu^2\right)$. Virtually, the last term in the approximation can be dropped and just symbol F will be used further in the text. The transverse component of the force acting on the massive photon is very weak and can be neglected compared with (5). For the massless photon it turns into zero.

If $\nu = \text{constant}$, $F = 0$, that is in the absence of an external force there is no time-dependent change in frequency of the photon. The standard interpretation of this fact is usually given in the reverse order: no force is acting upon the photon because its frequency is constant. First, in this statement cause and effect are misplaced. And secondly, the frequency of the photon, traveling, for example, in the expanding Universe, cannot be constant in the observer's reference frame.

Let's talk about (5) in more detail by going back to well-known facts in physics. In the standard Doppler effect scenario, if the relative velocity between an emitter and an observer remains constant, the difference between the observed and emitted frequencies of radiation remains constant as well. But if the relative velocity is a time dependent variable (the recession velocity of a galaxy depends on the cosmological epoch, no matter the expansion rate of the Universe is constant or time dependent), then the change in the difference of frequencies with time is equal to the change of the observed frequency with time because the emitted frequency is assumed to have a constant value. This is the point when the force, acting upon the photon, comes into play. We can say that the time dependent change in frequency of the photon is equivalent to the change in its energy and can be resulted by a fictious or real force acting on the photon. It can formally be explained in simple terms of quantum mechanics.

Let $\boldsymbol{p}$ be the vector of the linear momentum of the massive photon, and E is its energy. From standard relations $\boldsymbol{p} = \hbar \boldsymbol{k}$, $E = h\nu$ we can write an expression of the force in the form

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \hbar \frac{\mathrm{d}k}{\mathrm{d}t} \boldsymbol{e}, \tag{6}$$

where $\boldsymbol{e}$ is the unit vector in the same direction. As we stated above, the optical dispersion in vacuum can be derived from the Proca equation [2], and is usually written in terms of cyclic frequencies

$$\omega^2 = (kc)^2 + \omega_0^2, \quad \omega = 2\pi\nu, \quad \omega_0 = 2\pi\nu_0. \tag{7}$$

Substituting k from (7) into (6), we can obtain the equation (3). For massless photons $p = E/c$ and the magnitude of the force is

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{c}\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{h}{c}\frac{\mathrm{d}\nu}{\mathrm{d}t} = 2.209 \times 10^{-42} \frac{\mathrm{d}\nu}{\mathrm{d}t} \,\mathrm{N} \cdot \mathrm{s}^2 \tag{8}$$

as it is in (5). Thus, despite the velocity of the massless photon always equals the invariant velocity c and the acceleration is zero, expression (8) can be interpreted as a force acting on the photon. In the case of the massless photon the change in energy takes place without the change in the speed of motion—the situation impossible for massive particles. The acceleration of the massive photon takes place in a very short domain of velocities—between $(c-\Delta c, c]$, where Δc is the variation in the speed of electromagnetic waves. According to multiple physical experiments and astrophysical observations, $\frac{\Delta c}{c}$ varies between 10^{-21} and 10^{-4} [2] [21]-[23].

3. Time-Dependent Doppler Effect

Since we will be dealing with astronomical aspects of the Doppler effect, it makes sense (to some extent) to talk about just a relative radial speed between an emitter and an observer, ignoring the contribution of a transversal component of the velocity into the effect.

3.1. Redshift

Let V be the relative speed along the line of sight between the source and the observer, ν_e, ν are emitted and observed frequencies correspondingly. With these terms the relativistic redshift z is described by the well-known formula:

$$z=\sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}}-1, \quad \nu_e=\nu\sqrt{\frac{c+V}{c-V}}, \quad \frac{\nu_e}{\nu}=1+z \tag{9}$$

where V is assumed to be a time-dependent variable. The time derivative of the observed frequency is defined by

$$\frac{\mathrm{d}\nu}{\mathrm{d}t}=-\frac{\nu_e^2 c}{\nu(c+V)^2}\frac{\mathrm{d}V}{\mathrm{d}t}=-\nu_e(1+z)\frac{c}{(c+V)^2}\frac{\mathrm{d}V}{\mathrm{d}t} \tag{10}$$

For a non-relativistic case ($V \ll c \quad (z \ll 1)$) we have

$$\frac{\mathrm{d}\nu}{\mathrm{d}t}\approx-\frac{\nu_e}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t}\approx-\nu_e(1-z)\frac{\mathrm{d}z}{\mathrm{d}t}, \quad z=\frac{V}{c} \tag{11}$$

It is evidently that $\frac{\mathrm{d}\nu}{\mathrm{d}t}<0$ because $z>0$, and we have a decrease in frequency.

3.2. Blueshift

In this case the speed V can be reversed to - V in (9). We have

$$z=\sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}}-1, \quad \nu_e=\nu\sqrt{\frac{c-V}{c+V}} \tag{12}$$

In the non-relativistic case, we have the same expression for the time derivative of the observed frequency:

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} \approx -\frac{\nu_e^2 c}{\nu\left(c-V\right)^2}\frac{\mathrm{d}V}{\mathrm{d}t} \approx -\nu_e\left(1-z\right)\frac{\mathrm{d}z}{\mathrm{d}t}, \quad z = -\frac{V}{c} \tag{13}$$

The derivative $\frac{\mathrm{d}\nu}{\mathrm{d}t} > 0$, because $z < 0$. Thus, we have an increase in frequency.

4. Physical and Astrophysical Implications

These formulas are applicable to astrophysical scenarios in flat space approximation. In the nearby region of the Universe ($z \ll 1$), as is easy to see, $\mathrm{d}z/\mathrm{d}t \approx -H_0$, where $H_0 = 2.271\times10^{-18}\ \mathrm{s}^{-1}$ is the current average value of the Hubble constant (70 km/(s Mpc)) in standard units according to [24]-[28]. Substituting this number into (11) and (13) and using (8), we can find the magnitude of the force acting on the photon of visible radiation with $\nu_e = 5.0\times10^{14}\ \mathrm{Hz}$: $F \sim 10^{-45}\ \mathrm{N}$. We have found the same value in section 4.5, referring to a more general scenario of the expansion of the Universe. In section 4.2, we are going to consider the blueshift effect linked to the Eddington group observations of a starlight deflection near the Sun, which has become the second confirmation of Einstein's predictions in general relativity (the first one was the perihelion shift of Mercury orbit).

When we discuss a gravitational influence on the shift of spectral lines (red or blue) we have to take into account that we are dealing with two-component phenomenon: the Doppler shift caused by the relativity of motion (assuming that the relative velocity between a gravitating emitter and an observer can be constant or time-dependent) and the shift in frequency caused by gravity, which may be opposite to the Doppler shift or supplementary to it, depending on how the emitter and the observer move with respect to each other, and if the emitted radiation travels in the vicinity of another gravitating mass on its way to the observer. In all sections below we assume (for simplicity) that the transversal Doppler effect is significantly weaker compared with its radial counterpart. The radial Doppler effect is assumed to have already affected frequency of the photon approaching the gravitating mass. This approximation is enough for our purposes, but the generalized consideration of the effect is not a challenging task.

4.1. The Photon Traveling in a Gravitational Field

We consider the photon, acted upon by a gravitational force, generated by mass M with a spherical distribution of matter density in the weak-field approximation. It can be described by the Hamilton-Jacobi equation in general relativity in the form of the relativistic dispersion equation:

$$g^{\alpha\beta}P_\alpha P_\beta = m_\gamma^2 c^2 \tag{14}$$

Here $g^{\alpha\beta}$ -contravariant components of the metric tensor of the gravitational field, P_α, P_β are components of 4-vector of the momentum. In the meantime, we have the Schwarzschild metric in spherical coordinates (ct, r, θ, φ) with r as the radial distance from the center [19] [29] [30]:

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{15}$$

The contravariant components in Equation (14) in the weak-field approximation ($r_g \ll r$ with $r_g = 2GM/c^2$ being a gravitational radius) are:

$$g^{00} = \left(1 - \frac{r_g}{r}\right)^{-1} \approx 1 + \frac{r_g}{r}, \; g^{11} = -\left(1 - \frac{r_g}{r}\right), \; g^{22} = -\frac{1}{r^2}, \; g^{33} = \frac{1}{r^2 \sin^2\theta} \tag{16}$$

If we choose $\theta = \pi/2$ as the plane of motion of the photon, then we can set $P_2 = P_\theta = 0$. Magnitudes of the next components of 4-vector momentum are $P_0 = E/c$, $P_1 = P_r$, $P_3 = P_\varphi$. The last one is the generalized angular momentum of the photon which in the weak-field approximation takes the form [14] $P_\varphi = \gamma m_\gamma v r$, where γ is the Lorentz -factor: $\gamma = 1\big/\sqrt{1 - v^2/c^2}$. Substituting (16) and the components of the 4-vector momentum into (14) we obtain:

$$\left(1 + \frac{r_g}{r}\right) \frac{E^2}{c^2} - \left(1 - \frac{r_g}{r}\right) P_r^2 - \frac{P_\varphi^2}{r^2} = m_\gamma^2 c^2 \tag{17}$$

This equation can be rewritten in the following form if we take $E = h\nu$:

$$\left(1 + \frac{r_g}{r}\right) \frac{(h\nu)^2}{c^2} - m_\gamma^2 c^2 = \left(1 - \frac{r_g}{r}\right) P_r^2 + \frac{m_\gamma^2 v^2}{1 - \frac{v^2}{c^2}} \tag{18}$$

Now we can apply equation (2) and definition $\nu_0 = \frac{m_\gamma c^2}{h}$ to (18) and bring it down to the form:

$$\left(\frac{h\nu}{c}\right)^2 \frac{r_g}{r} \left(1 - \frac{r_g}{r}\right)^{-1} \approx \left(\frac{h\nu}{c}\right)^2 \frac{r_g}{r} = P_r^2 \quad \text{or} \quad P_r = \sqrt{\frac{r_g}{r}} \frac{h\nu}{c} \tag{19}$$

The force acting on the photon can be found by differentiating (19) with respect to the coordinate time:

$$F(r) = \frac{dP_r}{dt} = \sqrt{\frac{r_g}{r}} \frac{h}{c} \left(\frac{d\nu}{dt} - \frac{\nu}{2r} \frac{dr}{dt} \right) \tag{20}$$

The first term in parentheses shows the rate of gravitational shift in frequency and the second one consists of the radial derivative—the term describing the rate of change in the radial displacement of the photon while bending its trajectory. Numerical values of the derivatives in (20) depend on specific astrophysical scenarios that we are going to consider below in the text.

If we set $P_\varphi = 0$ in Equation (17) the radial component of the momentum P_r, describing a "free fall motion" of the photon, can be found from the equation:

$$P_r = \sqrt{\left(1 + \frac{2r_g}{r}\right)\left(\frac{h\nu}{c}\right)^2 - \left(1 - \frac{r_g}{r}\right) m_\gamma^2 c^2} = \frac{h\nu}{c} \sqrt{1 + \frac{2r_g}{r} - \left(1 - \frac{r_g}{r}\right)\left(\frac{\nu_0}{\nu}\right)^2} \tag{21}$$

In the case of massless photons ($\nu_0 = 0$), it boils down to the much simpler approximate equation

$$P_r = \frac{h\nu}{c}\left(1 + \frac{r_g}{r}\right), \tag{22}$$

For the gravitational field near the Earth surface ($r = 6.374\times10^6$ m) with $r_g = 9.067\times10^{-3}$ m the ratio $r_g/r = 1.422\times10^{-9}$ can be ignored compared with 1 and we simply have $P_r = h\nu/c$ —the result that could have been used straightforwardly. Taking time derivative of the momentum, we can find the force of Earth gravity (weight) acting of the photon:

$$F = 2.209\times10^{-42}\frac{\mathrm{d}\nu}{\mathrm{d}t} \tag{23}$$

This formula is the same as (5) and (8).

4.2. Light Deflection by a Gravitating Mass

Historically that was the second confirmed prediction of general relativity. The testing was performed by two English groups of astronomers during the solar eclipse on May 29 of 1919. They have observed stars from Hyades cluster in Taurus constellation. The results obtained by both groups are summarized in the article [17]. We'll try to estimate the magnitude of the force acting on the photon emitted by the star κ^1 Tauri which is classified as A7 IV spectral type in The SkyLive [31] with the effective temperature T_{eff} = 8748 ^{0}K. According to Rhee [32], spectral type of the star is A7 IV-V and T_{eff} = 9000 ^{0}K, but Kaler [33] points to T_{eff} = 8290 ^{0}K. For the effective temperature in 9000 ^{0}K, the peak frequency of radiation (if the photon is massless) for the star, according to Wien's displacement law, is

$$\nu_{\max} = 5.879\times10^{10}T_{eff}\left(\mathrm{Hz/K}\right) = 5.291\times10^{14}\ \mathrm{Hz} \tag{24}$$

This frequency is used below in the text as the emitted frequency ν_e.

For our purpose we can consider the photons traveling from infinity to the Sun and passing it by at a distance r (impact parameter). This situation corresponds to the blueshift of the emitted radiation while it is entering the solar gravitational field despite the radiation itself could have been redshifted if an emitting source is moving away from the Sun (we consider the event in the reference frame centered in the Sun). The deflection angle for massive photons was calculated by Lowenthal [14] and can be obtained straightforwardly from the deflection formula for ultra-relativistic particles [34] as it is mentioned in our work [15].

Here we consider only weak-field approximation of general relativity near the gravitational center that leads to the Newtonian regime characterized by the condition $r_g \ll r$ ($r_g = 2GM/c^2$ is the Schwarzschild (or gravitational) radius, M is the mass of a gravitating center), and the slight blueshift in the frequency of an approaching massless photon, which can be described by the formula, well-known from any course of general relativity (see for example[19]):

$$\nu = \nu_e \left(1 - \frac{\Delta\varphi}{c^2}\right), \tag{25}$$

where ν, ν_e are observed and emitted frequencies, $\Delta\varphi = \varphi = -GM/r$ is the difference between gravitational potentials at a distance r from the center and at infinity where the potential is zero.

The light deflection (and gravitational lensing) can be considered as a scattering of the photon on a gravitational potential, approaching the center of gravity from infinity and going to infinity (or to the observer) restoring its initial frequency. When the photon approaches mass *M*, the change in frequency takes place on the short arclength Δl near the turning point where deflection of the photon's straight pathway occurs (detailed description of the light ray trajectories in Schwarzchild metric is given by [29] (fig. 6.1), [35]), and the particle displaces slightly toward the gravitating mass in a radial direction by Δr. If $\theta_E = 4GM/rc^2 = 2r_g/r$ is the Einstein's deflection angle, then $\Delta l \approx r\theta/2 \approx r_g$, and the characteristic time of change in frequency ($\Delta\nu = \nu - \nu_e = -\nu_e\varphi/c^2 = \nu_e r_g/2r$) is $\Delta t \sim \Delta l/c = r_g/c$. In the weak-field approximation $r_g \ll r$ and $\Delta\nu \ll \nu_e$. For the estimate of the first derivative in (20), we have:

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} \sim \frac{\Delta\nu}{\Delta t} = \frac{\nu_e c}{2r} \tag{26}$$

For the second derivative $\mathrm{d}r/\mathrm{d}t \approx \Delta r/\Delta t$ we have, considering that the deflection angle is extremely small, $\Delta r \approx r\theta_E^2/2$ and $\Delta r/\Delta t \approx 2r_g c/r$. Thus, $(\nu/2r)(\mathrm{d}r/\mathrm{d}t) = \nu r_g c/r^2$, where the observed frequency can be written as $\nu = \nu_e(1 - r_g/2r) \approx \nu_e$. We can see that the second term in (20) is significantly small compared with $\mathrm{d}\nu/\mathrm{d}t$ and we can ignore it. It enables us to rewrite (20) in the form

$$F = \sqrt{\frac{r_g}{r}}\frac{h}{c}\frac{\nu_e c}{2r}\,\mathrm{N}\cdot\mathrm{s}^2 = 2.209\times10^{-42}\sqrt{\frac{r_g}{r}}\frac{\nu_e c}{2r} \tag{27}$$

We use (x = 0.334, y = 0.472) coordinates of κ^1 Tauri star on the photographic plate (as it is measured by the Eddington's group in conventional units, unit = 50') to estimate its angular distance from the Sun. It turned out to be equal to 28'.911, or $r = 1.841R_0$, where $R_0 = 6.95\times10^8$ m is the radius of the Sun. Substituting all these numbers with the speed of light c and (24) for ν_e into (27), we obtain for the force acting on the massless photon in the visible part of the spectrum: $F = 1.444\times10^{-31}$ N.

4.3. Gravitational Redshift/Blueshift near the Earth Surface

The first time the effect of gravity on radiation frequency was experimentally confirmed by Pound, Rebka [18]. In their experiment the distance between the source and the receiver was taken to be $l = 22.5$ m in vertical direction. The radiation of 14.4 keV energy was emitted by the iron isotope ^{57}Fe. This is equivalent to the frequency $\nu_e = 3.482\times10^{18}$ Hz. The difference in gravitational potential between the two locations is gl, where $g = 9.81\ \mathrm{m/s^2}$ is the free fall acceleration. It produces the expected relative gravitational red/blueshift equal to $\Delta\nu/\nu_e \approx 2.45\times10^{-15}$ which has been successfully measured with 10% of accuracy. Then the accuracy was improved up to 1% [36] and 0.007% [37].

Despite the effect is conventionally interpreted in terms of gravitational time dilation there is no restriction to interpret it in terms of force acting on the photon in its free fall motion near the Earth (see also [16]). The time it takes for the light to travel the distance of 22.5 m is $\Delta t = l/c = 7.5\times10^{-8}\,\text{s}$. It gives us the estimate for the time derivative of the frequency:

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} \approx \frac{\Delta\nu}{\Delta t} = 1.137\times10^{11}\,\text{s}^{-2} \tag{28}$$

Because the Earth's gravitational field is weak and homogeneous we can estimate the force acting on a massless gamma-ray photon by substituting the number in (28) into (8). We obtain $F = 2.512\times10^{-31}\,\text{N}$. This force is practically immeasurable, making 10^{-8} of the experimental threshold 4.2×10^{-23} N [38]. For the frequency of the photon in the visible part of the spectrum used above, we find $F = 3.534\times10^{-35}\,\text{N}$.

The same "weight" of the photon can be found straightforwardly, if we formally multiply its mass equivalent $m = h\nu/c^2$ by the acceleration of gravity

$$F = \frac{h\nu}{c^2} g = 7.222\times10^{-50}\nu \tag{29}$$

For the same frequency of gamma radiation we obtain $F = 2.515\times10^{-31}\,\text{N}$ and for the visible radiation with $\nu_e = 5.0\times10^{14}\,\text{Hz}$ we find $F = 3.611\times10^{-35}\,\text{N}$ as above in the text with slight difference. Comparing (5) and (26), we can write for a photon near the Earth gravity:

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} = 3.27\times10^{-8}\nu \tag{30}$$

Formula (29) can be derived from the expression in 3-form of the force acting on a particle in the homogeneous gravitational field [19] (see also discussion in [16]):

$$F = \frac{m_\gamma c^2}{\sqrt{1-\frac{v^2}{c^2}}}\left\{-\nabla \ln\left(\sqrt{1+\frac{2\varphi}{c^2}}\right) + \sqrt{1+\frac{2\varphi}{c^2}}\,\frac{\boldsymbol{v}}{c}\times\left(\nabla\times\boldsymbol{g}\right)\right\} \tag{31}$$

The last term in (31) is equivalent to the Coriolis force and can be dropped in the first approximation. In the weak field approximation $2\varphi/c^2 \ll 1$ and $\ln\sqrt{1+2\varphi/c^2} \approx \varphi/c^2 = -GM/rc^2$. From the optical dispersion in vacuum, it follows that the Lorentz factor in (31) can be written simply as ν/ν_0, and because $m_\gamma/\nu_0 = h/c^2$, (31) takes the form:

$$F = \frac{h\nu}{c^2}\frac{GM}{r^2}. \tag{32}$$

which is the law of universal gravitation applied to the interaction between a photon of mass (mass equivalent) $h\nu/c^2$ and the Earth. For the Earth surface, if r is taken equal to the radius of the planet, (32) turns into (29).

4.4. Gravitational Blueshift near Galaxy Clusters

In this section we are going to consider a gravitational blueshift of the photon's frequency in the reference frame connected with the center of inertia of a galaxy cluster. In this case, while estimating the magnitude of the force, acting on the photon approaching the cluster, we can ignore the effect of cosmological expansion in further considerations. The frequency of the photon, approaching (or entering) the galaxy cluster, is determined by the relative radial velocity between an emitting source and the cluster and can easily be calculated for specified objects. In this paper it is assumed to be equal (for demonstration purposes only) to the standard frequency in the visible part of the spectrum we used above in the text.

The gravitational lensing, produced by galaxy clusters, is accompanied by a blueshift in the frequency of the radiation emitted by the background source subject to focusing. Regarding this problem, we must recall, that the dynamics of galaxy clusters is assumed to be determined by dark matter (DM) that constitutes a very significant fraction of the mass of clusters (80% - 90%) [39], whose density profile is traditionally described by NFW model (Navarro, Frenk, White [40] [41]; see also [42] for observations and models). DM is the main cause of gravitational lensing in these systems, and, as such, is the leading contributor in the gravitational potential of the cluster and the light frequency shift compared with the baryonic matter contribution. It is important to mention that NFW model was confirmed by other studies based on gravitational lensing methodology (see, for example, [43]-[45]).

As the standard emitted frequency, we choose $\nu_e = 5.0\times10^{14}$ Hz in the visible part of the spectrum. As an example of a typical cluster of galaxies we choose Abell 370 with $z = 0.375$ that shows we are dealing with the relativistic object (its recession velocity is $V = 92432$ km/s) [46]. For our estimates instead of the radius of the cluster (which is not clearly defined parameter yet) we are going to take the virial radius $R_{vir} = r_{200} \sim 2.55\ \text{Mpc}$ [47] [48] as the characteristic size of the system. The subscript "200" points to the ratio of the average density of the cluster to the critical density of the Universe ρ_{crit} at the shown distance and the cluster redshift [40]. Although the deflection of light by a galaxy cluster is usually explained in terms of spacetime curvature we can formally interpret this phenomenon in terms of gravitational attraction of the photon by the cluster in flat spacetime in compliance with the equivalence principle.

Gravitational potential of DM distribution with NFW density profile [41]

$$\rho(r) = \frac{\rho_{crit}\delta_c}{\frac{r}{R_s}\left(1+\frac{r}{R_s}\right)^2} \tag{33}$$

at the distance r from its center can be found by solving Poisson's equation $\Delta\Phi = 4\pi\varrho$. Here δ_c is a characteristic (dimensionless) density of the cluster,

R_s is a scale radius. For an infinitely extended halo in the case of spherical symmetry, the potential is defined by the integral relation

$$\Phi(r) = -4\pi G\left[\frac{1}{r}\int_0^r r'^2\rho(r')\mathrm{d}r' + \int_r^\infty r'\rho(r')\mathrm{d}r'\right] \tag{34}$$

Substituting (33) in (34) and performing simple integration one can obtain:

$$\Phi(r) = -\frac{4\pi G\rho_0 R_s^3}{r}\ln\left(1+\frac{r}{R_s}\right), \tag{35}$$

where $\rho_0 = \rho_{crit}\delta_c$ is called a characteristic density and R_s is the scale radius. From (35) it follows that

$$\lim_{r\to\infty}\Phi(r) = 0, \quad \lim_{r\to 0}\Phi(r) = -4\pi G\rho_0 R_s^2 \tag{36}$$

The mass within the virial radius R_{vir} can easily be obtained by direct integration:

$$M(R_{vir}) = 4\pi\rho_0 R_s^3\left[\ln(1+c) - \frac{c}{1+c}\right], \tag{37}$$

where c is the concentration parameter such that $c = R_{vir}/R_s$. From (35) and (37) we can deduce

$$\Phi(r) = -\frac{GM(R_{vir})}{r}\frac{\ln\left(1+\frac{r}{R_s}\right)}{\ln(1+c) - \frac{c}{1+c}} \tag{38}$$

For many values of the concentration parameter the logarithmic term in (37) is of the order of unity, varying between 0.5 and 5.5 [49]. As we stated in section 4.1, the change in frequency caused by a gravitating mass takes place along a short arclength where the light deflection takes place. Thus, the time derivative of the frequency of the upcoming radiation is described by the formula like (26).

We use the same description of the light deflection influenced by gravity near a spherically symmetric gravitating mass as we did in section 4.2 (ignoring microlensing effects) and assume that the largest $r \approx r_E \sim R_{vir}$, where r_E—the radius of the Einstein's ring [50]. Outside a sphere of this radius light travels in a straight line in compliance with the law of inertia. In this approximation we can apply formula (27) to estimate the force acting on the photon entering the galaxy cluster from a background source. The change in frequency due to the change in the gravitational potential from zero to $\Phi(R_{vir})$ is

$$\Delta\nu = 2\nu_e\frac{\Phi(R_{Vir})}{c^2} = \frac{2\nu_e GM(R_{vir})}{R_{vir}c^2} = \frac{\nu_e}{R_{vir}}r_g \tag{39}$$

Thus, we have for the estimate of the derivative $\mathrm{d}\nu/\mathrm{d}t$ in the weak-field approximation:

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} \approx \frac{\Delta\nu}{\Delta t} = \frac{\nu_e c}{2R_{vir}}, \quad \Delta t = 2r_g/c \tag{40}$$

As we have stated above, the positive sign of the derivative shows an increase in frequency of radiation (gravitational blueshift) as the background photon approaches and enters the galaxy cluster. If we take for the mass of the galaxy cluster typical number $M(R_{vir}) \sim 10^{15} M_0 \sim 10^{45}$ kg (M_0—mass of the Sun) [40] [43] [44] [51], we obtain $\frac{\mathrm{d}\nu}{\mathrm{d}t} \approx 0.954\ \mathrm{s}^{-2}$. By substituting all above mentioned numbers into (27) we find for the magnitude of the force $F \sim 10^{-44}$ N. Of course, a real situation with gravitational lensing is more complicated and uncertainties in the parameters we just used, can change the estimate of the magnitude of the force presented above.

4.5. Force Acting on the Photon in the Expanding Universe

The redshift of a galaxy is a time dependent variable for any model of the Universe, uniformly expanding or accelerating. The accelerated expansion may be caused by some type of a repulsive force generated, for example, by dark energy in the late-time accelerating models of the Universe [52] or another nature [53]. We can state that time dependence of the redshift is caused by time dependence of the observed frequency because an emitted frequency from recessing source can be considered constant. This situation can be interpreted as conditioned by a force acting on the photon even if its rest mass is zero. The force manifests itself not through the change in speed (which is impossible), but through the time-dependent change in frequency. Our goal is to estimate the magnitude of the force.

Let $a(t)$ be the cosmological scale factor presented in the Freedman-Robertson-Walker metric the numerical value of which is linked to the model of inflationary Universe [54]-[56]. As we know, $\dot{a}(t)/a(t) = H(t)$, where $H(t)$ is the Hubble parameter, the current value of which will be taken equal to $H_0 = 70$ km/s Mpc as we stated above in the text, and $a(t_0) = 1$, t_0—present time. If ν_e and ν are emitted and observed frequencies with the redshift z, then we can write the following well known relations

$$a(t) = \frac{1}{1+z}, \quad z = \frac{\nu_e}{\nu} - 1, \tag{41}$$

from which we have

$$\dot{a} = -\frac{\mathrm{d}z/\mathrm{d}t}{(1+z)^2} = -a(t)^2 \frac{\mathrm{d}z}{\mathrm{d}t} = a(t)^2 \frac{\nu_e}{\nu^2} \frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{1}{\nu_e} \frac{\mathrm{d}\nu}{\mathrm{d}t} \tag{42}$$

Hence it follows

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{\nu^2}{\nu_e} \frac{H(t)}{a(t)} = \nu H(t) \tag{43}$$

For an arbitrary cosmological epoch with $z = z(t)$ we have the well-known expression for the Hubble parameter, derived from Friedmann equations within standard ΛCDM—model of the Universe which is more preferable model [57], [58] [59], where the contribution of radiation is ignored:

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda} \tag{44}$$

Here H_0 is the current Hubble constant, $\Omega_m, \Omega_k, \Omega_\Lambda$ are correspondingly matter, curvature and dark energy density parameters, Λ is the cosmological constant. For the flat space-time $\Omega_k = 0$ (according to [24] $\Omega_k \approx -0.002$) and (44) can be reduced to

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda} \tag{45}$$

The density parameters satisfy the condition $\Omega_m + \Omega_\Lambda = 1$. As their estimates we will use the average of measurements conducted within [24] [26] [60]): $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. For $z \ll 1$ expression (45) can be simplified:

$$H(z) \approx H_0 \left(1 + \frac{3}{2}\Omega_m z\right) \tag{46}$$

Then we have from (43) and (8)

$$\mathrm{d}\nu/\mathrm{d}t = \nu H_0 \left(1 + \frac{3}{2}\Omega_m z\right), \quad F = 2.209 \times 10^{-42} \nu H_0 \left(1 + \frac{3}{2}\Omega_m z\right) \left(\mathrm{N} \cdot \mathrm{s}^2\right) \tag{47}$$

This formula shows that the magnitude of the force acting on the photon, as was previously stated, is proportional to its frequency and increases with an increase in redshift. For Markarian galaxy Mrk 421 $z = 0.031$ [61]. The galaxy is the BL Lac type object being a very strong source of γ-rays with the frequency $\nu \sim 10^{26}$ Hz [21] [62] [63]. By substituting these numbers into (47) we obtain $F \sim 10^{-34}$ N. For visible radiation with $\nu = 5 \times 10^{14}$ Hz $F \sim 10^{-45}$ N, which is of the same order of magnitude as we found in section 4.

It follows from the above that a fictious/(or real force) acting upon the photon (massive or massless) and causing the change in its frequency always exists in the expanding Universe, no matter if the Universe is accelerating or not. The radial speed of a recessing galaxy increases with time as the galaxy moves farther away from us according to Hubble's law.

4.6. Effect of Massiveness of the Photon on the Frequency Shift

In this section we are going to consider the effect of massiveness of a traveling photon on the change of its frequency. We are going to find the difference in change of frequency shift between the massless and massive photon.

Let ν_e be the frequency of radiation emitted by a galaxy with recession velocity V and ν is the frequency recorded by an observer. If the motion is relativistic then the redshift of the galaxy is defined by the standard expression (9) shown above, if the photon is massless. The massless photon always travels with the invariant speed of light, no matter if its source is at rest or is set into motion. It can easily be seen from the relativistic composition law of velocities. We are going to refer to this law for the massive photon. Let v_γ —the velocity of the emitted massive photon in the rest frame of the source, moving with a velocity V with respect to the observer. The velocity u_γ measured by the observer can be found from the relativistic composition law:

$$u_\gamma = \frac{V + v_\gamma}{1 + \frac{V v_\gamma}{c^2}} \tag{48}$$

We can write two vacuum dispersion relations:

$$v_\gamma = c\sqrt{1 - (\nu_0/\nu_e)^2}, \quad u_\gamma = c\sqrt{1 - (\nu_0/\nu)^2} \tag{49}$$

If we substitute these expressions into (48) we will arrive at the equation:

$$\sqrt{1 - \nu_0^2/\nu^2} = \frac{\sqrt{1 - \nu_0^2/\nu_e^2} - \frac{V}{c}}{1 - \sqrt{1 - \nu_0^2/\nu_e^2}\frac{V}{c}}, \tag{50}$$

which can be simplified if we consider that in most cases of astrophysical interest the following condition holds: $\nu_0 \ll \nu_e, \nu$. Thus, we can approximate

$$\sqrt{1 - \nu_0^2/\nu^2} \approx 1 - \frac{\nu_0^2}{2\nu^2} \quad \text{and} \quad \sqrt{1 - \nu_0^2/\nu_e^2} \approx 1 - \frac{\nu_0^2}{2\nu_e^2} \tag{51}$$

and rewrite (50) in the form

$$\frac{\nu_e^2}{\nu^2} \approx \frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}\left(1 - \frac{\nu_0^2}{2\nu_e^2}\frac{V}{c - V}\right) \tag{52}$$

The first fraction on the right side in (52 is simply $(1+z)^2$, where z is the redshift of a massless photon. If we introduce a variable z' for the redshift of a massive photon, then we can rewrite (52) in the following way

$$(z'+1)^2 \approx (z+1)^2\left(1 - \frac{\nu_0^2}{2\nu_e^2}\frac{V}{c - V}\right), \tag{53}$$

from which, after taking square root of both sides, we approximately obtain:

$$\Delta z \approx z' - z \approx -(z+1)\frac{\nu_0^2}{4\nu_e^2}\frac{V}{c - V} \tag{54}$$

We can conclude that the massiveness of the photons reduces the redshift (heavy photons are slightly more reluctant in getting reddish than massless photons). This effect is extremely weak and virtually unnoticeable. If we use extreme numbers (lowest and highest frequencies) for $\nu_0 \sim 10^{-3}$ Hz, $\nu_e \sim 10^{26}$ Hz, presented in publications [21] [64] [65], and the highest recession velocity ever recorded for galaxies (galaxy JADES-GS-z14.3 [66] [67]) with the redshift of 14.32 (which makes $V = 297454.44$ km/s) then we will come to the estimate of $|\Delta z| \sim 10^{-58}$. Despite this result points to the additional slight component in the observed Doppler redshifts caused exclusively by "massiveness" of the photons it is far below experimental and observational possibilities to be measured (existing uncertainties and nominal precision in redshifts of galaxies are ~10^{-4} and ~10^{-6} respectively [68] [69]). The fact that the difference in redshifts between massive and massless photons depends on the square of the ratio of the rest and emitted frequencies keeps the number unmeasurable even for the ratio of much higher order of magnitudes. The photon acceleration phenomenon works like a process of energy transfer from one part of the spectrum to another [9] much like the refraction angle (coefficient of refraction and the speed) of the radiation, passing through a spectral prism, depends on its own frequency.

5. Conclusions and Discussions

In this paper, the relativistic Newton's second law of motion is applied to a massive photon. This is possible if we consider the optical dispersion in vacuum (solution resulting from the Proca equation for vector bosons of spin 1), when the speed of the photon varies with its frequency. The force, responsible for the acceleration of the massive photon, is proportional to the first order of time derivative of its frequency. Thus, any change in frequency with time for the photon traveling in space can be interpreted as resulting from the action of a hypothetical (or real) force. Even if gravity is a manifestation of spacetime curvature, the change in frequency of the photon affected by the curvature can formally be interpreted in terms of a gravitational force acting upon the particle.

It turns out that if the rest mass of the photon is set to zero, Newton's second law of motion still makes sense for the massless photon if the force (or its component) is acting in the direction of motion of the particle. Because the effect of massiveness of the photon is extremely small, it can be ignored while estimating the magnitude of the force acting upon it. This approach was applied to different astrophysical scenarios where the change in frequency with time is a measurable effect. These scenarios include redshift/blueshift of electromagnetic radiation near the Earth, deflection of light by the Sun and galaxy clusters, and expansion of the Universe (uniform or accelerated). In all scenarios discussed in the text, the magnitude of the force varies between (10^{-45} - 10^{-31}) N. The force acting on the photon at the lower limit could result from the dark energy in the late-time accelerating models of the Universe. But in any scenario described above, the magnitude of the force falls much below the experimentally achieved record: ~10^{-23} N. The massiveness of the photon makes it more reluctant to get redder in the expanding Universe. At present, it seems to be impossible to give a more specific insight into the topic discussed above.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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