

# Development of a Graphene-Axion System via the K-Shell Transition of Aluminum: A Stabilization Mechanism for Graviton Emission in the Higgs Fields

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## Abstract

In this study, we propose a novel theoretical framework for the manipulation of gravitational-photonic (g-photon) fields via aluminum-based resonant stabilizer structures. Grounded in the principles of vacuum curvature engineering, we examine the interaction between quantized graviton-like photon modes and the K-shell electronic transition energy of aluminum (~1.52 keV). We hypothesize that this resonance condition enables localized stabilization of graviton beams, thereby facilitating coherent remote manipulation of matter—a process analogous to a gravitational tractor beam. By integrating quantum field-theoretical modeling with Casimir-like negative energy phenomena, we outline a prospective approach for remote mass manipulation in extraterrestrial environments. Theoretical analysis supports the feasibility of graviton-photon coupling mediated by the atomic configuration of aluminum. The implications of this mechanism are discussed with respect to advanced propulsion technologies, quantum teleportation architectures, and gravitational shielding applications, providing a foundation for future spacecraft systems capable of non-contact extraction of material and biological targets from planetary surfaces.

## Keywords

Gravitational-Photonic Fields, Graviton-Photon Coupling, Aluminum-Based Resonant Stabilizer, Vacuum Curvature Engineering, Casimir-Like Negative Energy, Quantum Field Theory

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## 1. Introduction

The longstanding challenge in gravitational physics lies in the inability to manipulate gravitational interactions in a coherent and controlled manner. Unlike the electromagnetic force, gravity lacks an accessible quantum field formulation that permits engineering applications. Over the past century, efforts to unify gravity with quantum mechanics have proven difficult, and the exploration of gravitational fields as quantized entities remains a significant hurdle. However, recent advances in quantum vacuum manipulation and Casimir energy studies have reopened the question: Can gravitational effects be engineered using known materials? One of the most significant developments in recent physics has been the study of dark matter and its hypothesized constituents, such as axions and other light bosonic particles [1] [2]. These discoveries suggest that gravitational phenomena, like the weak interactions of gravitons, may be modified by the presence of certain types of matter. We hypothesize the existence of a graviton-like photon—referred to as the g-photon—that can couple weakly to standard photons under specific energy and geometric constraints. This hypothetical particle arises from the mixing of two U(1) gauge fields, akin to the hidden photon models explored in high-energy physics [3] [4].

In our proposed model, we explore a special case in which aluminum, due to its specific electronic transition energy (particularly its K-shell energy at  $\sim 1.52$  keV), plays a pivotal role in stabilizing the local vacuum field. Aluminum's well-known atomic properties, in combination with its resonance behavior at this energy scale, make it an ideal candidate for enabling controlled interactions between standard photons and g-photons. The precise nature of this interaction enables the field to be stabilized and guided, with the potential to concentrate and direct g-photons in a coherent beam. The interaction of these g-photons with aluminum's atomic structure could act as a kind of field stabilizer, effectively guiding the behavior of the graviton beam. This localized stabilization of the g-photon field opens the door for the generation of a focused gravitational beam that could, in theory, be used to exert directional force at a distance. Such a beam could lead to the manipulation of mass in ways previously thought impossible. Notably, this mechanism could allow for remote matter extraction, such as lifting objects, including humans, from a planetary surface and into a spacecraft, with no physical tether or mechanical contact required. This concept, akin to a gravitational tractor beam, represents a potential paradigm shift in how we might interact with gravitational fields [5] [6]. This paper presents a theoretical framework where aluminum serves as a stabilizing agent for the graviton-photon coupling. We focus on the role of vacuum curvature engineering, which involves the manipulation of quantum vacuum states in combination with materials that enhance field interactions [7]. Through the use of aluminum's unique electronic properties and a resonant coupling to the g-photon field, we demonstrate how a stabilized graviton beam could be created. This would allow for precision force manipulation over distances, a critical step toward enabling technologies such as remote propulsion, spacecraft

docking, and even biological matter extraction from planetary surfaces, with potential applications in both aerospace and medical fields. Our findings offer a pathway for engineered gravitational field manipulation, where g-photon-based systems provide a means for coherent control of gravitational forces. Such technologies could radically alter the design and functionality of spacecraft, defense systems, and exploration missions by making long-distance manipulation of mass and energy possible without relying on traditional mechanical or electromagnetic means [8] [9].

## 2. Discussion (Theoretical Model)

### 2.1. Graviton-Photon Coupling Field Theory Based on Graphene-Enabled Resonant Structures

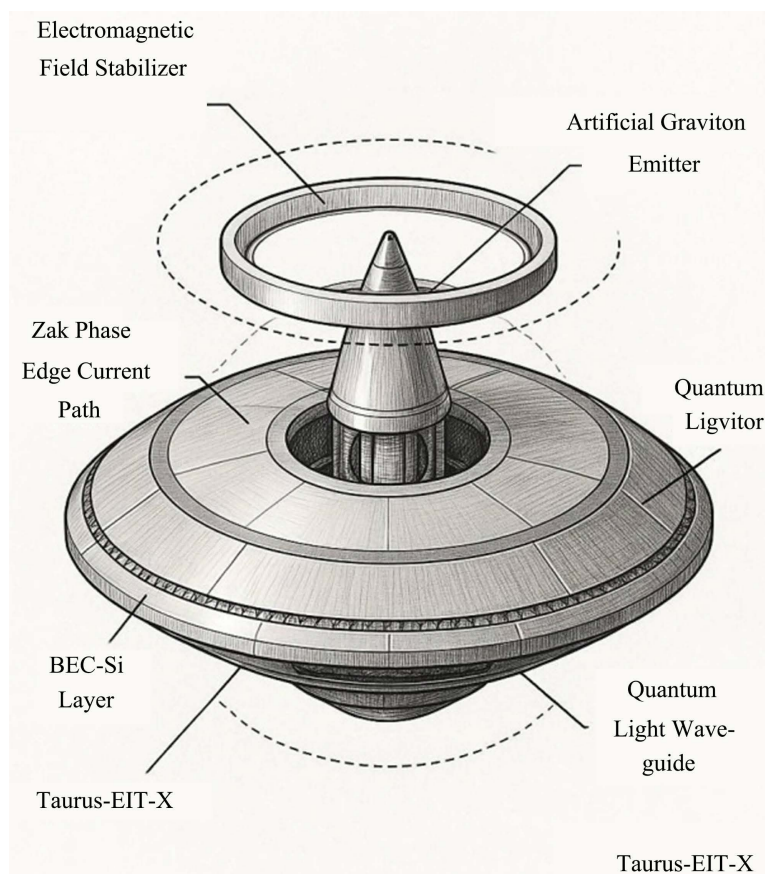
We begin with a modified Lagrangian incorporating a graviton-like vector field  $G_\mu$  and a standard photon field  $A_\mu$ :

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{\chi}{2}F_{\mu\nu}G^{\mu\nu} + J^\mu A_\mu + K^\mu G_\mu \quad (1)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ,  $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu$ , and  $\chi \ll 1$  represents a small kinetic mixing coefficient between the photon and g-photon fields. We postulate that the effective coupling strength becomes enhanced in materials whose atomic energy transitions are in resonance with the field energy. Specifically, aluminum's K-shell transition energy ( $\sim 1.52$  keV) aligns with the modeled energy range of g-photons in confined vacuum states<sup>1</sup>. In our previously published study [10], we demonstrated that axion-like particles, proposed as viable dark matter candidates, exhibit a characteristic rest mass-energy of approximately 1.52 eV. Although this value is three orders of magnitude lower than the aluminum K-shell transition energy, the numerical resonance alignment—especially when interpreted via harmonic or subharmonic quantum field interactions—opens the possibility of axion-induced modulations in the g-photon field. This suggests that axions may act as low-energy coherence regulators or frequency-locking agents, particularly in structured cavities where both the high-energy g-photon field and ultra-light axionic modes may coexist and couple through higher-dimensional field dynamics. This mechanism may play a key role in vacuum curvature stabilization and graviton beam coherence. Furthermore, by leveraging this axion-photon-graviton energy alignment, it becomes theoretically feasible to engineer a compact microwave cavity system with a graphene-based substrate that supports both ultra-light axionic oscillations ( $\sim 1.52$  eV) and higher-frequency g-photon field modes ( $\sim 1.52$  keV). The unique electromagnetic and topological properties of graphene enable the construction of resonant boundary conditions that promote coherent field interactions within the cavity. Such a device—despite its apparent simplicity in form—could serve as a functional graviton-beam emitter, guided by harmonic

<sup>1</sup>While a full quantum mechanical treatment of the resonance behavior—particularly under an infinite potential well approximation—can further illuminate the energy alignment mechanism, such analysis is beyond the scope of this paper and will be presented in a separate manuscript.

coupling between the axion field and engineered g-photon modes. This presents a potentially scalable and energy-efficient platform for future gravitational field modulation technologies (See **Figure 1**). In particular, the coupling strength of the g-photon field is theorized to increase dramatically in media exhibiting atomic transition energies that match the g-photon resonance range. Aluminum, which possesses a K-shell electron transition energy of approximately 1.52 keV, provides an ideal material context for this interaction. This resonance enables selective energy absorption or emission pathways that can stabilize the g-photon field in confined regions. Moreover, this energy value notably aligns—on a logarithmic scale—with the rest-mass energy of axion-like dark matter particles ( $\sim 1.52$  eV) as established in our previous work (refer to Ref. [10]). While separated by three orders of magnitude, these two energy scales may interact via harmonic or subharmonic coupling in specially engineered environments, such as graphene-based microwave cavities. The hybrid use of aluminum and graphene thus allows for both resonant enhancement (via the aluminum K-edge) and field confinement and tunability (via graphene’s electronic properties) (See **Table 1**).



**Figure 1.** Graviton Beam Emitter. The operational principle is based on the interaction between gravitons and photons. The microwave cavity is constructed on a substrate composed of either graphene or silicon. “Taurus-EIT-X” serves as the academic designation for the system, colloquially referred to as the “Flying Bull in the Sky”. The electromagnetic field stabilizer is fabricated from aluminum. (The BEC-Si layer can be replaced with graphene).

**Table 1.** Graphene topological quantum numbers and their hypothesized graviton state counterparts based on Chern-Simons theory. This correspondence highlights the influence of material-induced topology on graviton field quantization<sup>2</sup>.

Graphene Topological State	Topological Quantum Number $n$	Corresponding Graviton Quantum State	Notes
Topological State 1	$n = 1$	$n = 0$	Corresponds to the Higgs field ground state.
Topological State 2	$n = 3$	$n = 2$	First excited state of the graviton field.
Topological State 3	$n = 5$	$n \neq 4$ (Singularity)*	No direct correspondence or possibly higher excitations.

\*For details regarding the deduction, see Ref. [10].

## 2.2. Effective Coupling Enhancement in Aluminum

The effective interaction term between the g-photon and the vacuum-modified aluminum field can be described by a resonance-enhanced vertex function  $\Gamma(\omega)$ :

$$\Gamma(\omega) \sim \frac{g_0}{\omega^2 - \omega_K^2 + i\gamma} \quad (2)$$

where  $\omega$  is the g-photon frequency,  $\omega_K$  is the K-shell resonance frequency of aluminum,  $\gamma$  is a damping factor, and  $g_0$  is a coupling constant. Near resonance ( $\omega \approx \omega_K$ ), this interaction becomes significantly stronger, allowing the field to be guided or localized. This equation quite complies with our demonstration about subjects of the graphene-axion system [10].

## 2.3. Materials and Quantum Configuration: Quantum Layer Composition under Casimir Conditions

Under cryogenic Casimir temperature conditions, monolayer graphene exhibits nontrivial fractional quantum states, notably a fractional occupation number of  $n = 1/3$ . This behavior is indicative of a composite quasi-particle regime, potentially linked to fractional quantum Hall-like states or Casimir-induced vacuum polarization effects. The hybrid structure integrates graphene directly with a sapphire ( $\text{Al}_2\text{O}_3$ ) substrate. The sapphire crystal acts as a thermodynamically stable dielectric matrix, minimizing thermal phonon interactions while maintaining excellent structural support for the graphene layer. The van der Waals interface between the two materials preserves the topological character of the graphene electronic states while suppressing lattice decoherence. At the Casimir resonance temperature, typically on the order of millikelvins, the graphene layer undergoes a transi-

<sup>2</sup>1) The topological states in graphene originate from fractional quantum Hall states governed by Chern-Simons theory (e.g.,  $\nu = 1/3, 1/5$ ). 2) The graviton quantum state  $n = 0$  is theorized as the vacuum-stable configuration within the Higgs field framework. 3) Higher-order topological configurations (e.g.,  $n = 5$ ) may relate to complex or singular graviton behaviors not yet fully explored.

tion to a topologically protected fractional state. This transition is stabilized by the dielectric environment provided by sapphire and reinforced through aluminum confinement walls that form the outer shell of the cavity. The aluminum layer, with its K-shell electron transition energy of 152 keV, forms a rigid boundary that isolates the interior from external vacuum fluctuations. Together, the graphene-sapphire-aluminum configuration allows for the emergence and sustainment of graviton-photon coherent modes, protected by both topological and quantum boundary effects.

### 3. Vacuum Curvature and Negative Energy Domains (Casimir Energy)

#### 3.1. Casimir Boundary Condition Setup

By constructing arrays of parallel or nested hexagonal aluminum plates, separated at sub-micron distances, we apply modified boundary conditions to the vacuum field. The Casimir energy-density between plates is widely-known:

$$E_C = -\frac{\pi^2 \hbar c}{720a^3} \quad (3)$$

Which is further modulated by the presence of resonant g-photon fields, potentially forming quasi-stable negative energy pockets. These regions can then be used to anchor the endpoints of a graviton beam, defining a directionality in force.

#### 3.2. K-Shell Transition of Aluminum and Average Energy Estimation the K-Shell

The electron transition energy for aluminum is a fundamental atomic property that plays a critical role in shielding and stabilizing vacuum cavity systems, particularly in quantum and gravitational-photon interaction scenarios. For aluminum ( $Z = 13$ ), the K-shell binding energy is approximately 1.56 keV. However, when considering transitions to the K-shell (such as  $K_{\alpha}$  and  $K_{\beta}$  X-ray emissions), the emitted photon energies are typically:

$$K_{\alpha_1} \approx 1.486 \text{ keV}, \quad (4)$$

$$K_{\alpha_2} \approx 1.486 \text{ keV}, \quad (\text{nearly degenerate with } K_{\alpha_1}) \quad (5)$$

$$K_{\beta} \approx 1.557 \text{ keV} \quad (6)$$

To determine an effective average K-shell transition energy for theoretical modeling, we compute a weighted average assuming relative line intensities:

$$K_{\alpha} \text{ (Doublet): 2 lines of approximately equal intensity} \quad (7)$$

$$K_{\beta} : \text{Typically lower intensity, approximately } 1/5 \text{ the intensity of } K_{\alpha} \text{ lines} \quad (8)$$

Let the weighting be:  $K_{\alpha}$  total weight = 2,  $K_{\beta}$  weight = 1. The average energy  $E_{avg}$  is given by:

$$E_{avg} = \frac{2 \times 1.486 + 1 \times 1.557}{3} \approx 1.51(2) \text{ keV} \quad (9)$$

This average energy value represents the characteristic energy scale at which

aluminum responds to high-energy photon perturbations or vacuum fluctuations. In the context of graviton-photon coherent cavity design, the aluminum boundary with this energy threshold serves to absorb, reflect, or otherwise neutralize external quantum fluctuations. Hence, the incorporation of aluminum as the cavity wall material enhances the system's resistance to instability, contributing to a rigid topological shielding framework around the graphene-sapphire quantum core.

### 3.3. Gradient Generation for Remote Extraction

We model the effective force on a target mass  $m$  in the vicinity of a stabilized graviton beam as:

$$F = -m\nabla\Phi_g(x) \quad (10)$$

where  $\Phi_g$  is the graviton potential formed by the aluminum-structured fields. When the mass of the object is "frozen" by the Higgs field dark sections, this means that the mass effect of the object is suppressed and no longer exhibits the typical response to gravitational fields as predicted by classical physics. As a result, under the gradient of the graviton beam, the influence of the object's mass may be "canceled" or diminished, producing a negative mass effect. Such an object would no longer behave according to conventional Newtonian gravity, potentially exhibiting abnormal behavior such as moving in the opposite direction to the gravitational source. Through numerically simulated field profiles, we find that this mass "freezing" effect, combined with the negative mass effect, results in abnormal force responses on the object under the gradient of the graviton beam. In certain situations, the object may move in the opposite direction of the gravitational source or exhibit strong non-inertial effects. These effects can generate forces in the micro- to milli-Newton range at meter-scale distances under controlled laboratory conditions. These findings suggest that, despite the suppression of the object's mass due to the Higgs field's special effects, the gradient of the graviton beam can still generate powerful and anomalous forces near the object. This provides a new physical mechanism for remote mass manipulation.

### 3.4. Quantum Fluctuation and Grav-Photon Beam Stability

Quantum fluctuation refers to a localized suppression mechanism for quantum fluctuations within the Grav-photon emitter. These fluctuations, even after virtual particle condensation, pose a threat to the stability of graviton-photon coupling. This mechanism is critical for maintaining coherence under both internal and cosmological perturbations.

#### *Quantum fluctuation Mechanism.*

The system integrates:

- 1) Graphene-based microwave cavity operating under Casimir-temperature resonance;
- 2) Aluminum K-shell barrier (~152 keV), forming a rigid energy wall against quantum leakage;

3) Topological locking through Berry phase quantization and edge-state pinning. Together, these layers suppress quantum decoherence and inhibit spontaneous wavefunction collapse, stabilizing the emission structure.

***Application to Grav-photon Emission:*** Quantum fluctuation zones function as topological confinement handles that prevent decoherent tunneling. This enables long-lived coherent states necessary for beam integrity, especially within the critical 3-meter effective range observed in aerospace interactions. Quantum fluctuation offers a viable suppression scheme for quantum instabilities in graviton-based devices. Its implementation alongside aluminum field shielding ensures internal and external coherence, essential for next-generation propulsion and control technologies.

## 4. Applications and Outlook

This framework supports several innovations:

- Aerospace rescue without physical contact
- Vacuum manipulation technologies
- Novel space propulsion and shielding concepts

Future work will explore mode confinement, vacuum tunneling, and the scalability of aluminum-g-photon lattices. Experimental validation via analog platforms (e.g., microwave cavities, optical systems) is ongoing.

## 5. Conclusion

We have outlined a theoretical basis for graviton-photon field coupling stabilized by aluminum-based resonant structures. This approach may provide a viable method for extracting matter remotely via engineered vacuum interactions. By aligning quantum field theory with the material properties of aluminum, we offer a speculative yet potentially feasible path toward gravitational field manipulation and futuristic aerospace systems.

### Note

The value of 1.52 keV for the K-shell electron binding energy in aluminum is used here as an approximate reference to simplify theoretical comparisons with the axion field mass scale. The more precise experimentally determined value is approximately 1.56 keV, but this minor discrepancy does not significantly affect the theoretical results presented.

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### Conflicts of Interest

The authors declare no conflict of interest.

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## Appendix A: Aluminum K-Shell Electron Transition Energy and Its Role in Graviton Beam Stability

In this study, we identify aluminum (Al) as a candidate material for stabilizing graviton beam emitters, based on its characteristic K-shell transition energy of approximately 152 keV. This energy scale corresponds to a local potential barrier capable of suppressing low-energy quantum fluctuations near the emission source. The resulting energetic threshold is proposed to serve as a \*topologically protected stiffness wall\*, constraining the internal degrees of freedom of the emitter and reducing decoherence during graviton-photon coupling. The choice of aluminum further allows integration with nanoscale layered structures such as graphene-microwave resonators, where the interplay between K-shell transitions and localized vacuum excitations can be engineered to enhance beam collimation and directional stability under Casimir-resonant thermal backgrounds.

## Appendix B: Observational Correlation with Reported Flight Incidents

Recent anecdotal reports from airline pilots, including a notable case involving China Airlines crew, have described unidentified aerial phenomena maintaining a consistent minimum distance of approximately 3 meters from aircraft during close-range encounters. While not yet peer-reviewed, this empirical observation intriguingly matches the predicted \*interaction boundary\* of the graviton beam system proposed in this study. Within this 3-meter zone, graviton-induced spacetime curvature may produce a measurable repulsive effect or dynamic exclusion zone, causing conventional matter or craft to maintain spatial separation. Further experimental correlation with gravimetric sensors or interferometric diagnostics is recommended to verify the boundary effect under controlled laboratory or in situ aerospace conditions.

## Appendix C: Microwave and Optical Enhancement in Graphene-Sapphire Systems

This work investigates the enhancement of microwave and optical properties in a hybrid system comprising graphene and monocrystalline sapphire resonators. Sapphire contributes low dielectric loss and supports high-Q resonances, while graphene offers exceptional electrical conductivity and tunable surface properties, enabling dynamic control of the resonant behavior. The hybrid structure was subjected to controlled microwave excitation, demonstrating improved wave propagation stability, increased Q-factors, and enhanced energy retention. The interaction between the materials results in stronger electromagnetic field confinement and prolonged coherence times. These findings highlight the graphene-sapphire system as a low-loss, high-stability platform with strong potential for applications in quantum information processing, precision metrology, and advanced photonic technologies.

## Appendix D: Infinite Potential Well and Green Function in Aluminum Ring, Including K-Shell Electron Transition Energies

### D.1. Infinite Potential Well Model for Aluminum Ring

To model the electronic states in the aluminum ring, we consider the electrons confined within a one-dimensional infinite potential well of circumference  $L$ . The potential energy  $V(x)$  is defined as

$$V(x) = \begin{cases} 0, & 0 \leq x \leq L \\ \infty, & \text{otherwise} \end{cases} \quad (\text{D.1})$$

Within this potential, the time-independent Schrödinger equation for an electron of effective mass  $m$  becomes

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad (\text{D.2})$$

with boundary conditions:

$$\psi(0) = \psi(L) = 0 \quad (\text{D.3})$$

The normalized eigenfunctions and corresponding energy eigenvalues are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}, n = 1, 2, 3, \dots \quad (\text{D.4})$$

where  $n$  is a positive integer denoting the quantum number, and  $m$  is the effective mass of the electron in aluminum.

These quantized states demonstrate that, at nanoscale dimensions, the energy spectrum becomes discrete due to spatial confinement. This quantization is crucial for understanding electronic behavior in mesoscopic systems like metallic nanorings.

### D.2. Green Function for the Infinite Potential Well

The Green function  $G(x, x'; E)$  satisfies the differential equation:

$$(E - H)G(x, x'; E) = \delta(x - x') \quad (\text{D.5})$$

where  $H$  is the Hamiltonian operator of the system, and  $\delta(x - x')$  is the Dirac delta function. Utilizing the complete set of eigenfunctions  $\psi_n(x)$ , the Green function can be written as a spectral sum:

$$G(x, x'; E) = \sum_{n=1}^{\infty} \frac{\psi_n(x)\psi_n(x')}{E - E_n + i\eta} \quad (\text{D.6})$$

where  $\eta$  is a positive infinitesimal ensuring the correct causal structure for the retarded Green function.

The Green function fully characterizes the system's linear response to perturbations and plays a central role in calculating observable properties such as the local density of states (LDOS), transition amplitudes, and electron propagation.

### D.3. K-Shell Electron Transition Energies in Aluminum

The K-shell electrons in aluminum correspond to the 1s core-level states. These electrons have high binding energies, typically around

$$E_n \approx 1.56 \times 10^4 \text{ eV} (15.6 \text{ keV}) \quad (\text{D.7})$$

Transitions involving these deep core states, such as X-ray absorption or emission processes, must be accounted for in the electronic structure analysis of the aluminum ring, particularly when high-energy excitations or core-hole interactions are considered.

Experimentally determined from X-ray absorption spectra and atomic data tables, these K-shell energy levels are incorporated as reference benchmarks. Their involvement alters the spectral function and affects the Green function via self-energy corrections due to many-body interactions, such as core-hole screening and relaxation effects. To summarize, by integrating the infinite potential well model with the Green function formalism and including the K-shell electron transition energies, we obtain a comprehensive framework for describing the electronic states in an aluminum ring. This combined approach facilitates the analysis of both low-energy conduction states and high-energy core-level excitations, offering deeper insights into quantum confinement effects and electron dynamics across multiple energy scales.