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A Possible Solution of the Renormalization Paradox: From the Hypothesis of "Structure-Particle" to the Higgs Boson Mass Passing through a Finite Lattice of Propagators at Non-Divergent Integration

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Abstract

We show a possible "solution" to the paradoxical aspect of the renormalization by defining the mass m as the oscillation frequency ω assigned to a particle built by a closed set of coupled field oscillators (Structure-Particle). This eliminates divergences in calculations of QED and interactions with W bosons because the mass (frequency) no longer has infinite values, the structure-particle has the finite dimension of a Compton length, and Feynman diagrams are replaced by finite lattices of non-divergent propagators in wavelengths. After, we show that the coupling of the oscillators at vertices of a W-lattice builds the geometric structure of W-boson and Higgs's boson and, in final, by the geometric relations between them, we calculate the mass of Higgs boson, so demonstrating the validity of structure Hypothesis.

Keywords

IQuO, Renormalization, Electromagnetic Mass, Compton Wavelength, Lattice, Bosons

1. Introduction

For the discovery of a self-force (F_{self}) acting on the electron in accelerated motion, see Section 2.1, physicists introduced the concept of its "electromagnetic" mass. But divergences in the calculations of the values of this mass pushed physicists to consider a "physical" mass as the sum of an inertial mass with infinite negative values and an electromagnetic mass with infinite positive values. This procedure of sum of the infinite mass values has been called "renormalization" of the mass and this occurs in both classical and quantum electrodynamics (QED), where the interaction between the particles is perturbed by further interactions and "couplings" with virtual particles present in the quantum vacuum surrounding them. The divergences in the mass are reflected in the calculation of the integrals relating to the energy and the probability amplitude of the interaction processes, which thus assume divergent results or tending to infinity. To eliminate infinite-valued integrals in interaction processes, physicists have constructed a divergences' elimination procedure based both on the renormalization of the mass and on the "***regularization***" of the integrals (e.g., by transforming them into logarithmic divergences) and then on the "transformation" of the Feynmann diagrams (while maintaining the physical equivalence) with the help of additional diagrams capable of cancelling the divergent processes. The results achieved with the renormalization procedure are quite remarkable and the values of the various physical quantities calculated with this procedure approach the experimental values with infinitesimal differences. For this reason, physicists have been concerned with making the various theories describing the fundamental interactions "renormalizable". Despite the remarkable and indisputable success of the renormalization procedure, a fundamental problem remains in all renormalizable theories, which is of a physical and epistemological nature due to the presence of infinite values of mass and electric charge (in relativistic electrodynamics) and of negative values in the inertial mass.

Note that these divergences in the masses arise from placing an electron as a sphere of finite radius and then reducing it to zero to be compatible with relativity, which wants an elementary particle to be "point-like". Since the concepts of point-like particle and "infinite" are concepts relatively intuitive that "measurable", we think the renormalization as a "epistemological" mistake on the part of physicists, that derives from the not rigorous definition of mass in physics or from not knowing to which "intrinsic" characteristic of a particle it belongs to. This, while reiterating the physical validity of the renormalization procedure, leads us to consider renormalization as a physical "***paradox***". Reflecting on this aspect of the paradox of the renormalization, despite its successes in QED, we have followed an alternative path:

- We consider the relation formulated by Bethe to calculate the energy values relative to the frequencies of the atomic Lamb shift where he "fixes" the upper limit (k) of the integral present in the calculation, to the value ($k = k_c$), thus avoiding its ultraviolet divergence.
- Considering the undulatory aspect of a particle, we associate the proper mass (m_0) with a frequency (ω_0) of an internal intrinsic oscillation.
- Starting from the Compton effect, we have revealed, on closer inspection, an unexpectedly spatial size in the electron λ_e, stopping so to be a point-like particle.

However, in order not to conflict with relativity, it must be admitted that a massive particle cannot be "composed" of sub-parts but must be always a "unique" and individual system, even if it is endowed with an internal structure, an aspect already intuited by Dirac in his famous "Foundations of Quantum Mechanics" [1]. This uniqueness, combined with the quantum aspect of a particle, admits one and only one possibility: the particle as an inseparable structure of elastic couplings of field quantum oscillators. These two new aspects, such as the finite size of a particle with internal structure and its proper frequency of oscillation, make it possible to eliminate the divergences in the calculation of the electromagnetic mass of atomic phenomena (see the Lamb Shift) and to treat the Feynman diagrams in QED and weak interactions with massive W-bosons by propagators "***lattices***" having space steps (λ_c) at finite values and without divergences in integration calculates. The ideas of structure-particles and W-propagators' lattice led us to see a W-boson as a golden geometric shape of coupled quantum oscillators, where inside there is stuck Higgs boson as oscillators' couplings at square shape. This geometric aspect allows us to calculate in theoretical way the Higgs boson mass (125.35 GeV). We can thus consider the paradox of the renormalization of masses in the interactions between elementary particles, now understood as indivisible structures of coupled quantum oscillators, resolved just by the structure Hypothesis.

1.1. The Renormalization Problem into Classical Physics

The renormalization problem originates already in classical electromagnetism. An electron, accelerated by any external force F_{est}, causes a variation of the (A_μ) Electromagnetic Field (EMF) generated by it at a points x of space. This variation of the EMF generates an induced EMF (A_{ind}) [2], which can act, through a force defined F_{self} [3], on the same source electron:

$$\vec{F}(\vec{x},t)_{self} = \frac{-e}{3\pi^2}\int_{t_0}^{t} \mathrm{d}t'\,\dot{\vec{r}}(t')\int_0^\infty \mathrm{d}k\,k^2 f_k^2 \cos\left[kc(t-t')\right] \tag{1}$$

This equation expresses a divergence, see the integration in dk for have the Fourier development representing a wave packet. The form factor f_k describes instead the source charges of the induced EMF: for a point-like electron is ($f_k = 1$), and ($f_k \neq 1$) for an electron at spherical shape with radius (a) with the following definition:

$$f_k = \frac{\sin ka}{ka} \tag{2}$$

Substituting the Equation (2) into Equation (1), it is:

$$\begin{aligned}\vec{F}(\vec{x},t)_{self} &= \frac{-e}{3\pi^2}\int_{t_0}^{t} \mathrm{d}t'\,\dot{\vec{r}}(t')\int_0^\infty \mathrm{d}k\,k^2 f_k^2 \cos\left[kc(t-t')\right] \\ &= \frac{-e}{3\pi^2}\int_{t_0}^{t} \mathrm{d}t'\,\dot{\vec{r}}(t')\int_0^\infty \mathrm{d}k\,k^2 \left(\frac{\sin ka}{ka}\right)^2 \cos\left[kc(t-t')\right] \\ &= \frac{-e}{3\pi^2 a^2}\int_{t_0}^{t} \mathrm{d}t'\,\dot{\vec{r}}(t')\int_0^\infty \mathrm{d}k\,\sin^2 ka \cos\left[kc(t-t')\right]\end{aligned} \tag{3}$$

Note that divergence does not exist in this case. This result is, however, only apparent. In fact, integrating the Equation (3) also in time, one has after some calculations:

$$\vec{F}_{self} = -\frac{e^2}{3\pi^2 a^2 c}\left(\frac{\pi}{4}\right)\left[\dot{\vec{r}}\left(t-\frac{2a}{c}\right)-\dot{\vec{r}}(t)\right] \tag{4}$$

Developing in power series of a, one obtains the following force (self-induction):

$$\vec{F}_{self} = -\frac{e^2}{3\pi^2 c^2}\left(\frac{\pi}{2a}\right)\ddot{\vec{r}}(t)+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)+\frac{O(a^3)}{a^2} \tag{5}$$

The minus sign is due since F_{self} performs negative work on the source electric charge (see also Lenz's law). Consistently with the theory of relativity where the elementary particles are point-like, it is necessary to set $a \to 0$, in this way the first term diverges and the $F_{self} \to \infty$; in fact, it is:

$$\lim_{a\to 0}\left(\vec{F}_{self}\right) = -\frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\left(\frac{\pi}{2a}\right)\ddot{\vec{r}}(t)+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t) \tag{6}$$

Recall that:

$$\int_0^{k\to\infty}\mathrm{d}k = \int_0^{k\to\infty}\mathrm{d}k\left[\lim_{a\to 0}\left(\frac{\sin ka}{ka}\right)\right] = \lim_{a\to 0}\int_0^{\infty}\frac{\sin ka}{ka}\mathrm{d}k = \lim_{a\to 0}\left(\frac{\pi}{2a}\right)\to\infty \tag{7}$$

Note the dimensional coherence between two parts of this equation, which confirms the connection between EMF auto-induced and internal structure of the electron: [$(k = 2\pi/\lambda) \Leftrightarrow (\pi/2a)$]. By Equation (7) we will have:

$$\begin{aligned}\lim_{a\to 0}\left(\vec{F}_{self}\right) &= -\frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\left(\frac{\pi}{2a}\right)\ddot{\vec{r}}(t)+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)\\ &= \left[-\frac{e^2}{3\pi^2 c^2}\int_0^{\infty}\mathrm{d}k\right]\ddot{\vec{r}}(t)+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)\end{aligned} \tag{8}$$

Recalling the fundamental equation of dynamics $F = ma$, one can note that the coefficient of first term of Equation (8) represents the mass of source charge. In literature, this mass is defined as "*Electromagnetic Mass*" (m_{em}) of an electrically charged particle (e). The Equation (8) becomes:

$$\lim_{a\to 0}\left(F_{self}\right) = \left[-m_{em}\ddot{\vec{r}}(t)+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)\right]\to\infty \tag{9}$$

where the electromagnetic mass is:

$$m_{em} = \frac{e^2}{3\pi^2 c^2}\int_0^{k\to\infty}\mathrm{d}k = \frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\int_0^{\infty}\frac{\sin ka}{ka}\mathrm{d}k = \frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\left(\frac{\pi}{2a}\right) \tag{10}$$

Note the electromagnetic mass ($m_{em} > 0$) to depend by its internal distribution of component "points". The passage to the limit [$(a \to 0) \Leftrightarrow (k \to \infty)$] makes the m_{em} electromagnetic mass tending to infinity: "*this is the question*". The equation of motion would be:

$$m_{in}\ddot{\vec{r}}(t) = F_{self}(t)+F_{est}(t) = \left\{\left[-m_{em}\ddot{\vec{r}}(t)\right]_{\to\infty}+\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)\right\}_{a\to 0}+F_{est}(t) \tag{11}$$

This equation is called "*Lorentz-Dirac equation*". Note the F_{self} is given by two terms inside the braces. If one collects the mass terms, then it is:

$$m_{in}\ddot{\vec{r}}(t)+\left[m_{em}\ddot{\vec{r}}(t)\right]_{\to\infty}=(m_{in}+m_{em})\ddot{\vec{r}}(t)=m_{fis}\ddot{\vec{r}}(t) \tag{12}$$

In literature, m_{fis} is referred to as the "*physical mass*" of the electron (the experimental one) and m_{in} the one "*inertial*"; then, it is possible to admit ($m_{fis} = m_{in} + m_{em}$). It follows from Equation (11) that:

$$m_{fis}\ddot{\vec{r}}(t)=\left\{\frac{e^2}{6\pi c^3}\dddot{\vec{r}}(t)\right\}_{a\to 0}+F_{est}(t) \tag{13}$$

To have physical meaning, the physical mass m_{fis} must have finite values, as can be observed experimentally in electromagnetic phenomena related to self-induction, but here the electromagnetic mass m_{em} has instead infinite values. To balance the infinite value of the m_{em}, in literature it has been assumed that the inertial mass m_{in} must be as "negative" and "infinite": ($m_{in} \to -\infty$). This condition is known as "***renormalization***" of the mass. The renormalization is so given by the aspect of an electron of being at spherical shape with a radius. Recall the classical electromagnetism assign to set of point-like charged particles (m_j, e_j) a potential energy $U_{em} = (1/2)[\Sigma_j(e_j V_j)]$ where V is the Coulomb potential ($V_j = e_i/a$). In the case of an electron with electric charge (e) but no point-lake with radius a, its potential energy U is $U = eV = e^2/a$. If ($a \to 0$) then ($U \to \infty$). In the reference frame at rest of the electron, we can assign an electromagnetic proper energy $U_{em} = m_{em}c^2$ but having ($U \to \infty$) it follows ($m_{em} \to \infty$). Also, here in way experimental to electron one assigns a finite mass value m_{fis}, therefore, if m_{in} is the inertial mass, then it follows ($m_{fis} = m_{in} + m_{em}$), where the inertial mass could be ($m_{in} \to -\infty$). Here we find back the renormalization procedure of the mass. Note that if the radius has a finite value, then one has $U = eV = e^2/a = m_{em}c^2$ with $a_e = e^2/m_{em}c^2$. Physicists admit that the electromagnetism has a validity limit given by electron classical radius a_e. Note that for avoid difficult ideas as infinite negative masses would be enough stopping the a_e radius to finite value which is experimentally measurable. The Equation (7) becomes us in help: note the physical dimensions of the first term of Equation (7), with moment dimension k, and the second term, with radius a (length dimension). The physical dimensional coherence implies that the radius a must have the physical dimension of wavelength λ. Recall that ($k = 2\pi/\lambda$). In next section, we will see that if $\lambda = \lambda_c$ (Compton wavelength), that is placing a "cut" in the values of λ, one solves some problems in calculating integrals. Returning to the renormalization of mass in classical electromagnetism, it is my opinion that the renormalization is a "epistemological" mistake for a physicist: summing "infinities" is not mathematically proven to give us a finite value. To date, no one has given an exhaustive physical explanation of this procedure that is not mathematically exact. However, to have a physically sensible solution to the divergence problem in interactions, the physicists resorted to a perturbative solution with approximation calculations, see the Equation (2) in perturbative terms. This path is the one also that QM takes when an infinite electromagnetic mass reappears (see the QED). The effort to fix these bizarre mathematical aspects with perturbation theory was considered legitimate also because Equation (13), which mathematically has little meaning, has instead a physical meaning. In fact, it is found, see literature [4], that by elaborating Equation (13) with a "*finite value*" of m_{fis}, one obtains that the variation of kinetic energy ΔK of the electron (the electron is accelerated by an external force) is equal to the work W of the F_{est} minus the radiated electromagnetic energy (*Bremsstrahlung* or braking radiation):

$$\Delta K_{kinetic}(e) = W(F_{est}) - \frac{e^2}{6\pi c^3}\int_{t_1}^{t_2} \ddot{r}^2(t) \tag{14}$$

$$\left(\frac{\Delta\varepsilon_{rad}(e)}{\Delta t}\right) = -\frac{e^2}{6\pi c^3}\left(\ddot{r}^2(t)\right) \tag{15}$$

In this case, we find again the work of the F_{self} in the *Bremsstrahlung* ($\Delta\varepsilon_{rad}$), Equation (14). In quantum terms, we can say that an electron "*absorbs*" virtual photons from the external accelerating potential $V(r)$ and "*emits*" real photons, because in its (S_0) it is always at rest [5]. This is why an accelerated or slowed down electron emits photons by radiation (see braking radiation). The other conceptual difficulty in the "classical" renormalization procedure (and not only classical) is that of a point-like particle. This aspect expresses an intuitive concept that does not admit observables to be measured: it follows that we could admit, in physics, a point-like particle. It would therefore be necessary to definitively resolve such a contradiction, without however resorting to various stratagems, as has been done, and still happens, in physics. We believe that *the* "*renormalization*" *procedure* (*both classical and quantum*) *of the mass reveals to us*, *instead*, *that the concept of mass is not still well defined in a physical sense*. With courage, we should instead take an alternative path free from ad hoc artifices and obvious "non-physical" aspects.

1.2. The Renormalization in Quantum Mechanics

In Quantum Mechanics, we find F_{self} in the process in which an electron emits a "virtual" photon and after reabsorbs it. In fact, the ε_{self} is constituted by the virtual cloud of photons that surrounds an electron or a cloud of pions in the case of a source nucleon. When speaking about the electron, we distinguish between the two cases of a bound electron and a free one. In the case of a bound electron, let us recall the Bohr levels of the Hydrogen atom [6]:

$$\varepsilon_n = -\frac{\alpha^2/C^2}{2n^2} mc^2 \ {}_{(MKS)}, \quad \varepsilon_n = -\frac{\alpha^2}{2n^2} mc^2 \ {}_{(CGS)} \tag{16}$$

here, m coincides with the rest mass m_0 of the electron, and it is ($\varepsilon_n < 0$) because the electron is bound to the nucleus. To calculate the energy variation $\Delta\varepsilon_n$ in n-level eigenstate in the emission and reabsorption of a virtual photon of an electron in a particular energy eigenstate (see Bohr levels) one resorts to the perturbative and therefore non-local aspect of the photon in its k values (the emitted virtual photon can have any wavelength λ). The energy variation $\Delta\varepsilon_n$ in the emission and reabsorption will be [3], pages 147 ÷ 155:

$$\Delta\varepsilon_n = \frac{4\pi}{3}\left(\frac{e^2}{m_0^2 c^2}\right)\left(\frac{\hbar c}{(2\pi)^3}\right)\left[\sum_{n'}\left|p_{nn'}\right|^2 \int_0^\infty \mathrm{d}k \frac{k}{\varepsilon_n^0 - \varepsilon_{n'}^0 - \hbar ck}\right] \tag{17}$$

where ε^0 is the bound energy but with shape of "rest" energy. ***This integral is divergent for k ultraviolets***. Another infinity is found in the emission and absorption of a virtual photon in a free electron (in accordance with Heisenberg's principle). In this case, the Equation (17), with $\left[\left(\varepsilon_n^0 - \varepsilon_{n'}^0\right) = 0, \Delta\varepsilon_n \equiv \Delta\varepsilon_p\right]$, becomes:

$$\Delta\varepsilon_p = \left\{\frac{4\pi}{3}\left(\frac{e^2}{m_{in}^2 c^2}\right)\left(\frac{\hbar c}{(2\pi)^3}\right) p^2 \int_0^\infty \mathrm{d}k \frac{k}{-\hbar ck}\right\} \to \infty \tag{18}$$

where ($m_{in} = m_0$). Too here, we have a divergent value of the energy integral. Note the presence in this equation of electromagnetic mass m_{em}, see Equation (10):

$$\begin{aligned}\Delta\varepsilon_p &= -\frac{4\pi}{3}\left(\frac{e^2}{m_{in}^2 c^2}\right)\left(\frac{1}{(2\pi)^3}\right) p^2 \int_0^\infty \mathrm{d}k \\ &= -\left(\frac{p^2}{2m_{in}^2}\right)\left(\frac{e^2}{3\pi^2 c^2}\right)\int_0^\infty \mathrm{d}k = -\left(\frac{p^2}{2m_{in}^2}\right) m_{em}\end{aligned} \tag{19}$$

The free electron so "acquires" an energy $\Delta\varepsilon$ (with $\Delta\varepsilon < 0$ (!)) due to the interaction with photons; its energy will be [$\varepsilon(e) = K_{kin} + \Delta\varepsilon < K_{kin}$] that is, it loses energy by interacting with the virtual photons that surround it. This loss of energy due to interaction with the quantum "vacuum" is observed in the Lamb shift effect with the spectral lines of the $^2P_{1/2}$ level, slightly lowered compared to the same level calculated by the Schrödinger equation applied to the Hydrogen atom. From the Equation (19), we have:

$$\varepsilon_p = \left(\frac{p^2}{2m_{in}}\right) - \left(\frac{p^2}{2m_{in}^2}\right) m_{em} \tag{20}$$

Here, it is $\varepsilon_p > 0$ and [($m_{in} = m_0 > 0$), $K_{kin} > 0$)]. Algebraically we have:

$$\varepsilon_p = \left(\frac{p^2}{2m_{in}}\right) - \left(\frac{p^2}{2m_{in}^2}\right) m_{em} = \left(\frac{p^2}{2m_{in}}\right)\left[1 - \frac{m_{em}}{m_{in}}\right] \tag{20a}$$

If now we pass to the mass renormalization because ($m_{em} \to \infty$), we must admit ($m_{in} \to -\infty$), so having [($m_{em} > 0$), ($m_{in} < 0$)]. If we want $\varepsilon_p > 0$, see the Equation (20), and ε_p to be less than the kinetic energy ($\varepsilon_p < K_{kin}$) then the term (m_{em}/m_{in}) must be positive, but this is impossible because for the renormalization of the mass, the mass m_{in} is negative; not only, but it would result also $\varepsilon_p < 0$. We are, thus, in the presence of an incongruence that derives, in our opinion, from the ambiguous operation of adding infinity and setting a mass m_{in} to negative values. Instead, if we consider the Equation (20) with a development in series up to e^2, then we have the following equation:

$$\varepsilon_p = \left(\frac{p^2}{2m_{in}}\right) - \left(\frac{p^2}{2m_{in}^2}\right) m_{em} \approx \left(\frac{p^2}{2\left(m_{in} + m_{em}\right)}\right)_{2^\circ} \tag{20b}$$

In this case, we can set a physical mass of the electron ($m_{fis} = m_{in} + m_{em}$), with $m_{fis} > 0$, given by the sum of two infinite opposites [($m_{em} \to \infty$), ($m_{in} \to -\infty$)] and to so have $\varepsilon_p > 0$. Here, we have the first example of the "***paradoxical aspect***" of the renormalization procedure. Returning to LS, since the experimental values of the spectral lines are finite then physicists were forced to look for a particular procedure that was able to deal with the divergences and that gave results with finite values. This procedure was called mass Renormalization in QM. Continuing to follow the literature in QED, if in the calculations one works with infinite values of k, then it is necessary to make the substitution $m_{in} = m_{fis} - m_{em}$, for delete the m_{em} and so to delete the divergences.

1.3. The Bethe's Solution of the Lamb Shift

One aspect of interaction between vacuum and electron is found in the ***Lamb Shift*** of the atom, that is, in a (non-free) electron bound to a nucleus. Experimentally, a perturbation in the Bohr levels or Lamb Shift (LS) is detected [7]. Recalling the perturbation calculation, see the Equation (17), one note $\Delta\varepsilon$ is divergent. The ε of levels was assumed as [3]:

$$\begin{aligned}\varepsilon_n &= \varepsilon_n^0 + \left(\Delta\varepsilon_n\right)_{LS} \\ &= \left(-\frac{m_{in}\alpha c^2}{2n^2}\right) + \frac{4\pi}{3}\left(\frac{e^2}{m_{in}^2 c^2}\right)\left(\frac{\hbar c}{(2\pi)^3}\right)\left[\sum_{n'}\left|p_{nn'}\right|^2 \int_0^\infty \mathrm{d}k \frac{k}{\varepsilon_n^0 - \varepsilon_{n'}^0 - \hbar ck}\right]\end{aligned} \tag{21}$$

Experimentally, however, the $\Delta\varepsilon$ *observed by Lamb is finite, thus contradicting the divergence of* Equation (17). This aspect forced physicists to renormalize the calculation of $(\Delta\varepsilon)_{LS}$ by the substitution $m_{in} = m_{fis} - m_{em}$ up to the second order in e^2:

$$\begin{aligned}\varepsilon_n &= \varepsilon_n^0 + \left(\Delta\varepsilon_n\right)_{LS} \\ &= \left(-\frac{m_{fis}\alpha c^2}{2n^2}\right)_{1^\circ} - \left(-\frac{m_{em}\alpha c^2}{2n^2}\right)_{1^\circ} \\ &\quad + \frac{4\pi}{3}\left(\frac{e^2}{m_{fis}{}^2 c^2}\right)_{2^\circ}\left(\frac{\hbar c}{(2\pi)^3}\right)\left[\sum_{n'}\left|p_{nn'}\right|^2 \int_0^\infty \mathrm{d}k \frac{k}{\varepsilon_n^0 - \varepsilon_{n'}^0 - \hbar ck}\right]_{2^\circ}\end{aligned} \tag{22}$$

where the subscripts (1˚, 2˚) indicate the order of approximation. After some calculations, one obtains:

$$\left(\Delta\varepsilon_n\right)_{LS} = +\frac{1}{6\pi^2}\left(\frac{e^2}{m_{fis}^2 c^2}\right)\left[\sum_{n'}\left|p_{nn'}\right|^2 \int_0^\infty \mathrm{d}k \left(\frac{\varepsilon_n^0 - \varepsilon_{n'}^0}{\varepsilon_n^0 - \varepsilon_{n'}^0 - \hbar ck}\right)\right] \tag{23}$$

By integrating one obtains a logarithmic divergence and no longer linear, on which it is possible to work to reduce it, in approximation, to finite values. However, in according to Bethe "*the value of the definite integration does not depend on the value of the upper bound*" which cannot be greater than the energy to rest; so, it is possible to cut it to a finite value $\boldsymbol{k = k_c}$, for obtain a finite spectrum of LS. By Equation (23), we obtain:

$$\left(\Delta\varepsilon_n\right)_{LS(rin)} = +\frac{1}{6\pi^2}\left(\frac{e^2}{m_{fis}^2 c^2}\right)\left\{\sum_{n'}\left|p_{nn'}\right|^2\left[\left(\varepsilon_{n'}^0-\varepsilon_n^0\right)\ln\left(\frac{\hbar c k_c}{\left(\varepsilon_{n'}^0-\varepsilon_n^0\right)}\right)\right]\right\} \tag{24}$$

here, m_{fis} can be replaced (see Bethe) by the rest mass of the electron, $m_{fis} \equiv m_0$, where m_0 is the experimental value of the electron mass. We can thus calculate the value of the energy variation caused by the LS, which is coherent to experimental values.

1.4. Equivalence between the Compton Relation and Heisenberg Uncertainty

Note that *the experimental value of the LS is obtained if* $m_{fis} \equiv m_0 = m_e$ *and* k_c *is calculated with the Compton length of the electron* λ_c, *that is* $[\lambda_e = (h/m_e c)]$. We wonder why. As we know, the emission and reabsorption of a virtual photon is possible only within a scope of validity that satisfies the Heisenberg uncertainty relation. This must be valid for both free electrons and those bound to nuclei. Therefore, it must exist a connection between the Compton relation and that of Heisenberg. Now, *we properly show the physical equivalence between the two relations of Heisenberg and Compton*. Recall if Δt is the duration of the virtual process of emission and absorption of a photon with energy $\Delta\varepsilon$, then we have $[\Delta\varepsilon\,\Delta t \geq h]$, to minim of uncertainty → $[\Delta\varepsilon\Delta t = h]$. If we consider this process in reverse, but maintaining the same energy values, (absorption → emission) and suppose that the absorption and emission occur on the same line, we obtain a Compton process of the collision between a photon and an electron, in which the variation of the wavelength of the photon is equal in "length" to the "*Compton wavelength*" λ_e of the electron: $\Delta\lambda_\gamma = \lambda_e(1 - \cos\theta)\pi = 2\lambda_e$. ***We demonstrate that the*** (λ_e) ***coincides with the linear size*** Δx_e ***of an electron*** $(\lambda_e = \Delta x_e)$. The linear size Δx_e could tell us that *the photon* γ_1 "*penetrates*" *electron* (*absorption*), *stays* "*inside*" *the electron for a time* Δt_{stay} *and then exits* (*emission*) *from where it entered* (*that is, it resumes the same electromagnetic field line that was initially* "*incident*" *on the electron or* $(\theta = \pi)$). The connection with the initial line of EMF implies that the outgoing photon $(\gamma_1)_{out}$ is delayed compared to the photon that preceded γ_2 by a time Δt_{del} equal to the duration of the "stay" inside the electron ($\Delta t_{del} = \Delta t_{stay} = \Delta x_e/c$). But it is evident that Δt_{stay} coincides with the duration of the emission and absorption process Δt_γ with energy variation $\Delta\varepsilon_\gamma$ of the photon. We will then have that: $[\Delta\varepsilon_\gamma\Delta t_\gamma = h]$ → $[\Delta\varepsilon_\gamma\Delta t_{stay} = h]$ → $[\Delta\varepsilon_\gamma(\Delta x_e/c) = h]$. The $\Delta\varepsilon_\gamma$ will correspond to the variation $\Delta\omega_\gamma$ of the photon frequency during the absorption and emission process (and in the opposite process) and, therefore, we will have that $\Delta\varepsilon_\gamma = \hbar\Delta\omega_\gamma$. Here, we formulate a Hypothesis: *we consider the electron as a unique and indivisible* "***oscillating structure***" *composed by* "***coupled quantum oscillators***", *having a* "***proper frequency***" (ω_0) *of oscillation. We indicate this new aspect as a* "***Structure Hypothesis***". The only chance for the photon to come out from the electron is that to be perfectly in agreement with the frequency (ω_0) and phase of the structure oscillation. This means that the duration of the stay cannot be different from the duration of the proper time associated with the electron or from its oscillation period T_0, that is ($\Delta t_{stay} = T_0$). But a loss of time (equal to the period T_0) coincides with a loss in frequency respect to photon γ_2. Therefore, the loss in frequency $\Delta\omega_\gamma$ will coincide with the oscillation frequency ω_0 of the electron: $\Delta\omega_\gamma = \omega_0$. We will thus have that [$\Delta\varepsilon_\gamma = \hbar\Delta\omega_\gamma = \hbar\omega_0$]. *If we associate the rest mass to the proper frequency* ω_0, *we will have*: [$\varepsilon_0 = m_0c^2 = \hbar\omega_0 \rightarrow m_0 = (\hbar\omega_0/c^2)$].

It follows [$\Delta\varepsilon_\gamma = \hbar\omega_0 = m_0c^2$]. We could also say that the photon cannot subtract energy from the proper electron greater than the electron rest energy, because otherwise the electron mass would become negative. In the end, we will have:

$$\begin{aligned}&\left[\Delta\varepsilon_\gamma\left(\Delta x_e/c\right)=\left(h\right)\right]\rightarrow\left[m_ec^2\left(\Delta x_e/c\right)=\left(h\right)\right]\\&\rightarrow\left[\Delta x_e=\left(h/m_ec\right)\right]\rightarrow\left[\Delta x_e=\lambda_e\right]\end{aligned} \tag{25}$$

We obtain so that the Compton length correspond with the ***electron linear size****. But in this way, we also obtain the correspondence between Compton relation and the uncertainty relations.*

1.5. Two New Hypothesis

1) The relationship between the uncertainty in the interaction of the electron with the quantum vacuum (virtual photon cloud) and the Compton effect pushes us to say that ***any virtual photon absorbed and then emitted, and vice versa, by an electron cannot have a wavelength less than twice the Compton wavelength of the electron*****,** **($\Delta\lambda_\gamma = 2\lambda_e$).** *This means that the absorbed virtual photon and emitted* (*and vice versa*) *cannot have infinite energy but finite*. Note the relation ($\Delta\lambda_\gamma = 2\lambda_e$) just represents the Compton effect when the photon is deflected at an angle of π or it "bounces" back. This aspect represents a very important novelty in the processes analyzed by QED and therefore in Feynman diagrams. The "Compton-Heisenberg" relation places a maximum limit on the exchange energy between the electron and a virtual photon; this maximum value is a direct consequence of the finite "linear" size of the electron.

2) Another notable consequence of its finite size is that ***the electron can absorb one photon at a time*** (!). To demonstrate this, it is necessary to resort to the physical aspect that a photon (intermediary agent of interaction), whether virtual or not, during the coupling with the electron structure (free or bound) determines a "*first phase*" of "***delocalization***" of the wave state of the electron and then a "*second phase*" of "***energy exchange***", see the articles [8] [9]. A structural system (electron) in the delocalization phase (superposition states or non-local state) cannot absorb or emit photons since a non-local state is a "chaotic" phase of random phase shifts happening in the oscillating structure (*electron*). Recall that between two oscillator systems the energy exchange occurs in a state of reciprocal phase concordance. The absorption or emission of a photon can then only occur if there are phase agreements between the oscillations of the photon and electron. In the subsequent energetic exchange phase, the absorbed photon can only be one because an absorbed second photon (photon 2) would alter the phase concordance between the oscillation of the electron (local state) and that of photon 1, thus

opening a chaotic phase not local. In this case, there would be no absorption of photons by the electron which would thus remain in its undisturbed initial state (in its rest frame).

We have proof of all this in the clean spectral lines of the atoms (eigenstates or Bohr energy levels, Equation (16)) and in those of the LS itself:

- The clean line of the energy levels of the Bohr atom indicates the absorption of one and only one photon at a time
- The "sub" spectral lines of the LS are formed when the bound electron is already "fallen" into an eigenstate (a real photon incident γ_r on the atom has already been absorbed in its entirety) in which the oscillation is momentarily "stationary" and the de-excitation and emission phase of the previously absorbed photon γ_r has not begun.

Let us remember the phenomenon of resonance in oscillator systems which occurs because dissipative forces are present; these, in the atomic case, are represented precisely by the "vacuum" which causes de-excitation of excited states of an electron and the eventual separation of the spectral lines expressed in the LS: during the stationary phase, the electron can absorb a virtual photon γ_v, beginning so the de-excitation phase and emission of real photon γ_r for stimulated emission. However, if the energy of γ_v is not coincident with those of the Bohr energetic level, then the spectral line will present itself as LS line. The possibility of having a structure-electron (with finite space size and a proper frequency of internal oscillation ω_0) absorbing a photon and emitting, leads us to think that ***the electron cannot have mass sub-components***: the only possibility for a structure of finite size would be to admit oscillating elastic components in the electron, which, however, cannot be considered as sub-components or sub-particles. We have already indicated this aspect as "*structure hypothesis*", see also the references [10] [11]. It is evident that this hypothesis involves a revision of the concept of mass, (see the relation $m_0 = (\hbar\omega_0/c^2)$) and, thus, also of electromagnetic mass.

1.6. The Electromagnetic Mass m_{em} and Equivalence Principle $m_{em} \equiv m_{in}$

In the Equation (10), we suppose interrupting the passage to the limit of ($a \to 0$), that is the radius (a) reaches very small but finite values, $a \approx 0$. In this case, we would have:

$$\begin{aligned} m_{em} &= \frac{e^2}{3\pi^2 c^2}\int_0^{k\to\infty} \mathrm{d}k = \frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\int_0^{\infty}\frac{\sin ka}{ka}\mathrm{d}k \\ &= \frac{e^2}{3\pi^2 c^2}\lim_{a\to 0}\left(\frac{\pi}{2a}\right) \approx \frac{e^2}{3\pi^2 c^2}\left(\frac{\pi}{2a}\right)_{a\approx 0} \end{aligned} \tag{26}$$

If we suppose in Equation (26) ($a = \lambda_e = (h/m_e c)$, then it becomes:

$$\begin{aligned} m_{em}\left(a = \lambdabar_e\right) &= \frac{e^2}{3\pi^2 c^2}\left(\frac{\pi}{2\lambdabar_e}\right) = \left(\frac{e^2}{3\pi^2 c^2}\right)\left(\frac{\pi m_{in} c}{2\hbar}\right) \\ &= \left(\frac{e^2}{6\pi}\right)\left(\frac{m_{in}}{\hbar c}\right) = \left(\frac{1}{6\pi}\alpha\right) m_{in} \end{aligned} \tag{27}$$

and where α is the fine structure constant ($\alpha = e^2/\hbar c$). Note that this m_{em} results equal to one of the Equation (16) if [$m_{in} = m_0$, $\alpha^* = \alpha/6\pi$]; then we find

$$m_{em} = \alpha^* m_{in} \tag{28}$$

Since the structure hypothesis by the relation $m_0 = (\hbar\omega_0/c^2)$ involves that the mass inertial m_i is finite and positive, then also the m_{em}, is positive and finite, thanks also to the constant α. Therefore, also the F_{self}, needs to have finite values; in fact, in the case of presence of F_{self}, the Equation (12) would give us that:

$$m_{in}\ddot{\vec{r}}(t) + \left[m_{em}\ddot{\vec{r}}(t)\right]_{\to\infty} = (m_{in} + m_{em})\ddot{\vec{r}}(t) = \left[m_{in} + (\alpha^*)m_{in}\right]\ddot{\vec{r}}(t)$$
$$= m_{in}(1+\alpha)\ddot{\vec{r}}(t) = m_{fis}\ddot{\vec{r}}(t)$$

If there is not electromagnetic field acting on the electron, we will have $m_{fis} = m_{in}$. Note the fine structure constant α is dimensionless, then the Equation (28) could express a concept of "***equivalence***" between masses apparently of "different nature": $\boldsymbol{m_{em} \equiv m_{in}}$. Recall the quantization of the internal energy of the hydrogen atom, see Bohr Equation (16). In this equation, m stands for inertial mass m_{in}, as Bohr considered. We will then have that:

$$\begin{aligned}\varepsilon_n\big|_{(CGS)} &= -\frac{\alpha^2}{2n^2}m_{in}c^2 = -\frac{1}{2}\frac{\left[(m_{in}\alpha)\alpha\right]c^2}{n^2} \\ &= -\frac{1}{2}\frac{\left[(6\pi m_{em}/\alpha)\alpha^2\right]c^2}{n^2} = -3\pi\frac{m_{em}\alpha c^2}{n^2}\end{aligned} \tag{29}$$

inserting, in this case, a "bound mass" m_{bou} with $m_{bou} = \alpha m_{em}$. It follows that:

$$\varepsilon_n\big|_{(CGS)} = -3\pi\frac{m_{em}\alpha c^2}{n^2} = -\frac{3\pi m_{em}^{bou}c^2}{n^2} \tag{30}$$

We find again that a binding energy ε_n has the form of a "*rest energy*": $\varepsilon = m_n c^2$. Thus, obtaining that for the electron immersed in the electromagnetic field (coupling with a photon) we will have $m_{em} \equiv \alpha m_{in}$, while if it is bound, we obtain m_{em} (*bound*) $\equiv \alpha m_{em} \equiv \alpha^2 m_{in}$. In this way, we can assert that the coupling of the electron with the "atomic" electromagnetic field (electron bound to the nucleus) is "quadratic" due to the presence of the term α^2. *This aspect can make us think that the electron binds to the nucleus through a "double" coupling or a double exchange of photons*. In this case, considering a Feynman diagram, we would represent the binding interaction through a "***lattice***" of photons, in which the masses involved are all "*finite and positive*": we will about on a "***Feynman propagators lattice***". Here, we have the first clue of ***the equivalence between a set of Feynmann photonic diagrams*** $\{F_\gamma\}$ ***and a lattice of photonic propagators*** $\{D_\gamma\}$: $\{F_\gamma\} \equiv \{D_\gamma\}$. Note the energy in Equation (30) present itself as a "rest" energy. We ask ourselves how two different rest energies can exist, or two distinct rest masses of the same object, such as that of the free electron and that of a bound electron. The answer is not found in the corpuscular aspect but in the wave aspect of a "dual" particle. The bound electron was seen, in the dual treatment (Bohr representation), as a "vibrating string" in stationary state. Recall that a standing wave does not go anywhere, that is, it remains confined between two oscillation nodes. In this way, the energy of a standing wave is like rest energy because it does not progress in space. With this assertion, we move into an undulatory context where the mass appears to be related to the stationary oscillation associated with the atomic orbitals. This pushes us once again to consider ***the mass of a particle as a characteristic connected to the "oscillations" of the wave associated with it*** (in this case to the electron). An electron (bound to a nucleus via an internal EMF) becomes a "*forced oscillator*" (by internal EMF) for which its natural frequency (ω_0) passes to another value of frequency (ω), considering that a bound electron is always at rest. The relativistic difference in mass Δm [with $\Delta m = m_{em} - m_i = m_i(\alpha - 1)$] will thus be in direct relation with $\Delta\omega = \omega - \omega_0$. All this is a consequence of the *Structure Hypothesis* that is an electron is a structure of couplings between field oscillators having a proper frequency of oscillation which represent the mass of an electron. Also, in interactions the electron behaves as an oscillating structure of finite spatial dimensions, while remaining a single system and not composed of sub-parts.

2. The Mass

2.1. The Definition of Mass

We wonder how it is possible to consider the "*inertial mass*" or "*proper mass*" ($m_{in} = m_0$) of a particle as an oscillation frequency ω_0. To find an answer to this question, we need to go back to the theory of relativity. In relativity, a particle having inertial mass is an "object" to which we can associate a reference system where it is at "***rest***" and to which we can assign a proper time (τ), or a "*internal clock*". The Mass-Time connection is thus inseparable ($m \Leftrightarrow \tau$). We will say that "*time is inside massive particles*" and therefore any massive object can be seen as in "***moving in time***" (*the passage of internal time can be seen as a movement in time*). We recall [12] the imaginary velocity in Space-Time 4-dim. (x, ict) which is given by the constant (c) in [$p_4 = imc$], (you see the relativity) and even the concept of ε_0 energy a rest ($\varepsilon_0 = -ip_4c$). So, we can speak of mass-energy as the "*energy of movement in time*" in perfect coherence with the relativity. This uniform motion in time recalls once again the "clock" that exists inside every particle-object as a periodic "motion". with proper frequency (ω_0), which corresponds to [$\omega_0 \Leftrightarrow \tau$], where ($\tau$) is the proper time. To this periodic motion (ω_0) we could associate the "energy of movement in time" or energy at rest; it follows $\varepsilon_0 = \hbar\omega_0$. The proper characteristic of a massive particle, associated with the proper time (τ) of an object, coincides with the proper mass **(**m_0**)** of that object. Then, discussing the mass or mass-energy of a particle, it is the same that to consider the time of the clock which is inside them [$\omega_0 \Leftrightarrow \tau \Leftrightarrow m_0$]. Then, for QM [6]:

$$\begin{Bmatrix} \varepsilon_0 = mc^2 \\ \varepsilon_0 = \hbar\omega_0 \end{Bmatrix} \Rightarrow \left\{ m = \frac{\hbar\omega_0}{c^2} \right\} \tag{31}$$

If the frequency ω_0 generates the proper time τ of a massive particle [$\tau = \hbar/mc^2$], then for symmetry, there exists a wavelength Δ_c that originates the "***proper space***" of the particle [11]. Following De Broglie, we have:

$$\begin{cases} p_0 = mc \\ p_0 = \hbar \dfrac{2\pi}{\lambda_0} \end{cases} \Rightarrow \lambdabar_0 = \frac{\hbar}{mc} \equiv \lambdabar_c \tag{32}$$

We assert that [$\Delta_c = \hbar/mc$] (that is the Compton wavelength) defines the spatial step of the proper ST of the massive particle. Now, combining the equation of the relativistic energy with the equations of De Broglie and Einstein, we have:

$$\left\{ E^2 = m^2c^4 + p^2c^2 \quad \Leftrightarrow \quad \omega^2 = \omega_0^2 + k^2c^2 \right\} \tag{33}$$

The second equation is the dispersion relationship of waves, as described by the Klein-Gordon equation:

$$\left\{ \frac{\partial^2 \Psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \Psi(x,t)}{\partial x^2} - \omega_0^2 \Psi(x,t) \right\} \tag{34}$$

As is well known, this equation describes the oscillations in a set of pendulums coupled through springs [13] and scalar fields associated with massive particles with zero spin:

$$\begin{gathered} \left\{ \frac{\partial^2 \Psi(x,t)}{\partial x^2} - \frac{\partial^2 \Psi(x,t)}{c^2 \partial t^2} = \left(\frac{mc}{\hbar} \right)^2 \Psi(x,t) \right\} \\ \Leftrightarrow \left\{ \nabla^2 \Psi(x,t) = \left(\frac{mc}{\hbar} \right)^2 \Psi(x,t) \right\} \end{gathered} \tag{35}$$

We conjectured [11] that mass is a physical expression of the proper frequency (ω_0) related to a particular elastic coupling, which is in addition to the one already existing between the oscillators of the massless scalar field (Ξ). This "*additional coupling*", which produces the mass in a scalar field (Ξ), has been referred to as a "***massive coupling***". Then, we conjectured that the massive particle-field (Ξ) is originated by a "*transversal coupling*" (T_0) between the chains of oscillators of the scalar base field (Ξ). All that can be represented in way figurative as shown in **Figure 1**:

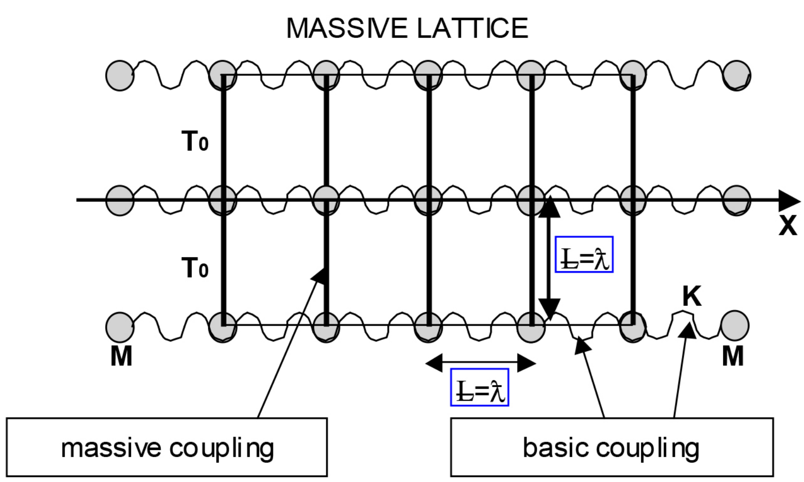

Figure 1. The massive scalar field as a lattice of "pendulums" with springs.

Therefore, a massive particle can be represented by lattice with transversal coupling on an Ξ-scalar field. This coupling can so originate a "structure" of "coupled oscillators", see **Figure 1**, on the base field Ξ. In this case the particle could be defined by a "***geometric structure***" of couplings of a set of field oscillators, which can be "extended" or well "delimited". Associating the mass with an additional coupling of field oscillators is so a new way of interpreting the mass of a particle. The massive particles are so constructed by the massive additional coupling which translates the "*internal structure*" of "coupled oscillators". Then we introduce a "***new paradigm***" in physical field theory: ***the mass as oscillation frequency of a structure of coupled quantum oscillators***. This structure could have also a "*geometric*" aspect, whether it be an extended lattice of field oscillators or a well-defined and space-bound geometric shape. The internal oscillators of the structure are indicated by acronym "**IQuO**" that is "**I**ntrinsic **Qu**antum **O**scillator". The model that is developing about this "geometric" aspect of a particle is referred to as the "***Geometric Model of Particles***", with acronym (***GMP***) [11]. Note this model is different from a pre-existing geometric model [14] in which the term "geometric" refers to Space-Time and the particle seen as a delimited curvature of Space-Time.

2.2. The Lagrange Description of the Propagation of a Structure-Particle

As is known, the K-G equation, Equation (35), is derived, via the Lagrange equation, from the density of the Lagrange function (£) describing the physical system Ψ of a Field-Particle [4]:

$$£=\left[\partial_{\mu}\Psi\left(x_i\right)\right]^2+m^2\Psi^2\left(x_i\right) \tag{36}$$

In Section 3.1, the massive particle with field Ψ_{KG} is created by a "transversal coupling" T_0 between the chains of oscillators of the no massive scalar base field (Ξ). When we observe only the oscillation with frequency ω_0 in all points x, then we are at rest with the massive particle ($m \Leftrightarrow \omega_0 \Leftrightarrow T_0$) and this aspect is coincident with that in which the springs do not are involved, see **Figure 1**. Instead, when the springs are involved, the wave becomes progressive with frequency ω and wavelength λ and represents a massive particle with velocity v, see Equation (33), in Laboratory Frame. Therefore, in the Lagrange function, see Equation (36), the term of mass ($m\Psi^2$) indicates an additional component to the field Ξ which so passes from a descriptive Ξ-function to the Ψ-function. So, note that in Lagrange theory of fields the Ξ-field is "hidden": therefore, *the Lagrange theory does not consider a possible internal structure of elementary massive particle*, *seeing this as a point particle*. We know that:

$$L=-\frac{c^2}{2}\int\left[\left(\frac{\partial\Psi^2}{\partial x_{\mu}}\right)+m^2\Psi^2\right]\mathrm{d}^3x=\int £\mathrm{d}^3x \Rightarrow L=\frac{1}{2}\sum_k\left(\dot{q}_k\dot{q}_{-k}-\omega^2 q_k q_{-k}\right) \tag{37}$$

where q_k is the generalized coordinate of an oscillation associated to a wave, with ($\lambda \Leftrightarrow k$), propagating along a line of oscillators (Ξ_i) coupled elastically, but with an additional coupling, each placed in a point (x_i) of space (the reference frame of laboratory S_{lab}). In this way we associate to a point (x_i) in S_{lab} an oscillator $[\Psi(x_i)]\Xi_i$. Recall that the quantum field Ψ-field is as a set of coupled quantum oscillators express by operators (a, a^+):

$$\Psi(x_i)=\left(\frac{1}{\sqrt{V}}\right)\sum_k q_k(t)\mathrm{e}^{i(\vec{k}\cdot\vec{x})}=\left(\frac{1}{\sqrt{V}}\right)\sum_k\left[\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left(a_k+a_{-k}^+\right)\right]\mathrm{e}^{i(\vec{k}\cdot\vec{x})}$$
$$\varepsilon_{(n_1,n_2,\cdots,n_k)}=\sum_k\left[\hbar\omega_k\left(n_k+\frac{1}{2}\right)\right],\quad P_{(n_1,n_2,\cdots,n_k)}=\sum_k\left[\hbar k_k\left(n_k\right)\right] \tag{38}$$

If now we introduce a particle with an "internal structure" of coupled oscillators, the operators (a, a^+) need to express this aspect. The unique possibility is to insert the geometric aspect in their component no time $\left(a_0,a_0^+\right)$:

$$\left[q_k(t)\right]_{op}=\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_k(t)+a_{-k}^+(t)\right]=\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_{0k}+a_{0(-k)}^+\right]\mathrm{e}^{i(\omega_k t)} \tag{39}$$

Therefore, the operators $\left(a_0,a_0^+\right)$ will be expressed by matrices $\left(A_0,A_0^+\right)$ with elements describing the structure of coupled quantum oscillators:

$$\left(\hat{A}_{0k}+\hat{A}_{0(-k)}^+\right)=\left[\begin{pmatrix}I_{11}&\cdots&I_{1n}\\\vdots&\ddots&\vdots\\I_{1m}&\cdots&I_{nm}\end{pmatrix}_k\left(a_{0k}+a_{0(-k)}^+\right)\right] \tag{40}$$

The term I_{ij} is a "projector" operator on the spatial axes (X, Y, Z) as well as a projector on the time axis (t). In n-dim spaces, we will have: $I_{ij}a_0 = I_{(X,Y,Z)}a_0 = (a_0)_{(x,y,z)}$. That is the operator $I_{(X,Y,Z)}$ places the quantum oscillator (a, a^+) in a space point of (X, Y, Z) [8]: $(a_0)_{(x,y,z)}$. Explicitly we will have:

$$\begin{aligned}\left(\hat{A}_{0k}+\hat{A}_{0(-k)}^+\right)&\equiv\begin{pmatrix}\hat{A}_{0k}\\\hat{A}_{0(-k)}^+\end{pmatrix}\\&=\left[\begin{pmatrix}\hat{I}_{11}a_0\\\vdots\\\hat{I}_{1n}a_0\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}\hat{I}_{1m}a_0\\\vdots\\\hat{I}_{nn}a_0\end{pmatrix}\right]_k\otimes\left[\begin{pmatrix}\hat{I}_{11}a_0^+\\\vdots\\\hat{I}_{1n}a_0^+\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}\hat{I}_{1m}a_0^+\\\vdots\\\hat{I}_{nn}a_0^+\end{pmatrix}\right]_{-k}\\&=\begin{bmatrix}\begin{pmatrix}(a_0)_{11}\\\vdots\\(a_0)_{1n}\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}(a_0)_{1m}\\\vdots\\(a_0)_{nm}\end{pmatrix}\\\begin{pmatrix}(a_0^+)_{11}\\\vdots\\(a_0^+)_{1n}\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}(a_0^+)_{1m}\\\vdots\\(a_0^+)_{nm}\end{pmatrix}\end{bmatrix}\end{aligned} \tag{41}$$

The matrix $\left(A_0,A_0^+\right)$ will indicate the distribution in the various points of space (X, Y, Z) of the IQuO quantum oscillators, that is the geometric structure of the coupled IQuOs. The symbol ⊗ formally indicates the operation of combining the elements of the set $\{I\}$ with the set $\{a, a^+\}$, highlighting the particular geometric structure assigned to a massive particle. The overall shape of the matrix $\hat{A}$

= [$I \otimes (a, a^{+})$] describes to the geometric structure of couplings between quantum oscillators (IQuO). We establish the matrix $\hat{A}$ indicates the set IQuO associated with an additional or massive coupling which builds the internal structure of a massive particle; thus, we will have, see the Equation (42):

$$\begin{aligned} \left[q_k(t)\right]_{op} &= \left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_k(t) + a^{+}_{-k}(t)\right] \\ \Rightarrow \left[\hat{q}_k(t)\right]_{op} &= \left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[\hat{A}_{0k} + \hat{A}^{+}_{0(-k)}\right]\mathrm{e}^{i(\omega_k t)} \end{aligned} \tag{42}$$

It has been demonstrated that a quantum oscillator, to be able to implement elastic couplings with other quantum oscillators, must be able to have a "***sub-oscillators structure with semi-quanta***" [8] [9]. The peculiarity of being an IQuO has its origin in the characteristic of being composed of "sub-oscillators" that can hook up with each other to realize the couplings of the structure which then has a natural frequency $m(\omega_0)$. In ref. [11] we demonstrate that ***the structure-particle propagate along an axis X following the Dirac Equation***. From the shape of the Lagrange function (its density), it is evident that the matrix ($\hat{A}$), the internal degree of freedom of the particle, is not involved in the interactions and, therefore, in the calculation of the *scattering matrix* T_{if} (for Bohr was the "external" behaviours of the particle). However, we can no longer maintain a positivist position and therefore only be interested in the external behaviour of a particle and ignore its internal structure. Even Dirac, one of the founders of QM, in his investigation about the spin, said that "*it indicates the existence of an internal degree of freedom in a particle*" [1]. Omitting an internal investigation would thus result in "minimizing" the physical knowledge of particles.

2.3. The Renormalization in QED and Its Solution Hypothesis

For simplicity, we consider a Coulomb Field $V(r)$ with potential energy $U = QV(r)$. If one introduces the perturbation action of quantum vacuum, then the ($m_{em} \to \infty$) if ($k \to \infty$). In this case, the propagators describing the diagrams diverge due to the presence of vacuum perturbation and, therefore, the interaction cannot be described. Also, in QED it is needed to recur to the renormalization. The Lagrange function of interaction between an electron and Coulomb Field, is expressed by substitution of the electron proper mass ($m_0 \equiv m_{inertial}$) with the physical mass m_{fis}:

$$\begin{aligned} \pounds = &-\left\{\Psi^{+}(x)\left[\gamma_\mu \partial_\mu + m_{fis}\right]\Psi(x)\right\} + m_{em}\Psi^{+}(x)\Psi(x) \\ &+ ieA_\mu(x)\Psi^{+}(x)\Psi(x) - \frac{1}{4}F^2_{\mu\nu}(x) \\ m_{fis} &= m_{in} + m_{em} \end{aligned}$$

Here, a mass term ($m_{em}\Psi^{+}\Psi$) has been added to the normal Lagrange function. The interaction term becomes:

$$\pounds_{\text{int}} = m_{em}\Psi^{+}(x)\Psi(x) + ieA_\mu(x)\Psi^{+}(x)\Psi(x)$$

The development in power series of e of the m_{em}, that is [$m_{em} = e^2 m_{em}^{(2)} + e^4 m_{em}^{(4)} + \cdots + e^n m_{em}^{(n)} + \cdots$] determines a set of Feynmann diagrams (divergences) which delete the diverging diagrams given by term $ieA\Psi^{+}\Psi$, only after to have regulated the correspondent integrals or propagators. However, in QED this is possible if we also "renormalize" the electric charge and the field functions present in the Lagrange function. At the end of the renormalization process, only convergent diagrams stay expressed by integrals with finite higher extreme k, see the Bethe integral, Equation (24). Note that the renormalization procedure is based on the hypothesis of the point form particles and virtual photons of vacuum with infinite energy. Therefore, if we change the two initial hypotheses, one could not consider the renormalization. In Section 2.5, we demonstrated that the virtual photon of vacuum emerges with finite energy and the electron has finite size ($\Delta x_e = \lambda_e$) and, therefore, finite m_{em}, that is $m_{em} = am_{in}$. In this case, the propagators admit integrals not diverging. Not only, but the set of Feynmann diagrams becomes a "***lattice***" of propagators not diverging. In the next section, we show this possibility.

3. Weak Interaction and the Renormalization

3.1. The Propagator of the Weak Interaction

Before the introduction of the intermediate W boson, the Beta Decay was described by the interaction Hamiltonian [3]:

$$\begin{gathered} H_{\text{int}} = -iG\int\left[\left(\bar{\Psi}_p \Gamma_\mu \Psi_n\right)_{J_{hadr}} \left(\bar{\Psi}_{e^-} \Gamma_\mu \Psi_\nu\right)_{J_{lept}}\right] \mathrm{d}^3 x \\ \to H_{\text{int}} = \frac{G}{\sqrt{2}}\left[J_\alpha^+ (hadr) J_\alpha (lept) + H.C\right] \end{gathered} \tag{43}$$

Whit (J_{hadr}) hadronic current and (J_{Lept}) leptonic current:

$$\begin{gathered} j_\mu(q)_{hadr} = iG\bar{\Psi}_p\left(x_\mu\right)\gamma_5 \hat{\tau}_q \Psi_n\left(x_\mu\right), \\ j_\mu(q)_{lept} = iG\bar{\Psi}_e\left(x_\mu\right)\gamma_\alpha\left(1+\gamma_5\right)\Psi_\nu\left(x_\mu\right) \end{gathered} \tag{44}$$

The corresponding diagram, to the first order, is punctual and will be defined as the Fermi's diagram, see **Figure 2**:

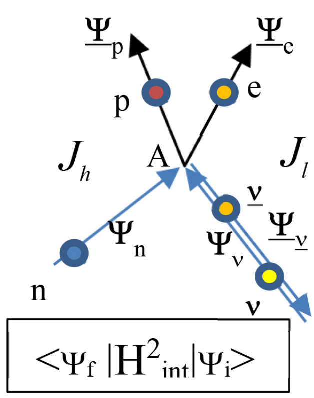

Figure 2. Fermi's diagram.

We also find the weak interaction in the decay of the pion:

$$\begin{aligned} H_{\text{int}}(weak) &= \frac{G}{\sqrt{2}}\left[J_\alpha^+ (quark) J_\alpha (lep.) \right] \\ &= \frac{G\cos\theta}{\sqrt{2}}\left\{ \left[\bar{\Psi}_{\underline{u}} \gamma_\alpha (1+\gamma_5) \Psi_d \right] \left[\bar{\Psi}_{\underline{\nu}_\mu} \gamma_\alpha (1+\gamma_5) \Psi_\mu \right] \right\} \end{aligned} \tag{45}$$

where ($\cos\theta$) is the Cabibbo angle. If instead we introduce an intermediary agent W, not yet well defined, then the H_{int} will be:

$$\begin{aligned} H_{\text{int}}(W) &= \frac{g}{2\sqrt{2}}\left[J_\alpha^C W_\alpha^+ + J_\alpha^{C+} W_\alpha \right] \\ &= \frac{g}{2\sqrt{2}}\left\{ \left[J_\alpha (hadr) + J_\alpha (lept) \right] W_\alpha^+ + \left[J_\alpha^+ (hadr) + J_\alpha^+ (lept) \right] W_\alpha \right\} \\ &= \frac{g}{2\sqrt{2}}\left\{ \left[\bar{u}\gamma_\mu (1+\gamma_5) d \right] W_\alpha^+ + \left[\bar{\mu}\gamma_\mu (1+\gamma_5) \nu \right] W_\alpha^+ \right\} \\ &\quad + \frac{g}{2\sqrt{2}}\left\{ \left[u\gamma_\mu (1+\gamma_5) \bar{d} \right] W_\alpha + \left[\mu\gamma_\mu (1+\gamma_5) \bar{\nu} \right] W_\alpha \right\} \end{aligned} \tag{46}$$

the superscript (c) indicates charged currents. The coupling constant g (with the presence of W) is related to the Fermi constant (G) of Equation (43). Besides, we have $J_\alpha = (J_{quark} + J_{lep})_\alpha$. The corresponding diagram, where the dashed line now represents the propagator of the W boson, is, see **Figure 3**:

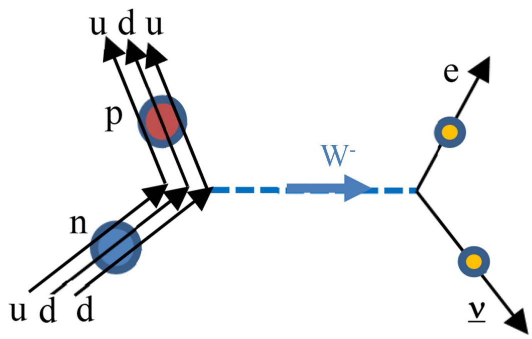

Figure 3. Diagram with W-propagator.

From this perspective, the graphical representation of the pion decay, involving quarks, is, see **Figure 4**:

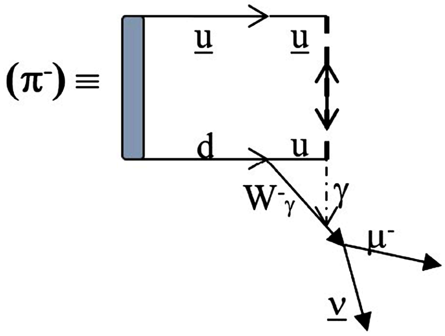

Figure 4. Pion decay.

The decay of the negative pion (π^-), composed of a *d*-quark and an *anti-u* quark, is described as follows:

$$\left(\underline{u},d\right)\to\left[\underline{u},\left(d\to \mathrm{W}^-+u\right)\right]\to\left[\left(\underline{u}+u\right)+\mathrm{W}^-\right]\to\left[\gamma_v+\mathrm{W}^-\right]\to\left(\mathrm{W}^-\right)_\gamma$$

Usually (in most cases) the virtual photon (γ_v), from annihilation of two quarks (u, $\underline{u}$), couples with the W boson, before decaying, and becomes excitation energy of W boson, $(\mathrm{W}^-)_\gamma$. The photon (γ_v), acts on the "***quantum not separated***" system $[(u, d) + \mathrm{W}]$ and reduces it; after it follows the decay of W boson. The mathematical representation of the propagator $D(W)$ is given by the following expression:

$$D(W)=\left[\frac{\delta_{\alpha\beta}+\left(q_\alpha q_\beta/M_W^2\right)}{q^2+M_W^2}\right],\quad q=\left(p_n+p_p\right)-\left(p_e+p_v\right)\tag{47}$$

where M_w is the mass of the W boson. If $q <<< M_w$ (q is the transferred 4-momentum), see the β-decay described by Fermi, the interaction becomes pointwise again:

$$D(W)=\left[\frac{\delta_{\alpha\beta}+\left(q_\alpha q_\beta/M_W^2\right)}{q^2+M_W^2}\right]\to D(W)_{(Fermi)}\approx\left[\frac{\delta_{\alpha\beta}}{q^2+M_W^2}\right]\tag{48}$$

$D(W)_{(Fermi)}$ indicates a propagator describing a β-decay close to description of Fermi. In truth, there are different degrees of approximations: the one that corresponds most closely to the experimental reality of β-decay is $[(q^2 + M^2) > (q^2/M^2)]$; we will then have:

$$D(W)_{(Fermi)}\approx\left(\frac{G}{\sqrt{2}}\right)\left[\frac{\delta_{\alpha\beta}}{q^2+M_w^2}\right]\to\left(\frac{G}{\sqrt{2}}\right)\left[\frac{\delta_{\alpha\beta}}{\left(q^2/M_w^2\right)}\right]=\left(\frac{GM_w^2}{\sqrt{2}}\right)\left[\frac{\delta_{\alpha\beta}}{q^2}\right]\tag{49}$$

with the condition that $\left[\left(G/2^{1/2}\right)=\left(g^2/8M_w^2\right)\right]$ we will have:

$$D_{(Fermi)}\approx\left(\frac{GM_w^2}{\sqrt{2}}\right)\left[\frac{\delta_{\alpha\beta}}{q^2}\right]=\left(\frac{g^2}{8M_w^2}M_w^2\right)\left[\frac{\delta_{\alpha\beta}}{q^2}\right]=\left(\frac{g^2}{8}\right)\left[\frac{\delta_{\alpha\beta}}{q^2}\right]\tag{50}$$

So, we obtain a propagator as that of **Figure 2**, without the W boson, which describes a β-decay compatible with the experimental data of the neutron decay. However, if we want to keep the intermediary agent, then we will have the Fermi propagator of Equation (48). With this approximation we will always have a diagram with an intermediate propagator, but with a massive "scalar" boson, called ***W scalar propagator or Fermi Propagator***. Instead, the weak interaction transmitted by a W boson can be highlighted only in processes where $q \approx M_w$. When this occurs, the weak interaction becomes comparable in intensity to the electromagnetic interaction, see CERN experiments [*experiment of detection of W*]. As can be highlighted in the cross-section calculations, the Fermi propagator of the neutron beta decay is a massive scalar field propagator renormalizable at any value of k. Instead, the weak interaction mediated by a vector boson W, Equation (47), is not renormalizable unless something new is introduced, see Weinberg's electroweak theory. Recall that due to the conservation of 4-momentum, the decay of a single W into a leptonic pair (fermions) requires that it be a massive vector field, unlike the non-massive photon that coupled with another photon generates a leptonic pair. That is, in describing an interaction with an intermediary agent W, it is necessary to assign it some particular property that makes the theory of weak interaction (QEWD) renormalizable like QED.

3.2. The Hypothesis of Massive Lattice of W

The decay of the pion prompts us to consider a particular question: How can a quark, which is an elementary particle, transform into another quark and emit a W-boson? The same is true of a vector boson which decays into a pair of spinor leptons. What drives the d-quark, its wave function, to turn into u-quark? What about the creation and annihilation of leptonic pairs where there are transformations of vector functions (bosons) into spinor functions (fermions) and vice versa. There must be a connection between bosons and fermions. We could think that a particle-field must "meet" something in its path, which transforms the form of its wave function into another. The same happens in a W boson when it breaks into two other forms, different as two fermions, the electron, and the anti-neutrino. These issues push us to change the descriptive perspective: we will no longer describe that a u-quark is transformed into a W boson and a d-quark, but we will say that the W boson transforms the d-quark into a u-quark. Exactly, in weak interactions, a W boson transforms a d-quark into a u-quark and vice versa, as also a W boson transforms a neutrino into a muon and vice versa. Note in Quantum Field Theory does not exit an operator which transform a wave function in another function. These issues lead us to believe that something is missing in quantum field theory: something that makes the Standard Model of particle-fields incomplete. In fact, we point out that if we instead consider particles as structures (even geometrically shaped structures of coupled oscillators, see the PGM) then it is possible to obtain descriptions of particles (structures) that transform "geometrically" into others (structures), see references [15]-[17]. The perspective change also tells us that W^- can be "decomposed" and therefore split into two fermions, the lepton pair, as if the field oscillator of the W boson is composed of the coupling of two field oscillators of the fermion type. In this perspective we could treat the β-decay with two coupled bosons (W^+, W^-). Furthermore, knowing that the W boson is massive, we could associate a lattice with it, see **Figure 1**. We can thus introduce the concept of lattice {W} ≡ (W^+, W^-). Just as we describe electrons surrounded by a virtual cloud of photons and nucleons surrounded by a virtual cloud of pions, we can also say that quarks are surrounded by a cloud of pairs of virtual bosons W, as well as by a cloud of gluons. So, we hypothesize that in the space around to quark could exist a ***lattice* {W}** composed by virtual pairs of ***$W^\pm$ boson*** (or ***W-dipole***): {W} ≡ (W^+, W^-). This just happen during the meson decay, such as a pion. So, we can think to another representation of the meson decay, see **Figure 5**:

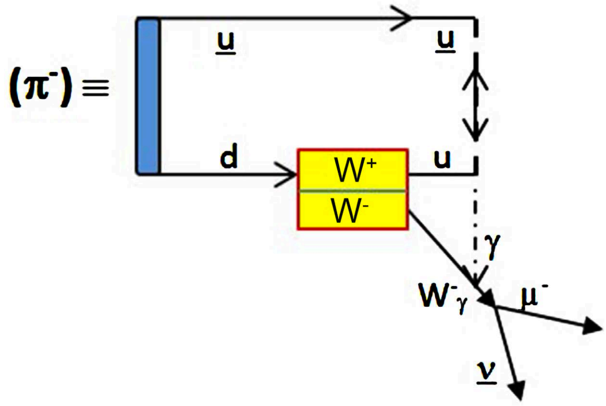

Figure 5. The W-lattice in beta decay of pion.

Here, the W^+ boson couples with *d*-quark of the π and transforms it into a *u*-quark, with the consequence of having an annihilation (u, $\underline{u}$) followed by emission γ ray which is absorbed by W^- boson; this absorption allows to the W^- boson of decay into a pair μ and an anti-ν (in rare cases also in an electron and an antineutrino).

3.3. The Lattice-Propagator {W}

Instead of going back to the well-known path of the Higgs mechanism and the Weinberg-Salaam solution into weak interaction with bosons W, we take a new path. Having introduced the idea that a massive particle can be represented by a lattice field with additional coupling (the massive one), see **Figure 1**, we modify the diagram of **Figure 3** and obtain the following **Figure 6**:

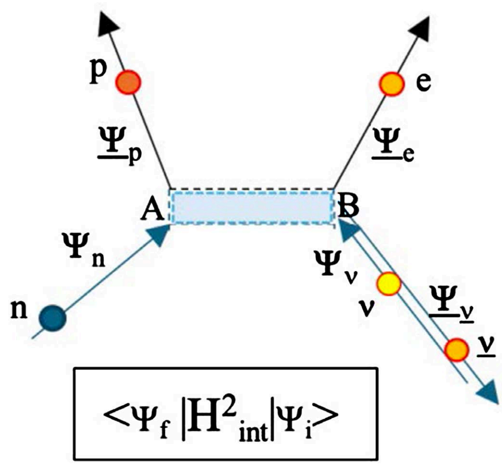

Figure 6. Interaction by lattice.

Here the dotted lines can represent the W lattice. Having said that the vector boson is electrically charged, we will have a lattice given by the pair (W^+, W^-). This involves the splitting of the propagator of **Figure 3**, and we will have the following representation, **Figure 7**:

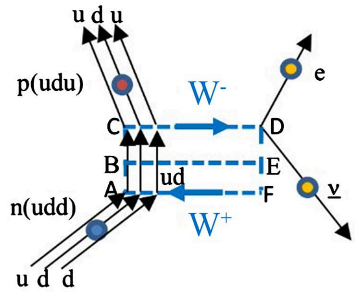

Figure 7. Lattice (W^+, W^-).

Where the arrow indicates the "direction" of the propagator. It is observed that the *u*-quark couples with the W^- and transforms itself into a *d*-quark. The W^+ boson remains uncoupled and decays into the leptonic pair. In this representation the propagators, see Equation (47), would thus become two:

$$D\{W\} \equiv \left[D\left(W^+\right), D\left(W^-\right)\right] = \left[\frac{\delta_{\alpha\beta} \pm \left(q_\alpha q_\beta / M_W^2\right)}{q^2 + M_W^2}\right]_{W^+}, \left[\frac{\delta_{\gamma\sigma} \pm \left(q_\gamma q_\sigma / M_W^2\right)}{q^2 + M_W^2}\right]_{W^-} \tag{51}$$

$$\left[q_{W^-} = \left(p_n + p_p\right) - \left(p_e + p_\nu\right)\right]_{W^-}, \quad \left[q_{W^+} = \left(p_e + p_\nu\right) - \left(p_n + p_p\right)\right]_{W^+}, \quad \left[q_{W^-} = -q_{W^+}\right]$$

Along any side BE of the pair (W^+, W^-), **Figure 7**, the oscillations in W^+ propagate with opposite phase and opposite charge respect to W^-, determining a propagation line of oscillation "quanta" with global zero electric charge; furthermore, the moments are opposite as also the spins, therefore a system of oscillations associated with an overall "scalar field" Φ_w is formed. So, *the* $\{W\}$ *lattice appears as a massive "lattice-field" with scalar typology and electrically neutral*. Thus, if we associate two propagators to the two (W^+, W^-) of {W} lattice, their superposition must correspond to a single massive propagator of zero charge of the type reported in Equation (50), equivalent to the Fermi propagator; operating mathematically on the superposition of the two propagators $D(W^+)$ $D(W^-)$, it is necessary to have at the end a scalar propagator, for which there is only one possibility, that is to have:

$$\begin{aligned} D\left(W^\pm\right) &= D\left(W^+\right) \oplus D\left(W^-\right) \\ &= \left[\frac{\delta_{\alpha\beta} + \left(q_\alpha q_\beta / M_w^2\right)_{W^+}}{q^2 + M_W^2}\right]_{W^+} + \left[\frac{\delta_{\gamma\sigma} + \left(q_\gamma q_\sigma / M_W^2\right)_{W^-}}{q^2 + M_W^2}\right]_{W^-} \\ &= \left[\frac{\delta_{\alpha\beta} + \left(\left(q_\alpha q_\beta\right)_{W^+} / M_w^2\right)_{W^+}}{q^2 + M_W^2}\right]_{W^+} + \left[\frac{\delta_{\gamma\sigma} + \left(\left(q_\gamma q_\sigma\right)_{W^-} / M_W^2\right)_{W^-}}{q^2 + M_W^2}\right]_{W^-} \\ &= \left[\frac{\left(\delta_{\alpha\beta} + \delta_{\gamma\sigma}\right) + \left(\frac{-q_\alpha q_\beta + q_\alpha q_\beta}{M_W^2}\right)}{q^2 + M_W^2}\right]_{\{W\}} = \left[\frac{\left(\delta_{\varepsilon\tau}\right)}{q^2 + M_W^2}\right]_{\{W\}} \end{aligned} \tag{52}$$

As is well known, this sum propagator coincides with the Fermi propagator described by Equation (48). If we then consider a {W} lattice of size (λ_W), the upper limit of integration becomes k_λ ($k = (2\pi/\lambda)$) and, therefore, the integration does not admit divergence. In a lattice representation an infinity has been eliminated. It is necessary to justify the minus sign in the numerator ($q_\alpha\ q_\beta$) of the W^+ propagator. In **Figure 7**, as already said, we have that the W^+ propagator is "entering" in the pion system (u, d), while W^- is exiting, see also the Equation (51). However, the minus sign in Equation (52) transforms an interval of genus time into an interval of genus space. We will thus have a W^+ that acts with intervals of type "space" while the other W^- acts with intervals of type "time". This action of the two W-bosons has already been shown in ref. [8] [9], where it is shown that the interaction between two particles takes place in two phases: the first phase expresses a reciprocal "action" of phase shifts brought to the wave functions representing the two particles, then once the phase concordance of the respective oscillations has been reached, a second phase begins in which exchanges of 4 energy-momenta occur. Well, the first phase operates with intervals of genus space (the phase shifts propagate with superluminal velocity) while the second with intervals of genus time. On the pion system (u, d) the W^+ boson acts non-locally with superluminal phase shifts while the other W^- boson, the one that splits into two leptons, propagates with a time-like interval. Note that globally the {W} lattice operates as a neutral scalar field, see already citated the Φ_w, with an aspect already noted in BE segment of **Figure 7**. A final consideration: in the treatment with a lattice of fields, k cannot tend to infinity, that is, we must cut the upper limit of the integral of the propagator with k_c such that $k_c = (2\pi/\lambda_c)$ and, therefore, cut values of wavelengths λ smaller than λ_c. For wavelengths λ smaller than ($\lambda_c = \lambda_W$), the number of waves k increases (increasing momentum) with values belonging to the series $(k_c, 2k_c, 3k_c, \ldots, nk_c, \ldots)_W$. In this case, we obtain all the propagators related to the Feynman diagrams, of the propagator of the massive vector boson W. This means that the each propagator (W^+, W^-) of various orders must have integrals cut with $(k_c, 2k_c, 3k_c, \ldots, nk_c, \ldots)_W$. It is intuitive to assume that there must be some connections between the weak interaction treated by Weinberg and that treated by a {W} lattice. A possible connection could arise by considering that ***the Higgs boson also has a lattice representation*** {H}. In this case, *the Weinberg mechanism would emerge where it is necessary to connect the two lattices to describe the weak interaction* in all its aspects. Looking carefully at **Figure 7** we can identify a "geometric" shape of the {W} lattice and therefore also of the individual bosons (W^+, W^-). This geometric representation has been presented in the papers reported in ref. [17]. The single W boson can so be represented by an additional elastic coupling between IQuO, quantum field oscillators (A, B, C, D, E, F), that is, we can have a structure given by the following **Figure 8**.

The arrows along the sides indicate the direction of the current of quanta and the sign of the electric charge. Massive couplings are distinguished (W^+, W^-). The presence of the diagonal inside the rectangles indicates the possibility that quanta also propagate along lines joining oscillators in opposite vertices (F, B) and (B, D), see **Figure 9**.

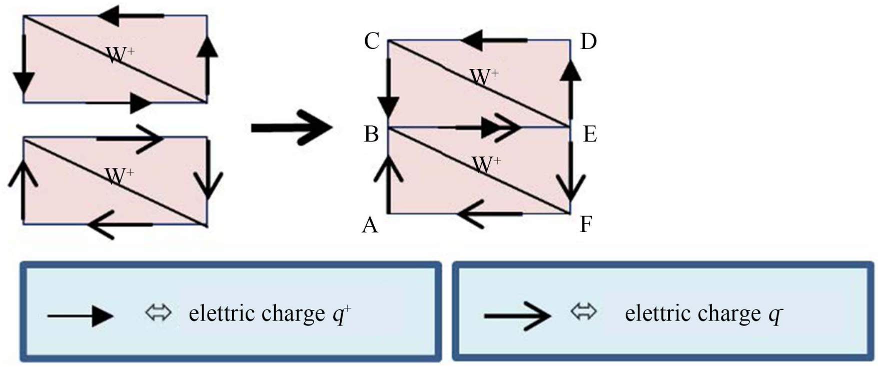

Figure 8. Representation of Bosons lattice with Bosons at quadrangular geometric form.

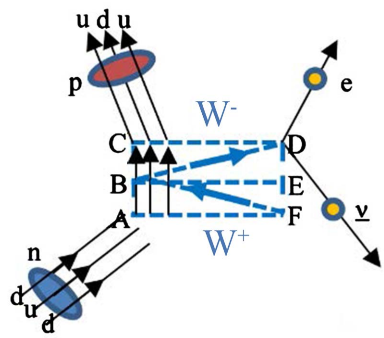

Figure 9. Neutron decay by W-lattice.

In weak decays of other hadronic particles with quark flavors other than the (u, d) flavors, we have to compute the integrals of the various propagators with upper bounds cut at the values $nk_c \equiv (k_c, 2k_c, 3k_c, \ldots, nk_c)$. Each value of the upper bound indicates the propagator of order 1, 2, … and so on. As the k increases, the spatial (linear) dimension of the various W components of the lattice decreases but the number of "rectangles" W_i components of the lattice {W}, increases, see **Figure 10**:

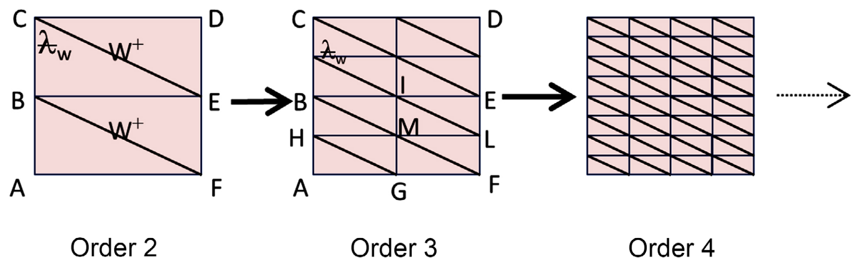

Figure 10. The various orders of {W} lattice.

Looking **Figure 9**, we consider that the lines [(BF), (BD)] are the propagators of two bosons (W^+, W^-) and, therefore, we assign the Compton wavelength λ_W to these lines. Speaking of the Higgs boson lattice {H}, we could consider a representation with the two lattices [{W}, {H}] connected, that is [{W} ⇔ {H}]; seeing **Figure 8** and **Figure 10**, we have the following representation, **Figure 11**:

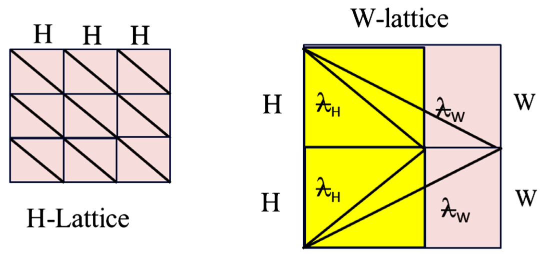

Figure 11. Intersection between two lattices [{W}, {H}].

See the ref. [17], we have the following **Figure 12**:

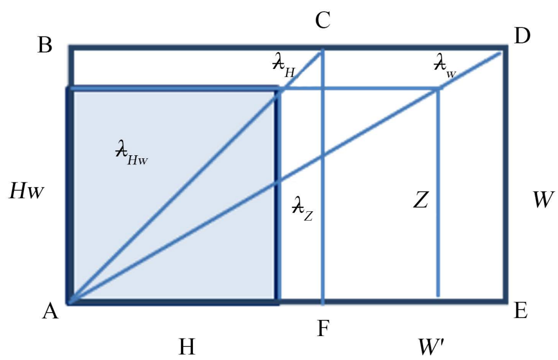

Figure 12. The H-Boson with (λ_H), the H_w-boson with (λ_{Hw}) and golden W-boson (λ_W).

If we consider the rectangle (ABDE) of the "***golden type***", we will have that:

$$BD = \phi AB = (1.618) AB \tag{53}$$

In this case, the H boson (ABCF) will be called the "golden" H-boson (H), because it is stuck inside a golden W and its geometric shape would be that of a "***square***". Note that the CDEF rectangle is also golden and therefore W' is a golden boson. Note the geometric shapes (H, W') of **Figure 12** are given by golden properties of the golden rectangle W (ABDE). We will have *AD*/*AC*:

$$AD/AC = \left[\phi^2 + 1\right]^{1/2} / 2^{1/2} = (1.345) = (\lambda_W / \lambda_H) \tag{54}$$

With $\lambda_H < \lambda_W$. Thus, it is [m(H)/m(W)] = (1.345). That is:

$$m(\mathrm{H}) = m(\mathrm{W})(1.345) \tag{55}$$

From Equation (52), the D($W^{\pm}$) present itself as a neutral electrically lattice of W-bosons, see **Figure 9**: a first possibility is given by two contiguous rectangles ((ABEF), (BCDE)), represented by operation ($W_{(ABEF)} \oplus W_{(BCDE)}$), see **Figure 9** and **Figure 10**, while the second possibility by the superposition of two states (Ψ_{W+}, Ψ_{W-}), that is with two super placed rectangles ((ABEF) ⊗ (BCDE)), where the ⊗ indicates the operation of "***interpenetration***" of two W-bosons. So, we can have two neutral bosonic states (Z′, Z″) given by the two different combinations of two bosons (W^+, W^-), also see the ref. [17], that is Z = ($W^+ \underline{\otimes} W^-$), where the $\underline{\otimes}$-operation of combination is given by $\underline{\otimes} = (\oplus, \otimes)$. This Z-boson could be the neutral vector boson Z_w of weak interactions: both bosons are a linear combination of two bosonic states [18] [19]. We indicate by H_w the Higgs's boson discovered at CERN, which was identified and measured in mass by the reaction with leptonic decay in 4*l* [20]:

$$\left\{H_w \to (Z+Z) \to \left[\left(l^+ + l^-\right)_Z + \left(l^+ + l^-\right)_Z\right]\right\} \tag{56}$$

We think that H_w is in relation with $H_{(ABCF)}$ of **Figure 12**: ($H_w \Leftrightarrow H_{(ABCF)}$). The reaction (56) tells us that:

$$\left[H = \left(Z' \underline{\otimes} Z''\right) = \left(W^+ \underline{\otimes} W^-\right), \left(W^+ \underline{\otimes} W^-\right)\right] \tag{57}$$

Considering $\underline{\otimes} = (\oplus, \otimes)$, it is easy to understand that we can have the different cases of decay of the H_w-boson observed at CERN [20]; it is facile one can have:

$$\left[\left(W^+ \oplus W^-\right), \left(W^+ \oplus W^-\right)\right] \to \gamma\gamma,$$
$$\left[\left(W^+ \otimes W^-\right), \left(W^+ \otimes W^-\right)\right] \to 4l,$$
$$\left[\left(W^+ \oplus W^-\right), \left(W^+ \underline{\otimes} W^-\right)\right] \to W^+W^-$$

This establishes the equivalence ($H \equiv H_w$). Therefore, considering the Equation (57), to calculate the H_w-Boson mass, with size the λ_{Hw}, we must consider the *Z* boson mass, see the Equation (57). Since Z is composed of electrically charged bosons ($W^\pm$) while H is neutral, we need to add an electromagnetic missing mass Δm to get the H_w boson mass. Then, we will have:

$$\left[m\left(H_w\right) = m(Z)(1.345) \pm \Delta m\right] \tag{58}$$

Recall that the wave function Ψ_Z of Z-Boson is a vector wave function with spin $s = 1$, while Ψ_H is a scalar wave function with spin $s = 0$. To compose a vectorial function, we must combine two scalar functions: it follows two oscillating H-bosons (H_z, H_y) can be in correspondence with a Z-boson in rotation ($s = 1$): [(H_z, H_y) ⇔ Z]. See **Figure 13**:

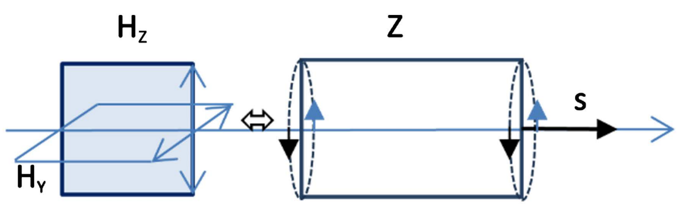

Figure 13. Correspondence between H-boson and Z-boson.

The decay H → (Z + Z) it will be given:

$$H_{(Hz,Hy)} \underline{\otimes} H_{(Hz,Hy)} \to \left[Z_{(Zz,Zy)} \underline{\otimes} Z_{(Zz,Zy)}\right] \tag{59}$$

So, we will have 4H ⇔ 4Z ⇔ 4W. If the mass difference between Z-boson and

W-boson is

$$\Delta m\left(Z,W^{\pm}\right)=\left[M\left(Z\right)-M\left(W^{\pm}\right)\right]=\left(10.81\right)\text{GeV}/c^{2} \tag{60}$$

Then, the total mass difference $\Delta m(Z, W^{\pm})$ must be divided into each of the 4 components (H_z, H_y) of the $H_{(Hz,Hy)} \otimes H_{(Hz,Hy)}$. The Equation (58) allows us of calculate the theorical value of Higgs boson mass:

$$\begin{aligned} m\left(H_{w}\right) &= \left(1.345\right)m\left(Z\right)+\Delta m\left(Z,W^{\pm}\right)/4 \\ &= \left(1.345\right)\left(91.19\right)\text{GeV}/c^{2}+\left(2.71\right)\text{GeV}/c^{2} \\ &= \left(125.35\right)\text{GeV}/c^{2} \end{aligned} \tag{61}$$

In all experiments done at CERN, the average value of the mass values found at CMS and ATLAS is given by **$<m(H_w)> \approx 125.35$ GeV/c^2.**

We thus find that the theoretical value m_H calculated in our geometric model (PGM) can be considered, within the experimental errors, to coincide with the experimental mean value of the Higgs boson H_w.

A note is due: the hypothesis of the golden form of the W boson derives from the hypothesis that quarks are golden triangles of IQuO coupled particles, see ref. [10] [15] [21] and that a *d-quark* is transformed into a *u-quark* by the W-boson [17].

4. Conclusion

The possibility of seeing a particle in undulatory way and as an inseparable system of coupled oscillators having a well-defined structure, allows us to resolve the "*renormalization paradox*". In fact, considering the undulatory aspect of a particle, we have associated the proper mass (m_0) with a proper frequency (ω_0) of an internal intrinsic oscillation, which opens the via to the "Structure Hypothesis". The relativistic condition is that the single "component" oscillator of the structure, defined as **IQuO** (acronym for **I**ntrinsic **Qu**antum **O**scillator), has no physical meaning outside the structure and therefore cannot be individually detected through an experimental observation process. Associating a structure to massive particles implies that a finite spatial dimension can be associated with them, see the Compton wavelength λ_c. This allows us to truncate at the moments' value $k = \lambda_c$, in the calculations of the integrals present in the cross sections and in the equations that define the electromagnetic mass. This operation cancels the mass renormalization procedure and makes the propagators of the intermediary agents, whether photons or massive bosons, non-divergent. But the final solution to the renormalization paradox is given by replacing the Feynman diagrams with a lattice of propagators with finite k values. This substitution allows us to treat the weak interaction mediated by W-bosons in geometric terms: by assigning a geometric structure to the W, Z and H bosons, we have the possibility of determining the theoretical value of the mass of the Higgs boson, very close to the "average" experimental one. Thanks to the idea of internal structure with IQuO, that is the PGM, we can predict new particles and new behaviors and, moreover, clearly explain aspects that are not yet understood and solve problems that are still unsolved by Standard Model, see in this last study the renormalization. In all articles where one speaks of a structure-particle emerge so many connections and correspondence with Standard Model which push us to consider very valid the PGM and see it as a Physical Model well able to represent the particles' phenomenology.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Dirac, P.A.M. (1930) The Principles of Quantum Mechanics. Oxford University Press.

[2] Purcell, E.M. (1963) Electricity and Magnetism. McGraw-Hill Book Company.

[3] Morpurgo, G. (1992) Introduzione alla Fisica delle Particelle. Zanichelli.

[4] Sommerfeld, A. (1931) Über die Beugung und Bremsung der Elektronen. *Annalen der Physik*, **403**, 257-330. (In German) https://doi.org/10.1002/andp.19314030302

[5] Weinberg, S. (2019) Soft Bremsstrahlung. *Physical Review D*, **99**, Article ID: 076018. https://doi.org/10.1103/physrevd.99.076018

[6] Wichmann, E.H. (1971) Quantum Physics. McGraw-Hill Book Company.

[7] Wikipedia, "Lamb Shift". https://en.wikipedia.org

[8] Guido, G. (2025) A New Paradigm in Quantum Fields: The Quantum Oscillator with Semi-Quanta (IQuO) (Part One). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **11**, 61-95. https://doi.org/10.4236/jhepgc.2025.111008

[9] Guido, G. (2025) A New Paradigm in Quantum Fields: The Quantum Oscillator with Semi-Quanta (IQuO) (Part Two). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **11**, 138-164. https://doi.org/10.4236/jhepgc.2025.111012

[10] Guido, G. (2019) The Bare and Dressed Masses of Quarks in Pions via the of Quarks' Geometric Model. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **5**, 1123-1149. https://doi.org/10.4236/jhepgc.2019.54065

[11] Guido, G. (2023) The Geometric Model of Particles (the Origin of Mass and the Electron Spin). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **9**, 941-963. https://doi.org/10.4236/jhepgc.2023.94070

[12] Goldstein, H. (1950) Classical Mechanics. Addison-Wesley Publishing Company, Inc.

[13] Crawford Jr., F.S. (1968) Waves. McGraw-Hill Book Company.

[14] Battey-Pratt, E.P. and Racey, T.J. (1980) Geometric Model for Fundamental Particles. *International Journal of Theoretical Physics*, **19**, 437-475. https://doi.org/10.1007/bf00671608

[15] Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic about Standard Model (Geometric Model of Particle: The Quarks). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **10**, 854-874. https://doi.org/10.4236/jhepgc.2024.102053

[16] Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic of the Standard Model "the Particle's Geometric Model" (Nucleons and K-Mesons). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **10**, 1054-1078. https://doi.org/10.4236/jhepgc.2024.103065

[17] Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic about Standard Model: "the Particles' Geometric Model" (Leptons and Bosons). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **10**, 1424-1449. https://doi.org/10.4236/jhepgc.2024.104080

[18] Glashow, S.L. (1961) Partial-Symmetries of Weak Interactions. *Nuclear Physics*, **22**, 579-588. https://doi.org/10.1016/0029-5582(61)90469-2

[19] Weinberg, S. (1967) A Model of Leptons. *Physical Review Letters*, **19**, 1264-1266. https://doi.org/10.1103/physrevlett.19.1264

[20] Aad, G., Abbott, B., Abbott, D.C., Abeling, K., Abidi, S.H., Aboulhorma, A., *et al*. (2023) Measurement of the Higgs Boson Mass in the $H \to ZZ^* \to 4\ell$ Decay Channel Using 139 fb^{-1} of $\sqrt{s}$ = 13 Tev Pp Collisions Recorded by the ATLAS Detector at the LHC. *Physics Letters B*, **843**, Article ID: 137880. https://doi.org/10.1016/j.physletb.2023.137880

[21] Guido, G. (2019) The Origin of the Color Charge into Quarks. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **5**, 1-34. https://doi.org/10.4236/jhepgc.2019.51001