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Classical Cosmology V. The Average Absolute Magnitude of Galaxies versus Redshift

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Abstract

The Euclidean environment of the generalized tired light hypothesis is applied to the analysis of the average absolute magnitude of galaxies versus redshift. To this end, two truncated luminosity functions (LFs) for galaxies are reviewed: the truncated Schechter LF and the truncated generalized gamma LF. The application is done to the SDSS DR12 catalog for galaxies.

Keywords

Galaxy Groups, Clusters, Superclusters, Large Scale Structure of the Universe Cosmology

1. Introduction

An history of the tired light hypothesis in standard cosmology for the period 1929-1939 can be found in [1]. We now report some recent developments. The hypothesis of curvature pressure has been used to derive a static and stable cosmology with a tired-light redshift [2]. Zwicky's tired light mechanism based on gravitational redshift for a blurring-less and a frequency-independent dispersion-less redshift mechanism was re-analysed by [3]. In a new tired light (NTL) hypothesis, the photons of light are absorbed and re-emitted by electrons which recoil, leading to a loss of these photons' energy and an increase in the wavelength of the redshift [4].

These cosmological theories can be tested on the samples of Supernova (SN) of type Ia once the distance modulus is provided. The first sample to be used to derive the cosmological parameters contained 7 SNs, see [5], the second one contained 34 SNs, see Figure 4 in [6] and the third one contained 42 SNs, see [7]. The above historical samples allowed deriving a first evaluation of the cosmological parameters for the expanding and accelerating universe. At the moment of writing, astronomical research is focused on the value of the distance modulus versus redshift: the Union 2.1 compilation contains 580 SNs, see [8], the joint light-curve analysis (JLA) contains 740 SNs, see [9] and the Pantheon sample contains 1048 SNs [10] [11]. Combining the two above arguments is possible to see the differences between the tired light cosmology and the standard cosmology. The aim of this paper is to test the reliability of a new version of the tired light cosmology derived in 2024 by [12]. In particular, we pose two questions:

1) Can we replace the formula for the absolute magnitude as given in the SDSS catalog for galaxies with that given by the tired light hypothesis?

2) Can we model the average magnitude of galaxies as a function of the redshift with the help of two truncated luminosity functions?

In order to answer the above questions, Section 2 reviews the standard cosmology and Section 3 reviews the generalized tired light (GTL) cosmology. Sections 4.2 and 4.3 review the standard and the truncated Schechter luminosity functions for galaxies. Section 5 applies the results of the GTL cosmology to the average magnitude for galaxies as a function of the redshift.

2. The Standard Cosmology

In the ΛCDM cosmology, the *Hubble distance* D_{H} is defined as

$$D_{\mathrm{H}} \equiv \frac{c}{H_0}, \tag{1}$$

where c is the speed of light and H_0 is the Hubble constant. We then introduce a parameter Ω_{M}

$$\Omega_{\mathrm{M}} = \frac{8\pi G \rho_0}{3H_0^2}, \tag{2}$$

where G is the Newtonian gravitational constant and ρ_0 is the mass density at the present time. Another parameter is Ω_Λ

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}, \tag{3}$$

where Λ is the cosmological constant, see [13]. Once Ω_Λ and H_0 are found, the numerical value of the cosmological constant is derived, $\Lambda \approx 1.2 \frac{1}{m^2}$.

The two previous parameters are connected with the curvature Ω_K by

$$\Omega_{\mathrm{M}} + \Omega_\Lambda + \Omega_K = 1. \tag{4}$$

The comoving distance, D_{C}, is

$$D_{\mathrm{C}} = D_{\mathrm{H}} \int_0^z \frac{\mathrm{d}z'}{E(z')}, \tag{5}$$

where $E(z)$ is the "Hubble function"

$$E(z) = \sqrt{\Omega_{\mathrm{M}} (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda}. \tag{6}$$

The above integral cannot be evaluated in analytical terms, except for the case of $\Omega_\Lambda = 0$, but the Padé approximant, see Appendix A in [14], allows us to derive an approximation for the indefinite integral. We now report the above approximation for the modulus of the distance as a function of the redshift in the ΛCDM cosmology with data as in **Table 1**.

Table 1. Numerical values of parameters for the Pantheon sample, H_0 is expressed in km·s^{-1}·Mpc^{-1}; 1048 SN Ia.

Cosmology	Parameters
ΛCDM	$H_0 = (68.209 \pm 0.2)$; $\Omega_M = (0.278 \pm 0.02)$; $\Omega_\Lambda = (0.651 \pm 0.02)$
GTL	$H_0 = (68.04 \pm 0.2)$; $\phi = 3.07$

$$m - M = 25 + \frac{5\ln\left(13252.03816\sinh\left(A + 0.2351783727 - 0.07716606044\ln(B) + C\right)(1+z)\right)}{\ln(10)}, \quad (7)$$

where

$$A = 0.4589749955\arctan\left(0.6717400132z + 0.3694609176\right), \quad (8)$$

$$B = 68.645582665z^2 + 75.51120852z + 172.8946930, \quad (9)$$

$$C = 0.08779875496z, \quad (10)$$

m is the apparent magnitude, M the absolute magnitude and z the redshift.

3. The Generalized Tired Light Model

The generalized tired light model has been developed in [12] and [15].

We report an analytical expression for the distance modulus in the GTL cosmology.

$$(m - M) = 25 + \frac{5\ln\left(-\frac{\left(-(z+1)^\phi + z + 1\right)c}{H_0(-1+\phi)(z+1)}\right)}{\ln(10)}, \quad (11)$$

here, ϕ is a parameter of the theory and c is the speed of light, see Equation (22) in [12] or see Equation (23) in [15]. The numerical expression for the above distance modulus is obtained by inserting the parameters of the GTL cosmology reported in **Table 1**:

$$(m - M) = f(z) = 25 + \frac{5\ln\left(-\frac{2128.560764\left(1 - (z+1)^{3.07} + z\right)}{z+1}\right)}{\ln(10)}. \quad (12)$$

The absolute magnitude will therefore be

$$M = m - f(z). \quad (13)$$

4. The Adopted LFs

This section reviews the adopted statistics, the Schechter LF, the truncated version of the Schechter LF, and the truncated version of the generalized gamma LF.

4.1. The Adopted Statistics

The merit function χ^2 is computed as

$$\chi^2 = \sum_{j=1}^{n} \left(\frac{LF_{theo} - LF_{astr}}{\sigma_{LF_{astr}}} \right)^2 , \tag{14}$$

where n is the number of bins for the LF of the galaxies and the two indices $theo$ and $astr$ stand for "theoretical" and "astronomical", respectively. The residual sum of squares (RSS) is

$$\text{RSS} = \sum_{j=1}^{n} \left(y(i)_{theo} - y(i)_{astr} \right)^2 , \tag{15}$$

where $y(i)_{theo}$ is the theoretical value and $y(i)_{astr}$ is the astronomical value.

A reduced merit function χ^2_{red} is evaluated by

$$\chi^2_{red} = \chi^2 / NF , \tag{16}$$

where $NF = n - k$ is the number of degrees of freedom and k is the number of parameters. The goodness of the fit can be expressed by the probability Q, see Equation (15.2.12) in [16], which involves the number of degrees of freedom and χ^2. According to [16], the fit "may be acceptable" if $Q > 0.001$. The Akaike information criterion (AIC), see [17], is defined by

$$\text{AIC} = 2k - 2\ln(L), \tag{17}$$

where L is the likelihood function and k is the number of free parameters in the model. We assume a Gaussian distribution for the errors, then the likelihood function can be derived from the χ^2 statistic $L \propto \exp\left(-\frac{\chi^2}{2}\right)$ where χ^2 has been computed by Equation (14), see [18] [19]. Now the AIC becomes

$$\text{AIC} = 2k + \chi^2 . \tag{18}$$

The mean percentage error (MPE) is defined as

$$\text{MPE} = 100 \times \frac{1}{n} \times \sum_{j=1}^{n} \left\| \frac{y(i)_{theo} - y(i)_{astr}}{y(i)_{theo}} \right\| . \tag{19}$$

4.2. The Schechter LF

Let L be a random variable taking values in the closed interval $[0, \infty]$. The Schechter LF of the galaxies, after [20], is

$$\Phi(L; \Phi^*, \alpha, L^*) \mathrm{d}L = \left(\frac{\Phi^*}{L^*} \right) \left(\frac{L}{L^*} \right)^{\alpha} \exp\left(-\frac{L}{L^*} \right) \mathrm{d}L, \tag{20}$$

where α sets the slope for low values of L, L^* is a characteristic luminosity, and Φ^* is the number of galaxies per Mpc[3]. The normalization is

$$\int_0^\infty \Phi\left(L;\Phi^*,\alpha,L^*\right)\mathrm{d}L = \Phi^*\Gamma\left(\alpha+1\right), \tag{21}$$

where

$$\Gamma\left(z\right) = \int_0^\infty \mathrm{e}^{-t}t^{z-1}\mathrm{d}t, \tag{22}$$

is the gamma function. An equivalent form in absolute magnitude of the Schechter LF is

$$\Phi\left(M;\Phi^*,\alpha,M^*\right)\mathrm{d}M = 0.921\Phi^*10^{0.4(\alpha+1)\left(M^*-M\right)}\exp\left(-10^{0.4\left(M^*-M\right)}\right)\mathrm{d}M, \tag{23}$$

where M^* is the characteristic magnitude.

4.3. The Truncated Schechter LF

We assume that the luminosity L takes values in the interval $\left[L_l,L_u\right]$, where the indices l and u mean "lower" and "upper"; the truncated Schechter LF, S_T, is

$$S_T\left(L;\Psi^*,\alpha,L^*,L_l,L_u\right) = \frac{-\left(\frac{L}{L^*}\right)^{\alpha}\mathrm{e}^{-\frac{L}{L^*}}\Psi^*\Gamma\left(\alpha+1\right)}{L^*\left(\Gamma\left(\alpha+1,\frac{L_u}{L^*}\right)-\Gamma\left(\alpha+1,\frac{L_l}{L^*}\right)\right)}, \tag{24}$$

where $\Gamma\left(a,z\right)$ is the incomplete Gamma function defined as

$$\Gamma\left(a,z\right) = \int_z^\infty t^{a-1}\mathrm{e}^{-t}\mathrm{d}t, \tag{25}$$

see [21]. The normalization is the same as for the Schechter LF, see Equation (21),

$$\int_0^\infty S_T\left(L;\Psi^*,\alpha,L^*,L_l,L_u\right)\mathrm{d}L = \Psi^*\Gamma\left(\alpha+1\right). \tag{26}$$

The average value is

$$\left\langle L\left(\Psi^*,\alpha,L^*,L_l,L_u\right)\right\rangle = \frac{N}{L^*\left(\Gamma\left(\alpha+1,\frac{L_u}{L^*}\right)-\Gamma\left(\alpha+1,\frac{L_l}{L^*}\right)\right)} \tag{27}$$

with

$$\begin{aligned} N = \Psi^*\Bigg(&L^{*2}\Gamma\left(\alpha+1,\frac{L_u}{L^*}\right)\alpha - L^{*2}\Gamma\left(\alpha+1,\frac{L_l}{L^*}\right)\alpha + L^{*2}\Gamma\left(\alpha+1,\frac{L_u}{L^*}\right) \\ &-L^{*2}\Gamma\left(\alpha+1,\frac{L_l}{L^*}\right) - L^{*-\alpha+1}\mathrm{e}^{-\frac{L_l}{L^*}}L_l^{\alpha+1} + L^{*-\alpha+1}\mathrm{e}^{-\frac{L_u}{L^*}}L_u^{\alpha+1}\Bigg)\Gamma\left(\alpha+1\right). \end{aligned} \tag{28}$$

The four luminosities L, L_l, L^* and L_u are connected with the absolute magnitudes M, M_l, M_u and M^* through the following relationship

$$\frac{L}{L_\odot} = 10^{0.4\left(M_\odot-M\right)}, \frac{L_l}{L_\odot} = 10^{0.4\left(M_\odot-M_u\right)}, \frac{L^*}{L_\odot} = 10^{0.4\left(M_\odot-M^*\right)}, \frac{L_u}{L_\odot} = 10^{0.4\left(M_\odot-M_l\right)} \tag{29}$$

where the indices u and l are inverted in the transformation from luminosity to absolute magnitude and $L_\odot$ and $M_\odot$ are the luminosity and absolute magnitude of the sun in the considered band. The equivalent form in absolute magnitude of the truncated Schechter LF is therefore

$$\Psi\left(M;\Psi^*,\alpha,M^*,M_l,M_u\right)\mathrm{d}M$$
$$=\frac{-0.4\left(10^{0.4M^*-0.4M}\right)^{\alpha}\mathrm{e}^{-10^{0.4M^*-0.4M}}\Psi^*\Gamma(\alpha+1)10^{0.4M^*-0.4M}\left(\ln(2)+\ln(5)\right)}{\Gamma\left(\alpha+1,10^{-0.4M_l+0.4M^*}\right)-\Gamma\left(\alpha+1,10^{0.4M^*-0.4M_u}\right)}. \quad (30)$$

The average absolute magnitude is

$$\left\langle M\left(\Psi^*,\alpha,L^*,L_l,L_u\right)\right\rangle=\frac{\int_{M_l}^{M_u}M\left(M;\Psi^*,\alpha,L^*,L_l,L_u\right)M\mathrm{d}M}{\int_{M_l}^{M_u}M\left(M;\Psi^*,\alpha,L^*,L_l,L_u\right)\mathrm{d}M}. \quad (31)$$

More details can be found in [22].

4.4. Generalized Gamma LF

The magnitude version of the generalized gamma LF with truncation is

$$\Psi\left(M;a,M^*,c,M_l,M_u,\Psi^*\right)=\Psi^*\frac{LD}{LN}, \quad (32)$$

$$LD=0.4c(c+1)a\mathrm{e}^{-\mathrm{e}^{0.921a\left(M^*-M\right)}+ac\left(0.921M^*-0.921M\right)}\left(\ln(2)+\ln(5)\right), \quad (33)$$

$$\begin{aligned}LN=&\,\mathrm{e}^{-\mathrm{e}^{-0.921a\left(M_l-M^*\right)}+\left(0.921M^*-0.921M_l\right)ca}c-\mathrm{e}^{-\mathrm{e}^{0.921a\left(M^*-M_u\right)}+ac\left(0.921M^*-0.921M_u\right)}c\\&+\mathrm{e}^{-0.5\mathrm{e}^{-0.921a\left(M_l-M^*\right)}+\left(-0.46M_l+0.46M^*\right)ca}M_{c/2,c/2+1/2}\left(\mathrm{e}^{-0.921a\left(M_l-M^*\right)}\right)\\&-\mathrm{e}^{-0.5\mathrm{e}^{0.921a\left(M^*-M_u\right)}+\left(0.46M^*-0.46M_u\right)ca}M_{c/2,c/2+1/2}\left(\mathrm{e}^{0.921a\left(M^*-M_u\right)}\right)\\&+\mathrm{e}^{-\mathrm{e}^{-0.921a\left(M_l-M^*\right)}+\left(0.921M^*-0.921M_l\right)ca}-\mathrm{e}^{-\mathrm{e}^{0.921a\left(M^*-M_u\right)}+ac\left(0.921M^*-0.921M_u\right)}.\end{aligned} \quad (34)$$

In the above formulae, a and c are two positive parameters, Ψ^* is the number of galaxies per Mpc^3, M_l, M_u and M^* are, respectively, the lower, the upper and the characteristic absolute magnitudes. The average absolute magnitude is

$$\left\langle\Psi\left(M;a,M^*,c,M_l,M_u,\Psi^*\right)\right\rangle=\frac{\int_{M_l}^{M_u}\Psi\left(M;a,M^*,c,M_l,M_u,\Psi^*\right)M\mathrm{d}M}{\int_{M_l}^{M_u}\Psi\left(M;a,M^*,c,M_l,M_u,\Psi^*\right)\mathrm{d}M}. \quad (35)$$

More details can be found in [23].

5. Astrophysical Applications

We processed the SDSS Photometric Catalogue DR 12, see [24], which contains 10,450,256 galaxies (elliptical + spiral) with redshift and rest frame u' absolute magnitude. The lower absolute magnitude is fixed at $M_l=-30$ and the upper absolute magnitude is the maximum absolute magnitude of the selected bin in redshift. The SDSS DR12 gives the measured value of the apparent magnitude, m, and the derived value of the absolute magnitude, M. The first test is done on the reliability of the absolute magnitude given by the GTL cosmology, see Equation (13). The percent average error between absolute magnitude as given by the SDSS and that given by the GTL cosmology is $\mathrm{MPE} \approx 7.46\%$. After this test, all the absolute magnitudes will be expressed according to the tired light hypothesis. The second test is done on the Malmquist bias, see [25] [26], which was originally applied to the stars and later on to the galaxies by [27]. We now define the Malmquist bias as the systematic distortion in luminosity or absolute magnitude for the effective range of galaxies due to a failure in detecting those galaxies with fainter luminosity or high absolute magnitude at large distances. We now introduce the concept of limiting apparent magnitude and the corresponding completeness in absolute magnitude of the considered catalog as a function of redshift. A practical implementation of the Malmquist bias can be obtained by introducing into Equation (13) $m = m_l$, where m_l is the limiting magnitude, see **Figure 1**.

We now evaluate the average absolute magnitude as a function of the redshift. The lower absolute magnitude is fixed at $M_l = -30$ and the upper absolute magnitude is the maximum absolute magnitude of the selected bin in redshift. The above choice adopts the GTL cosmology for the absolute magnitude, see Equation (13). We now report the results for the two LFs here reviewed. **Figure 2** displays the astronomical average absolute magnitude, the theoretical average absolute magnitude as given by the truncated Schechter LF, the lower and the upper limits in absolute magnitude, as functions of the redshift. **Figure 3** reports the results for the truncated generalized gamma LF.

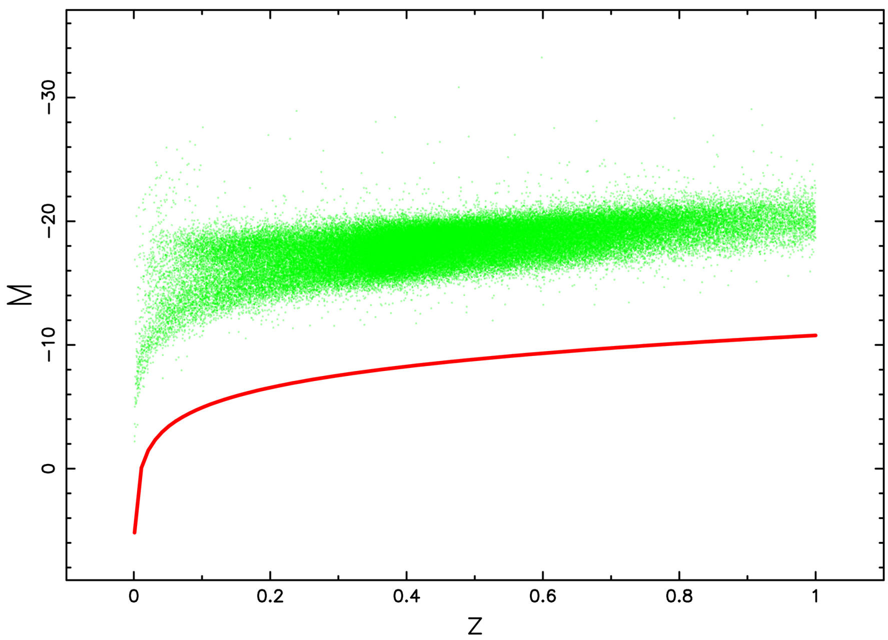

Figure 1. The absolute magnitude M evaluated with Formula (13) for 100,000 galaxies randomly selected from the SDSS DR12 (green points). The upper theoretical curve as represented by Equation (13) is shown as a red thick line when $m = m_l = 33.4$.

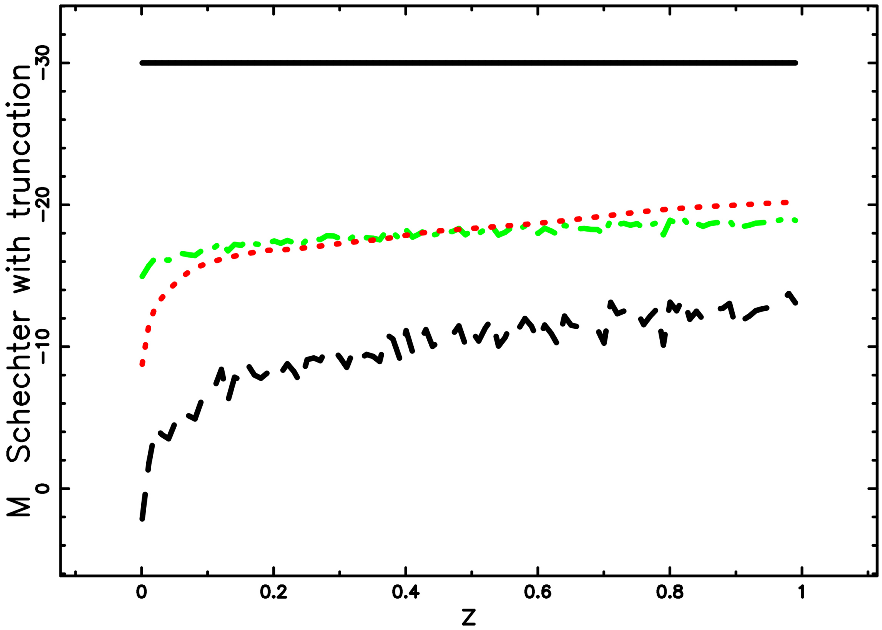

Figure 2. Average observed absolute magnitude versus redshift for SDSS galaxies (red points), average theoretical absolute magnitude for the truncated Schechter LF as given by Equation (31) (dot-dash-dot green line), the lowest absolute magnitude is $M = -30$ (full black line) and the highest absolute magnitude at a given redshift as given by the maximum value of the selected sample (dashed black line); RSS = 147.16. The other parameters for the truncated Schechter LF are: $M^* = -23.49$, $\alpha = -0.9$.

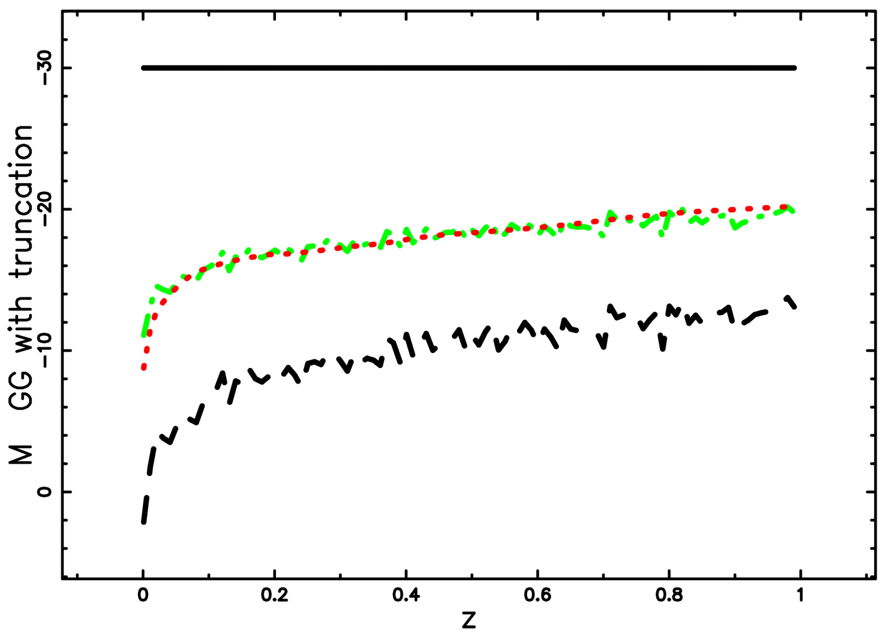

Figure 3. Average observed absolute magnitude versus redshift for SDSS galaxies (red points), average theoretical absolute magnitude for the truncated generalized gamma LF as given by Equation (35) (dot-dash-dot green line), the lowest absolute magnitude is $M = -30$ (full black line) and the highest absolute magnitude at a given redshift as given by the maximum value of the selected sample (dashed black line); RSS = 33.94. The other parameters for the truncated generalized gamma LF are: $M^* = -21$, $a = 0.1$ and $c = 0.2$.

6. Conclusions

Absolute magnitude: We derived the absolute magnitude as given by Equation (13) for the SDSS galaxies. The MPE turns out to be approximately 7.46% when a comparison between the above absolute magnitude and that reported in the SDSS DR12 catalog is done.

Average Magnitude versus redshift: The average absolute magnitude of the SDSS galaxies is reasonably fitted by the average absolute magnitude as given by the truncated Schechter LF, see Equation (31) and by the truncated generalized gamma LF, see Equation (35). In order to perform a reasonable fit, we inserted as lower and upper absolute magnitudes those given by the minimum and maximum values of the selected bin in redshift. The RSS turns out to be smaller when the truncated generalized gamma LF is used.

Euclidean Universe: All the formulae here adopted are deduced in the framework of an Euclidean Universe with minimum difference in respect to the standard cosmology.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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