

# How to Use Starobinsky Inflationary Potential Plus Argument from Alder, Bazin, and Schiffer as Radial Acceleration to Obtain First Order Approximation as to Where/When Cosmological Constant May Form

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## Abstract

Using the Klauder enhanced quantization as a way to specify the cosmological constant as a baseline for the mass of a graviton, we eventually come up and then we will go to the Starobinsky potential as a replacement for the term  $N$  used in Equation (3) and Equation (4). From there we will read in a way to describe conditions allowing for where the cosmological constant may be set. The idea also is to describe a regime of space-time where the initial perturbation/start to inflation actually occurred, as is alluded to in the final part of the document.

## Keywords

Radial Acceleration, Cosmological Constant, Raviton

## 1. Input from General Relativity First Integral From [1] [2] [3] [4]

We use the Padmanabhan 1<sup>st</sup> integral [1] [2] of the form, with the third entry of Equation (1) having a Ricci scalar defined via [9] and usually the curvature  $\mathfrak{R}$  set as extremely small, with the general relativity [3] [4]

$$\begin{aligned} S_1 &= \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda) \\ &\& - g = -\det g_{uv} \\ &\& \mathfrak{R} = 6 \cdot \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{\mathfrak{S}}{a^2} \right) \end{aligned} \quad (1)$$

Also, the variation of  $\delta g_n \approx a_{\min}^2 \phi$  as given by [5] [6] will have an inflaton,  $\phi$  given by [6] [7] [9] leads to [7] [8] to the inflaton, which is combined into other procedures for a solution to the cosmological constant problem. Here,  $a_{\min}$  is a minimum value of the scale factor [8] [9].

### Next for the Idea from Klauder

We are going to go to page 78 by Klauder [4] of what he calls on page 78 a restricted Quantum action principle which he writes as:  $S_2$  where we write a one to one equivalence as in [1], which is also seen in [3]

$$S_2 = \int_0^T dt \cdot [p(t)\dot{q}(t) - H_N(p(t), q(t))] \approx S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda) \quad (2)$$

Our assumption is that  $\Lambda$  is a constant, hence we assume then the following approximation from [3]

$$\begin{aligned} \frac{p_0^2}{2} &= \frac{p_0^2(N)}{2} + N; \text{ for } 0 < N \leq \infty, \text{ and } q = q_0 \pm p_0 t \\ V_N(x) &= 0; \text{ for } 0 < x < 1 \\ V_N(x) &= N; \text{ otherwise} \\ H_N(p(t), q(t)) &= \frac{p_0^2}{2} + \frac{(\hbar \cdot \pi)^2}{2} + N; \text{ for } 0 < N \leq \infty \end{aligned} \quad (3)$$

Our innovation is to then equate  $q = q_0 \pm p_0 t \sim \phi$  and to assume small time step values. Then

$$\Lambda \approx \frac{-\left[ \frac{V_0}{3\gamma-1} + 2N + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2} \right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} + \left( 6 \cdot \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right) \right) \Bigg|_{t=\tilde{t}} \quad (4)$$

These are terms within the bubble of space-time given in [1] using the same inflaton potential. The scale factor is presumed here to obey the value of the scale factor in [9].

## 2. Modification of Equation (4) If the Term of $N$ in Equation (3) We Replace $N$ by the Starobinsky Potential

In order to do this, we will replace the term,  $N$ , as appearing in Equation (3) and Equation (4) by the Starobinsky early inflation potential.

From [10] pp 152 to 153, where we assume  $q \sim 1$

$$N \approx V(\phi) = V_0 \cdot (1 - \exp[-q \cdot \phi/m_p])^2 \quad (5)$$

Also, making use of [7] and

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 \quad (6)$$

And

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{-t \cdot \gamma \cdot (3\gamma-1)}{m_p G} \cdot \sqrt{\frac{1}{8\pi} + \frac{8\pi G}{3}} \cdot [V_0 \cdot (1 - \exp[-q \cdot \phi/m_p])^2] \quad (7)$$

Where we have used

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \equiv \frac{8\pi G}{3} \cdot \left[ V_0 \cdot (1 - \exp[-q \cdot \phi / m_p])^2 \right] \tag{8}$$

And so we obtain if we have a scale factor behaving as

$$\begin{aligned} \Lambda &\approx \frac{-\left[ \frac{V_0}{3\gamma-1} + 2N + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2} \right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} + \left( 6 \cdot \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right) \right) \Big|_{t=\tilde{t}} \\ &\approx \frac{-\left[ \frac{V_0}{3\gamma-1} + 2V_0 \cdot (1 - \exp[-q \cdot \phi / m_p])^2 + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2} \right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} \\ &\quad + 6 \cdot \frac{-t \cdot \gamma \cdot (3\gamma-1)}{m_p G} \cdot \sqrt{\frac{1}{8\pi} + \frac{48\pi G}{3} \cdot \left[ V_0 \cdot (1 - \exp[-q \cdot \phi / m_p])^2 \right]} \end{aligned} \tag{9}$$

The time interval we are specifying in Equation (9) is of about Planck time.

We can write an expression for  $V_0$  from [10], page 153 taking the form of

$$V_0^{1/4} = \frac{0.022 m_p}{\sqrt{q N_{e\text{-folds}}}} \tag{10}$$

This value will be important when we make a linkage to Gravitons in the end of this analysis.

To complete our analysis, we will go to [11], and an interesting piece of analysis, about the cosmological constant. Before doing that we will say a bit about evaluation of a term in one of the denominators of Equation (9).

### 3. Addressing the Term of $\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x$ in Equation (9)

Here  $-g$  as in  $\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x$ .

We will be assuming, also that as from [7] that

$$\begin{aligned} a(t) &= a_{\text{initial}} t^\nu \\ \Rightarrow \phi &= \ln \left( \sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu-1)}} \cdot t \right)^{\sqrt{\frac{\nu}{16\pi G}}} \\ \Rightarrow \dot{\phi} &= \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\ \Rightarrow \frac{H^2}{\dot{\phi}} &\approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{1.66^2 \cdot g_*}{m_p^2} \approx 10^{-5} \end{aligned} \tag{11}$$

The numbers here are extremely small, hence we will be considering a restricted version of  $g$  in  $\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x$  with regards to [11].

From [11], page 399, in what is called considering the dynamics of a non vanishing cosmological constant, namely if  $\phi$  is the classical mechanics potential

for how a particle travels along a geodesics of a metric so considered

$$\begin{aligned} g &\rightarrow -g_{00} = -1 - 2\varphi/c^2 \\ \varphi &= -\frac{\kappa M}{r} - \frac{1}{6} \cdot \Lambda \cdot c^2 r^2 \end{aligned} \quad (12)$$

The claim from page 399 of [11] is that then there would be a net acceleration of a test particle, we could write as

$$a_{\text{radial}} = \frac{\Lambda}{3} \cdot c^2 r \quad (13)$$

Our supposition that such a small initial acceleration would be pertinent to the production/acceleration of Gravitons from the initial part/place of expansion of the universe, *i.e.* what would be the starting gun for inflation.

If this were the initial nudge of space-time this would be commensurate with the following interpretation of  $\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x$  with regards to [11] will to a certain degree presuppose not paying attention to the following initial line metric

$$\begin{aligned} dS_{\text{Schwarzschild}} \Big|_{\text{Spatial part line element}} \\ \approx dJ^2 \Big|_{M \rightarrow 0} = (1 - \Lambda r^2/3)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \end{aligned} \quad (14)$$

I am pre supposing that in the initial stages of space-time that the term  $g$  appearing in  $\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x$  with regards to [11] would be treatable as being a constant, so as to place the initial configuration of the cosmological “constant” parameter as a straight out inter-play between a positive set of terms, and a negative set of terms, so as to nearly cancel, which is such that if the initial acceleration as to gravitons would be, at its initial inception very small.

However, once perturbed, *i.e.* a barrier of sorts breached, inflation would commence as a violent universe creating phenomenon.

For the record, the term  $r$  in Equation (13) would be assumed to be very small, of the magnitude of Plancks length, whereas the driving force to expansion would be doable as linked to Equation (10)

Note plank mass would be presumed to be about  $2.1764 \times 10^{-5}$  grams whereas a common mass of the graviton, if it has mass would be about  $10^{-65}$  grams, whereas, if we have by [12] [13] compared with Equation (10) the mass-energy equivalent of  $10^{62}$  gravitons, which may be in the situation specified as to the cosmological constant

#### 4. Conclusion: Do We Have a One to One Correspondence via This “Cosmological Constant” Argument in Magnitude with the Mass of a Massive Graviton?

Klauder’s program [4] is to embed via Equation (6) as a quantum mechanical well for a Pre Planckian-system for inflaton physics as given by Equation (3). As given in Klauder’s treatment of the action integral as of page 87 of [4] where Klauder talks of the weak correspondence principle, where an enhanced classical Hamiltonian is given one to one correspondence with quantum effects, in a non-

vanishing fashion. If so, by Novello [3] [12]

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{15}$$

This formula needs to be confirmed via likely CMB radiation as could be ascertained by interaction of the CMB spectra, and gravitational waves. However, if Equation (15) were true, it may mean that the initial regime of cosmological constant creation may be able to give a description as to what exactly initiated the transition to inflation, to begin with. And that is an open question.

The author has presupposed all of this using the following Uncertainty principle [13] [14] [15] [16]

$$\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_p^2 \frac{\Delta E}{\hbar} \Rightarrow (\Delta E)^2 - \frac{\hbar \Delta t}{\gamma t_p^2} (\Delta E) + \frac{\hbar^2}{\gamma t_p^2} = 0$$

$$\Rightarrow \Delta E = \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left( 1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_p^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_p^2}\right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left( 1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}} \right) \tag{16}$$

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left( 1 \pm \left( 1 - \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2} \right) \right) \tag{17}$$

$$\Rightarrow \Delta E \approx \text{either } \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}, \text{ or } \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left( 2 - \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2} \right)$$

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2} \equiv \frac{4\hbar}{\Delta t} \tag{18}$$

Our supposition is that the initial radial distance regime of creation of space-time allowing for a cosmological constant at the beginning of the universe is commensurate with a time interval  $\Delta t$ . Again, if say  $10^{62}$  gravitons may be produced as of the time when a cosmological constant is produced, and if some of the gravitons are created by the decay of mini black holes, this says a lot about initial conditions which may be verifiable later as to suitable data sets in gravitational wave astronomy

In particular, Equation (10) is directly from Starobinsky as reported in [10] and it would be of stunning import if the following is confirmed

$$V_0^{1/4} \approx \frac{5.4 \times 10^{16} \text{ GeV}}{\sqrt{q N_{\text{efolds}}}} \tag{10}$$

Also see reference [17]-[22].

The term  $q$  is of about 1 in magnitude, where as  $N_{\text{efolds}}$  is the number of evoids in inflation. Tying this in, say if a graviton were of  $10^{-65}$  grams would be of huge importance as far as GW astronomy is concerned.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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