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Dark Matter: An Unforeseen Gravitational Property of Relativistic Space-Time?

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Abstract

If the wave functions of matter expanded with time dilation for an outside observer in the same way as photons do in gravitational redshift; with some modifications the general relativity might alone explain dark matter, galaxy rotation curves, and part of the energy released in supernova explosions. Also, the event horizons of black holes couldn't be formed when packing matter more and more densely together. Essentially, if the time dilation increases enough, the particles turn less localized to outside observers and the mass distribution of the same particles would expand into larger volume of space. Small particles deep inside a black hole might seem like dark matter instead by their gravitational influence if the time dilation alters their size enough for outside observers. At the same time, the surface particles of the black hole would be less dispersed, creating the Newtonian gravitational potential we see closer to black holes. The following research doesn't attempt to reformulate the general relativity itself, but only proposes the idea while approximating the Milky way gravity profile to compare the hypothesis with measurements. Therefore, actually proving the hypothesis is still far off while the idea is sound at its core.

Keywords

Dark Matter, Black Holes, Gravitation, Supernova

1. Introduction

Albert Einstein's general relativity is often described as something rather difficult to comprehend and master, but it explains the properties of the spacetime extremely well. Phenomena like gravitational time dilation, gravity lensing of the light, Shapiro delay [1] and orbit of Mercury have all been explained by it. One of the basic formulae we learn when studying relativity simplifies into the fol-

lowing form

$$v_x^2 + v_y^2 + v_z^2 + \tau^2 c^2 = c^2, \qquad (1)$$

where v_x is the velocity scalar in x-direction, v_y in y-direction, v_z in z-direction; τ is the speed of time with c being the speed of light. The formula explains that the speed of time depends on the relative velocity v. With few leaps of deduction, one can also say any ordinary electromagnetic wave function must travel at the speed of light (whether it propagates in time or space or in both). For example, the simplest electromagnetic wave we know, the photon, follows this formula by travelling only in space, and not even the slightest tick in time. A moving observer doesn't see any change in speed of photon because the photon does not propagate in time ($\tau = 0$). Therefore, a photon must always travel at the speed of light independent of any observer. Being inside a gravity well doesn't affect its speed either, but only its relative energy to the observer. However, as Shapiro time delay indicates, high gravity compresses space-time in such a way that the photon travelling inside a big gravitational potential well must propagate a longer distance than what outside observer sees, making it look like it was moving with speeds lower than c.

Consider next our simplest known wave, the photon. When it exits a gravity well, its energy seems to decrease from outside observer perspective. The photon wavelength increases in gravitational redshift [2]. The photon stretches both in time and space. However, does the photon redshift, while leaving the gravitational potential or is the photon always the same, its energy just being relative to the observer?

The gravitational redshift was first explained by Einstein back in 1916 [3]. Observe a single photon being emitted from gravitational potential well. The photon arises from the potential well. We observe the photon outside the well and we see it redshifts only by the time dilation factor itself. As Einstein stated, it is logical to assume the photon does not change its wavelength nor its frequency as it travels. Only observers see it differently according to their own environmental properties of relativistic space-time.

How about the charged particle which creates the photon deep inside the gravitational potential well? It would be easy to assume that this particle also looks larger and time dilated for the outside observer. However, we also know from general relativity that the space itself compresses in such a place and therefore, while the particle is larger, it looks regular sized for an outside observer (at least while observing electromagnetically).

The next thought chain supports the conclusion: Particles expand in space if inserted into a high time dilation zone. We know we can create matter by colliding ordinary photons in proper conditions [4]. Therefore, it is safe to assume the wave function of the created matter to also travel at the speed of light. The oscillation frequency and the waveform of this wave function are unknown, but let us just assume the frequency is finite. If the observed time dilation increased in the location of the particle, the observer would see this wave function fre-

quency shift equally. If frequency drops and wave function still travelled at the speed of light, it would mean that the path length of a single wave function oscillation would increase with a similar factor (at least if we consider that the other geometry of the wave function would stay similar and the wave function itself is not infinitely small). As mentioned earlier, general relativity takes this into account by introducing spatial compression inside deep gravitational potential wells. The spatial compression counters the earlier wave function path length increase. Observers can't directly see any change in the particles.

Now the same again with more details: How the gravitational field forms for a group of particles? Current understanding of quantum singularities implies that when we bring enough particles together in a small, dense zone, an event horizon forms as the summed gravitational field from all the particles crush the particles in the same point. In this zone, gravitational potential well is so deep that even light can't escape from inside—every direct line in space eventually hits the same singularity. How would such an object form? What happens when time dilation starts to increase faster and faster.

2. The Mass Distribution of Wave Function and Gravity

How should we approximate the gravitational potential of particles like neutrons or protons? Often in quantum mechanics we assume wave packets are Gaussian distributed and with that information we can expect that the mass creating the space-time stress is also Gaussian distributed at the location of the particle. We are next evaluating a test gravitational potential for a subatomic particle. We can assume the radius of a neutron is roughly 8×10^{-16} meters (it is roughly the measured size of the particle). Gravitational acceleration for stationary object far enough away from this neutron is approximated with the familiar simple Newtonian formula

$$a_g = \gamma \frac{m_{\text{neutron}}}{r^2}, \tag{2}$$

where a_g is the scalar of gravitational acceleration, γ is the gravitational constant and r is the distance from the center of the neutron. Now, what is the gravitational field inside the neutron? If we assumed the mass was Gaussian distributed inside the particle, we can write the equation

$$a_g = \gamma \frac{m_{\text{neutron,eff}}}{r^2}, \tag{3}$$

with the new $m_{\text{neutron,eff}}$ being the expression of the neutrons encircled mass inside the radius r. We need to calculate a single integral to find out the expression for the encircled mass. Assuming the wave function and mass are Gaussian distributed inside the sphere in 4-dimensional space-time:

$$m_{\text{neutron,eff}} = \frac{3}{2\pi r_p} m_{\text{neutron}} \int_0^r \frac{4\pi \left(\frac{x}{r_p}\right)^3}{3} e^{-\frac{x^2}{r_p^2}} \, dx. \tag{4}$$

Here x is the axis with its zero being the center of the mass distribution and r_{p} is the defined size of the particle (or neutron in our case). The factor in front of the expression is just a normalizing term as the integral should reach $m_{\mathrm{neutron}} \cdot 1$ when x (distance from the center of the sphere) is nearing $+\infty$. It is good to note we are here assuming all the mass resides in flat space-time with no gravitation. The assumption should suffice well enough for our purposes. Assuming the mass fills a 4-dimensional space-time yields very similar results when compared with the assumption of it residing only in 3-dimensional space. We are assuming the mass is distributed in 4-dimensional space-time. After evaluating the integral, we get a simple function for the enclosed mass:

$$m_{\mathrm{neutron,eff}} = m_{\mathrm{neutron}} \frac{r_{\mathrm{p}}^2 - \mathrm{e}^{\frac{-r^2}{r_{\mathrm{p}}^2}}\left(r_{\mathrm{p}}^2 + r^2\right)}{r_{\mathrm{p}}^2}. \tag{5}$$

This is the mass of our particle inside the r-radius sphere. From here, it is a simple task to approximate how much gravitational acceleration the particle produces at the distance of r from the center of the particle. The gravitation depends only on the encircled mass. From Newtonian approximation, we can write the gravitational acceleration inside and outside the neutron to be

$$a_{\mathrm{g,n}}\left(r\right) = \gamma m_{\mathrm{neutron}} \frac{r_{\mathrm{p}}^2 - \mathrm{e}^{\frac{-r^2}{r_{\mathrm{p}}^2}}\left(r_{\mathrm{p}}^2 + r^2\right)}{r_{\mathrm{p}}^2 \cdot r^2}. \tag{6}$$

The gravitational acceleration reaches the greatest value close to the imagined "surface" of the neutron and then begins to drop drastically like our usual $\frac{1}{r^2}$ force dependency dictates.

A measurement device inside the particle can't see the gravitational acceleration because most of the particle's mass is around the measurement device like shown in **Figure 1**. Next, we are considering high density particle groups and approximating gravity near them.

3. Rotation Curves around a Quantum Singularity

Einstein's general relativity dictates that the speed of light is locally a constant for all the reference frames while the speed of time and curvature of space is free to change with respect to gravitational fields and movement. Every possible test that we have made for general relativity has so far proven the theory correct from time and again. Almost every possible test has however been electromagnetic at their core. For example, we have barely been able to get experiments to register gravitational waves [5].

The next idea is the big leap into the unknown in this research. What if the mass distributions of particles are observer dependent? In other words, what if the mass distributions are not compressed gravitationally for outside observers like the electromagnetic fields are. Essentially, this would require the reformulation
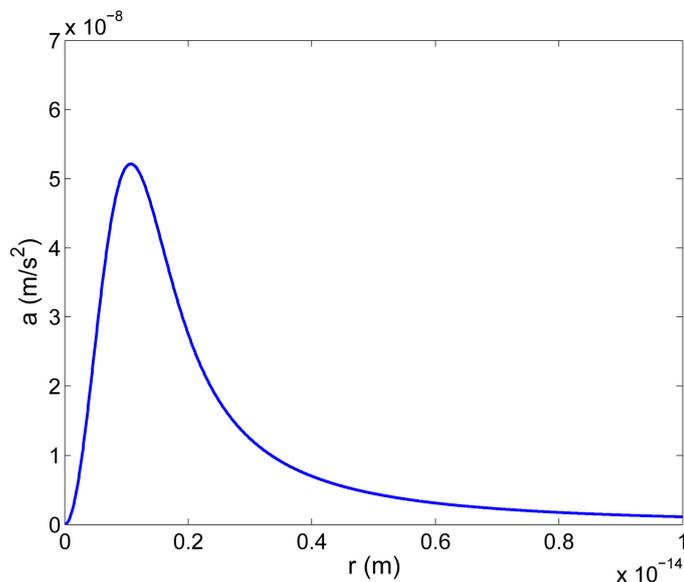
Figure 1. Gravitational acceleration caused by a regular neutron. The field strength is zero close to the center of the particle.

of the general relativity, but it shows a large difference for calculating gravitation in locations where there are particles with mass in nearby deep gravitational potential wells.

Let's test what happens in the gravitational field of an object that is highly compressed—compressed enough to otherwise produce a quantum singularity. Earlier in Equation (5) we calculated the mass enclosed in a finite size sphere with Gaussian mass distribution in 4-dimensional space-time.

Let us assume the mass distribution is symmetrical, for otherwise our equations would turn out complex. How should we start to calculate the gravitational field from the object? We know that mass (or energy) generates the gravitational field by stressing the space-time. We also know that we can approximate the gravitational field by Newton's Equation (2) if there is no movement or very little movement and field strength is reasonably low. We only need to know how much there is mass in the "sphere" we are either standing on or observing from afar. If we are inside the sphere, we can ignore the layers of the sphere that is above us.

Let us continue with this knowledge and observe our dense object. Let's assume the particles are just so close to each other they are overlapping, and the gravitational field is strong enough to keep them that way. When we start to escape the object, the time dilation increases for all the mass inside the object because of high gravity.

The speed of time slows down by following expression

$$T_d = \mathrm{e}^{\frac{1}{c^2} \cdot \int_{h_1}^{h_2} g(h)\mathrm{d}h},$$ (7)

where T_d equals the time dilation factor, h_1 and h_2 the distances from the mass to the object and the observer, $g(h)$ the gravitational field strength and

dh represents the integration direction.

Like we discussed earlier in Chapter 2, the wave functions of the matter also stretch into space because the matter resides in the gravitational potential well. If the static space-time stress is not compressed with gravity along electromagnetic energy and what we usually call empty space, we can see that the particles grow inside the gravitational potential well if observed through their gravitational field. Ordinarily, the space itself stretches just like time inside the gravitational potential well. This would mean that the mass distribution of the particles also gets diluted into a larger zone of space (even for an outside observer). If the gravitational field is strong enough, it becomes clear that the particles are gravitationally big enough to exist mostly outside the zone of space the observer assumes the gravitation to emanate from.

Now, let us insert more particles in the same position, their total mass being 1×10^{40} kg. We can easily deduct with our current understanding of physics how that would form a quantum singularity, massive enough to look like what is at the center of most galaxies. Let us observe how the gravitational field looks like if the particles were in a higher time dilation. Say the time dilation would slow down time by a factor of 1×10^{36}. What is the size of the particles in this case if it is relative to the observer? If the particle radius is around 8×10^{-16} m for observers in the same space-time, the particles expand with the factor of time dilation for the observer outside the potential well. Why? Like discussed earlier, the wave function travels at the speed of light, so if the wave function frequency shifts, it must travel a longer route for an outside observer because the speed of light is constant for everyone. Therefore, the gravitational acceleration outside the gravitational potential well of the object should be dictated roughly by following expression

$$a_{\text{g,o}}\left(r \right) = \gamma m_{\text{obj}} \frac{r_{\text{p}}^2 \cdot T_d^2 - \mathrm{e}^{\frac{-r^2}{r_{\text{p}}^2 \cdot T_d^2}}\left(r_{\text{p}}^2 \cdot T_d^2 + r^2 \right)}{r_{\text{p}}^2 \cdot T_d^2 \cdot r^2}. \tag{8}$$

From Figure 2 we can easily see the resemblance to dark matter halo gravity in most galaxies. The only real difference here is that calculated mass distribution is not high enough further away from the center of the black hole to create enough gravitational pull to account for dark matter [6]. Also, it is not realistic to say the gravitation would disappear near the mass concentration itself. Even recent reliable measurements for Sagittarius A* tell us otherwise [7]. Also, it is arrogant to assume all the angular momentum collected by the black hole could reside in a singular spot with the particle group. Most likely we can assume the mass is residing on a disc or ring-like distribution, rotating at rapid speeds as many other research articles have hypothesized [8].

Consider a following situation where the mass resides on a stable distribution (possibly on a rotating ring-like structure). Part of the mass is then located deeper inside the gravitational potential well and some part not as deep inside the gravitational potential well. How will the gravitational field look now? Let's also
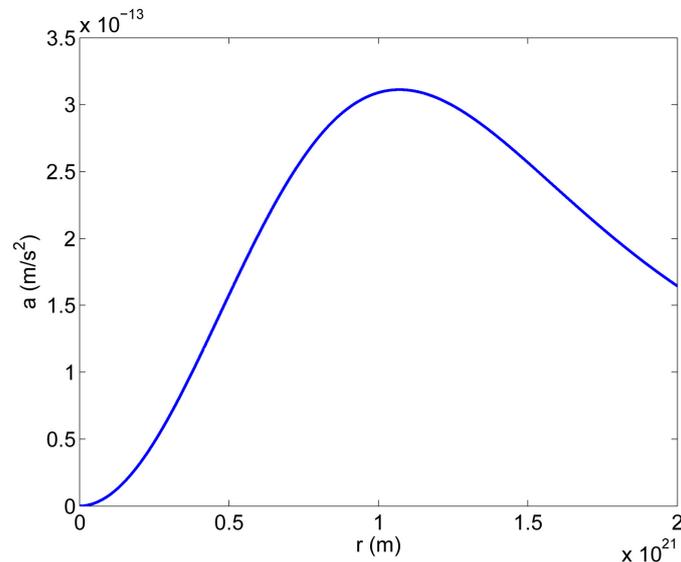
Figure 2. Gravitational acceleration caused by an imaginary black hole with all matter residing in the same place. The field strength is zero close to the simplified black hole if all the particles are inside a zone where the time-dilation is high. Here we do not consider having the time dilation decrease when approaching the object.

calculate how the time-dilation shifts from low to high, when moving further away from the dense construct of matter. The only thing left is to compare the computational gravitational field of the object with measurements at close ranges to black hole Sgr A* and rotational curves of the Milky way (the dark matter distribution would consist of all the black hole cores in the Milky way, but this approximation proves the point of this research well enough).

Next, simplify our calculation for this more complex object by assuming all the mass fills a certain sphere and by forgetting all movement. We don't really have to know the details of black holes yet to prove the viability of the idea of having dark matter just be time-dilated regular matter inside black holes. Let us assume the matter distribution in the actual black hole is such that most of the mass resides on high time dilation zones and less mass on the zones closer to the surface of the object. Here with the surface, we are referring to the location where there is no time dilation before escaping the object.

The gravitational acceleration is the sum of all the mass distributions that are inside the r-radius sphere. Solving the differential equation for gravitational acceleration is difficult analytically, because the distance travelled from the black hole affects the time dilation, which affects the gravity and then again gravity is what causes the time dilation. Therefore, it is easier to input the problem for a computer to solve. The problem consists of almost the same equations seen in the text earlier. First, the gravitation is the sum of all the gravitational effects of all the particles. There is a great number of particles, so we can ease out the computation by assuming there is mass only at discrete time dilation zones (just to make the numeric modelling computationally light enough for practical purposes. The particles can exist in any time dilation zone, depending on their loca-

tion inside the object). For our approximation, let's model the black hole by having mass at 100 different layers that have all a specific time dilation

$$a_{obj}(r) = \sum_{i=1}^{100} a_{\mathrm{g},i}\left(r, T_{d,i}^2, m_i\right). \tag{9}$$

Here, the gravitational field strength is a sum of all the gravitational fields from all those earlier 100 different layers of mass. Moving away from the black hole in such high gravity, we start to observe an increase in time dilation. Equation (7) dictates the change. When we are using a computer to calculate the change in time dilation, we are moving in very small steps and can also use the approximation

$$T_d(r+\mathrm{d}r) = T_d(r) \cdot \mathrm{e}^{\left(a_{obj}(r)\cdot\mathrm{d}r\right)}. \tag{10}$$

After each step, we must re-calculate the gravitational acceleration until we are far enough from the black hole. All in all, the problem is straightforward to solve, but calls for great care with numerical calculation accuracy, especially when using short step lengths near the black hole.

Now, as we can calculate approximate gravitational field strengths from black holes, we are free to use the program to search for mass profiles that roughly match the dark matter halos of galaxies. After finding one, it is a simple process to use a least square method or similar to tune the mass profile to exactly match wanted gravitational profile. Let's take the mass profile of the Milky way supermassive black hole at Sgr A* [9] and the dark matter halo of the Milky way [6] combined and assume the radius of the black hole is 100,000 km. We get a mass profile for the supermassive black hole and it doesn't look particularly odd either in Figure 3.

As we can see, the mass is mostly in high time dilation zones and only a small fraction is near the surface. The form of the distribution can possibly be explained by assuming that the matter falling inside is slowing down because of the general relativity and can't reach the center of the object in practical time limits. This might possibly mean that the supermassive dark matter halo part of the black hole is very ancient and hasn't grown in mass since the beginning of the universe (black hole collisions might be an exception, but that aspect is not considered in this research). Total mass for all the discrete mass zones is 2.0×10^{42} kg. Best to keep in mind that this is a very crude estimate without taking account most of general relativity. With the static mass profile we get the following gravitational field, matching fairly well the measurements in Figure 4.

Black hole mass distribution seems to produce a matching gravitation if compared with the dark matter measurements. Can we assume static space-time stress is not compressed inside a gravitational potential well for an outside observer? The preliminary results of this research are indeed promising, but the results were obtained by blatantly dismissing most of general relativity. One would have to reformulate general relativity for most part to actually test the idea thoroughly.
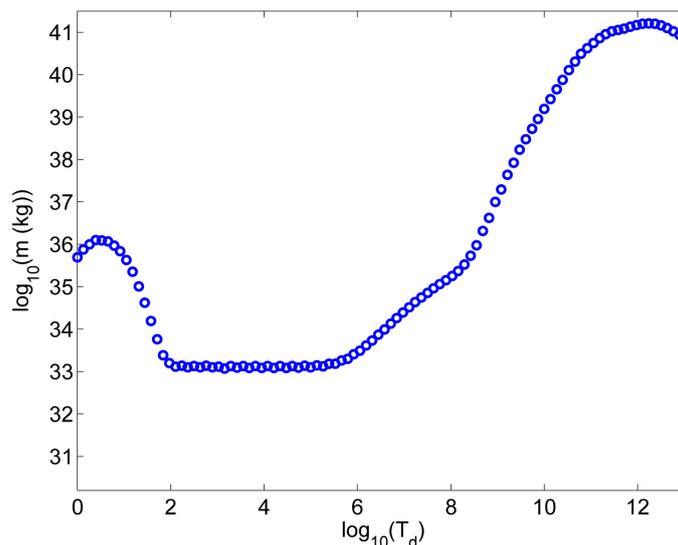
Figure 3. The gravitational acceleration matches the combined dark matter halo and Sgr A* measured gravity. Black hole mass is discretely distributed to 100 different time dilation zones in this model. The complete dark matter halo should be modelled by considering all the black holes in the galaxy, producing a small error in this research. The observer is on the surface of the black hole where the closest matter is in the same space-time as the observer (with no gravitational time dilation). After climbing out from the high gravity, the time dilation has increased (on the surface and otherwise inside the object) roughly by a factor 3.3×10^{24}. The form of the smaller mass distribution closer to the surface can't be estimated accurately with current galactic gravitational measurements and could have a different shape in reality. Also, the mass of the accretion disc is assumed to be zero in this research.
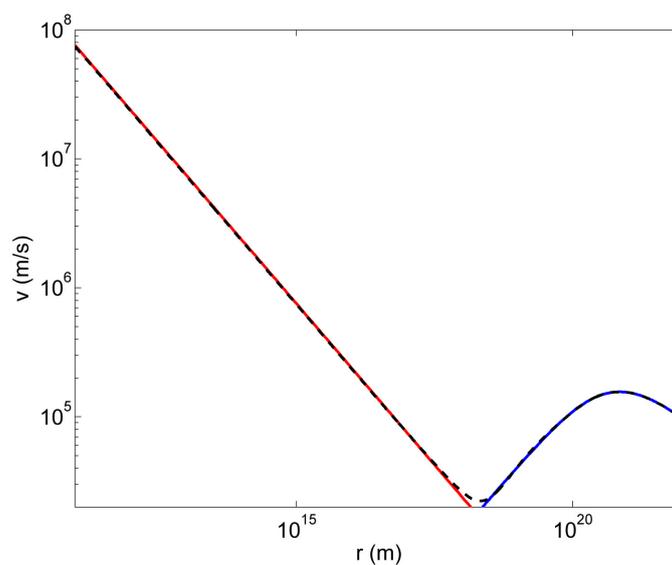

Figure 4. The measured rotational curves at the Milky way from the Sgr A* (red) and the dark matter (blue) in comparison to the modelled rotational curves of the black hole with the calculated mass profile (black, dashed). The rotation speeds are the rotation speeds of objects in circular, stable orbits. The gravitational potential is almost completely Newtonian near the black hole and the distribution of mass in the high time dilation shows itself as the dark matter halo.

4. Conclusions and Thoughts

Do the wave functions of matter expand gravitationally to a larger volume of space for an outside observer if the source of the space-time stress is inside a gravitational potential well? While the tests in the previous chapter look compelling, it is best to remember that this research based on rather many approximations. For example, general relativity would need almost complete re-writing to include the idea. Would it be possible without breaking all the other fully functional aspects in Einstein's masterpiece? That is not an easy task to test.

There are already multiple competing theories trying to explain dark matter. Any of the extended gravitational theories or the suggested particle candidates may still explain dark matter correctly. This research only explores an alternative approach to explain dark matter if none of the earlier theories can be verified in the coming years.

This research didn't cover any movement of mass in black holes and any other black holes of the Milky way aside than Sgr A*. Therefore, the shape of the dark matter halo in this research is strictly spherical while realistic halos can possibly be slightly flattened. Also, gravitational waves travel at the speed of light [5] and it is not obvious whether this is possible while assuming that the gravitational size of particles is observer dependent. Equally, the shape of the emitted gravitational waves might be affected by this assumption, possibly disproving it [10].

The basis for this research could be condensed into the following statement:

"Static gravitational fields form in a way that the particle generating the field is seen in the Cartesian coordinate frame of the stationary observer. When the time dilation increases for the particle, it starts to appear gravitationally bigger to the outside observer".

According to the idea presented earlier, the matter of black holes shouldn't exist in infinite time dilation. This would also mean that there are no event horizons of the kind previously assumed. Before time dilation could increase enough, the gravitation near the big dense object would vanish because the particles would start to appear less localized to outside observers. This would mean some radiation should escape black holes, but with a huge redshift. Then again, the spectrum of a rotating black hole is somewhat difficult to grasp with intuition alone and would also need more studying. Also, the time dilation on the surface of such an object is very high and the black body radiation intensity escaping outside should be very small (almost non-existent).

The interesting result of space-time stress being observer dependent can possibly be seen in supernova explosions. When a massive star starts to collapse on its own gravity at the end of its lifespan, it often forms a black hole and shines very brightly, very briefly. If a small black hole formed from a considerable amount of the star's mass, a significant amount of the near gravity would also disappear in a very brief instant, leaving the remaining hot plasma free to explode in space with the photons that are on their route to the surface of the star. This might account for some of the energy released in such explosions as the hot

plasma is just contained in the high gravitational field itself and would be further heated by gravity shortly before the time dilation increases enough at the center of the black hole to alter the gravitational profile.

The surface matter mass of a black hole is depending more on the actual size of the black hole than the amount of the inner time dilated mass. Therefore actually recognizing how much dark matter is one black hole holding is not an easy task to measure. Does the inertial mass of the black hole differ from the mass that generates its near gravity? There is a good chance; many of the black holes may have more inertial mass than we had previously assumed just by observing their near gravity. However, the complete answer to the posed question is still outside the scope of this research and additional measurements are called for.

The idea of static gravity forming in a gravitationally non-curved space-time of the observer should also be possible to disprove if it is incorrect. For example, if we find galaxies with dark matter and no supermassive black holes, this research couldn't be true (as an example in M33, smaller intermediate-size black holes might carry far more invisible dark matter mass in their inner cores than it seems for an outside observer). The same would hold for finding a galaxy with a black hole with a supermassive inner core, but no dark matter. Then again, one research claims finding a galaxy (NGC1052-GR2) which lacks the dark matter [11]. However, it is still unclear whether this galaxy also lacks the central black hole or has dark matter after all. The hypothesis of this research would link all the black holes and the dark matter together. Also, this research would claim that "the dark matter cores" of the black holes shouldn't grow in mass any further, but they might change their gravitational geometry if more matter drops in the surface of the supermassive black hole and increases the time dilation at the center of it (although, black hole collisions might be an exception). That is also consistent with the observed behaviour of dark matter—we can see different shapes of dark matter halos [12] and relatively young galaxies with plenty of dark matter [13]. However, if we were to observe that dark matter halos are not more compact in very young galaxies on average when compared with old ones, we might also surmise that the hypothesis in this research is incorrect.

The hypothesis in this research is not confirmed in the previous chapters, but rather found to be consistent with measurements so far in the few angles approached from. There are still many ways to try to disprove it. While the research looks good in principle, the author of this research has not yet made any serious attempt to modify how space-time stress is calculated in general relativity itself. It could be very well so that the general relativity can't be formulated to include this idea while keeping the other aspects of the theory intact. Also, movement in multiple black hole systems has not been tested. In addition, the formation process of the supermassive inner cores of the black holes is also a mystery. Finally, if we cannot find stable black hole structures with appropriate mass profiles that fulfil the rules of mathematics, this is all just philosophy with no link to reality. I leave the deduction for whether this hypothesis has any merit

to you, the readers. The idea is basically only a plan to modify the gravitational theory (which was found through a theoretical approach). Including the idea in the general relativity without breaking its fundamental parts is much more complex an issue to tackle on and may ultimately prove to be impossible.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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