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Numerical Method: Heat Conductions of White Dwarfs, Neutron Stars, Black Holes and Derivation of Perfect Black Body Radiation of Black Holes

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Abstract

In this paper, the authors plan to derive the famous perfect black body radiation formula in order to suit the statement for black holes that M. Hawking had ever desired. Based on the calculation resulted by numerical method, a black hole's thermal conductivity is $k = 1.909\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$, in a condition of a black hole undergoes on thermal conduction and information saved or escaped such complies with AdS/CFT correspondence. This can be generalized to the study of electronic engineering considering that the material where computer information can conserve maximum capability (Bakenstein bound) can tolerate and contribute to this exploration returned as to reference study in cosmology and astrophysics. In another respect, neutron stars possess interestingly the critical point of phases: ^{4}He (I/II) and ^{3}He superflow or superglass in our calculations by the same physical principle worked out and this quite supports the other studies.

Keywords

Thermal Conduction, Conductivity, Bekenstein Bound, White Dwarf, Neutron Stars, Black Holes

1. Introduction

Heat transfer is an important subject in science and is one of the cores of classical physics. This subject is not only used in chemical and mechanical engineering,

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but also widely used in astrophysics [1]-[9]. Heat transfer is a discipline of thermal engineering that concerns the generation, use, conversion, and exchange of thermal energy between physical systems. It is classified into various mechanisms, such as thermal conduction, thermal convection, thermal radiation, and transfer of energy by changes of phases. Engineers also consider the transfer of mass of differing chemical species (mass transfer in the form of advection), either cold or hot, to achieve heat transfer. While these mechanisms have distinct characteristics, they often occur simultaneously in the same system. The famous physician M. Hawking once submitted the idea of black hole evaporation, which later became known as the Hawking radiation formula: $T_H = \frac{\hbar c^3}{8\pi G M k_B}$, but this formula only shows the relationship between the black hole temperature and its gravity parameters. Upon investigation, it was found that there is no statement of any Boltzmann-Planck black body radiation formula in references [10]-[16]. This paper uses deductive methods to improve the discussion of many previous studies that used numerical analysis methods to find the key magic number "4", thereby connecting it with **Bekenstein Bound** and finding conductive photometric transfer to be used in a descriptive general equation for heat conduction in compact stars (e.g. the equation for thermal convection applies to neutron stars; the equation to heat conduction applies to black holes). This article is a numerical method and not a theoretical/phenomenological method; astronomy and physics bias a few chemical engineering.

2. Discussions

2.1. Bekenstein Bound

The discussion sections begin with the famous Bekenstein bound, as to be addressed to black-body radiation, eventually. The advantage is quickly forced Hawking radiation connecting with thermal conduction handling object of black holes. Of course, where Newton method is used to unfold the frustration of numerical method when undergoing approximately acquired the routine of Triple-Alpha Process. The black hole's entropy is denoted as well-known:

$$S_{BH} = \frac{k_B A}{4 l_p^2} \tag{1}$$

Via entropy derivations, we obtain that the parameter $\mathbf{4T}$ represents in Equation (1). This origins from the thermal conductivity possesses 4-directions $a_{r_1}, a_{r_2}, a_{r_3}, a_{r_4}$:

$$S = \frac{|\Delta Q|}{T'} = \frac{1}{4\Delta T}\left|\sum_{i=1}^{4} Q_i(r_0) - Q_0(R)\right| = \frac{E_{B.E.}}{4(T - T_0)} \propto \frac{1}{4} \tag{2}$$

where $E_{B.E.} = Mc^2$ and $T' \overset{up}{=} 4\Delta T$. Followings resulted in

$$S_{BH} = \frac{Mc^2}{4\Delta T} \tag{3}$$

Apply $M = \frac{rc^2\hbar}{2G\hbar}$ substituted into Equation (3). Therefore

$$\begin{aligned} S_{BH} &= \frac{\overbrace{4\pi r^2}^{=A_{BH}}}{4\Delta T}\frac{\hbar c/(8\pi r)}{G\hbar/c^3} = \frac{4\pi r^2}{4}\frac{\overbrace{\hbar c/(8\pi r\Delta T)}^{=[k_B]}}{\sqrt{G\hbar/c^3}^{\,2}}, \\ S_{BH} &= \frac{k_B A}{4l_p^2} \end{aligned} \tag{4}$$

This reproduction is very important since it helps one to think the physical sense of form of black hole that required main sequence stars ($\geq 25M_\odot$) raised-up the temperature $T \to 4T$ in the evolution process of collapse.

2.2. The Heat Transfer of White Dwarf

The material in a white dwarf no longer undergoes fusion reactions, so the star has no source of energy. As a result, it cannot support itself by the heat generated by fusion against gravity-collapse, but is supported only by electron degeneracy pressure, causing it to be extremely dense. The physics of degeneracy yields a maximum mass for a non-rotating white dwarf, the Chandrasekhar limit, approximately $1.44M_\odot$, beyond which it cannot be supported by electron degeneracy pressure.

White dwarf masses < $10M_\odot$ is located on in general case. The main mode of heat transfer belongs to **thermal radiation**, but to a certain extent, the problem of white dwarfs can be regarded as through conductive **photometric transfer**.

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{1}{k}Q_r = -\frac{1}{k}\frac{l}{4\pi r^2} \tag{5}$$

Therefore

$$\begin{aligned} &\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{1}{k}\frac{l}{4\pi r^2}, \\ &\int_{T_0}^{T} 4\mathrm{d}T = -\frac{l}{k\pi}\int_{r=R}^{r=r_0}\frac{1}{r^2}\mathrm{d}r \end{aligned} \tag{6}$$

Obviously, we get "**four times original temperature** expanded in radial directions"

$$4(T - T_0) = \frac{l}{k\pi}\left(\frac{1}{r_0} - \frac{1}{R}\right) \tag{7}$$

This equation contained **magic number 4** such actually appeals one that black hole Hawking radiation could be derived the corresponding equation of black hole's black body spectrum (*i.e.*, Stephan-Boltzmann function) M. Hawking desired. As such, this equation is flavored and helpful. Of course above deduction is based on the conclusion that resulted in **Bekenstein bound**.

2.3. The Heat Transfer of Neutron Stars

The majority mode of heat transfer on the neutron star $(1.5M_\odot < M \leq 3.2M_\odot)$

surface belongs to the thermal convection. Indicated local heat flux is **heat convention** and is expressed as

$$\dot{Q} \equiv \nabla^2 Q_r = h(T_s - T_\infty) = h(T - T_0), T > T_0 \tag{8}$$

Therefore

$$h(T - T_0) = l \cdot \nabla^2 \frac{1}{4\pi r^2} = \frac{-l}{2\pi r^3}, r = r_0 \to 0(\text{core}), r = R(\text{surface}) \tag{9}$$

And [2][2],

$$\lim_{T \to T_0} \bar{h}(T - T_0) \overset{(L.H.)}{=} \frac{\dot{Q}}{6\pi r^2} \tag{10}$$

Followings,

$$\bar{h} \equiv \lim_{T \to T_0} \frac{\dot{Q}}{4 \cdot (3\pi r^2/2)(T - T_0)} = \frac{1}{4(T - T_0)} \lim_{T \to T_0} \frac{\dot{Q}}{3\pi r^2/2},$$

$$\frac{2}{3}(T - T_0) \equiv \frac{\dot{Q}}{\bar{h}(4\pi r^2)}, 0 < r \le R \tag{11}$$

$$\frac{1}{3}(T - T_0) \equiv \frac{\dot{Q}}{h(4\pi r^2)}, 0 < r \le R$$

Due to $4\pi \approx 0.5 + 4 \cdot 3!/2$, $4\pi h = h(0.5 + 4 \cdot 3!/2) = 1/3$, thus we have **Heat Transfer Coefficients** $h = \frac{1}{3(0.5 + 4 \cdot 3!/2)} \approx 0.027$ [W/m^{2}K][1]. By this thread, see **Triple-Alpha Process**. And due to Equation (10), such both lead that the surface ***R*** to the inner-core r_0 of neutron stars the heat convention corresponds to **SUPERFLOW** coefficients (*i.e.*, **SUPERFLUIDITY**). Supposed that its physical behaviors are like **^{4}He (I/II)** [or superglass/supersolid] in regions of strong magnetic fields, otherwise **^{3}He** in the outer core through 9 km of neutron stars (*i.e.*, regions of weak magnetic fields) [1] [2]. This mathematical deduction by factorial ($n!$) for cosmology-physics is well to use, because it can reduce the complexities of deduction process.

2.4. The Heat Transfer of Black Holes

Based on the previous deductions and the logics, we can think that the one of the three mode heat transfers such can be worked out the famous black hole's perfect black body radiation formula that Hawking desired. Here the **thermal conduction** is the definite option as deduction and does not violate any fundamental physical principle. In detailed:

$$4(T - T_0) = \frac{l}{k\pi}\left(\frac{1}{r_0} - \frac{1}{R}\right), r_0 \to 0(\text{singularity}), R = r_s \tag{12.1}$$

[1] *i.e.*, Newton method is used to solve ***h*** approximately.

[2] Interestingly, **Equation (10)** can be rewritten as $D = \lim_{\eta \to \infty} \frac{k_B T}{6\pi \eta r^2} r = \lim_{\eta \to \infty} \frac{k_B T}{6\pi \eta r}$ (the famous **Einstein relation**) where $Q = k_B T$. And the indicated diverging viscosity $\eta \to \infty$ simply gives the superglass phase.

$$4\left(T-T_0\right)=\lim_{\substack{r_0\to 0\\ M\to\infty}}\frac{l}{k\pi}\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right), \tag{12.2}$$

$$4\cdot\pi\cdot k\left(T-T_0\right)=\lim_{\substack{r_0\to 0\\ M\to\infty}} l\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right), \tag{12.3}$$

$$\left.\begin{aligned}
&4!\int_{T_0}^{T}\int_{T_0}^{T}\int_{T_0}^{T}\int_{T_0}^{T}\left(T-T_0\right)\mathrm{d}^4T=\int_M\int_M\int_M\int_M\lim_{\substack{r_0\to 0\\ M\to\infty}} l\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right)\mathrm{d}^4M,\\
&K\left(T^4-T_0^4\right)=\int_M\int_M\int_M\int_M\lim_{\substack{r_0\to 0\\ M\to\infty}} l\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right)\mathrm{d}^4M, K=const=A_{BH}\varepsilon\sigma\\
&A_{BH}\sigma\varepsilon\left(T^4-T_0^4\right)=\int_M\int_M\int_M\int_M\lim_{\substack{r_0\to 0\\ M\to\infty}} l\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right)\mathrm{d}^4M
\end{aligned}\right\} \tag{12.4}$$

For black holes $l=0$ and via observation, the terms inside this bracket resulted in $\infty-\infty$. As such, we obtained $\int_M \lim_{\substack{r_0\to 0\\ M\to\infty}} l\left(\frac{1}{r_0}-\frac{c^2}{2GM}\right)\mathrm{d}M = finite$ (e.g. power of back-body radiation). So far, the famous black hole's **perfect black body radiation formula** Hawking desired is now derived:

$$P_{BH}=A_{BH}\sigma\varepsilon\left(T^4-T_0^4\right) \tag{13}$$

Based on the above calculation, via investigation of tables of material thermal conductivities, we found that $\boldsymbol{k}$ **= 1.909 W·m^{-1}·K^{-1}** (thermal conductivity) corresponds to material made of china/porcelains. As a result, we can think that black holes if were in touch, would be felt like ordinary porcelains, and only if gravity were not taken into consideration (actually this is impossible). As such, it would be such interesting. This thermal property is exactly corresponding to Ads/CFT correspondence since as a material of porcelains, it saves information or lets information escape is not strange.

3. Conclusion

Although this article does not discuss heat transfer in chemical engineering, the paper uses a lot of knowledge in this field as a research subject. We eventually derived the perfect black body radiation formula of black holes that Hawking desired. This formula is very important because it not only reproduces Planck's blackbody radiation formula but also marks the correctness of the beginning of modern physics, and combines this correctness with new physics, adding further impetus to the creation of new physics. Furthermore, this paper deduced that the material of the surface thermal convection of the neutron star is ^4He, and found that it is a superglass, which is consistent with the research results of other studies.

4. Justification

Indicated Equation (13), P_{BH} where $\boldsymbol{B}$ is denoted as Boltzmann/Bekenstein/Black, and $\boldsymbol{H}$ is denoted as Hawking/Henry/Hole, in attitudes of solemnity. Honorifics orders of names are stepped by the year-times and then as to list matters.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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