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Fermions: Dirac Internal Exchange Frequencies

Doron Kwiat

Mazkeret Batyia, Israel
Email: doron.kwiat@gmail.com

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Abstract

Using real fields instead of complex ones, it was recently claimed, that all fermions are made of pairs of coupled fields (strings) with an internal tension related to mutual attraction forces, related to Planck's constant. The solution to Dirac equation gives four, real, 2-vectors solutions $\psi_1 = \begin{pmatrix} U_1 \\ D_1 \end{pmatrix}$ $\psi_2 = \begin{pmatrix} U_2 \\ D_2 \end{pmatrix}$ $\psi_3 = \begin{pmatrix} U_3 \\ D_3 \end{pmatrix}$ $\psi_4 = \begin{pmatrix} U_4 \\ D_4 \end{pmatrix}$ where (ψ_1, ψ_4) are coupled via linear combinations to yield spin-up and spin-down fermions. Likewise, (ψ_2, ψ_3) are coupled via linear combinations to represent spin-up and spin-down anti-fermions. Here, a deeper investigation of the free fermion internal frequency is discussed, hinting to an exchange interaction between the two components of which a fermion is made of. An upper limit estimate is given to the strength of this interaction.

Keywords

Fermions, Internal Frequency, Exchange Energy, Coupling

1. Dirac Equation with Real Wave Functions

The relativistic Dirac Equation, describing a free Fermion of mass m is given by [1] [2]:

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\right)\Psi = 0 \tag{1}$$

One may separate the Dirac operator $i\hbar\gamma^{\mu}\partial_{\mu} - mc$ and the complex wave function Ψ into their real and imaginary parts [3] [4]

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

With $\psi_1 = \begin{pmatrix} U_1 \\ D_1 \end{pmatrix}$ $\psi_2 = \begin{pmatrix} U_2 \\ D_2 \end{pmatrix}$ $\psi_3 = \begin{pmatrix} U_3 \\ D_3 \end{pmatrix}$ $\psi_4 = \begin{pmatrix} U_4 \\ D_4 \end{pmatrix}$ all real components.

As will be shown, these 4 components represent two fermions and two anti-fermions. Each pair is the source for two opposing spin states.

After some work and boosting to a system moving with the particle along the +x axis ($p_y, p_z = 0 \to \partial_y, \partial_z = 0$),

the Dirac equations take the form:

$$-\frac{mc^2}{\hbar}\Psi_1 = +\partial_t\Psi_4 - c\sigma_x\partial_x\Psi_4 \tag{2}$$

$$-\frac{mc^2}{\hbar}\Psi_4 = -\partial_t\Psi_1 - c\sigma_x\partial_x\Psi_1 \tag{3}$$

$$-\frac{mc^2}{\hbar}\Psi_2 = +\partial_t\Psi_3 + c\sigma_x\partial_x\Psi_3 \tag{4}$$

$$-\frac{mc^2}{\hbar}\Psi_3 = -\partial_t\Psi_2 + c\sigma_x\partial_x\Psi_2 \tag{5}$$

This shows, that, Ψ_1 is coupled with Ψ_4 and Ψ_2 is coupled with Ψ_3. As will be shown later, linear combinations of these represent spin-up and spin-down fermion and anti-fermion.

2. Solution with 8 Real Components

Each Ψ_i is a 2-vector with real components. Thus, the Dirac Equation is actually 8 equations of real components with coupled pairs (Ψ_1, Ψ_4), and (Ψ_2, Ψ_3).

Applying a time derivative to the first equation of each pair and using the second component of each pair, leads to:

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_1 + c\partial_x\partial_t U_1 = -\frac{mc^3}{\hbar}\partial_x U_4 \tag{6}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_1 - c\partial_x\partial_t D_1 = +\frac{mc^3}{\hbar}\partial_x D_4 \tag{7}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_4 - c\partial_x\partial_t U_4 = +\frac{mc^3}{\hbar}\partial_x U_1 \tag{8}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_4 + c\partial_x\partial_t D_4 = -\frac{mc^3}{\hbar}\partial_x D_1 \tag{9}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_2 + c\partial_x\partial_t U_2 = -\frac{mc^3}{\hbar}\partial_x U_3 \tag{10}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_2 - c\partial_x\partial_t D_2 = +\frac{mc^3}{\hbar}\partial_x D_3 \tag{11}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)U_3+c\partial_x\partial_tU_3=-\frac{mc^3}{\hbar}\partial_xU_2 \tag{12}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)D_3-c\partial_x\partial_tD_3=+\frac{mc^3}{\hbar}\partial_xD_2 \tag{13}$$

These 8 real components equations demonstrate the existence of coupled pairs: ($U_1 \Leftrightarrow U_4$), ($D_1 \Leftrightarrow D_4$), ($U_2 \Leftrightarrow U_3$) and ($D_2 \Leftrightarrow D_3$).

These equations show, that every fermion is composed of 4 real fields which are coupled in a yet to be explored manner.

The solutions are described in the following:

$$\Psi_1=\begin{pmatrix}U_1\\D_1\end{pmatrix}=\begin{pmatrix}\cos\left(px-\left(\omega_0+cp\right)t\right)\\\sin\left(px-\left(\omega_0-cp\right)t\right)\end{pmatrix}$$

$$\Psi_4=\begin{pmatrix}U_4\\D_4\end{pmatrix}=\begin{pmatrix}\sin\left(px+\left(\omega_0+cp\right)t\right)\\\cos\left(px+\left(\omega_0-cp\right)t\right)\end{pmatrix}$$

$$\Psi_2=\begin{pmatrix}U_2\\D_2\end{pmatrix}=\begin{pmatrix}\cos\left(px-\left(\omega_0+cp\right)t\right)\\\sin\left(px-\left(\omega_0-cp\right)t\right)\end{pmatrix}$$

$$\Psi_3=\begin{pmatrix}U_3\\D_3\end{pmatrix}=\begin{pmatrix}\sin\left(px+\left(\omega_0-cp\right)t\right)\\\cos\left(px+\left(\omega_0+cp\right)t\right)\end{pmatrix}$$

Here $p\equiv\frac{p_x}{\hbar}$, where p_x is the x component of the momentum.

In order to better understand the nature of the Equations (6)-(13), one can boost the coordinates to the fermion's rest frame along a random axis, say the x-axis.

When boosted to the fermion's rest frame, where $p_x=0$, one can omit all ∂_x terms ($\partial_x=0$).

Equations (2)-(5) then read (for all four wave functions):

$$\left[\partial_t^2+\left(\frac{mc^2}{\hbar}\right)^2\right]\Psi_i=0 \tag{14}$$

And likewise for all 8 components U_i and D_i:

$$\left[\partial_t^2+\left(\frac{mc^2}{\hbar}\right)^2\right]U_i=0 \tag{15}$$

$$\left[\partial_t^2+\left(\frac{mc^2}{\hbar}\right)^2\right]D_i=0 \tag{16}$$

Solving this equation by setting $\Psi=\cos\left(\omega t\right)$ or $\Psi=\sin\left(\omega t\right)$ shows that all components of the fermion at its rest frame, are oscillating at a rate given by $\omega_0=\frac{mc^2}{\hbar}$.

Inserting the values in MKS units one obtains, for all fermions $\omega_0 = 1.36554 \times 10^{32} \times m$.

For an electron $m = 0.511$ MeV/c^2 and thus $\omega_0 \approx 7.76 \times 10^{11}$ GHz. (=0.512 MeV). For a proton, $m = 939$ MeV/c^2 and $\omega_0 \approx 7.296 \times 10^{6}$ GHz and for the τ quark $m = 173$ GeV/c^2 and $\omega_0 \approx 2.63 \times 10^{17}$ GHz (**Table 1**).

Table 1. Masses and equivalent electromagnetic range Properties of electron, proton and tau-quark.

particle	ω_0	EM range	Mass
electron	7.77×10^{11} GHz	Soft X-ray	0.512 MeV
Proton	14.28×10^{14} GHz	Gamma	939 MeV
τ Quark	2.63×10^{17} GHz	Gamma	173 GeV

3. A Model for the Fermion Internal Structure

Previously [3] it was shown that the description of Quantum Mechanics can be done by use of real fields and operators. The Schrodinger equation was then shown to be describing a double string model with particle exchange between the two strings. It was also shown, that all known quantum mechanical unique phenomena, such as interference and entanglement, can be explained based on real fields approach [3] [4].

The same argumentation was applied to Dirac equation, demonstrating a double string coupling.

Dirac equation shows us, that the internal frequency of each component of the fermion equals the rest mass of that fermion.

Even though a fermion is assumed to be made of 2 components, *U* and *D*, each one inherits its oscillation frequency ω_0 from the same total fermion mass *m*, and not from *U's* or *D's* individual masses (if any).

As shown earlier, each Ψ_i is a 2-vector with real components.

We will assume next that for a reference frame near static with respect to the fermion

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_1 = \varepsilon U_1 \tag{17}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_1 = \varepsilon D_1 \tag{18}$$

even in the non-boosted system. Same is true for

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_4 = \delta U_4 \tag{19}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_4 = \delta D_4 \tag{20}$$

Therefore,

$$c\partial_x\partial_t U_1 = -\frac{mc^3}{\hbar}\partial_x U_4 + \varepsilon U_1 \tag{21}$$

$$-c\partial_x\partial_t U_4 = +\frac{mc^3}{\hbar}\partial_x U_1 + \varepsilon U_4 \tag{22}$$

$$-c\partial_x\partial_t D_1 = +\frac{mc^3}{\hbar}\partial_x D_4 + \delta D_1 \tag{23}$$

$$c\partial_x\partial_t D_4 = -\frac{mc^3}{\hbar}\partial_x D_1 + \delta D_4 \tag{24}$$

This makes the components U_1 coupled to U_4 and same for D_1 to D_4. One may write

$$\partial_t U_1 = -\frac{mc^2}{\hbar}U_4 + \frac{\varepsilon}{c}\int U_1 \mathrm{d}x \cong -\frac{mc^2}{\hbar}U_4 + \frac{\varepsilon}{c}\Delta x\langle U_1\rangle \tag{25}$$

$$\partial_t U_4 = -\frac{mc^2}{\hbar}U_1 + \frac{\varepsilon}{c}\int U_4 \mathrm{d}x \cong -\frac{mc^2}{\hbar}U_1 + \frac{\varepsilon}{c}\Delta x\langle U_4\rangle \tag{26}$$

$$\partial_t D_1 = -\frac{mc^2}{\hbar}D_4 + \frac{\varepsilon}{c}\int D_1 \mathrm{d}x \cong -\frac{mc^2}{\hbar}D_4 + \frac{\varepsilon}{c}\Delta x\langle D_1\rangle \tag{27}$$

$$\partial_t D_4 = -\frac{mc^2}{\hbar}D_1 + \frac{\varepsilon}{c}\int D_4 \mathrm{d}x \cong -\frac{mc^2}{\hbar}D_1 + \frac{\varepsilon}{c}\Delta x\langle D_4\rangle \tag{28}$$

where the integration in x is over the negligible size of the fermion, and where $\langle\ \rangle$ represents the average amplitude over that extent.

This allows us to neglect the extra terms on the r.h.s of the equations and obtain, even in a non-stationary frame of reference (relative to the fermion):

$$\partial_t U_1 = -\frac{mc^2}{\hbar}U_4 = 0 \tag{29}$$

$$\partial_t U_4 = -\frac{mc^2}{\hbar}U_1 = 0 \tag{30}$$

$$\partial_t D_1 = -\frac{mc^2}{\hbar}D_4 = 0 \tag{31}$$

$$\partial_t U_4 = -\frac{mc^2}{\hbar}D_1 = 0 \tag{32}$$

These equations indicate to a coupling by exchange mechanism between U_1 and U_4. When looking at the change in amplitude of U_1 we get:

$$\Delta U_1 = \partial_t U_1 \Delta t = -\frac{mc^2}{\hbar}U_4\Delta t = -\omega_0 U_4 \Delta t \tag{33}$$

Internal fluctuations of U_1 and U_4 at rate ω_0 indicate an energy exchange between the two components. Same argument is valid for the internal fluctuations of D_1 and D_4

So, during the exchange, U_1 gains $\frac{mc^2}{\hbar}$ in amplitude per second if U_4 is negative, and loses amplitude if U_4 is positive.

Recall that they oscillate at a rate of 1.24 × 10^{11} GHz for an electron. This is a picture of two adjacent strings, oscillating in anti-phase, at said frequency (see **Figure 1**). This ant-phase is a must, in order to keep the particle's momentum zero in the non-x (perpendicular) direction.

We assume that at most, the whole mass participates in the kinetic energy transfer mechanism. This assumption puts an upper limit on E_k, the kinetic energy transfer.

An upper limit estimate of the kinetic energy transfer between U_1 and U_4 in the electron (change in amplitude per second is velocity) will then be given by:

$$E_k < \frac{1}{2} m \left(\partial_t U_1\right)^2 = \frac{1}{2} \frac{m^3}{\hbar^2} U_4^2 = 2.12 \times 10^{-4} \text{ eV} \times U_4^2$$

If $U_4 = \cos\left(\omega_0 t\right)$ then the time average of U_4^2 is $\left\langle U_4^2 \right\rangle = \frac{1}{2\omega_0}$.

So, the average kinetic energy transfer between the coupled strings in the electron, E_k is less than 2.73 × 10^{-25} eV/sec. For a proton the upper limit is 9.2 × 10^{-19} eV/sec and 3.12 × 10^{-14} eV/sec for the tau quark. All cases have values far below our detection capabilities.

Same arguments hold for the kinetic energy transfer between D_1 and D_4.

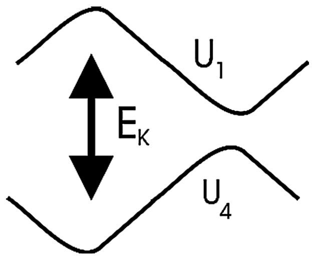

Figure 1. Energy transfer model based on anti-phase oscillations between the two components U_1 and U_4 of a fermion.

The interaction may be due to some yet unknown particles, where possible candidates may well be those suggested by Harrari [5] and Hubsch [6].

4. Conclusions

All fermions are made of pairs of coupled fields (strings) with an internal tension related to mutual attraction forces, affected by Planck's constant. The solution to Dirac equation gives rise to four, real, 2-vector fields $\psi_1 = \begin{pmatrix} U_1 \\ D_1 \end{pmatrix}$, $\psi_2 = \begin{pmatrix} U_2 \\ D_2 \end{pmatrix}$, $\psi_3 = \begin{pmatrix} U_3 \\ D_3 \end{pmatrix}$, and $\psi_4 = \begin{pmatrix} U_4 \\ D_4 \end{pmatrix}$ where (ψ_1, ψ_4) are coupled via linear combinations to yield spin-up and spin-down fermions. Likewise, (ψ_2, ψ_3) are coupled via linear combinations to represent spin-up and spin-down anti-fermions.

An investigation of the free fermion internal frequency is discussed, hinting to an exchange interaction between the two components of which a fermion is made of. An upper limit estimate is given to the strength of this interaction.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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