Journal of High Energy Physics, Gravitation and Cosmology, 2024, 10, 1424-1449
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

An Original Didactic about Standard Model: "The Particles' Geometric Model" (Leptons and Bosons)

Giovanni Guido[1], Abele Bianchi[2], Gianluigi Filippelli[3]

[1]EEE Project, High School "C. Cavalleri", Parabiago, Italy
[2]EEE Project, High School "G. Gandini", Lodi, Italy
[3]INAF Astronomic Observatory "Brera", Milan, Italy
Email: gioguido54@gmail.com, abelebia@gmail.com, gianluigi.filippelli@brera.inaf.it

How to cite this paper: Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic about Standard Model: "The Particles' Geometric Model" (Leptons and Bosons). *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 1424-1449.
https://doi.org/10.4236/jhepgc.2024.104080

Received: May 10, 2024
Accepted: September 24, 2024
Published: September 27, 2024

Abstract

This work shows a didactic model representative (GPM) of the particles described in the Standard Model (SM). Particles are represented by geometric forms corresponding to geometric structures of coupled quantum oscillators. From the didactic hypotheses of the model emerges an in-depth phenomenology of particles that is fully compatible with that of SM. Thanks to this model, we can calculate "geometrically" the mass of Higgs's Boson and the mass of the pair "muon and muonic neutrino", and, by the geometric shapes of leptons and bosons, we can also solve crucial aspects of SM physics as the neutrinos' oscillations and the intrinsic chirality of the neutrino and antineutrino.

Keywords

Golden Particle, Quark, Leptons, IQuO, Lattice, Boson, Higgs, Chirality

1. Introduction

Despite the clarity and systematic nature of the Standard Model (SM), the greatest difficulty for students (and not only) in understanding the physical reality of particles is that of not being able to "see" the particles if they are represented as mere mathematical objects described by wave functions and equations. Since we cannot see a space "point", we think of giving the particle a representation in which it is not seen as a point particle but becomes an "object" with a spatial dimension, identified in the Compton wavelength. Then, we therefore decide, for educational and speculative purposes, to assign an internal geometric structure to a particle without compromising its elementary nature. This is possible only if we consider a massive particle as a structure of coupled quantum oscillators. In this way, the particle object would also acquire the dimension of a well-defined "structure" in space. Starting then from some peculiar aspects, such as analogies and some indicative phenomena, we proceed to formulate a representation in which the particles have a structure with a "geometric" shape. In this way, we build a descriptive didactic model of the phenomena that is compatible with MS and avoids some problematic aspects of its basic theories, the Theory of Relativity (point particle) and Quantum Mechanics (renormalization), because some fundamental concepts, even if defined in a didactic way, they instead have a more physical meaning, such as mass, electric charge, and spin. The educational model (and not only) is defined as the Particles' Geometric Model (PGM).

In the present article, we will show the geometric relationships between massive bosons and leptons. In Sections 2.1 and 2.2, we introduce the first "Didactic Idea" which presents a massive particle as a triangular structure of coupled quantum oscillators propagating along an axis X with a rotation around this or "spin": the quarks' pair (u, d) is described by of isosceles golden triangles while the leptons' pair (e^+, e^-) by of isosceles rectangular rectangles. It is shown, in Section 3.1, that the mass is defined precisely by the "additional coupling" between quantum oscillators (IQuO) that allows us of constructing the geometric shapes. This aspect introduces a "new paradigm" in physics: the particles' phenomenology can be described using geometric structures or a "Particles' Geometric Model" (PGM). In this way, one can think that a square geometric structure can represent massive bosons as the H-boson (a square) and W-boson (a rectangle). Instead, in Section 3.2, we show the structure without massive coupling of a massless boson, as the photon (a segment without additional coupling). In Section 3.3, we represent the processes of a pair creation mediated by an "electromagnetic" boson H_{em} with square geometric shape, which couples with the two incident photons originating so two rectangular triangles which constitute the pair (e^+, e^-). In Section 3.4, the annihilation process of pair mediated by H_{em} is descripted. Tanks to the H_{em}, one can describe, see Section 3.5, the decay of the "Higgs Boson" into two photons detected at CERN: in this way, we can think that the Higgs Boson can have different shapes or states, and one of these can be that of the H_{em} boson expressed in PGM. In Section 4, we attempt of represent the weak interaction (mediated by vector bosons ($W^\pm$, Z)) by of the quadrangular geometric shapes as the H boson and W boson. In Section 4.1, we show the decay of the pion through the $\{W^\pm\}$ lattice of W rectangular bosons, $\{W^\pm\}$, and a quadrangular "weak" boson H_w, inserted into a W boson. This happens because the (u, d) quarks can be inserted into the W boson (this is so defined as golden), and through W transform into each other. In Section 4.2, the decay of the neutron is shown geometrically, where the H_w boson gives rise to the electrons while the golden W gives rise to the neutrino. This aspect "breaks" the geometric symmetry between leptons charged and neutral: the electrons are rectangular isosceles triangles while the electron neutrinos are golden isosceles triangles. In Section 5, one shows the geometric shape of the other leptons (μ, τ) and the respective neutrinos, having the same shape. In Section

5.1, we represent the decays of the pion in the pair (μ, ν_μ) while, in Section 5.2, that of D meson, in the pair (τ, ν_τ). In this way, it is demonstrated that weak neutrinos can oscillate in their three savours because all three have the same shape; the oscillation between savours does not happen for the charged leptons because the electron has a different shape from the other two. In Section 5.3, we show the possible geometric shape of the "sterile" neutrinos (same geometric shape as electrons but electrically neutral), which is so different from that of the "weak" neutrinos. In Section 6.1, we represent the geometric shape of the Z boson ($Z = (W^+ \otimes W^-)$) and its possible decay in a pair of neutrinos: $Z \to (\nu_L + \underline{\nu}_R)$. By the geometric representation of Z-decay one can show the intrinsic difference between the neutrino and antineutrino because they have opposite "chirality" and "helicity" due to the asymmetry of the geometric shape in a golden triangle. We so demonstrate, thanks to the geometric structure of particles, that the weak interactions violate the P-Symmetry and not the CP-Symmetry. In this way, we demonstrate that the existence of only left-handed neutrinos and right-handed antineutrinos is the consequence of a geometric structure possessed by bosons and neutrinos. Finally, in Section 6, by the mass of Z boson, we can calculate, using the geometric shapes of the H boson and golden W Boson, the mass of Higgs's boson H_w, see the Section 6.2; in Section 6.3 we can also calculate the mass value of the system (μ, $\underline{\nu}_\mu$) given by the value (105.84) Mev/c^2.

2. Hypothesis of Structure

2.1. The Geometric Structure Hypothesis into Quarks, Hadrons, and Bosons

In precedent articles, we formulated the hypothesis that the massive particles are geometric structures of coupled oscillators (***IQuO***) of quantum field [1]-[3] composed of elementary "*sub-oscillators*" with "*semi-quanta* (*sq*)" (*full sq*(•) = $h\omega/2$, *empty sq*(**o**) = $h\omega/4$) of energy moving along the sides of the structure. To adequately represent a particle, we formulated the following ***Didactic Idea*** **(1)**: **"*The triangular structures can propagate along a 'guide' and rotate around this line, thus physically carrying out a spin*"**. *S*ee the following **Figure 1**:

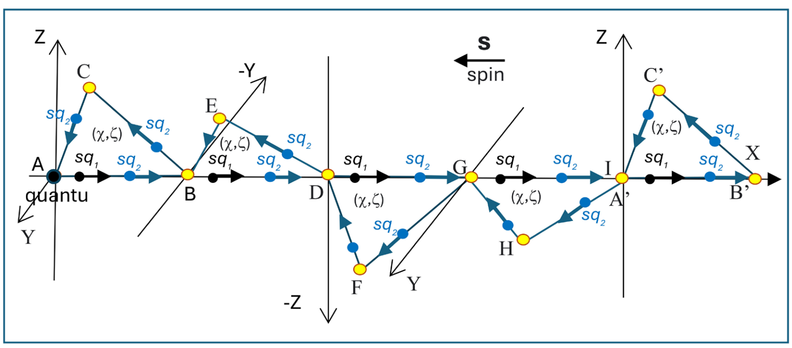


Figure 1. The configurations of triangle-particle in the propagation along X-axis.

The arrows indicate the movement direction of two sq_i(•) which compose the energy "quantum" (•), with [• = (•, •)], along the sides (flow vector **Φ**). We assume that: "***The value of the electric charge is given by the probability of detecting a quantum*** (•) ***along the propagation side***" (***Didactic Idea*** **(2)**), see **Figure 2**. In the case of quarks (*u*, *d*), we will have the following representation [4]:

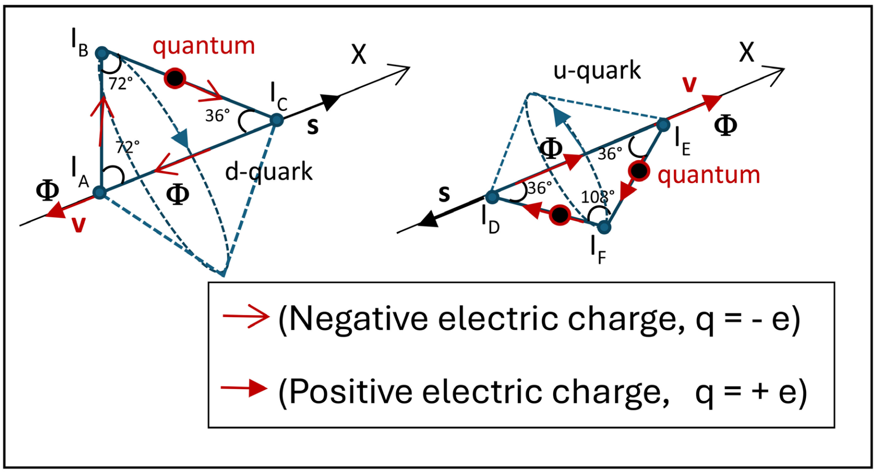


Figure 2. Geometric Structure of quark (*u, d*).

The triangles are "***goldens***": (I_A, I_B, I_C) are IQuO vertices, while (I_{AB}, I_{BC}, I_{CA}) are Junction vertices.

2.2. The Basic Lepton

We remember that the massive coupling builds the quarks-structure and the hadrons [5] [6]. The electrons, like the quarks, have a triangular structure but cannot be golden triangles; it follows the ***Didactic Idea*** **(3): "*The electron* (*positron*) *can be represented by a rectangular isosceles triangle*"**, see **Figure 3**:

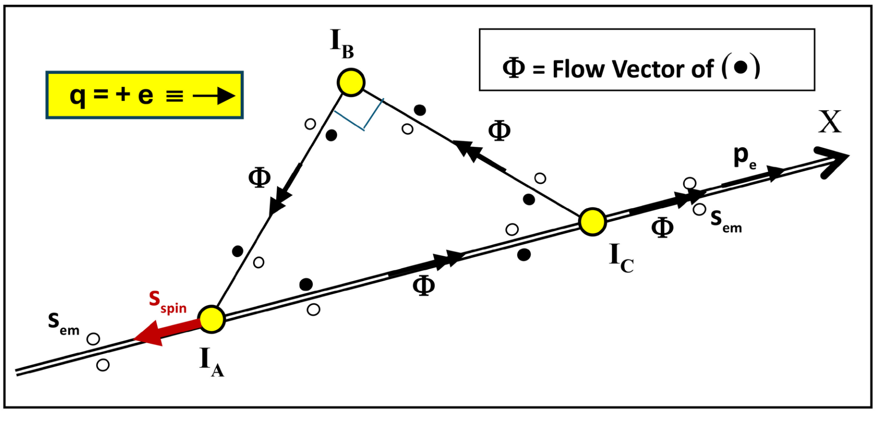


Figure 3. The IQuO-structure with semi-quanta of a positive electron and its electromagnetic guideline.

The two arrows indicate the Φ-flow of two *sq*(•) along the sides. Matter and antimatter are represented by the same shape geometric; but the negative electric charge by "empty" arrows while the positive charges by "full" arrows, see **Figure 4**:

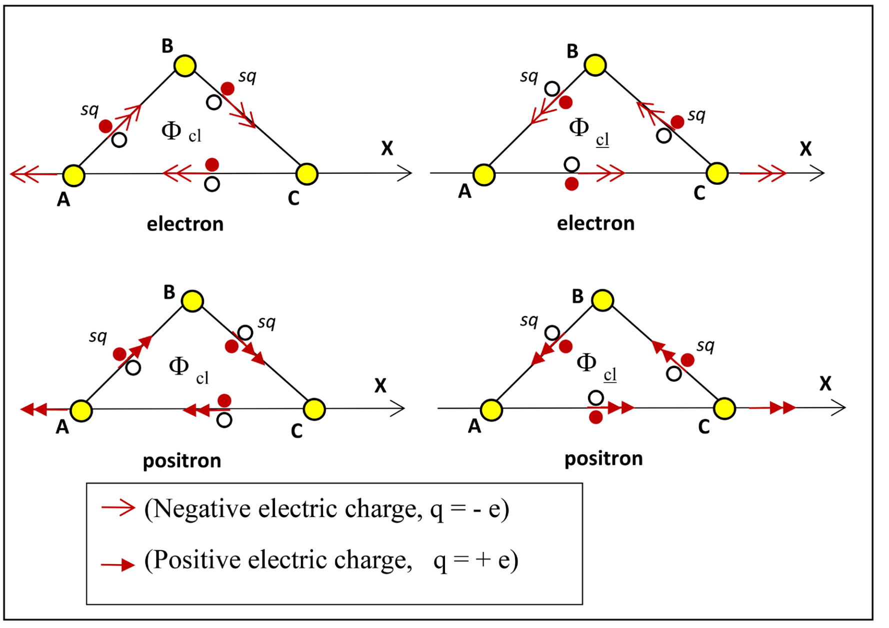


Figure 4. The configuration of the electron and positron, and *sq*-flow vector Φ.

Note: (***Didactic Idea*** (4)) "***The structure-particle propagates in the same direction of Φ flow vector along the side AC***".

Here, in the electrons, the probability of detecting the quantum (•) along the propagating side (vertices A, C) is: $q(e) = P(\bullet)_e = 1$. This is because a photon (***gauge boson***) along the guideline always operates a phase shift on the *sq*(•), such that in two vertices (A, C) there are always full *sq*(•). The geometric representation so is coherent to the representation of the ***Gauge Field*** [7].

3. The Bosons

3.1. The Massive Bosons

We showed [3] [8] the mass is defined by the "***additional transversal coupling***" of IQuO, referred as a "***massive coupling***", see Figure 5:

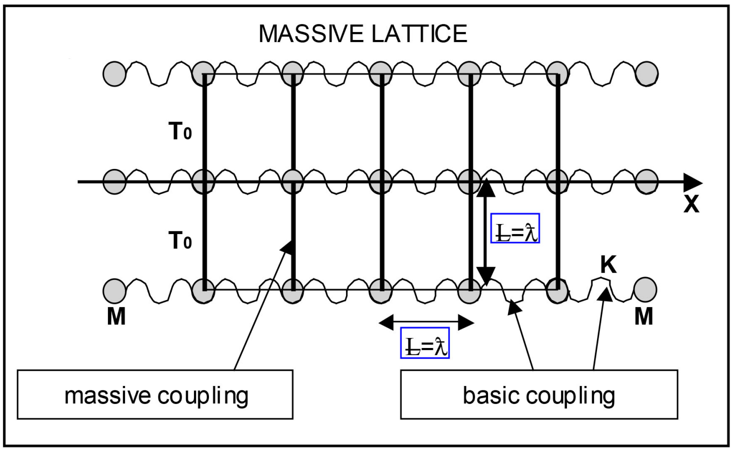
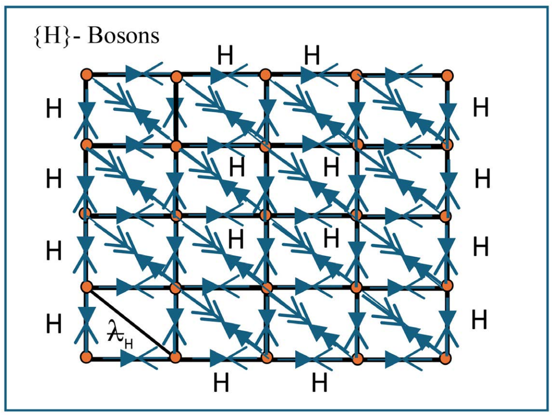


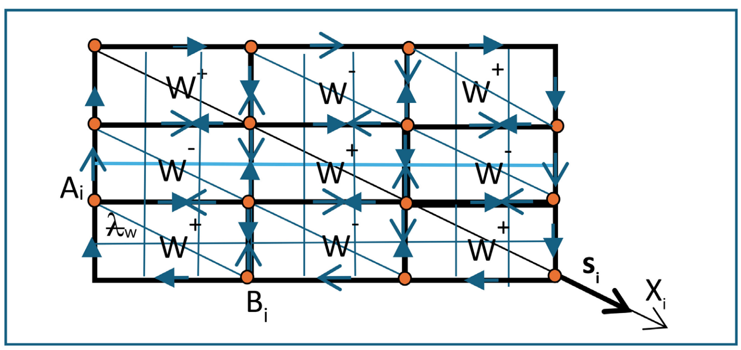
Figure 5. The massive scalar field as a lattice of "pendulums" with springs.

"***The additional coupling between field oscillators can build triangular structures (Fermions) and quadrangular (Bosons)*" (***Didactic Idea* (5))**. In a lattice "***the bosons appear adjacent to each other but are in an "entanglement" quantum state***" (***Didactic Idea* (6)**). We can build two types of lattices: ({H}, {$W^{\pm}$}), see Figure 6.

Note in lattice, the quadrangular bosons can be in a state of "superposition" state of eigenstates and in a state of mutual interpenetration. If we consider the *W* bosons of a lattice as vector Bosons (***W***) having a spin **s**, see Figure 6, then we think that "***The W-bosons can be 'inserted' a {G} lattice of scalar G-bosons: the W bosons can so rotate because they are pivoted at certain points of the G lattice***" (***Didactic Idea* (7))**. The G-bosons could be the "***Goldston Bosons***" but with mass***;*** we pose: [{*W*}⊗{*G*}], where the symbol ⊗ indicates a combination operation of fields. The representation at "lattice-particle" is so correspondent to Fields' representation: ***Field ⇔ Lattice***.



(a)



(b)

Figure 6. (a) The {H}-lattice with Compton wavelengths (λ_H). (b) The {$W^{\pm}$}-lattice with Compton wavelengths (λ_w).

3.2. The Massive Boson (Photon)

If the oscillations involving the additional "transversal" coupling (or massive) represent massive particles, the oscillations involving only the "***longitudinal***" X-axis, see Figure 5, (the axis where the springs are extended), instead represent massless particles. Thus, photons will involve "longitudinal" but not transverse oscillations. This longitudinal oscillation takes on the role of "***guide track***" of the photon itself, that is, it is a "***waveguide***" [9]. The geometric representation of the photon (boson) cannot thus be a "flat" quadrangular figure; it follows the ***Didactic Idea* (8) "*The* "*geometric*" *representation of the photon is a longitudinal* "*segment*" *along the X-axis of propagation***". Then, we have, Figure 7:

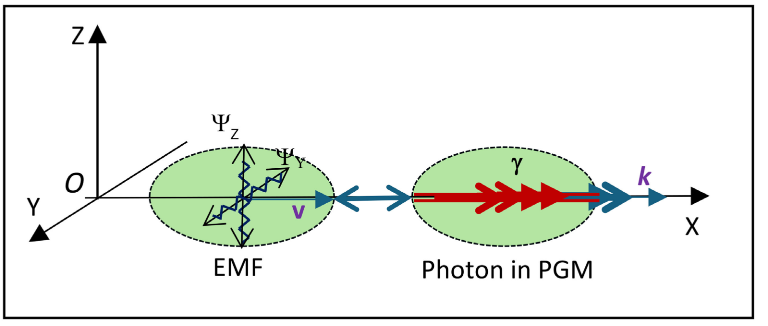


Figure 7. Representation in PGM of a photon (EMF).

Recall [10] photon is electromagnetic wave (EMF) with transversal oscillations (Ψ_Y, Ψ_Z) to the X-direction of propagation moment $\boldsymbol{k}$. Then, we can conventionally represent the photon, with a segment having "***six arrows***": two for each axis of transversal oscillation or two sq(o, •) per axis (the **four red arrows**) and **two blue arrows** for the oscillation-guide (***waveguide***), along the axis X, with two sq(o, •)$_{wg}$; it follows: (γ) ≡ [sq(o, •)$_Y$, sq(o, •)$_Z$, sq(o, •)$_{wg}$]. If we instead consider "bosons" having a "transversal" additional, see the Figure 5 and Figure 6, then we add the massive oscillation Ψ_m, with the frequency ($\boldsymbol{\omega_0}$ ⇔ $\boldsymbol{m_0}$). Thus, in summary, we will have: Photon ⇔ {[(Ψ_Y, Ψ_Z), $\boldsymbol{v}_X$]}, massive Boson ⇔ {[(Ψ_X, Ψ_Y, Ψ_Z), $\boldsymbol{v}_X$], Ψ_m}.

3.3. The Pair Creation Process

We recall the pair creation process: [$(\gamma_1 + \gamma_2) \rightarrow (e^+ + e^-)$]. Each photon has an elliptical polarization, with oscillation components in Z and in Y, that is: [$(\gamma_1)_Z$, $(\gamma_1)_Y$], [$(\gamma_2)_Z$, $(\gamma_2)_Y$]. We will have, Figure 8 and Figure 9:

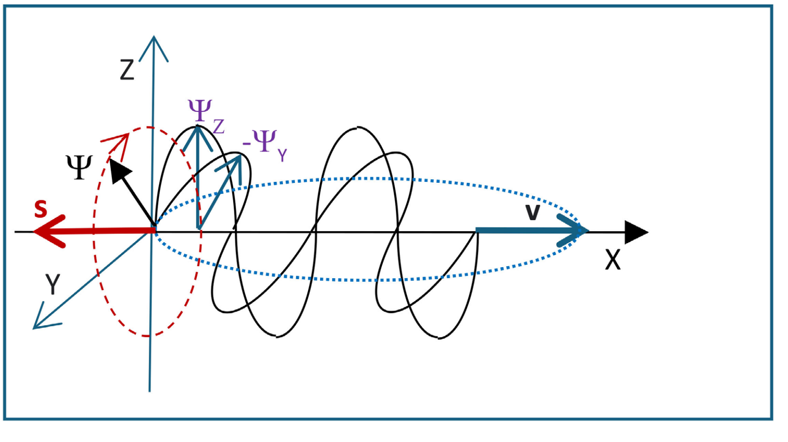


Figure 8. Photon with spin s opposite to the velocity v.

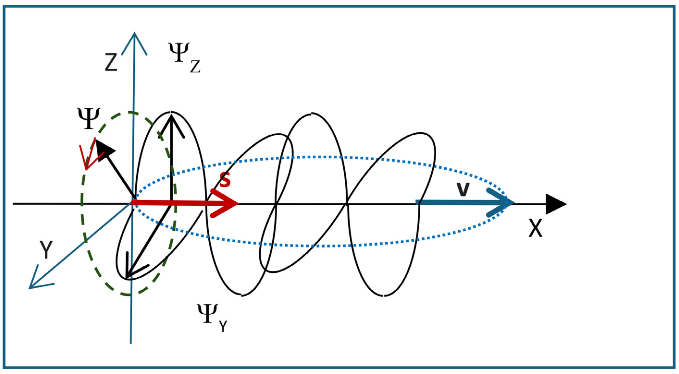


Figure 9. Photon with spin *s* parallel to the velocity *v*.

In literature [11] [12] a pair creation process is represented by **Figure 10**:

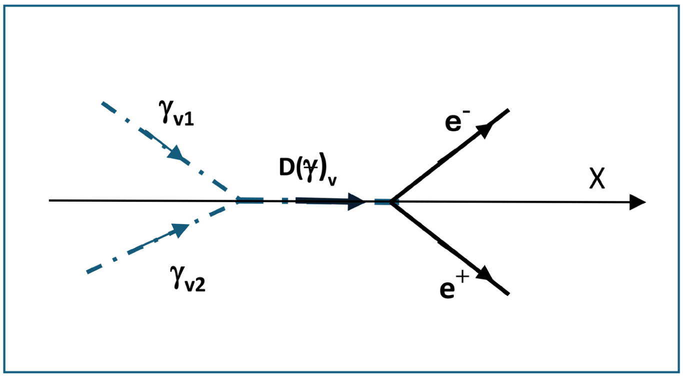


Figure 10. A pair creation process by the Photon Propagator $D(\gamma)$.

A note: as we well know, photons have no mass while electrons and positrons are massive particles, that is defined by an additional coupling building a lattice. This means that the additional coupling cannot arise from by coupling of two photons (having only longitudinal oscillations). We need to think the following ***Didactic Idea* (9): "*A background massive field H_{em} intervenes in the pair creation process*"**. We then will have:

$$\left[\left(\gamma_1+\gamma_2\right)+H_{em}\right]\rightarrow\left(e^{+}+e^{-}\right) \quad (1)$$

Then, the representation of H_{em} in PGM is, **Figure 11**:

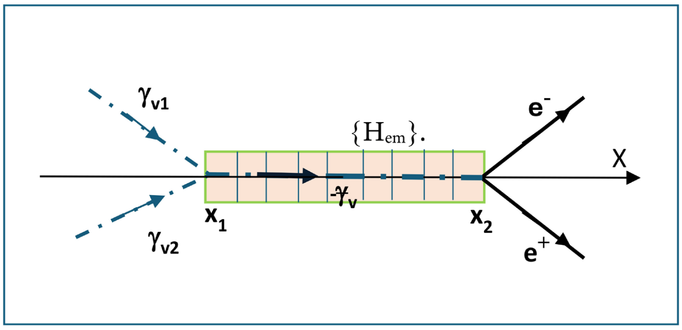


Figure 11. Representation of the coupling Photon-H_{em}.

1) The background field will have a reticular shape, see Figure 5 and Figure 6

2) H_{em} refers to a field that couples with photons

3) H_{em} has the shape of a quadrangular structure of quantum oscillators (IQuO) with massive coupling

4) Spin of H_{em} is zero $\boldsymbol{s}(H_{em}) = 0$

This because it is: $[(\boldsymbol{s}(\gamma_1) + \boldsymbol{s}(\gamma_2) + \boldsymbol{s}(H_{em})] \rightarrow [(\boldsymbol{s}(e^+) + \boldsymbol{s}(e^-)]$

Therefore, since it is $\boldsymbol{s}(\gamma_1) + \boldsymbol{s}(\gamma_2) = 0$, and $[(\boldsymbol{s}(e^+) + \boldsymbol{s}(e^-)] = 0$ it follows $\boldsymbol{s}(H_{em}) = 0$

It follows that, Figure 12:

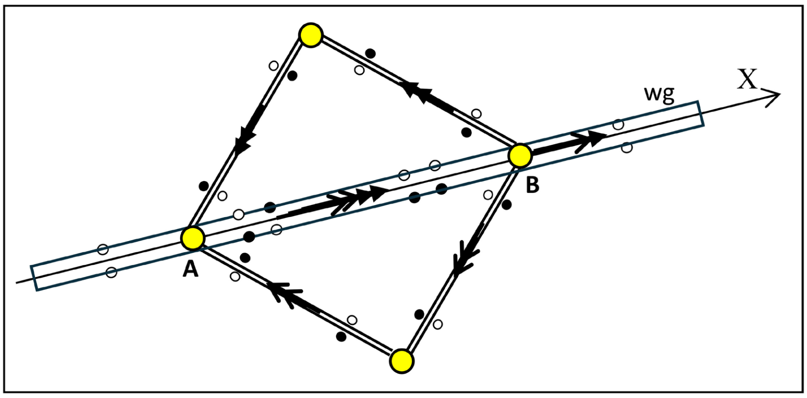


Figure 12. The IQuO-structure of the bosons H_{em}.

Note the line of H-square is double since it is $[(H_{em})_X, (H_{em})_Y]$ which the involving dimensions. Since the H_{em} boson is electrically neutral, we could conjecture the following configurations of the H_{em} Boson, see Figure 13:

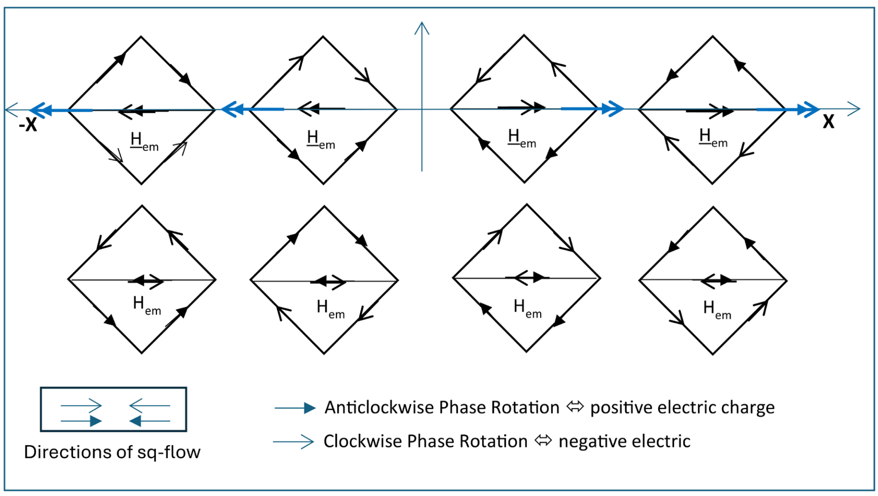


Figure 13. Representation of the $(H_{em}, \underline{H}_{em})$ Bosons.

We observe that two different forms of configurations can exist. Exactly, $\underline{H}_{em}$ is a boson that can propagate along an axis lying along the diagonal of the square, since it has a non-zero flux vector $\boldsymbol{\Phi}$ of $sq(\bullet)$ along this diagonal, while H_{em} cannot propagate because it has a zero flux vector $\boldsymbol{\Phi}$ along the diagonal. We now need represent two photons in coupling ($\gamma_1 + \gamma_2$), **Figure 14**:

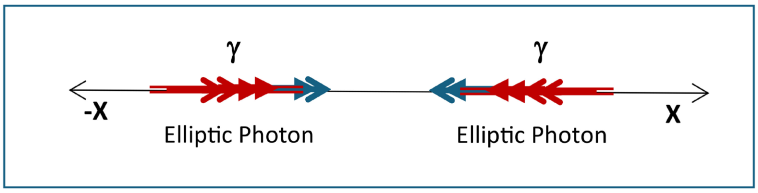


Figure 14. Representation of the γ Elliptic photons.

Also, in this representation of a photon, the arrows would indicate the sign of the electric charge of the particle, which in the **Figure 14** would be electrically neutral. We now can represent the two processes of a pair creation and annihilation, see **Figure 15**; we have (***s*** ≡ spin, ***p*** ≡ impulse), ($\underline{\otimes} \equiv (\otimes, \oplus)$, with $\otimes$ ≡ operation of interpenetrating and coupling, and $\oplus$ ≡ operation of exchanging of $sq(\bullet)$:

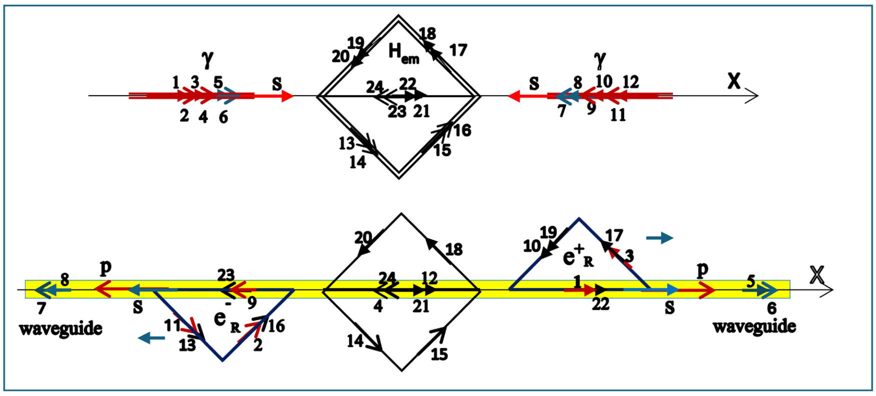


Figure 15. Pair creation by of H_{em}.

Where numbers indicate $sq(\bullet)$. Here H_{em} has two oscillation planes (see the double line of sides): there are $[(H_{em})_Y + (H_{em})_Z]$ as also the photons have oscillation along the axes (Y, Z). Now it is:

$$
\begin{aligned}
&\left\{\left[(\gamma_{v1})_Z \otimes (\gamma_{v1})_Y\right]_{(+s)} \underline{\otimes} \left[(\gamma_{v2})_Z \otimes (\gamma_{v2})_Y\right]_{(-s)}\right\} \underline{\otimes} (H_{em})_{(Z,Y)} \\
&= \left\{\left[(\gamma_{v1})_Z \otimes (\gamma_{v1})_Y\right]_{(+s)} \underline{\otimes} (H_{em})_{(Z,Y)}\right\}_1 + \left\{\left[(\gamma_{v2})_Z \otimes (\gamma_{v2})_Y\right]_{(-s)} \underline{\otimes} (H_{em})_{(Z,Y)}\right\}_2 \quad (2) \\
&\to \left(e_R^+ + e_R^-\right)
\end{aligned}
$$

We can think that:

- the massive lepton (electron triangle) propagates in the same direction as the flux vector **Φ**.
- there is the formation of an IQuO line which acts as a waveguide, again thanks to the presence of the H_{em} "square" boson.
- To the end of the reaction of pair creation stays a virtual H_{em}.
- The system ({H_{em}}, γ) is physically equivalent to a "lattice" of Feinman diagrams.
- The H lattice with finite (λ) Compton wavelength of the H boson could

eliminate the problem of renormalization of the mass (recall the problem of infinites).
- The geometric structure with spatial dimension (λ) of the electron eliminates the problem of renormalization of the electric charge.

3.4. The Annihilation Process by H_{em}

We formulate the following ***Didactic Idea* (10) "*The H-boson also intervenes as an intermediary agent in the annihilation process*".** We will have:

1) Along the waveguide (line AB), combining the *sq*-operators of the two F-IQuO (leptons) having the opposite phase rotation, two lines of B-IQuO (photons) along AB are generated.

2) The *sq*(•) of the additional sub-oscillators of the oblique sides of the two leptons pass into the sub-oscillators of the Higgs Boson, remaining trapped there.

3) Leptons thus lose mass.

4) Real photons are formed which propagate along the diagonal passing through AB.

We will have in Figure 16:

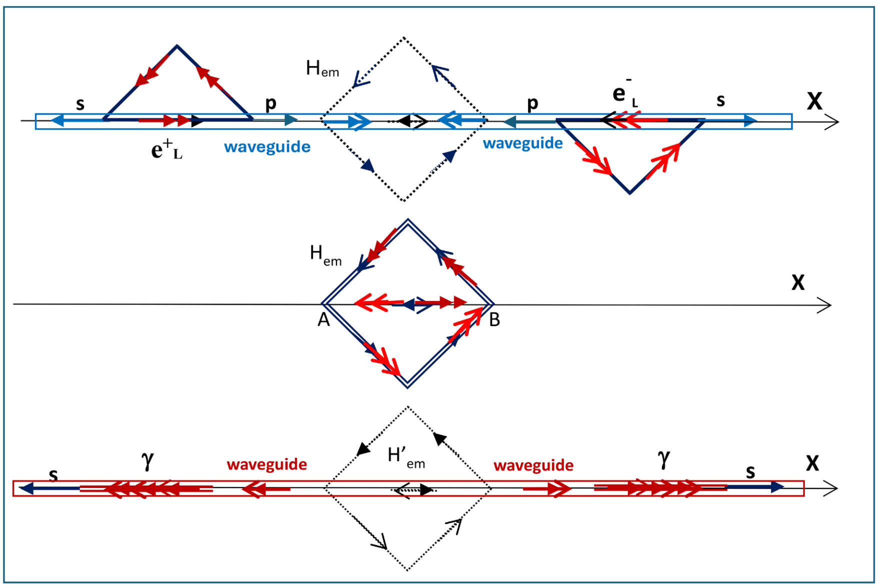


Figure 16. Annihilation process by H_{em}.

Where it is

$$\left[(\gamma_1+\gamma_2)+H_{em}\right]\rightarrow\left[\left(e^{+}+e^{-}\right)+H'_{em}\right]\rightarrow(\gamma_1+\gamma_2) \quad (3)$$

Note the presence of the line of waveguide. It follows that the two photons are "**correlated**" or in an "**entanglement state**".

3.5. The Reaction $\underline{H}\rightarrow(\gamma+\gamma)$

If we consider H_{em} Boson (see Figure 13), propagating along X-axis, that is ($\underline{H}_{em}$), we could have the "*electromagnetic*" reaction observed at CERN [13], see Figure 17: $\underline{H}\rightarrow(\gamma+\gamma)$

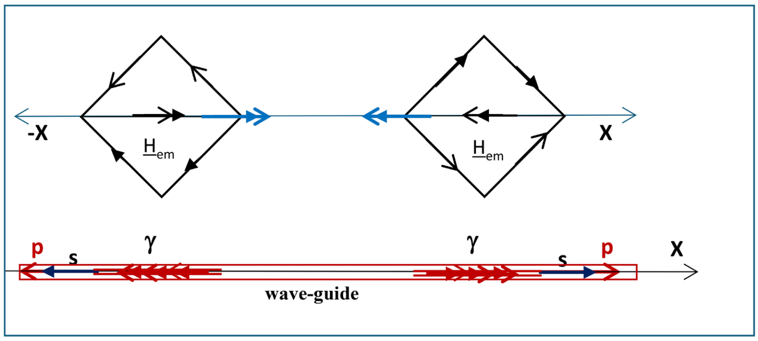


Figure 17. Representation of the reaction $\underline{H} \rightarrow (\gamma + \gamma)$.

Where it is $(\underline{H}_{em} + \underline{H}_{em}) \rightarrow (\gamma_1 + \gamma_2)$. Note the boson type H presents itself as a lattice $\{H\}$, Figure 18:

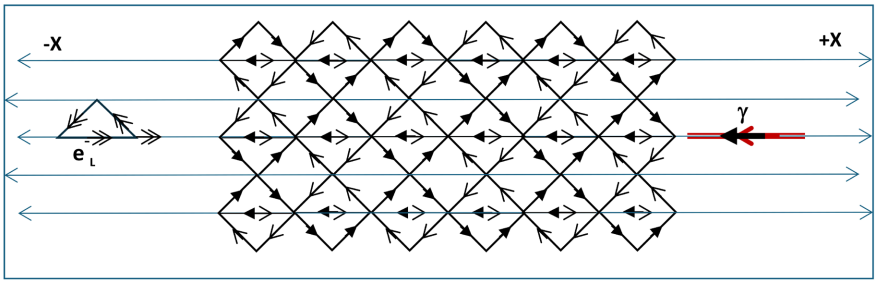


Figure 18. Lattice $\{H_{em}\}$.

4. The Weak Interactions with H_w Boson

4.1. The Charge Pion Decay

We consider a "golden" rectangle [14]; it follows the ***Didactic Idea*** **(11)** "A "*golden*" *rectangle of coupled oscillators could represent the W-boson of weak interactions* W_w". Since the pion decay occurs by weak interaction, we could assume that (***Didactic Idea*** **(12)**): "*A* $W^{\pm}$ *pair of golden W-boson can contain geometrically a d-quarks*". We could then insert the (u, d) quarks in a lattice $\{W^{\pm}\}$ bosons, see Figure 19:

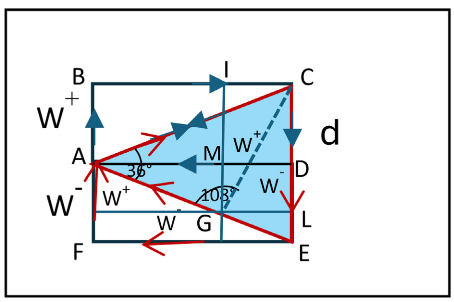


Figure 19. A d-quark embed in a lattice $\{W^{\pm}\}$.

In the new perspective, we could treat the Feynman diagrams of the β-decay with two coupled bosons $(W^+, W^-) \in \{W\}$: so, one introduces the "**lattice**" $\{W\} \equiv \{(W^+, W^-)\}$ in the theory of weak interactions, see Figure 20:

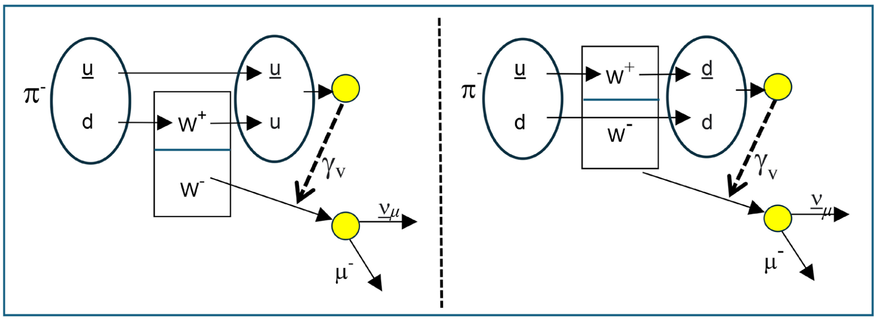


Figure 20. Pion decay.

Recall the reactions $[(W\underline{\otimes}\, u)\to d, (W\underline{\otimes}\, d)\to u]$, $\{W\} = (W^{+}\underline{\otimes}\, W^{-})$ and $\pi^{\pm} = (u\underline{\otimes}\, d)$; using the matrix form of $(\{W\}, \pi)$ and the spin $(\downarrow, \uparrow)$ we have, see the refer [14]:

$$\begin{aligned}
\begin{pmatrix} W^{-}_{\uparrow\uparrow} \\ W^{+}_{\downarrow\downarrow} \end{pmatrix} \underline{\otimes} \begin{pmatrix} \underline{u}_{\downarrow} \\ d_{\uparrow} \end{pmatrix}_{m(\pi)} &= \begin{pmatrix} W^{-}_{\uparrow\uparrow}\, \underline{\otimes}\, \underline{u}_{\downarrow} \\ W^{+}_{\downarrow\downarrow}\, \underline{\otimes}\, d_{\uparrow} \end{pmatrix}_{m(\pi)} = \begin{pmatrix} W^{-}_{\uparrow\uparrow}\, \underline{\otimes}\, \underline{u}_{\downarrow} \\ u_{\downarrow} \end{pmatrix}_{m(\pi)} \equiv \left[\left(W^{-}_{\uparrow\uparrow}\, \underline{\otimes}\, \underline{u}_{\downarrow}\right) \underline{\otimes}\, u_{\downarrow}\right]_{m(\pi)} \\
&= \left[\left(u_{\downarrow}\, \underline{\otimes}\, \underline{u}_{\downarrow}\right)_{\varepsilon(u,\underline{u})} \underline{\otimes} \left(W^{-}_{\uparrow\uparrow}\right)_{\Delta m^{*}}\right] = \left[\left(\gamma_{\downarrow\downarrow}\right)_{\varepsilon(u,\underline{u})} \underline{\otimes} \left(W^{-}_{\uparrow\uparrow}\right)_{\Delta m^{*}}\right] \\
&= \left[\left(\gamma_{\downarrow\downarrow}\right)_{\varepsilon(u,\underline{u})} \underline{\otimes} \left(\mu^{-}_{\uparrow} \underline{\otimes} \nu_{\uparrow}\right)_{\Delta m^{*}}\right] = \left[\left(\gamma_{\downarrow\downarrow}\, \underline{\otimes}\, \mu^{-}_{\uparrow}\right) \underline{\otimes} \left(\nu_{\uparrow}\right)\right] \\
&\to \begin{pmatrix} \underline{\nu}_{\uparrow} \\ \mu^{-}_{\downarrow} \end{pmatrix} \equiv \begin{pmatrix} \underline{\nu}_{R} \\ \mu^{-}_{R} \end{pmatrix}
\end{aligned} \tag{4}$$

Note that the spins of two leptons of decay are opposites because the spin of initial pion is zero; since the movement directions are opposite, for the moment conservation, the leptons have the same helicity.

4.2. The Neutron Decay

We consider the free neutron decay [7] and that of neutron inside a nucleus N. Using the matrices, we have:

$$\begin{aligned}
&\begin{pmatrix} W^{-}_{\uparrow\uparrow} \\ I_{(W^{+}\otimes W^{-})} \\ W^{+}_{\downarrow\downarrow} \end{pmatrix} \underline{\otimes} \begin{pmatrix} d_{\uparrow} \\ u_{\downarrow} \\ d_{\uparrow} \end{pmatrix}_{\uparrow} = \begin{pmatrix} W^{-}_{\uparrow\uparrow} \otimes d_{\uparrow} \\ I \otimes u_{\downarrow} \\ W^{+}_{\downarrow\downarrow} \otimes d_{\uparrow} \end{pmatrix} = \begin{pmatrix} W^{-}_{\uparrow\uparrow} \otimes d_{\uparrow} \\ I \otimes u_{\downarrow} \\ u_{\downarrow} \end{pmatrix} = \left(W^{-}_{\uparrow\uparrow}\right) + \left[\begin{pmatrix} d_{\uparrow} \\ u_{\downarrow} \\ u_{\downarrow} \end{pmatrix}_{\downarrow}\right]_{p} \\
&\qquad = \begin{pmatrix} \underline{\nu}_{\uparrow} \\ e^{-}_{\uparrow} \end{pmatrix} + \left[\begin{pmatrix} d_{\uparrow} \\ u_{\downarrow} \\ u_{\downarrow} \end{pmatrix}_{\downarrow}\right]_{p} = p_{\downarrow} + \left(e^{-}_{\uparrow} + \underline{\nu}_{\uparrow}\right) \\
&\begin{pmatrix} W^{+}_{\uparrow\uparrow} \\ I_{(W^{+}\otimes W^{-})} \\ W^{-}_{\downarrow\downarrow} \end{pmatrix} \underline{\otimes} \begin{pmatrix} d_{\uparrow} \\ u_{\downarrow} \\ d_{\uparrow} \end{pmatrix}_{\uparrow} = \begin{pmatrix} W^{+}_{\uparrow\uparrow} \otimes d_{\uparrow} \\ I \otimes u_{\downarrow} \\ W^{-}_{\downarrow\downarrow} \otimes d_{\uparrow} \end{pmatrix} = \begin{pmatrix} u_{\uparrow\uparrow\uparrow} \\ I \otimes u_{\downarrow} \\ W^{-}_{\downarrow\downarrow} \otimes d_{\uparrow} \end{pmatrix} = \left(W^{-}_{\downarrow\downarrow}\right) + \left[\begin{pmatrix} u_{\uparrow\uparrow\uparrow} \\ u_{\downarrow} \\ d_{\uparrow} \end{pmatrix}_{\uparrow\uparrow\uparrow}\right]_{p^{*}} \\
&\qquad = \begin{pmatrix} \underline{\nu}_{\downarrow} \\ e^{-}_{\downarrow} \end{pmatrix} + \left[\begin{pmatrix} u_{\uparrow\uparrow\uparrow} \\ u_{\downarrow} \\ d_{\uparrow} \end{pmatrix}_{\uparrow\uparrow\uparrow}\right]_{p^{*}} = \left[p^{*}_{\uparrow\uparrow\uparrow} + \left(e^{-}_{\downarrow} + \underline{\nu}_{\downarrow}\right)\right]_{N}
\end{aligned} \tag{5}$$

Since the antineutrino is right-handed ($\underline{\nu}_R$), the electron will be left-handed (e_L)

if the two particles are opposite in movement direction. This is consequence of the following (***Didactic Idea*** **(13)):** "*In the decay of the neutron there is a coupling between the* {*H*}*-bosons lattice, see the* H_w *and* H_w'*, and golden bosons lattice* {*W*}: {*H*} $\underline{\otimes}$ {*W*}". Let us consider the geometric representation of the decay of a free neutron [10], Figure 21:

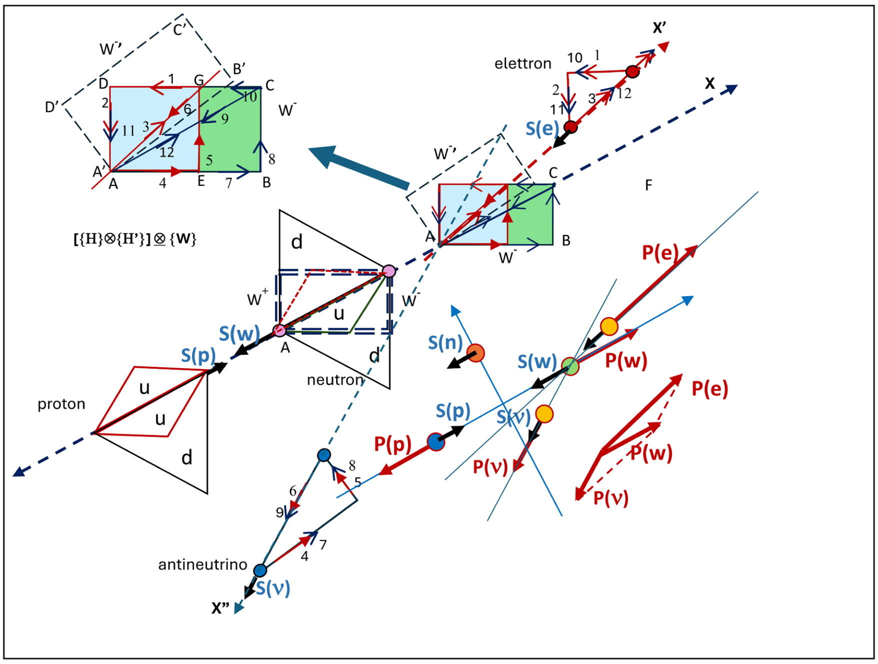


Figure 21. The neutron β-decay.

The numbers represent the $sq(\bullet)$ of the system ($\{H_w\}$, W^-) which we find again in the two leptons. In Figure 21 are represented the two phases of decay: the first phase of production $[n \to (p + W^-)]$ along the axis X and in the two phase the decay of W boson $[W^- \to (e^- + \underline{\nu})]$, along the axes X' and X", with Δt the decay time. Note that the proportions between λ_n and λ_e are not respected in Figure 21. Note that:

1) the $[sq_1(\bullet), sq_2(\bullet), sq_3(\bullet)]$ couple with $[sq_{10}(\bullet), sq_{11}(\bullet), sq_{12}(\bullet)]$ and are trapped in the triangle (AGD), forming the electron which propagates along the axis X'.

2) the $[sq_4(\bullet), sq_5(\bullet), sq_6(\bullet)]$ couple with $[sq_7(\bullet), sq_8(\bullet), sq_9(\bullet)]$ and are trapped in triangle $(A'B'C')_{w'}$ which is rotated respect to the triangle $(ABC)_w$, forming the antineutrino which propagates along the axis X", see the Figure 21.

The two axes (X', X") are different due to conservation of momentum, see the Figure 21.

Note that the bosons (H, W) are in a virtual state with empty $sq_1(\mathbf{o})$ and full semi-quanta $sq(\bullet)$. The H-Boson is in an excited vacuum state. This excitation induces rotations in the W-Boson (quadrilateral ABCD), which becomes the W'-boson (A'B'C'D'). In the quadrilateral (A'B'C'D') occur the couplings of the semi-quanta that will form the antineutrino.

Note that only when the couplings constructing the two triangles (AGD) and

AB'C') have been realized, then the splitting of W-boson can occur by the quadrilaterals H(AEDG) and W(ABCD).

Let us also remember that the W^- boson has a spin ($s = \pm 1$) so, in neutron decay, the spins of the two leptons agree. Since the directions of the two leptons are opposite, the two decay particles will have opposite helicities. Note that the electron should emerge from an axis prolongation of the side AG, but this would violate the total moment conservation. This does not happen. Thanks to the excitement of the system $(H_w \underline{\otimes} W^-) \to (H_w' \underline{\otimes} W^-')$, the electron is compatible with the conservation of momentum, also if H_w' "***breaks***" the golden geometric symmetry of the system $(H_w \underline{\otimes} W^-)$. We will say that the system $\{H_w \otimes H_w'\}$ has "broken" symmetry. The two H boson (H_w, H_w') can be considered as two excited states of a generic H boson: in fact, the "transformations" $[(n \to p) \equiv (d \to u), (W \to (e + \nu))]$ imply the "acquisition" of more $sq(\bullet)$ of virtual excitation from the system $[\{H_w \otimes H_w'\} \underline{\otimes} W^-]$. This is possible if the $\{H_w \otimes H_w'\}$ lattice is of course in an "excited vacuum state". The systems $[\{H_w \otimes H_w'\} \underline{\otimes} W^-]$ and $[\{W\} \underline{\otimes} \{W'\} \equiv \{W\}_G]$, see the sect. 3.1, recall [7] [10] the "***Higgs Mechanism***". The "broken" symmetry of system $\{H_w \otimes H_w'\}$ introduces a geometric asymmetry in the geometric model of Weak interactions because the triangular structures of the two leptons of decay are different: the electron is an isosceles right-angled triangle while the neutrino is a golden triangle with angles (90˚, 72˚, 18˚). Given the presence of the angles (72˚, 36/2), we will formally speak of a "golden" neutrino. The oscillations between H_w and H_w' and the rotation $(W \to W')$ delay the simultaneous emission of the pair $(e^-, \underline{\nu})$: this determines the considerable decay time of the neutron, just under 10 minutes. The decay time is determined by the probability $P(e, \underline{\nu})$ that all the $sq(\bullet)$ of the system $[\{H_w \otimes H_w'\} \underline{\otimes} \{W^- \otimes W^-'\}]$ are simultaneously distributed in the two particle triangles $(e^-_{(ADG)}, \underline{\nu}_{(A'B'C')})$. However, the first structure to form is the electron while the second structure, that of the neutrino, is slow to form. Let us remember that the electron is originated in the transition time from H_w to H_w', while the neutrino is formed from the time in which the surviving $sq(\bullet)_H$ of H_w transfer to W' to combine with the $sq(\bullet)_w$ of W and thus originate the neutrino structure. This explains the difference between the Compton length of the electron ($\boldsymbol{\lambda}_e$) and that of the neutrino ($\boldsymbol{\lambda}_\nu$). In PGM the probability $P(e, \underline{\nu})$ will determine a scale factor (κ) in the space dimension between the neutrino and electron. We will have $[\kappa \Leftrightarrow P(e, \underline{\nu})]$; since the $(e, \underline{\nu})$ pair originates from the (H_w, W) system it follows:

$$\bar{\lambda}_W / \bar{\lambda}_H = k\left(\bar{\lambda}_\nu / \bar{\lambda}_e\right) \quad (6)$$

5. The Leptons

5.1. The Muon

The available energy in pion decay is greater than that in neutron decay. The μ-muon, however, cannot be an electron since the energy involved is greater. Besides, in the pion there are two quarks which, via the W lattice, are transformed into two particles; the number of $sq(\bullet)$ present in two quarks will thus be double, as depicted in **Figure 22**. Even in the pion decay the W boson lattice must couple with the H-boson lattice: $\{[\{H_w\} \underline{\otimes} (\{W\} \underline{\otimes} \{\gamma\})] \rightarrow (\mu, \underline{\nu})$. Thanks to the $sq(\bullet)$ of photons the μ-muon and antineutrino take form. A possible decay, see the neutron decay, can be the following, Figure 22:

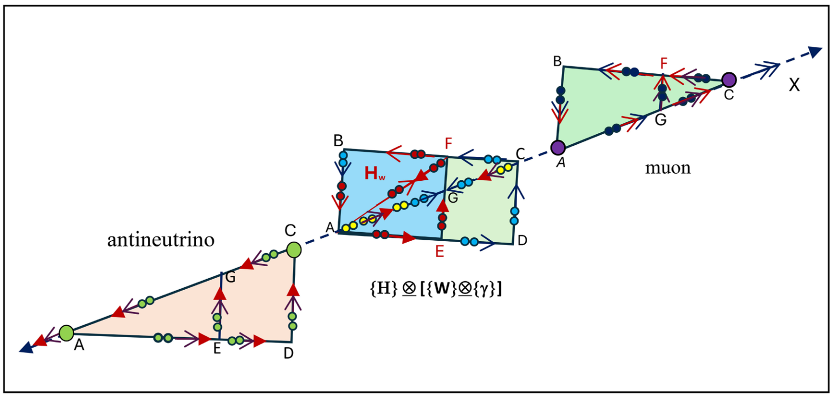


Figure 22. The $\{H\} \underline{\otimes} \{W\}$-decay.

Now the energy is sufficient to admit that the $sq(\bullet)$ of H-boson and γ-photon enter into the pathway (ACB), and they couple with the $sq(\bullet)$ of W boson to give the structure of the muon: at point A the $sq(\bullet, \text{o})$ of $\{H\}$ and $\{W\}$ converge and in A point the exchange takes place. The same happens to build the antineutrino. Since the decay occurs simultaneously for the two particles of decay, the muon has to wait a time Δt for the formation of the antineutrino to leave the $\{H \underline{\otimes} W\}$ lattice. Note that the muon is "golden" and, thus, is different from the electron even if it is a lepton. If the electron originates from the H_w' vibrational eigenstate of the $\{H\}$ lattice in the excited vacuum state, the muon originates from the H_w eigenstate coupled with the W^- boson.

Now we describe the muon decay, see Figure 23:

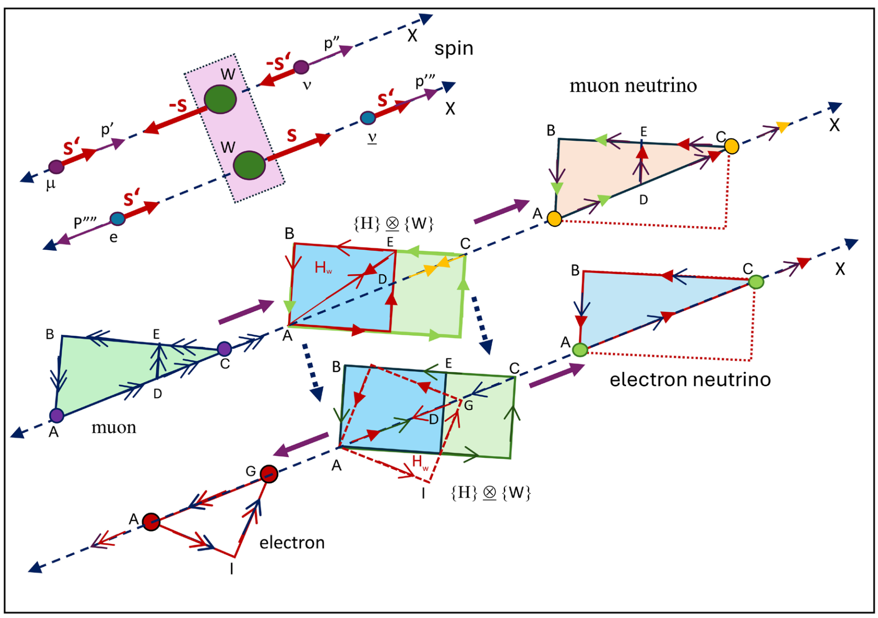


Figure 23. Muon decay.

5.2. The Tau Lepton (τ)

The (c, $\underline{c}$) mesons' pair [15], such as (J/Ψ), can have the following structure into a W-lattice, see **Figure 24**:

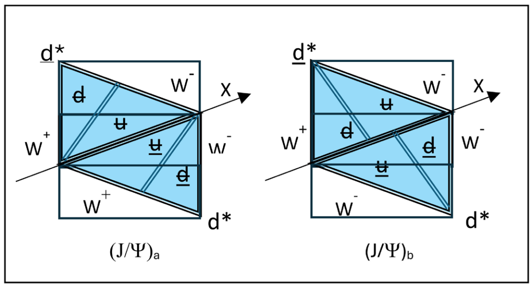


Figure 24. (J/Ψ) setting in four W bosons.

A meson with charm, see ($D^{\pm}$, D^0), could generate not only kaons and pions but also a $\boldsymbol{\tau}$ lepton. We can imagine the following decay generating a $\boldsymbol{\tau}$ lepton, see **Figure 25**:

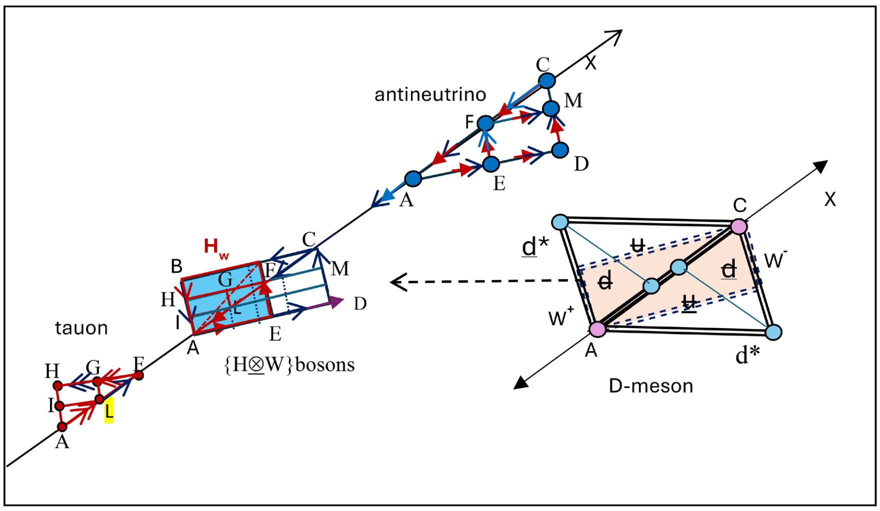


Figure 25. Decay of the D-meson with a final tau lepton.

On the diagonal AC of D-meson one can build the {$H \underline{\otimes} W$} lattice (the image to the left in **Figure 25**) then, we have formulated the following ***Didactic Idea*** **(14)** "*From the D-meson decay one can draw the* $\boldsymbol{\tau}$*-lepton structure*". In the experiments, the discovery of the τ lepton was made from various decay products of heavy mesons as D-mesons. The compatibility between the tau lepton and the W boson leads us to believe that $\tau^{\pm}$ can have the origin, like the muon, from a lattice [{H} $\underline{\otimes}$ {W}]. the τ lepton, like the muon, also originates from the coupling of the degenerate eigenstate H_w' (H_{w1}', H_{w2}') with the W boson. So, the [{H} $\underline{\otimes}$ {W}] lattice acting on the D-meson (c, $\underline{d}$) allowing the decay into a pair of leptons (τ, ν_τ).

5.3. The Leptons

We can thus give the structures of the charged Leptons, **Figure 26**:

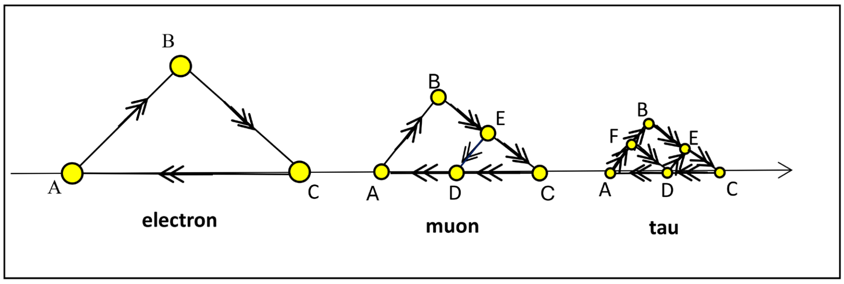


Figure 26. The three possible configurations of leptons (e, μ, τ).

Note that no oscillation is possible between the electron and muon because they have different geometric structures. Same between the electron and tau. This is because the electron is a rectangular isosceles triangular structure while the muon is a golden triangle, the same as the tau. Instead, oscillations between the muon and tau could occur. However, it is found that the transition from tau to muon manifests itself as tau decay. Let's move on to neutral leptons. We can show that in PGM the neutral leptons differentiate into "weak" neutral lepton and an "electromagnetic" neutral. In fact, the elementary charged lepton, the electron, can be associated with two neutral leptons of different nature, Figure 27:

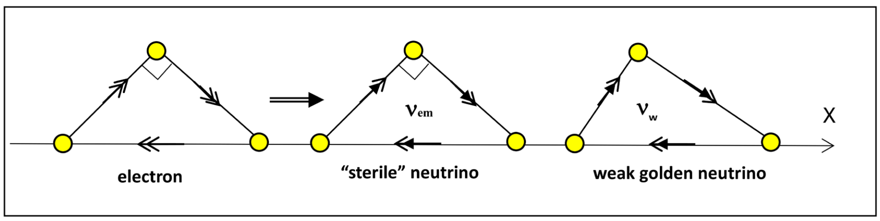


Figure 27. The electron and two neutrinos to it associated.

The weak neutrino (ν_w) would be the one associated with the weak interaction or Beta decay. The electromagnetic neutrino ν_{em} would instead be associated with a coupling between two photons and two "electromagnetic" bosons H_{em}. Nevertheless, this neutrino could appear as a "neutral" electron, see Figure 28:

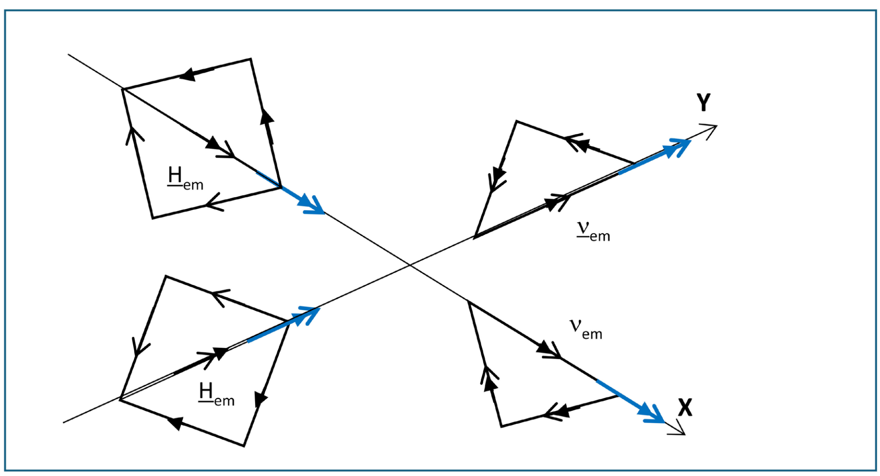


Figure 28. Representation of the reaction $\underline{H} \rightarrow (\nu_{em} + \underline{\nu}_{em})$.

In the Dirac Theory, this neutrino $\boldsymbol{\nu}_{em}$ would be given by a spinor Ψ with two components (Ψ_1, Ψ_2) because represents a neutral particle. But here, see **Figure 28**, the two neutrinos ($\boldsymbol{\nu}_{em}$, $\underline{\boldsymbol{\nu}}_{em}$) appear as particles with the same helicity (right-handed) and mass (they could also have the same mass as the electron one). As it happens for the electrons, these neutrinos can be particles left-handed or right-handed, and thus they could be "sterile" neutrinos [16]. We ask, even, if they could be the Majorana neutrinos, since is ($\boldsymbol{\nu}_{em} = \underline{\boldsymbol{\nu}}_{em}$). Nevertheless, see the Section 4.2, in the theory of the PGM is ($\boldsymbol{\nu}_{em} \neq \boldsymbol{\nu}_{w}$), where $\boldsymbol{\nu}_{w}$ is the weak neutrino which has the shape of a golden triangle while $\boldsymbol{\nu}_{em}$ has the shape of a rectangular isosceles triangle. Besides, recall that in the β-decay it is ($\boldsymbol{\nu}_{w} \neq \underline{\boldsymbol{\nu}}_{w}$) because the neutrino experimentally is always left-handed while the antineutrino is right-handed. This aspect affirms that, in PGM, the $\boldsymbol{\nu}_{em}$ cannot be present in β-decay. It follows that the electromagnetic neutrino $\boldsymbol{\nu}_{em}$ or "sterile" neutrino cannot be a "Majorana's neutrino" [17], supposed this last present in the "double" β-decay. Recall that the "double" β-decay can happen only if ($\boldsymbol{\nu} = \underline{\boldsymbol{\nu}}$). If in PGM we show that the weak neutrinos $\boldsymbol{\nu}_{w}$ are different from weak antineutrinos $\underline{\boldsymbol{\nu}}_{w}$, then we will show that the "double" β-decay with weak neutrinos does not exist. The weak neutrino would instead have the following configurations, **Figure 29**:

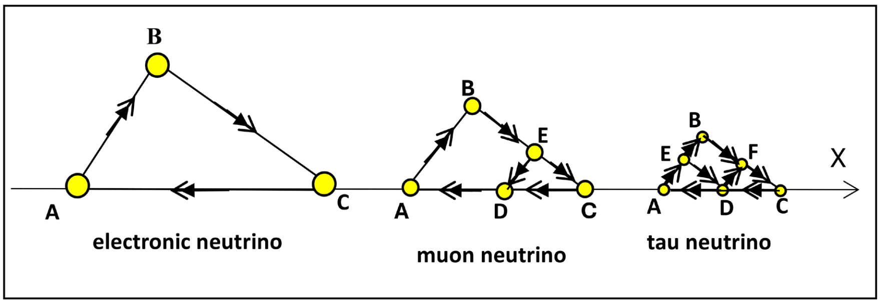


Figure 29. The three possible configurations of neutrinos (ν_e, ν_μ, ν_τ).

Perhaps this can only happen if the flavours [18] differ slightly in mass, **Figure 30**:

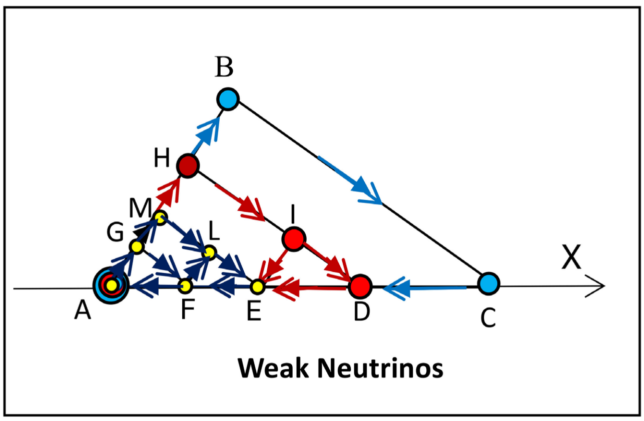


Figure 30. The three states of oscillations of the three neutrinos (ν_e, ν_μ, ν_τ).

In **Figure 30**, the three neutrinos appear as unique particle.

A note due: the weak neutrinos cannot decay like partner leptons, but they can only oscillate into their three savours.

6. The *Z* Boson

6.1. The Geometric Shape of the *Z* boson

Neutrinos could be produced by a particular state of the lattice $\{W\}$ where the W bosons instead of being adjacent, see **Figure 6**, may be superposed, and interpenetrated with each other. In this case, the geometric superposition gives rise to the Z boson [4] given by $Z = W^+ \underline{\otimes} W^-$. The combination $\underline{\otimes}$ implies a state in which there are two aspects ($\oplus, \otimes$):

1) Configuration of the **Figure 19**, where the two W-bosons (W^+, W^-) exchange $sq(\bullet) \to Z = W^+ \oplus W^-$

2) Where the W-bosons are overlapped and in interpenetration $\to Z = W^+ \otimes W^-$

As it has been said [8], for the neutral pion $\pi^0 = \pi^+ \underline{\otimes} \pi^-$, where the mass is given by $m(\pi^0) = m(\pi^\pm) \pm \Delta m$, we will also have for the Z-Boson, with $Z = W^+ \underline{\otimes} W^-$, that $m(Z) = m(W^\pm) \pm \Delta m$. In the pion we had $m(\pi^0) = m(\pi^\pm) - \Delta m$, instead in Z-Boson it is $m(Z) = m(W^\pm) + \Delta m$; this happens because in Z-Boson there are two possible coupling configurations of two $W^\pm$-bosons. Regarding the spin, the interpenetration, as happens in the pions, admits $\boldsymbol{s}(Z) = \boldsymbol{s}(W^\pm)$, that is the spin of the Z-boson will be the same as that of one of two $W^\pm$. However, by the intermediation of H_w boson, we could also have $[W^+ \underline{\otimes} W^- \equiv \gamma]$, so as it happens in the process of annihilation of pair (e^+, e^-) where the H_{em}-boson intermediates the annihilation, see the sect. 3.4. Note this is compatible with the electroweak theory. Finally, we note that Z, being a "combination" and "interpenetration" of two bosons $[Z = W^+ \underline{\otimes} W^-]$, could decay in a pair (matter-antimatter) $[(l^+, l^-), (\nu, \underline{\nu}), (q, \underline{q})]$; in the case of a pair neutrinos, it is, see **Figure 31**:

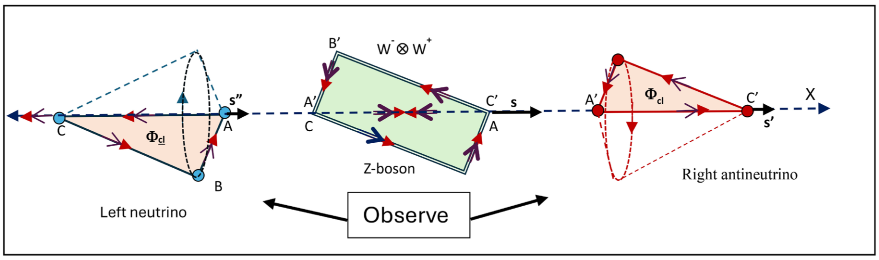


Figure 31. Decay of the Z boson in two neutrino-structures.

Remember that in SM the two particles originating from the β-decay are two fermions of a matter-antimatter pair: if to the left there is a particle, to the right there is its antiparticle. Even here, see **Figure 31**, if to the left there is a particle, to the right there is its antiparticle. Note, in **Figure 31**, that the two particles always have opposite helicity. Therefore, in PGM as it happens in SM, the Z boson could decay into a pair of golden neutrinos: $[Z \to (\nu_L + \underline{\nu}_R)]$.

6.2. The Chirality in PGM

In the mirror we will have, Figure 32:

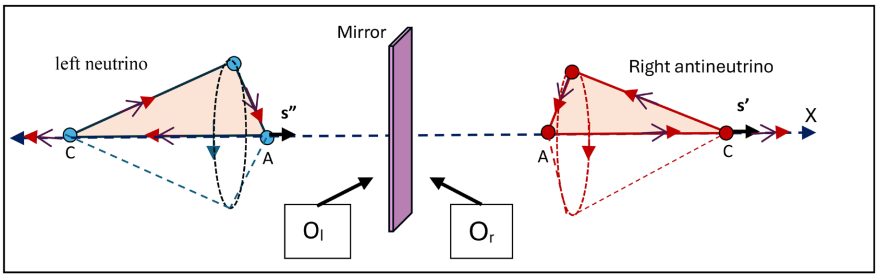


Figure 32. Mirror reflection in P mirror of a neutrino and antineutrino.

Note that each of them is the mirror image of the other (they have opposite rotations, both around the propagation axis and that of the internal flow of $sq(\bullet)$, and opposite directions of motion ($\boldsymbol{p}' = -\boldsymbol{p}''$)) except for the spin vector which is invariant in direction ($\boldsymbol{s}' = \boldsymbol{s}''$). This determines that the mirror image of a particle has opposite helicity. It follows that in PGM the reflected image into mirror P of a neutrino would be an antineutrino. If neutrino and antineutrino were the same particle (like the neutral pion) then the physical phenomenology determined by each would be identical for P transformations, that is, they would give rise to the same reactions. However, this does not happen: the weak interactions involving neutrinos and antineutrinos are different (see decays) and undergo variations for P transformations, that is the weak interactions do not maintain "parity" [7] [10]. This unequivocally means that "weak" neutrinos and antineutrinos are two distinct particles. Remember that, if we observe one of the two particles from moving reference systems, the helicity will change, because the spin of a particle is a relativistic invariant, while the helicity is not.

In fact, in a Reference Frame S' that surpasses one of the two particles the helicity changes ($\boldsymbol{v}' > \boldsymbol{v}$), see Figure 33:

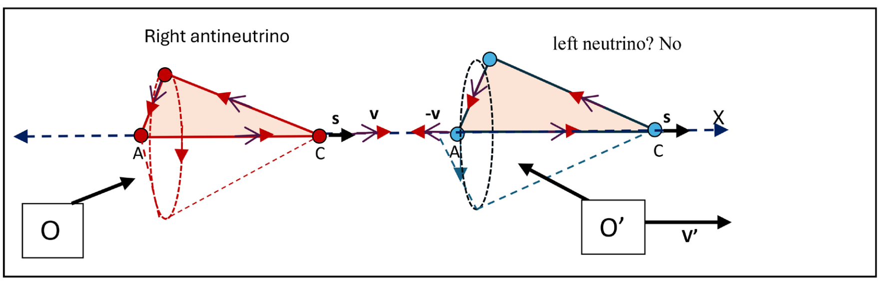


Figure 33. The right-handed antineutrino characteristic does not change for observers in relative motion.

It seems, in PGM, that by surpassing a right-handed antineutrino it appears to us as a left-handed particle. If this happens then being a neutrino or being an antineutrino would be relative to the observer. Instead, there must thus be an intrinsic characteristic of the neutrino and antineutrino that makes them such for any observer. This characteristic cannot be helicity, which instead depends on the motion of the observer and which changes upon *P*-reflection. The matter-antimatter distinction, however, must be absolute and not relative (Lorentz Invariance). It is therefore necessary that there is something that makes the neutrino and the antineutrino different for all observers in relative motion. This indication must be found within the geometric model, where particles are geometric structures of coupled quantum oscillators. Even in the wave equation there must be a variable, other than helicity, which remains unchanged for the Lorentz transformations, and which makes it possible to say that a left-handed neutrino cannot become a right-handed neutrino. This variable is "***chirality***" (χ). Recall that chirality [19] is connected to the left-handed property of the left hand and the right-handed property of the right hand. Recall that in Dirac theory, chirality is defined by the γ^5 array, which allows us to talk about right-handed and left-handed particles. The mathematical meaning is known but not the physical meaning in depth. In PGM, instead, we show that the variable χ is closely connected to the possibility that a particle has a geometric structure with an internal flow vector Φ. Look carefully at **Figure 34**, where one can highlight the correspondence between the hand and a triangular structure:

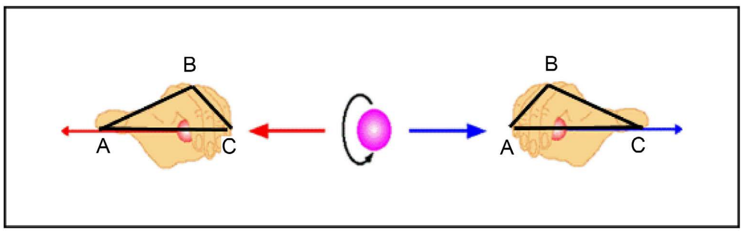


Figure 34. The Chirality of the hands.

Note that:

• each hand can be considered as a "triangular structure" in which the thumb direction is given by the AC(→) side for the right hand, and CA(←) side for the left hand

• each triangle-particle (not isosceles) has a "thumb" side [(AC), (CA)] or chirality indicator (I_χ): {[($I_{\chi r}$) ⇔ AC(→)] ⇔ right-hand}, {[($I_{\chi l}$) ⇔ CA(←)] ⇔ left-hand}

• the two chirality indicators [($I_{\chi l}$), ($I_{\chi r}$)] defined in PGM are coincident to two chirality operators (Γ_l, Γ_r) built with the γ^5 array of the Dirac theory

• a neutrino will be a triangular structure with a left thumb [($I_{\chi l}$) ⇔ AC(→)] while the antineutrino will be a triangular structure with a right thumb [($I_{\chi r}$) ⇔ CA(←)]

We observe that the right-handed particle (**Figure 32**) has a spin parallel to the displacement vector and therefore helicity equal to one $[\varepsilon = \boldsymbol{s} \bullet \boldsymbol{p} = +1] \rightarrow (\Psi_R)_{\varepsilon=+1}$

Instead, the left-handed particle has a spin antiparallel to the displacement vector and therefore has negative helicity: $[\varepsilon = \boldsymbol{s} \bullet \boldsymbol{p} = -1] \rightarrow (\Psi_L)_{\varepsilon=-1}$.

Thus, we will have: $[(\Psi_R)_{\varepsilon=+1}, (\Psi_L)_{\varepsilon=-1}]$

We have thus demonstrated that the existence of only left-handed neutrinos and right-handed antineutrinos is a consequence of a geometric structure possessed by neutrinos.

6.3. The Mass of the Higgs's Boson (H_w)

Now, we insert a H boson with "square" shape of side AB into a "golden" W boson, see the Figure 6; then we have the Figure 35:

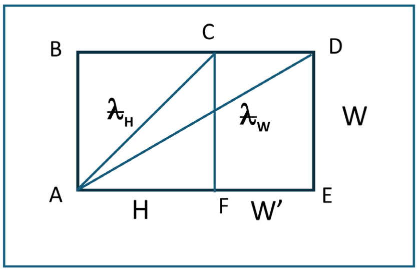


Figure 35. The H-Boson with (λ_H) and golden W-boson (λ_W).

If we consider the rectangle (ABDE) of the golden type, we will have that:

$$BD = \phi AB = (1.618) AB \tag{7}$$

In this case, the H boson (ABCF) will be called the "golden" H boson ($\underline{\underline{H}}$) because it is stuck inside a golden W. Note that the CDEF rectangle is also golden and therefore W' is a golden boson. We will have that AD/AC will be:

$$AD/AC = \left[\phi^2 + 1\right]^{1/2} / 2^{1/2} = 1.345 = \lambda_W / \lambda_{\underline{\underline{H}}} \tag{8}$$

Thus, it is $m(\underline{\underline{H}})/m(W) = (1.345)$. That is:

$$m\left(\underline{\underline{H}}\right) = m(W)(1.345) \tag{9}$$

We remember that $Z = (W^+ \underline{\otimes} W^-)$. Given the presence of both interpenetration ($\otimes$) and energy exchange ($\oplus$), we could suspect that there is a mass difference, Δm, not zero as in the neutral pion and, therefore, it is $m(Z) = m(W) \pm \Delta m$. The Higgs's boson discovered at CERN is a H_w and was identified and measured in mass by the reaction $H_w \rightarrow W^+ + W^-$ [13]. In PGM the reaction could be ($H \underline{\otimes} H \rightarrow W^+ + W^-$). Therefore, when we calculate the mass of the H_w Boson, we must consider the Z boson: $Z = (W^+ \underline{\otimes} W^-)$. We could thus have that $m(H_w) \approx m(Z) \pm \Delta m$. From the equation 9, we will have $\boldsymbol{m(H_w)} \approx (1.345)\ (m_Z)$, then, it follows:

$$m\left(\underline{\underline{H}}\right) = (1.345)(m_Z) = (1.345)(91.1876)\,\text{GeV}/\text{c}^2 = (122.647)\,\text{GeV}/\text{c}^2$$

The H boson discovered at CERN is a weak boson of Higgs H_w: $H \rightarrow W^+ + W^-$
Thus, we will have that $m(H_w) = \iota m(\underline{\underline{H}})$. It involves identifying the value of ι.

6.4. The Muon Mass (μ)

Let us return to the decay of the charged pion, see Figure 20. The $\{W\}$ lattice induces the pion decay by transforming its quarks; in this process, it gives virtual energy ε_w to the pion in the transformation ($u \to d$) or receives energy in the transformation ($d \to u$). Recall that the bonded masses of the quarks are:

$$\left\{\left[m(d)_{\pi^\pm} = (86.26)\,\text{MeV/c}^2\right],\left[m(u) = (53.31)\,\text{MeV/c}^2\right]\right\} \quad (10)$$

with m the bonded masses by gluons of $(u, d)_g$ quarks into pion. The difference in absolute value is:

$$\left|\Delta m(u,d)\right|_g = \left|m(u) - m(d)\right|_g = \left|53.31 - 86.26\right|\text{MeV/c}^2 = (32.95)\,\text{MeV/c}^2 \quad (11)$$

The transformation ($d \to u$) introduces a massive energy of *32* Mev/c² into {W} but takes it (−32 Mev/c²) away from the pion (in the reference frame of pion is all massive energy or initial total energy); instead, the transformation ($u \to d$) removes a massive energy of 32 Mev/c² from the {W} lattice. In the first case, since there is a pion that decays, 32 Mev/c² of massive energy will be missing from the mass of the system that replaces the pion, while in the second case, 32 Mev of massive energy will be missing from the W^- boson that after decays in the leptonic pair. Ultimately, in both cases, to the end of the decay process, a massive energy Δm = 32 Mev/c² will always be missing from the initial mass (only that of the pion): $(m(\pi^\pm) - \Delta m)$. Thus, in the reference frame at rest of pion, coincident with that of the W boson, the final massive energy of system {$W^- \underline{\otimes} (\mu, \underline{\nu}_\mu)$} will be:

$$m(W)_{(\mu,\underline{\nu}_\mu)} = m(\pi^\pm) - \Delta m = (139.57 - 32.95)\,\text{Mev/c}^2 = (106.62)\,\text{Mev/c}^2 \quad (12)$$

This massive energy is next to the muon mass. We consider some aspects:

1) The energy (−32 Mev/c²) will be used as kinetic energy (K) in the (μ, ν_μ) system.

2) The annihilation photon γ, in both cases, flows inside the W^- producing a phase shift between the electrically charged sector and the neutral one of the systems {$W \underline{\otimes} H_w$}; this is because the photon acts only on its electric sector.

3) This phase shift separates the two sectors which will free themselves from each other only when they reach the completeness of a quantum oscillating system (that is they will have an integer n number of quanta).

4) In this separation, a part $\Delta\varepsilon_{spl}$ of massive energy of system {$W^- \underline{\otimes} (\mu, \underline{\nu}_\mu)$} could be used for the separation of two triangular sectors; it follows $\boldsymbol{m(\mu, \underline{\nu}_\mu) < m(W)_{(\mu, \underline{\nu}_\mu)}}$.

5) The splitting energy $\Delta\varepsilon_{spl}$ for separate the system is:

$\Delta\varepsilon_{spl} = m(W)_{(\mu,\underline{\nu}_\mu)} - m(\mu,\underline{\nu}_\mu)$.

Note that in the neutron decay, we have the difference between the masses of neutron and proton is: $(\Delta\varepsilon_m)_{(n,p)} = (1.29)\,\text{MeV/c}^2 = m(W)_{(e,\underline{\nu}_e)}$. Also in the neutron decay there is the splitting energy $\Delta\varepsilon_{spl}(n,p) \to m(e,\underline{\nu}_e) < m(W)_{(e,\underline{\nu}_e)}$.

If we have $m(e)$ = (0.511) MeV/ c² and $m(\nu) \ll m(e)$, then it could be:

$$(\Delta\varepsilon_m)_{(n,p)} - m(e) = \Delta\varepsilon_{spl}(n,p) = (1.29 - 0.51)\,\text{Mev/c}^2 = (0.78)\,\text{Mev/c}^2$$

If $\boldsymbol{\Delta\varepsilon_{spl}(e) = \Delta\varepsilon_{spl}(\mu)}$ then it is:

$$\Delta\varepsilon_{spl}(\mu) = m(W)_{(\mu,\underline{\nu}_\mu)} - m(\mu,\underline{\nu}_\mu) \to m(\mu,\underline{\nu}_\mu) = m(W)_{(\mu,\underline{\nu}_\mu)} - \Delta\varepsilon_{spl}(\mu)$$

It follows:

$$m\left(\mu,\underline{\nu}_{\mu}\right)=m\left(W\right)_{\left(\mu,\underline{\nu}_{\mu}\right)}-\Delta\varepsilon_{spl}\left(\mu\right)=\left(106.62-0.78\right)\mathrm{Mev/c^2}=\left(105.84\right)\mathrm{Mev/c^2}$$

This is the mass of system (μ, $\underline{\nu}_{\mu}$). It follows: $m\left(\mu\right)=m\left(\mu,\underline{\nu}_{\mu}\right)-m\left(\underline{\nu}_{\mu}\right)$

If we use the experimental value of muon mass [(105.65) Mev/c^2], then we can approximately calculate the $\boldsymbol{\nu}_{\mu}$. muon neutrino mass:

$$m\left(\underline{\nu}_{\mu}\right)=m\left(\mu,\underline{\nu}_{\mu}\right)-m\left(\mu\right)=\left(105.84-105.65\right)\mathrm{Mev/c^2}=\left(0.19\right)\mathrm{Mev/c^2}$$

7. Conclusion

Reading the article, it can be noted that PGM is a "simple" and "faithful" representation of the SM, with the advantage of being a representative model of the phenomenology of particles which deepens their physical knowledge and also manages to make predictions both on the structures of the particles same as also on the physical values of some physical quantities such as the mass of the muon or the mass of the Higgs boson. Other representative insights of PGM are very fruitful in understanding electroweak phenomena: from the structures of leptons, we derive fundamental aspects such as neutrino oscillations while for charged leptons it is shown that there are no possible oscillations between flavors. From an in-depth reading of the article, it is also possible to see the descriptive potential of the model such as the interactions between hadrons, the calculation of their masses (using the structural equations), and the interactions relating to the weak bosons (W, Z) and the Higgs's boson. As also can be seen in this article, PGM introduces, didactically and otherwise, new descriptive paradigms of particle phenomenology as the interactions between particles through intermediary lattices (bosons) and the quantum oscillator at sub-oscillators and semi-quanta (IQuO). Thanks to these new paradigms, it is possible to see that mass, electrical charge, and color charge find "geometric" expressions that facilitate the understanding of these fundamental concepts of physics. Thus, in PGM, it is possible to avoid basic problematic aspects of SM such as the "renormalization" of mass and electric charge and some issues of symmetries and non-symmetries. In conclusion, in representing didactically the physics of SM through geometric figures, we have had the notable surprise that the didactic model created (PGM) was a model exhaustive, in-depth analysis, and predictive of the phenomena described mathematically by the SM. Not only, but perhaps, in this article, we could also glimpse a further effort to broaden the idea of geometrization of physics, begun a century ago by Einstein with the geometrization of the gravitational field: PGM could so constitute an unprecedented and original attempt to "geometrize" all the particle-fields. We therefore ask ourselves whether by PGM it is possible to finally solve the age-old problem of the quantization of the gravitational field or graviton.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Guido, G. (2012) The Substructure of a Quantum Oscillator Field. arXiv: 1208.0948. https://doi.org/10.48550/arXiv.1208.0948

[2] Guido, G. (2019) The Origin of the Color Charge into Quarks. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **5**, 1-34. https://doi.org/10.4236/jhepgc.2019.51001

[3] Guido, G. (2023) The Geometric Model of Particles (The Origin of Mass and the Electron Spin). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **9**, 941-963. https://doi.org/10.4236/jhepgc.2023.94070

[4] Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic about Standard Model (Geometric Model of Particle: The Quarks). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **10**, 854-874. https://doi.org/10.4236/jhepgc.2024.102053

[5] Guido, G. (2020) A New Descriptive Paradigm in the Physics of Hadrons, and Their Interactions. *Global Journal of Science Frontier Research*, **20**, 41-50. https://doi.org/10.34257/GJSFRAVOL20IS13PG41

[6] Guido, G. (2021) Theoretical Spectrum of Mass of the Nucleons: New Aspects of the QM. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **7**, 123-143. https://doi.org/10.4236/jhepgc.2021.71006

[7] Quigg, C. (1997) Gauge Theories of the Strong, Weak, and Electromagnetic Interactions. Perseus Westview Press.

[8] Guido, G. (2019) The Bare and Dressed Masses of Quarks in Pions via the of Quarks' Geometric Model. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **5**, 1123-1149. https://doi.org/10.4236/jhepgc.2019.54065

[9] Crawford Jr., F.S. (1968) Waves. McGraw-Hill Book Company.

[10] Morpurgo, G. (1992) Introduzione alla Fisica delle Particelle. Zanichelli.

[11] Feynman, R.P. (1961) Quantum Electrodynamics. W.A. Benjamin, Inc.

[12] Greiner, W. and Reinhardt, J. (2009) Quantum Electrodynamics. Springer.

[13] Aad, G., Abbott, B., Abbott, D.C., Abeling, K., Abidi, S.H., Aboulhorma, A., *et al.* (2022) A Detailed Map of Higgs Boson Interactions by the ATLAS Experiment Ten Years after the Discovery. *Nature*, **607**, 52-59. https://doi.org/10.1038/s41586-022-04893-w

[14] Guido, G., Bianchi, A. and Filippelli, G. (2024) An Original Didactic about Standard Model. (Geometric Model of Particle: The Quarks). *Journal of High Energy Physics*, *Gravitation and Cosmology*, **10**, 854-874. https://doi.org/10.4236/jhepgc.2024.102053

[15] Köpke, L. (1989) J/Ψ Decays. *Physics Reports*, **174**, 67-227. https://doi.org/10.1016/0370-1573(89)90074-4

[16] Volkas, R.R. (2002) Introduction to Sterile Neutrinos. *Progress in Particle and Nuclear Physics*, **48**, 161-174. https://doi.org/10.1016/s0146-6410(02)00122-9

[17] Bilenky, S. (2020) Neutrinos: Majorana or Dirac? arXiv: 2008.02110. https://doi.org/10.48550/arXiv.2008.02110

[18] Samuel Reich, E. (2012) Neutrino Oscillations Measured with Record Precision. *Nature*. https://doi.org/10.1038/nature.2012.10202

[19] Peskin, M.E. (2019) Concepts of Elementary Particle Physics. Oxford University Press. https://doi.org/10.1093/oso/9780198812180.001.0001