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The BTFR and MOND with Redshifts of Graviton Energy

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Abstract

This report is about the graviton redshift theory (GRST) which hypothesises the redshift of the energy of gravitons traveling in fields. A new source of energy loss in galaxy dynamics is introduced. Due to the hypothetical interactions of gravitons with the expansion of the universe, which causes an energy loss of the gravitons due to cosmological redshift, the rotation equation for galaxies, which previously had the Newtonian potential energy and the graviton gravitational redshift energy loss, is now updated with the graviton cosmological redshift energy loss. From the galaxy rotation equation, the baryonic Tully-Fisher relation (BTFR) and the modified Newtonian dynamics (MOND) are defined in radial distribution form. Fits to galaxy rotation motion are detailed. A cosmic connection for the BTFR is defined. The result is that galaxy rotation curves are fully accounted for with the GRST rotation equation and the BTFR and MOND theories are incorporated into a unified framework.

Keywords

Gravitons, Gravitational Redshift, Cosmological Redshift, Graviton Coupling Coefficients, Spiral Galaxies

1. Introduction

This paper is a development of earlier papers [1] and [2]. There the concept was introduced that if gravitons exist, and given that they are bosonic relativistic particles, like photons but of spin 2, then, like photons traveling in a gravitational field, the gravitons traveling in the field would experience a gravitational redshift as they go from a lower negative potential near the source mass to a higher potential going toward an orbiting mass. This energy loss $\delta\xi_g$ due to the gravitational redshift is expressed by,

$$\delta\xi_g(r) = -\frac{GM(r)m}{r^2}\delta r, \tag{1}$$

where G is Newton's gravitational constant, $M(r)$ is the baryonic source mass at r, m is the relativistic mass of the gravitons and δr is a small change in the position of the gravitons. Integrating Equation (1) from the origin of the system of masses to the position r of the gravitons and multiplying by a coupling coefficient $K_g(r)$ of the gravitons to the mass at position r yields the total energy loss $\Delta\Xi_g(r)$ given by,

$$\begin{aligned}\Delta\Xi_g(r) &= K_g(r)\int_0^r \delta\xi_g(s) \\ &= -k_f \frac{M(r)}{M_b}\int_0^r \frac{GM(s)m}{s^2}\delta s,\end{aligned} \tag{2}$$

where the coupling coefficient

$$K_g(r) = k_f \frac{M(r)}{M_b}, \tag{3}$$

where k_f is a coupling constant, $M(r)$ is the baryonic mass within radial position r and M_b is the total baryonic mass of the galaxy.

In addition to gravitons experiencing reduction of their energy due to gravity, it is hypothesised that even in galaxies, gravitons are also susceptible to the expansion of the universe and undergo an energy loss $\delta\xi_e$ due to cosmological redshift of their energy, which is assumed to take the form,

$$\delta\xi_e = -mc^2 k_e \frac{\delta r}{R_0}, \tag{4}$$

where m is the relativistic mass of the gravitons, c is the speed of gravitons (speed of light) in vacuum, k_e is a coupling constant, δr is the change in distance from the galaxy center and $R_0 = c/H_0$ is the radius of the universe where H_0 is the Hubble constant at the epoch of interaction. Integrating Equation (4) from the origin of the system of masses to the position r of the gravitons yields the total energy loss $\Delta\Xi_e(r)$ given by,

$$\Delta\Xi_e(r) = \int_0^{\Delta\Xi_e(r)} \delta\xi_e = -\int_0^r mc^2 k_e \frac{\delta s}{c/H_0} = -mk_e cH_0 r. \tag{5}$$

Adding $\Delta\Xi_g(r)$ from Equation (2) and $\Delta\Xi_e(r)$ from Equation (5), while treating them as potential energies, they can be combined with the gravitational potential energy in the form,

$$-\frac{mG\mathcal{M}(r)}{r} = -\frac{mG}{r}\left(M(r) + \frac{k_f M(r) r}{M_b}\int_0^\infty \frac{M(s)}{s^2}\mathrm{d}s + \frac{k_e cH_0 r^2}{G}\right), \tag{6}$$

where the total galaxy mass $\mathcal{M}(r)$ is defined by,

$$\mathcal{M}(r) = M(r) + \frac{k_f M(r) r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s + \frac{k_e cH_0 r^2}{G}. \tag{7}$$

Then, since by the virial theorem the kinetic energy is minus half the potential energy, with Equation (6), the standard form for the equation of circular motion in a galaxy becomes,

$$\frac{1}{2}mv^2(r) - \frac{mG\mathcal{M}(r)}{r} = -\frac{mG\mathcal{M}(r)}{2r}, \tag{8}$$

which expresses the graviton redshift theory (GRST) version of the energy equation of motion of an object of mass m in a circular orbit in the galaxy,

$$\begin{aligned} &\frac{1}{2}mv^2(r) - \frac{GM(r)m}{r} - k_f\frac{M(r)}{M_b}\int_0^r \frac{GM(s)m}{s^2}\delta s - mk_e cH_0 r \\ &= -\frac{GM(r)m}{2r} - k_f\frac{M(r)}{2M_b}\int_0^r \frac{GM(s)m}{s^2}\delta s - m\frac{k_e cH_0}{2}r. \end{aligned} \tag{9}$$

Moving all terms except $v^2(r)$ from the left hand side (l.h.s.) to the right hand side (r.h.s.) of Equation (9), while changing δs to $\mathrm{d}s$, and simplifying, yields the expression for the rotational velocity at the radial distance r from the galaxy center, given by,

$$v^2(r) = \frac{GM(r)}{r} + \frac{k_f M(r)}{M_b}\int_0^\infty \frac{GM(s)}{s^2}\mathrm{d}s + k_e cH_0 r. \tag{10}$$

Equation (10) is the new equation which will be used to fit the rotation curves of spiral galaxies and to derive a distribution form for the baryonic Tully-Fisher relation (BTFR).

This paper relates to the study of the dynamics of spiral galaxy rotation curves found in [3] and will access the data provided by the SPARC data base [4] to which that study refers.

2. Obtaining the Formulas for k_f and k_e

Assuming that the galaxy rotation velocity is constant in a neighborhood of radial position r_0 which is near the furthest radial position of the galaxy, take the derivative with respect to r of the velocity in Equation (10), assuming the mass is constant so $\mathrm{d}M(r)/\mathrm{d}r = 0$ at r_0, and setting the result to zero at r_0 gives,

$$0 = -\frac{GM(r_0)}{r_0^2} + \frac{k_f G(M(r_0))^2}{M_b r_0^2} + k_e cH_0, \tag{11}$$

which can be solved for k_e, expressed by

$$k_e = \frac{GM(r_0)}{cH_0 r_0^2}\left(1 - \frac{k_f M(r_0)}{M_b}\right). \tag{12}$$

To obtain a formula for k_f, solve for it using Equation (10) evaluated at $r = r_0$, expressed by,

$$k_f = \frac{v^2(r_0) - \frac{GM(r_0)}{r_0} - k_e cH_0 r_0}{\frac{M(r_0)}{M_b}\int_0^{r_0}\frac{GM(s)}{s^2}\mathrm{d}s}, \tag{13}$$

which after substitution for k_e using Equation (12) reduces to,

$$k_f = \frac{v^2(r_0) - \dfrac{2GM(r_0)}{r_0}}{\dfrac{M(r_0)}{M_b}\left(\displaystyle\int_0^{r_0}\frac{GM(s)}{s^2}\mathrm{d}s - \frac{GM(r_0)}{r_0}\right)}. \tag{14}$$

The next section will define the updated quadratic equation for the baryonic mass distribution which includes the cosmological term.

3. Quadratic Equation for the Baryonic Mass $M(r)$

With the additional cosmological term, an updated quadratic equation for the baryonic mass distribution $M(r)$ can be obtained, as first described in [2]. The second term on the r. h. s. of Equation (10) can be split into two parts as follows,

$$\frac{k_f M(r)}{M_b}\int_0^r \frac{GM(s)}{s^2}\mathrm{d}s = \frac{k_f M(r)}{M_b}\int_0^{r_i}\frac{GM(s)}{s^2}\mathrm{d}s + \frac{k_f M(r)}{M_b}\int_{r_i}^r \frac{GM(s)}{s^2}\mathrm{d}s, \tag{15}$$

where $r_i < r$ and in the second term on the r.h.s., in the integrand, it is assumed that $M(s) = M(r)$ for $r_i \le s \le r$. In other words, for a small enough interval $\Delta r = r - r_i$, the galaxy mass at that interval is the mass at radial position r. Thus in the above Equation (15), in the second term on the r.h.s., the mass $M(s)$ in the integrand can be moved out of the integral, giving the square of mass $M(r)$ in the form,

$$\frac{k_f M(r)}{M_b}\int_{r_i}^r \frac{GM(s)}{s^2}\mathrm{d}s = \frac{k_f (M(r))^2}{M_b}\int_{r_i}^r \frac{G}{s^2}\mathrm{d}s. \tag{16}$$

Combining the results of Equations (15) and (16) into Equation (10), after simplification, yields the equation quadratic in baryonic mass $M(r)$,

$$\left(\frac{k_f G}{M_b}\int_{r_i}^r \frac{\mathrm{d}s}{s^2}\right)(M(r))^2 + \left(\frac{G}{r} + \frac{k_f G}{M_b}\int_0^{r_i}\frac{M(s)}{s^2}\mathrm{d}s\right)M(r) + k_e c H_0 r - (v(r))^2 = 0. \tag{17}$$

By defining the three parameters $a(r)$, $b(r)$ and $c(r)$ in the form,

$$a(r) = \frac{k_f G}{M_b}\int_{r_i}^r \frac{\mathrm{d}s}{s^2}, \tag{18}$$

$$b(r) = \frac{G}{r} + \frac{k_f G}{M_b}\int_0^{r_i}\frac{M(s)}{s^2}\mathrm{d}s, \tag{19}$$

$$c(r) = k_e c H_0 r - (v(r))^2, \tag{20}$$

we obtain solutions for $M(r)$ in the familiar form,

$$M(r) = \frac{1}{2a(r)}\left(-b(r) + \sqrt{(b(r))^2 - 4a(r)c(r)}\right), \tag{21}$$

where the positive square root was chosen to keep the mass positive and it is apparent that the quantity under the square root is always non-negative since $a(r)$ is positive and $-c(r)$ is positive since $(v(r))^2 > 2k_e c H_0$ probably in most physical cases[1].

Knowing the galaxy total baryonic mass M_b and the observed rotation curve with radial positions and velocities (r_i, v_i), Equation (10) along with Equations (12) and (14) for k_e and k_f, respectively, and with the distribution of the baryonic mass $M(r)$ given by Equations (18) to (21), an iterative fit of the equation of motion to galaxy data can be obtained.

4. The BTFR

From Equation (10), move the cosmological term to the l.h.s. of the equation, factor out the baryonic mass $M(r)$ from the first and second terms on the r.h.s., divide the l.h.s. by the resulting factor to $M(r)$, multiply the l.h.s. by the unity factor v^4/v^4, and factor out $v^2(r)r$, with some simplification, to obtain the baryonic Tully-Fisher relation (BTFR),

$$\begin{aligned} M(r) &= \left(\frac{v^2(r)r - k_e c H_0 r^2}{G v^4(r)\left(1 + \frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s\right)} \right) v^4(r) \\ &= \left[\frac{r\left(1 - \frac{k_e c H_0 r}{v^2(r)}\right)}{G v^2(r)\left(1 + \frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s\right)} \right] v^4(r) \\ &= A(r) v^4(r), \end{aligned} \tag{22}$$

where the BTFR normalization $A(r)$ [5] [6] is defined by,

$$A(r) = \frac{r\left(1 - \frac{k_e c H_0 r}{v^2(r)}\right)}{G v^2(r)\left(1 + \frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s\right)}. \tag{23}$$

Although the standard BTFR is applied only at the flat rotation part of the galaxy at location R_f with velocity V_f, the GRST version of the BTFR Equation (23), after fitting $v(r) = v_{obs}(r)$ for any r of the galaxy, is extended to the entire observed rotational velocity and baryonic mass range of the galaxy.

It has been observed in numerous reports that when the BTFR is expressed in the form,

$$V_f^4 = \frac{M_b}{A} = a_0 G M_b, \tag{24}$$

where A is the normalization, a_0 is an acceleration, and M_b is the total baryonic mass of the galaxy, then the values of a_0 for many spiral galaxies falls into the approximate range 1×10^{-10} m·s^{-2} to 1.5×10^{-10} m·s^{-2}, which is about $0.15cH_0$ to $0.3cH_0$. This forms the basis of the modified Newtonian dynamics theory

[1]Addendum: In [2] there was an extra factor of the gravitation constant G which must be removed in one of the terms of both Equation (13) and Equation (15) of that paper.

(MOND) [7]. Inverting Equation (23) to solve for the velocity $v^4(r)$ in terms of $M(r)/A(r)$, the cosmological acceleration cH_0 can be factored out in the form,

$$v^4(r) = \left(\frac{1}{A(r)}\right) M(r) = a_0(r) G M(r), \tag{25}$$

where the acceleration $a_0(r)$ is given by,

$$a_0(r) = \left(\frac{v^2(r)}{r}\right) \frac{1 + \frac{k_f r}{M_b} \int_0^r \frac{M(s)}{s^2} \mathrm{d}s}{1 - \frac{k_e c H_0 r}{v^2(r)}}. \tag{26}$$

Although the cosmic acceleration cH_0 appears in the acceleration $a_0(r)$ of Equation (26), because k_e is inversely proportional to cH_0, the cosmic acceleration drops out from $a_0(r)$ and from any term in any equation that contains the factor $k_e cH_0$.

5. MOND

The GRST can generate a distributed version of the fundamental modified Newtonian dynamics (MOND) acceleration constant a_0 which is the foundation of that theory [7] [8]. MOND theory is defined using an interpolating function $\mu(x)$ which transforms the real rotation acceleration v^2/r in a galaxy into the Newtonian acceleration GM/r^2 based on the galaxy baryonic mass M, expressed by,

$$\mu\left(\frac{g(r)}{a_0}\right) \mathbf{g}(r) = \mathbf{g}_N(r), \tag{27}$$

where

$$\begin{aligned} \mathbf{g}(r) &= \frac{v^2(r)}{r} \hat{\mathbf{r}}, \\ \mathbf{g}_N(r) &= \frac{GM(r)}{r^2} \hat{\mathbf{r}}, \\ g(r) &= |\mathbf{g}(r)| = \frac{v^2(r)}{r}, \\ a_0 &= 1.22 \times 10^{-10}\ \mathrm{m \cdot s^{-2}}, \end{aligned} \tag{28}$$

where $g(r)$ is the acceleration of an orbit in a galaxy, g_N is the Newtonian acceleration and $M(r)$ is the baryonic mass. A sticky point is that a_0 is not a constant but varies around $(1.27 \pm 0.30) \times 10^{-10}\ \mathrm{m \cdot s^{-2}}$ for the standard $\mu(x)$ in the analysis of different galaxies [8]. Using the GRST, now with a cosmological term, the acceleration a_0 becomes a variable applicable to any distance r in a galaxy and settling to around the MOND value for any particular galaxy. This can be shown by using Equation (10), setting the galaxy acceleration $g(r) = v^2(r)/r$. Dividing Equation (10) by r yields,

$$g(r) = \frac{v^2(r)}{r} = \frac{GM(r)}{r^2} + \frac{2k_f M(r)}{r M_b} \int_0^r \frac{GM(s)}{s^2} \mathrm{d}s + 2k_e cH_0. \tag{29}$$

The Newtonian acceleration is the standard and using Equation (29) becomes,

$$g_N(r)=\frac{GM(r)}{r^2}=\frac{\frac{v^2(r)}{r}\left(1-\frac{k_e cH_0 r}{v^2(r)}\right)}{1+\frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s}. \tag{30}$$

The MOND theory can be expressed in terms of the BTFR, dividing Equation (25) by r^2,

$$\frac{v^4(r)}{r^2}=a_0(r)\frac{GM(r)}{r^2}, \tag{31}$$

which, using Equations (29) and (30) can be put into the form,

$$\left(g(r)\right)^2=\left(\frac{v^2(r)}{r}\right)^2=a_0(r)\frac{GM(r)}{r^2}=a_0(r)g_N(r). \tag{32}$$

From Equation (32) we solve for $a_0(r)$ in the form,

$$a_0(r)=\frac{\left(g(r)\right)^2}{g_N(r)}, \tag{33}$$

which, substituting for $g_N(r)$ from Equation (30), after simplification, becomes,

$$\begin{aligned} a_0(r)&=\frac{\left(g(r)\right)^2}{g_N(r)}=\left(g(r)\right)^2\frac{1+\frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s}{\frac{v^2(r)}{r}\left(1-\frac{k_e cH_0 r}{v^2(r)}\right)} \\ &=\left(\frac{v^2(r)}{r}\right)\frac{1+\frac{k_f r}{M_b}\int_0^r \frac{M(s)}{s^2}\mathrm{d}s}{1-\frac{k_e cH_0 r}{v^2(r)}}, \end{aligned} \tag{34}$$

where substitution for $g=v^2(r)/r$ was made using Equation (28).

The standard form of the interpolating function $\mu(x)$ equation in MOND is defined,

$$\mu(x)=\frac{x}{\sqrt{1+x^2}}, \tag{35}$$

where

$$x=\frac{g}{a_0}=\frac{v^2(r)/r}{a_0}, \tag{36}$$

where the magnitude of the galaxy radial acceleration $g(r)$ at position r is related to the Newtonian radial acceleration magnitude $g_N(r)$ by the definition,

$$\mu\left(\frac{g(r)}{a_0}\right)g(r)=g_N(r). \tag{37}$$

For the GRST the acceleration a_0 takes on a continuous property, Equation (34), where $a_0=a_0(r)$ at radial position r. For this analysis, the acceleration distribution $a_0(r)$ will be used instead of constant a_0 in the MOND interpolating function Equation (37).

For the GRST, the interpolating function $\mu_g(x)$ is defined using Equations (27) and (34),

$$\mu_g\left(\frac{g(r)}{a_0(r)}\right)g(r)=\left(\frac{g(r)}{a_0(r)}\right)g(r)=g_N(r), \tag{38}$$

where,

$$\mu_g\left(\frac{g(r)}{a_0(r)}\right)=\frac{1-\dfrac{k_e cH_0 r}{v^2(r)}}{1+\dfrac{k_f r}{M_b}\displaystyle\int_0^r\frac{M(s)}{s^2}\mathrm{d}s}. \tag{39}$$

6. Demonstration of the Theory Using Data from the Spiral NGC 3198

With the BTFR baryonic mass input of $M_b=2.464\times10^{10}\,\mathrm{M}_\odot$ for NGC 3198, which is matched by the GRST, the total graviton redshift (gravitational and cosmological) relativistic mass at $r_0=40.1\,\mathrm{kpc}$ determined by the model is $M_g=18.513\times10^{10}\,\mathrm{M}_\odot$ (see Table 1). The BTFR acceleration at normalization $A=50$ is $a_0=1.507\times10^{-10}\,\mathrm{m\cdot s^{-2}}$. Table 2 shows the parameters determined by the iterative fitting using total rotation energy Equation (10), coupling constants Equations (12) and (13), and the quadratic equations to determine the baryonic mass, Equations (18) to (21). The normalization parameter for the fit was $48.68\,\mathrm{M}_\odot\cdot\mathrm{s}^4\cdot\mathrm{km}^{-4}$ which compares well to the default BTFR normalization. The GRST acceleration at the r_0 is $a_0=1.548\times10^{-10}\,\mathrm{m\cdot s^{-2}}$.

Table 1. SPARC data for galaxy NGC 3198 using the graviton model (10) with masses from the SPARC mass model. D is the distance to the galaxy. r_0 is the radial position used for the flat curve velocity. V_f is the flat velocity. M_b BTFR is the baryonic mass used in the fit computed using the flat velocity $V_f=150\,\mathrm{km\cdot s^{-1}}$ and a normalization of 50. M_{total} is the total galaxy mass at r_0.

D	r_0	V_f	M_b BTFR	M_{total}
Mpc	kpc	km·s^{-1}	$10^{10}\,\mathrm{M}_\odot$	$10^{10}\,\mathrm{M}_\odot$
0.8	40.1	150	2.464	20.977

Table 2. Fit results with SPARC data for galaxy NGC 3198 using the graviton model Equation (10). The galaxy baryonic mass M_b is the fitted mass at the final flat velocity radial position $r_0=40.1\,\mathrm{kpc}$. k_f and k_e are the coupling constants for the graviton gravitational and cosmological redshifts, respectively. MAE is the mean absolute error of the rotation curve fit. A is the BTFR normalization value for the fit at r_0.

M_b	k_f	k_e	MAE	A
$10^{10}\,\mathrm{M}_\odot$			km·s^{-1}	$\mathrm{M}_\odot\cdot\mathrm{s}^4\cdot\mathrm{km}^{-4}$
2.464	0.655	0.00108	0.0036	48.68

Figure 1 shows the rotation curves for the disk, gas and graviton gravitational and cosmological redshifts and the sum of these fitting to the galaxy rotation velocity. Figure 2 shows the baryon mass distribution for $M(r)$ which was derived by iteration of the suite of equations including the rotation Equation (10) and the mass quadratic Equation (21). Figure 3 shows the masses for the baryons and graviton gravitational and cosmological redshifts, the sum of these masses equal to the galaxy mass, and also equal to the galaxy mass determined using the observed rotation velocities and positions.

Comparing with the dark matter paper [9] which determined for NGC 3198, for a maximum disk mass model, a disk mass $M_{disk} = 3\times10^{10}\,\mathrm{M}_\odot$ and a spherical halo model dark matter mass $M_{dm} = 12\times10^{10}\,\mathrm{M}_\odot$ for a total galaxy mass of $M_{total} = 15\times10^{10}\,\mathrm{M}_\odot$ at radius $R_f = 30\text{ kpc}$. On the other hand, with the GRST model the total galaxy fit to radius 44.08 kpc predicts a baryon mass $M_{b30} = 2.321\times10^{10}\,\mathrm{M}_\odot$ at 30 kpc radius and a graviton redshift (gravitational and cosmological) relativistic mass of $M_{g30} = 12.587\times10^{10}\,\mathrm{M}_\odot$ for a total galaxy mass at 30 kpc of $14.908\times10^{10}\,\mathrm{M}_\odot$. This is a fair match to the dark matter study considering the slight difference in baryonic mass between the two models.

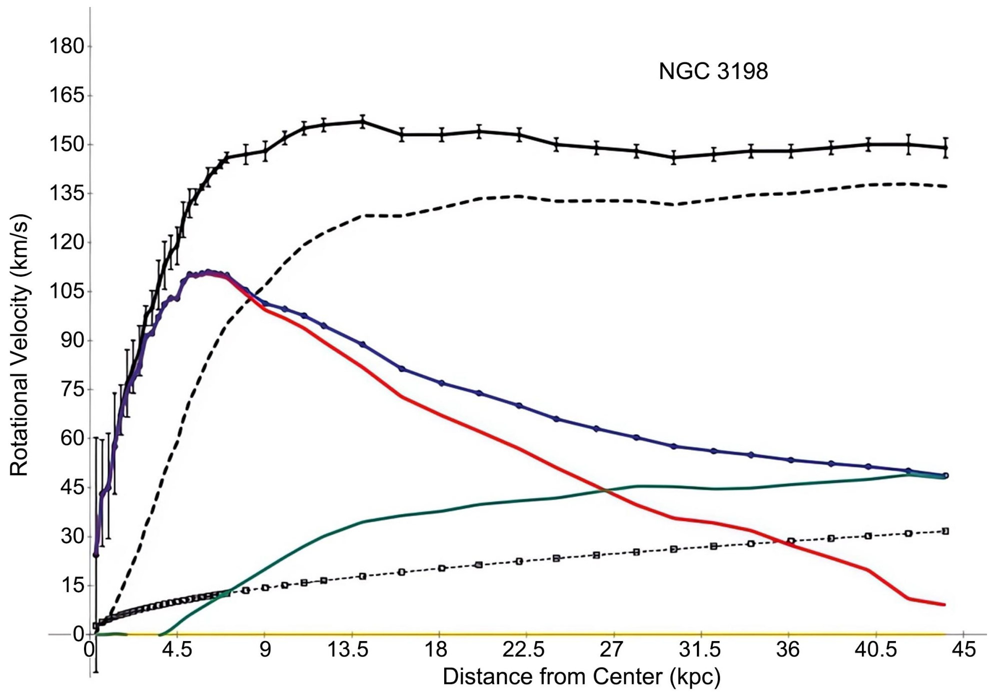

Figure 1. Fit made to NGC 3198 with SPARC data with velocity profiles for gas, disk and bulge. The plot shows the rotation velocity (solid black line) and the data points (solid points with error bars.) The curve of black circles is the velocity due to Newtonian theory with baryon mass, $GM(r)/r$. The black dashed line is the velocity due to the graviton gravitational redshift energy loss. The black dashed line with open squares is the velocity due to the graviton cosmological redshift energy loss. The solid red line represents the velocity due to the stellar mass, which is the baryonic velocity less the gas velocity. The solid yellow line is the velocity due to the bulge mass (zero since bulge is not present.) The solid green line is the velocity for the gas mass. The solid blue line is the sum of the stellar, bulge and gas velocities.

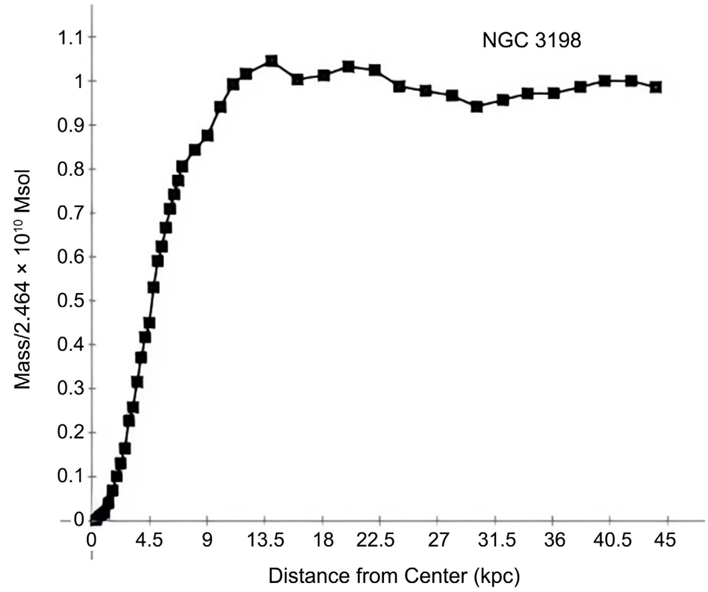

Figure 2. Baryonic mass distribution for NGC 3198, (Mass / Mass NGC 3198). The baryonic mass of NGC 3198 is computed using the BTFR at the flat galaxy velocity $V_f = 150\ \text{km}\cdot\text{s}^{-1}$ by the formula $M_{NGC3198} = 50V_f^4 = 2.464\times10^{10}\ \text{M}_\odot$. The black squares are the baryonic mass determined by solving the quadratic Equations (18) to (21). The solid black line is the mass determined by the GRST version of the BTFR Equations (22) and (23).

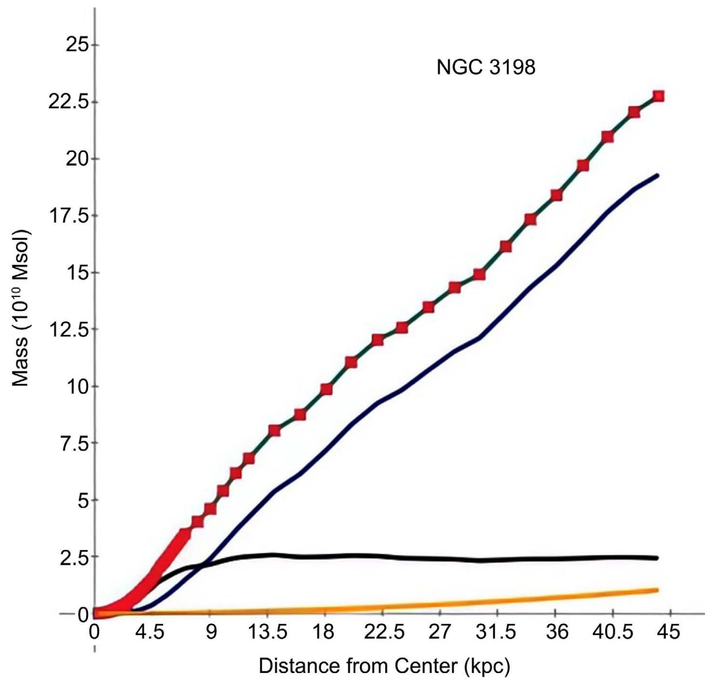

Figure 3. Fit made to NGC 3198 with SPARC data masses derived from rotation velocity fit. The solid black line is the fitted baryon mass. The solid yellow line is the relativistic mass due to the graviton energy cosmological redshift loss. The solid blue line is the relativistic mass due to the graviton energy gravitational redshift loss. The solid green line is the total galaxy mass $\mathcal{M}(r)$ of Equation (7), the sum of baryon mass, graviton gravitational and cosmological redshift relativistic masses. The red squares are the total galaxy mass obtained from the rotation velocity and position data.

Comparing with the MOND paper [8] where the acceleration constant $a_0 = 0.9\times10^{-10}\ \text{m}\cdot\text{s}^{-2}$ for a NGC 3198 distance of $D = 11.2\ \text{Mpc}$, the GRST acceleration $a_0\left(r = 30\ \text{kpc}\right) = 1.475\times10^{-10}\ \text{m}\cdot\text{s}^{-2}$. **Figure 4** displays a plot for the GRST version of the MOND standard interpolating function $\mu\left(x\right)$ of Equation (35) where $x = g\left(r\right)/a_0\left(r\right)$ along with the GRST interpolating function $\mu_g\left(r\right) = g\left(r\right)/a_0\left(r\right)$ given by Equation (39). The mean absolute difference (MAE)[2] between the GRST version of the standard MOND $\mu\left(x\right)$ and the GRST $\mu_g\left(x\right)$ for the fit to NGC 3198 is 2.974×10^{-4}.

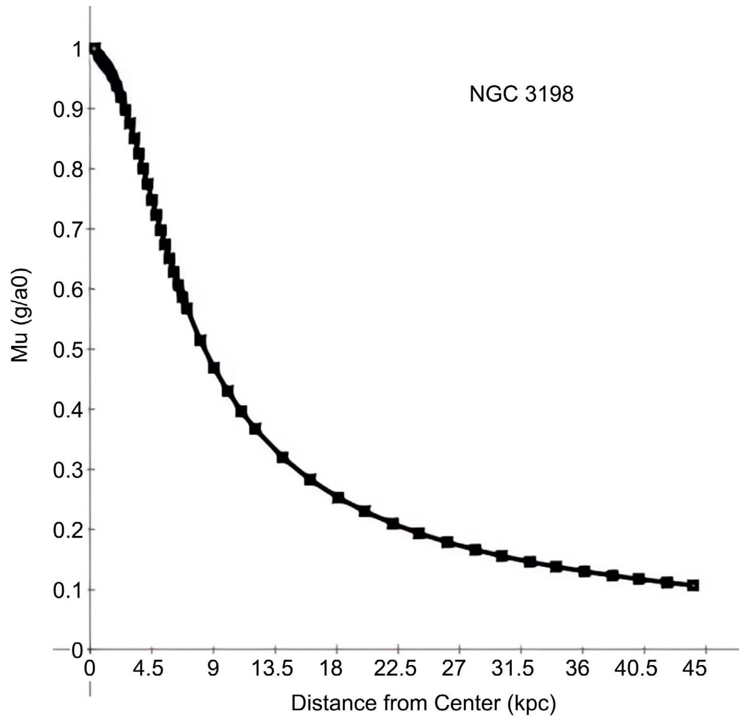

Figure 4. MOND interpolating function $\mu\left(x\right)$ plots using NGC 3198 SPARC rotation data. The black squares are for the GRST version of MOND $\mu\left(x\right) = x/\sqrt{1+x^2}$ where $x = g\left(r\right)/a_0\left(r\right)$ where $g\left(r\right) = v^2\left(r\right)/r$ and $a_0\left(r\right)$ is given by Equation (34). The solid black line is the graviton redshift energy loss theory for $\mu_g\left(r\right) = g\left(r\right)/a_0\left(r\right)$ given by Equation (39).

In [8], in Fig. 8 of that paper at radial distance 30 kpc, the Newtonian velocity for the stellar disk is about 50 $\text{km}\cdot\text{s}^{-1}$ with a mass to light of 0.76 whilst the gas velocity is about 49 $\text{km}\cdot\text{s}^{-1}$ implying a total baryonic mass of baryonic mass $M_{mond} = 3\times10^{10}\ \text{M}_\odot$. With an acceleration $a_0 = 0.9\times10^{-10}\ \text{m}\cdot\text{s}^{-2}$ and using the simple interpolating function, $\mu_s\left(x\right) = x/\left(1+x\right)$, $\mu_s\left(g\left(r = 30\ \text{kpc}\right)/a_0\right) \approx 0.202$, implies a rotation velocity $v\left(r = 30\ \text{kpc}\right) = 145.6\ \text{km}\cdot\text{s}^{-1}$, which is within the range of the observed rotation data. Using the GRST version of the standard interpolating function $\mu_g\left(x\right)$ of Equation (39), with the same mass M_{mond}, with

[2]The mean absolute error (MAE) is defined as the average of the sum of the absolute values of the corresponding pairwise differences of two sets of variables $\{x_i\}$ and $\{y_i\}$.

an $a_0 = 1.12\times10^{-10}\ \text{m}\cdot\text{s}^{-2}$, the rotation velocity at 30 kp is 146.1 km·s^{-1} which agrees with the observed rotation velocity at that position.

Figure 5 shows the distribution of the acceleration a_0 for NGC 3198. The value of the acceleration at near the end of the galaxy at $r_0 = 40.1\ \text{kpc}$ is $a_0(r_0) = 1.548\times10^{-10}\ \text{m}\cdot\text{s}^{-2}$.

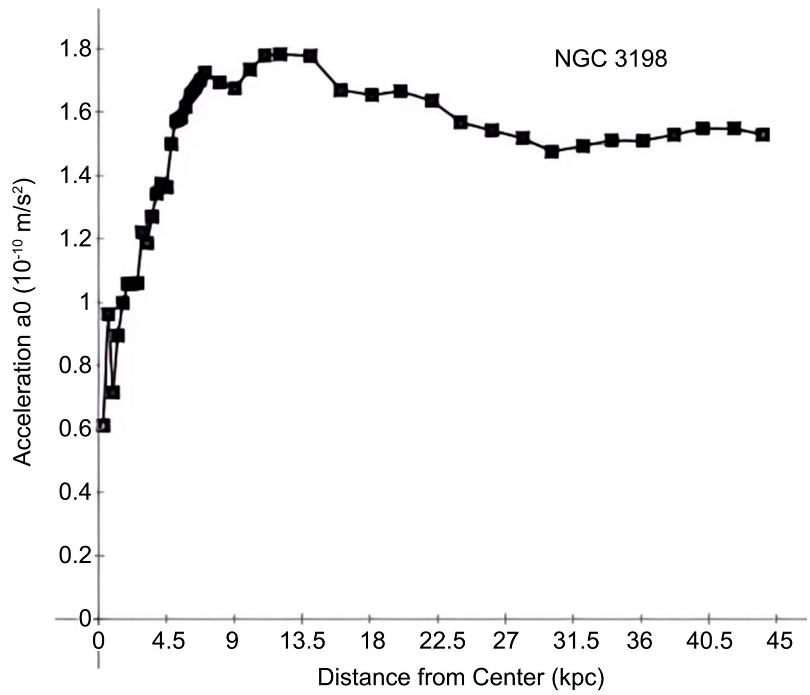

Figure 5. Acceleration $a_0(r)$ for NGC 3198 with SPARC data rotation velocity fit. The filled black squares are $a_0(r)$ derived using the BTFR normalization $A(r)$ of Equation (23). The solid black line is for $a_0(r)$ from Equation (34). For this paper the chosen value for the Hubble constant is $H_0 = 70\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$.

7. Discussion

The addition of the graviton interaction with the expanding universe in the form of a cosmological redshift to its energy as it travels in the galaxy was discovered in the process of assessing the value for the graviton gravitational redshift coupling constant k_f. If the graviton cosmological redshift constant k_e is dropped in Equation (11), since $M(r_0) = M_b$, this would imply that $k_f = 1$, a constant, which is problematical because fits do not work well for NGC 3198 using this value of the coupling constant. The value for k_f is peculiar to each galaxy. Thus, the idea of the graviton cosmological redshift energy loss for galaxies was born. And, the remarkable aspect is that the role of these two types of graviton redshifts in galaxies is analogous to the roles of graviton energy redshifts in the expansion of the universe. The GRST, with gravitational and cosmological graviton interactions, seems to solve the mystery of dark matter and dark energy in spiral galaxies and the expanding universe.

A Possible Cosmic Connection for A and a_0

It has been a well noted feature that the acceleration a_0 in Equation (24), with the normalization $A = 50\ \mathrm{M}_\odot \cdot \mathrm{km}^{-4} \cdot \mathrm{s}^4$ [6], has the value,

$$a_0 = \left(\frac{1}{G50}\right)\left(\frac{10^{12}}{\mathrm{M}_\odot}\right) = 1.507 \times 10^{-10}\ \mathrm{m} \cdot \mathrm{s}^{-2}, \tag{40}$$

where the factor $10^{12}/\mathrm{M}_\odot$ converts from km^4 to m^4 and from solar mass to kg, and it is evident that a_0 has a value close to the cosmic acceleration $cH_0^{75} = 7.287 \times 10^{-10}\ \mathrm{m} \cdot \mathrm{s}^{-2}$ where $H_0^{75} = 75\ \mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{Mpc}^{-1}$. Looking for a possible cosmic connection, it is perhaps coincidental that considering the Lambda Cold Dark Matter cosmology, for a matter density parameter $\Omega_M = 0.286$, the dark energy mass density parameter $\Omega_{DE} = 0.714$, and a baryonic mass density parameter $\Omega_B = 0.049$, with H_0^{75}, make the following definition of a cosmic normalization parameter $\mathcal{A}$, expressed by,

$$\mathcal{A} = \left(\frac{\Omega_M - \Omega_B}{\Omega_B}\right)\left(\frac{1}{cH_0^{75}G}\right)\left(\frac{10^{12}}{\mathrm{M}_\odot}\right) = 50.015\ \mathrm{M}_\odot \cdot \mathrm{km}^{-4} \cdot \mathrm{s}^4, \tag{41}$$

where $\Omega_{DM} = \Omega_M - \Omega_B$ is the so called dark matter density parameter. Recall that in the GRST the dark matter density parameter Ω_{DM} and the dark energy density parameter Ω_{DE} are due to graviton gravitational and cosmological redshift energy losses, respectively, as a result of the expansion of the universe [1]. In a recent BTFR article [10] of a study of ~10,000 galaxies, the Hubble constant was determined to have a value $H_0 = 75.5 \pm 2.5\ \mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{Mpc}^{-1}$, which supports the value H_0^{75} used in the SPARC data analysis.

The cosmic acceleration α_0 related to cosmic normalization parameter $\mathcal{A}$, is defined in the form of Equation (40), given by,

$$\alpha_0 = \left(\frac{1}{G\mathcal{A}}\right)\left(\frac{10^{12}}{\mathrm{M}_\odot}\right) = \left(\frac{\Omega_B}{\Omega_M - \Omega_B}\right)\left(cH_0^{75}\right) = 1.507 \times 10^{-10}\ \mathrm{m} \cdot \mathrm{s}^{-2}, \tag{42}$$

which agrees with a_0 of Equation (40). This might be used as a model for a derivation of the acceleration a_0 from first principles.

From the standard form of the BTFR which is stated for the flat part of the rotation curve where the velocity is V_f and the position is R_f, assuming it to be near the edge of the galaxy, Equation (24) can be put into the forms,

$$\begin{aligned} V_f^4 &= a_0 G M_b, \\ \left(\frac{V_f^2}{R_f}\right)\left(\frac{V_f^2 R_f}{G}\right) &= M_b a_0, \\ M_{tot}\left(\frac{V_f^2}{R_f}\right) &= M_b a_0, \end{aligned} \tag{43}$$

where the total galaxy mass is $M_{tot} = V_f^2 R_f / G$. And, to take a step further, assuming that the cosmic acceleration α_0 is a real acceleration of the expanding universe and that it is equal to acceleration a_0, then combining Equations (42) and (43) gives,

$$M_{tot}\left(\frac{V_f^2}{R_f}\right)=M_b a_0=M_b \alpha_0=M_b\left(\frac{\Omega_B}{\Omega_M-\Omega_B}\right)\left(cH_0^{75}\right). \tag{44}$$

If Equation (44) is considered a balance of force equation, where the galaxy inward attractive force is balanced by an outward force due to the expanding universe, then the particles having orbital velocity V_f just beyond distance R_f will be unbound to the galaxy. This could explain why the edge of the galaxy is where it is at.

8. Conclusion

The GRST equation of motion does well at explaining the extra mass found in a spiral galaxy. In algorithmic form, the equation of motion giving $v^2(r)$, the formulas to determine the coupling coefficients k_f and k_e and the quadratic equation to determine the baryonic mass distribution $M(r)$ provide a powerful tool to handle the dynamics of spiral galaxies. It is notable that both the BTFR and the MOND theory are derivable in continuous form with this theory. The mass in the relation $M(r)=A(r)v^4(r)$ represents the baryonic mass throughout the galaxy, not just at the galaxy edge. Likewise for the MOND relation, the acceleration $a_0(r)=v^4/GM(r)$ is continuous for the galaxy and there is no need to determine a transition radius from high to low acceleration. Finally, the new cosmic normalization parameter $\mathcal{A}$ was introduced in the discussion section as a possible model for the BTFR acceleration a_0.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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