

Proton Decay Reaction in Massive White Dwarfs

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Abstract

Two magnetic monopole models (*i.e.*, model (I, II)) are presented to discuss the energy resources problem based on magnetic monopole catalytic nuclear decay in massive white dwarfs. We find that the luminosities for most of massive white dwarfs increase as the temperature increases. The luminosities of model (II) are agreed well with those of the observations at relativistic high temperature (e.g., $T_6 = 1, 10$), However, the luminosities of the observations can be five orders of magnitude larger than those of model (I).

Keywords

White Dwarfs, The Energy Source, Magnetic Monopoles

1. Introduction

As well known, white dwarfs (hereafter WDs) has no thermal nuclear burning in their Interior. The temperature in the interiors of WDs can be about 10^6 K with total thermal energy less than 10^{47} ergs. The radius of WDs can be about 10^4 km with surface temperature $T \approx 3 \times 10^3 \sim 4 \times 10^4$ K. From the Stefan-Boltzmann law, the radiation luminosity of WDs is

$$L_{\text{rad}} = 4\pi R^2 \sigma T_{\text{eff}}^4 \approx 7.1 \times 10^{30} \left(\frac{R}{10^4 \text{ km}} \right)^2 \left(\frac{T_{\text{eff}}}{10^4 \text{ K}} \right)^4, \quad (1)$$

where R is the radius of the star, and $\sigma = 5.6704 \times 10^{-5} \text{ ergs} \cdot \text{s}^{-1} \cdot \text{cm}^{-2} \cdot \text{K}^{-4}$ is the radiation constant from Stefan's law. This surface temperature is defined in astronomy as the effective temperature T_{eff} by means of Stefan's law. For WDs with temperature $3 \times 10^3 \sim 4 \times 10^4$ K, so that $L \approx 5.6 \times 10^{28} \sim 1.8 \times 10^{33} \text{ ergs} \cdot \text{s}^{-1}$.

The WDs energy sources problem has always been an interesting issue. Refs.

[1]-[4] studied the WDs energy sources problem and showed that the burning of ^{22}Ne in WDs may be an extra heating source. However, Ref. [5] discussed this issue and showed that ^{22}Ne can not be a possible cause of the heating in WDs. In this paper, we present two models to solve this problem for 25 typical massive WDs selected from Ref. [6]. Our model is based on the magnetic monopoles (hereafter MMs) catalytic proton decay (RC effect) [7] [8]. The issues on MMs are forefront subjects in astrophysics (e.g., Refs. [9]-[13]). We also studied the problem of MMs and other related issues (e.g., Refs. [14]-[20]).

This paper is arranged as follows. In Section 2, we study the number of possible MMs captured by WDs, and the luminosity by RC effect. In Section 3, the MMs model and RC luminosity in WDs are discussed. Results and discussions are given in Section 4. We obtain some conclusions in Section 5.

2. The Numbers of the MMs Captured and the Luminosity by Catalytic Proton Decay in Stars

The number of MMs captured in space at the surface of stars is given by follows [22]

$$N_{m(\text{sur})} = 5.7 \times 10^{-25} m_9 v_{-3} n_B \frac{\zeta_m^0}{\zeta_s}, \quad (2)$$

where $v_{-3} = v_m/10^{-3}c$, $m_9 = m_m/10^9 m_p$, v_m, m_m are the velocity and the mass of the MMs, respectively. The MMs mass is about $m_m \sim 10^{16} m_p$ [11], and $m_p c$ are the mass of the proton and the speed of light, respectively. ζ_m^0 is the content of MMs in space. $\zeta_s \approx 1.9 \times 10^{-32} m_9$ is the maximum number of MMs, which is defined as Newton saturation value. $n_B = (10^4 \sim 10^6) \text{cm}^{-3}$ is the number density of baryons in the space of the Milky Way galaxy [22]. In Equation (2), the number density of nucleons can be written by [22]

$$n_B = 2.90 \times 10^{16} \left(\frac{R}{R_g} \right)^{-3} (M_{12})^{-2} = 2.242 \times 10^{24} M_* R_*^{-3} \text{cm}^{-3}, \quad (3)$$

where $R_g = 2.96 \times 10^5 M_*$ is the Schwarzschild radii, and $M_{12} = M_*/10^{12}$, $M_* = M/M_\odot$, and $R_* = R/R_\odot$.

According Equations (2~3), the total number of MMs trapped in space after the formation of stars (or planets) is estimated to be

$$\begin{aligned} N_{m(\text{tot})} &= 4\pi R^2 t N_{m(\text{sur})} = 3.463 \times 10^7 v_{-3} n_B t_9 m_9 \left(\frac{\zeta_m^0}{\zeta_s} \right) \left(\frac{R}{R_\odot} \right)^2 \\ &= 3.463 \times 10^7 \xi v_{-3} n_B t_9 R_*^2, \end{aligned} \quad (4)$$

where $R_* = R/R_\odot$, $t_9 = t/10^9 \text{yr}$, and $\xi = m_9 F/\zeta_s$. R , t , and F are the radius, the age of the star, and the MMs flux in space, respectively.

$pM \rightarrow e^+ \pi^0 M + \text{debris}(85\%)$, and $pM \rightarrow e^+ \mu^\pm M + \text{debris}(15\%)$ are named proton decay catalyzed by MM, which proposed by Callen and Rubacov (RC effect) [7] [8]. The luminosity due to the RC effect is [22]

$$L_m \approx \frac{4\pi}{3} r_c^3 n_m n_B \langle \sigma_m v_T \rangle m_B c^2 = N_m n_B \langle \sigma_m v_T \rangle m_B c^2, \quad (5)$$

where $v_T = \sqrt{kT/m_B} \approx 8.691 \times 10^3 T^{1/2}$ cm/s is the thermal movement speed of the nucleus relative to the MM. n_m , T and k are the number density of MMs, the temperature and the Boltzmann constant, respectively. $m_B \approx 1.78 \times 10^{-24}$ g and r_c , are the nucleons mass and the radius of the stellar central region, respectively. σ_m is the reaction cross section, whose range is about $10^{-26} - 10^{-24}$ cm². By using the SU(5) grand unification theory, Ref. [23] gave the value of $\sigma_m \approx 4.28676 \times 10^{-24}$.

3. The MMs Model and RC Luminosity in WDs

3.1. MMs Catalytic Proton Decay Model (I) in WDs

The mass-radius relation of WDs is one of interesting issue for astrophysicist and is given by [24]

$$R_{*1} = 1.080 \times 10^{-2} \mu_e^{-5/3} M_*^{-1/3} \quad (6)$$

where $\mu_e = A/Z$ is the molecular weight per electron.

Based on Equation (3) (6), Equations (4) (7), and Equations (5)-(8) the number density of nucleons, the numbers of MMs captured, and the total luminosity in WDs by model (I) are given by

$$n_B(\text{I}) = 2.242 \times 10^{24} M_* (R_{*1})^{-3} = 1.7794 \times 10^{30} \mu_e^5 M_*^2 \text{ cm}^{-3}, \quad (7)$$

$$N_1 = N_{m(\text{tot})}(\text{I}) = 3.463 \times 10^7 \xi v_{-3} n_B(\text{I}) t_9 (R_{*1})^2 \quad (8)$$

$$\begin{aligned} L_1 &\approx \frac{4\pi}{3} r_c^3 n_m n_B(\text{I}) \langle \sigma_m v_T \rangle m_B c^2 = N_2 n_B(\text{I}) \langle \sigma_m v_T \rangle m_B c^2 \\ &= 5.541 \times 10^4 n_B^* v_{-3} n_B(\text{I}) \langle \sigma_m v_T \rangle \xi t_9 (R_{*1})^2 \end{aligned} \quad (9)$$

3.2. MMs Catalytic Nuclear Decay Model (II) in WDs

Izawa (1986) discussed the Equation of the state in WDs by considering the RC effect as an energy release process. He gave an expression of the mass-radius relation [25]

$$R_{*2}^0 \approx 2 \times 10^{-10} \left(\frac{M}{M_\odot} \right)^{-2.35} N_2^{0.45} = 2 \times 10^{-10} M_*^{-2.35} N_2^{0.45}, \quad (10)$$

According to Equations (3) (4) (10), and Equations (5) (10) (11), the number density of nucleons, the numbers of MMs captured, and the total luminosity in WDs are given as, respectively

$$n_B(\text{II}) = 2.242 \times 10^{24} M_* R_{*2}^{-3} = 2.803 \times 10^{53} N_2^{-1.35} M_*^{8.05} \text{ cm}^{-3}, \quad (11)$$

$$\begin{aligned} N_2 &= N_{m(\text{tot})}(\text{II}) = 3.463 \times 10^7 \xi v_{-3} n_B(\text{II}) t_9 R_{*2}^2 \\ &= \exp \left[\frac{\ln \left(1.36719 \times 10^{31} F t_9 M_*^{-1.35} v_{-3}^{-2} \right)}{0.55} \right], \end{aligned} \quad (12)$$

$$\begin{aligned}
 L_2 &\approx \frac{4\pi}{3} r_c^3 n_m n_B (\text{II}) \langle \sigma_m v_T \rangle m_B c^2 = N_2 m n_B (\text{II}) \langle \sigma_m v_T \rangle m_B c^2 \\
 &= 5.541 \times 10^4 n_B (\text{II}) v_{-3} n_B (\text{II}) \langle \sigma_m v_T \rangle \xi t_9 R_{*2}^2.
 \end{aligned}
 \tag{13}$$

4. Results and Discussions

The MMs flux has been considerable interest issue. Parker (1970) gave the MMs flux as $F \leq 10^{-16} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$ [21]. Ref. [26] gave a limit on the flux by $F(\sigma v)_{-28} \leq 10^{-21} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$ in neutron star. Ref. [27] discussed the MMs flux, which may be $F(\sigma v)_{-28} \leq 2 \times 10^{-18} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$ in WDs. Ref. [28] also shown that the bound was stated as $F(\sigma v)_{-28} \leq 10^{-28} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$. In this paper, we select the MMs flux of $F = 1.9 \times 10^{-23} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$ and the other parameters are selected as $n_B^0 = 10^5 \text{ cm}^{-3}$, $T_6 = 0.01, 0.1, 1, 10$, $m_m = 10^{16} m_p$, $\xi = 10^2$. We selected 25 typical high mass WDs from [6]. Some main parameters can be reference in Table 3 of Ref. [6]. **Figure 1** and **Figure 2** show the number of magnetic monopoles captured from space in the lifetime of the O + Ne(C + O) core high mass WDs for model (I), and (II) as a function of M_* when $\xi = 10^2$, and $n_B^0 = 10^6 \text{ cm}^{-3}$ at $T_6 = 0.01, 0.1, 1, 10$. As the temperature increases, the number of MMs captured increases by about two, and three orders of magnitude for model (I), and (II), respectively. However, the number of MMs captured increases by about three orders of magnitude for model (II).

When the temperature is certain, the number of MMs captured can be $N_2 > N_1$ and the higher the mass, the larger the number of MMs captured becomes from **Figure 1**. It is because that according to Equations (6) (8) (10) (11), we know that

$$N_1 \propto (1/M_*)^{2/3}, \text{ but } N_2 = \exp \left[\frac{\ln \left(1.36719 \times 10^{31} F t_9 M_*^{-1.35} v_{-3}^{-2} \right)}{0.55} \right].$$

The effect of mass

radius relation on the number of the MM captured is ignored in model (I), Model (II) considers the effect of mass radius relation and RC effect on the number of captured monopoles, so the data is relatively accurate.

In WDs, monopoles captured can catalyze proton decay and provide the internal heating. A monopole which passes through WDs can also lose enough energy. Ahlen and Kinoshita (1982) calculated these energy, which given by $dE/dx \approx 100 \rho \beta \text{ GeV/cm}$, where ρ is the density of the WDs (in $\text{g}\cdot\text{cm}^{-3}$), and β is the velocity of the MM as it passes through the WDs [29]. The energy loss in traveling through the WDs can be about $5 \times 10^{17} \text{ MeV}$. If the MM is captured, it sinks toward the center of the WDs and the time scale for the MM to fall from rest to the center can be estimated to be about 1000 s.

Figure 2 display that the luminosities as a function of M_* for WDs at the temperature of $T_6 = 0.01, 0.1, 1, 10$ when $\xi = 10^2$, $n_B^0 = 10^6 \text{ cm}^{-3}$. The luminosities increase as the temperature increases. By comparing the luminosities of the observations with those of model (I), and (II), we find that the luminosities of model (II) are agreed well with those of the observations at relativistic high temperature (e.g., $T_6 = 1, 10$), However, the luminosities of the observations can be five orders of magnitude larger than those of model (I).

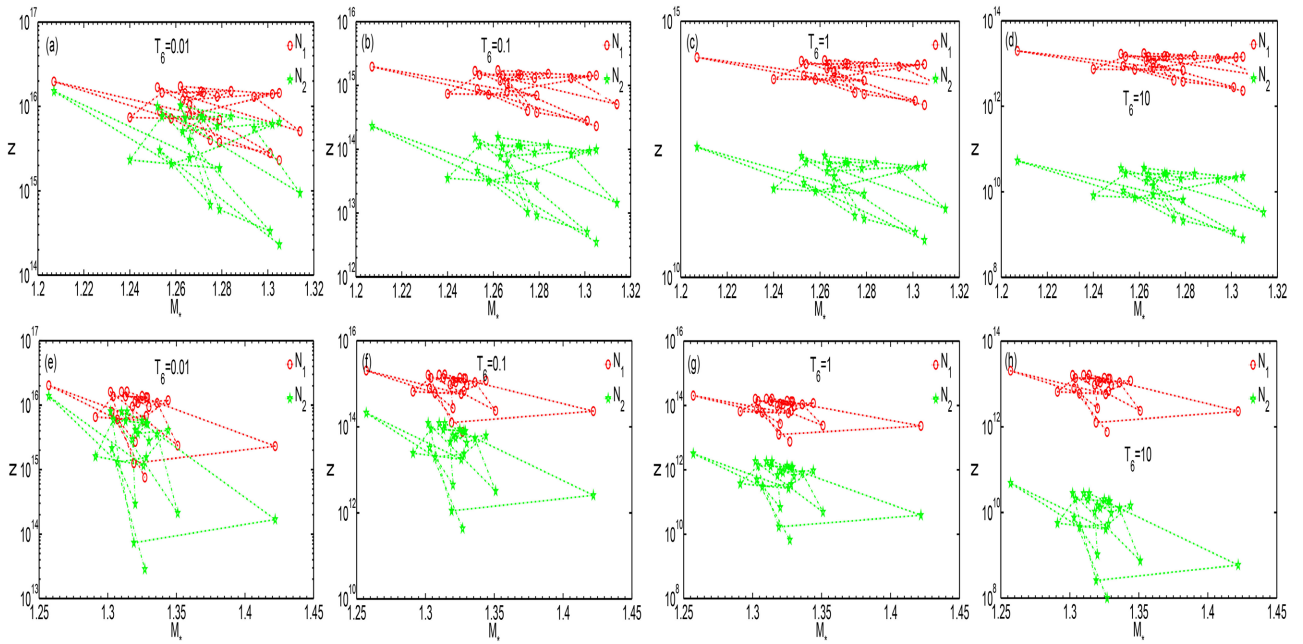


Figure 1. The number of magnetic monopoles captured from space in the lifetime of the O + Ne core high-mass WDs ((a)-(d)) and the C + O core high-mass WDs ((e)-(h)) ([6]) for model (I), and (II) as a function of M_* when $\xi = 10^2$, and $n_B^0 = 10^6 \text{ cm}^{-3}$ at the temperature of $T_6 = 0.01, 0.1, 1, 10$, respectively.

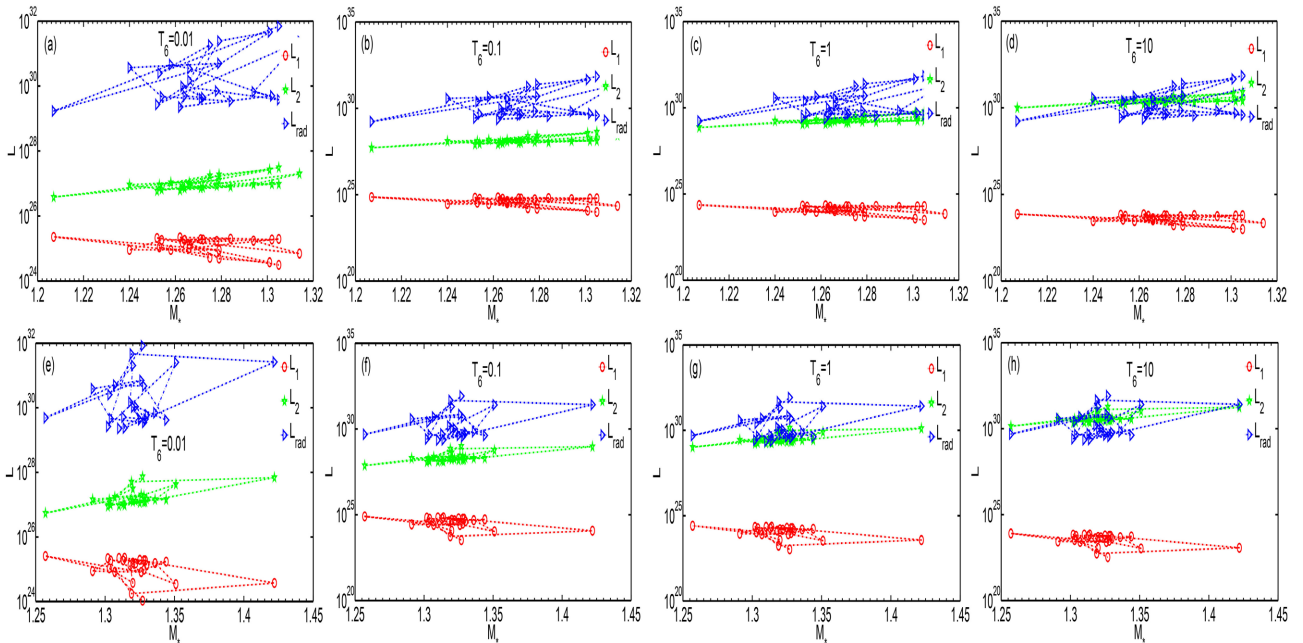


Figure 2. The luminosity as a function of M_* for the O + Ne core high-mass WDs ((a)-(d)) and the C + O core high-mass WDs ((e)-(h)) ([6]) when $F = 1.9 \times 10^{-23} \text{ cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$, $\xi = 10^3$, and $n_B^0 = 10^6 \text{ cm}^{-3}$ at the temperature of $T_6 = 0.01, 0.1, 1, 10$, respectively.

Based on the above analysis, one can conclude that with the increasing of the number of MMs captured by the WDs, the luminosity of MMs catalyzed nuclear decay increases linearly with time until it becomes the main contribution to the

total luminosity. Even one can observe that for some of the oldest white dwarfs, the luminosity may have passed its minimum, and some reheating may have occurred. The annihilation of MM and anti-MM may make a significant reduction in the number of MMs and the catalytic luminosity of the monopole in the WDs. Ref. [30] calculated the annihilation cross sections of MMs and anti-MMs caused by two-body and three-body recombination. Their results showed that the annihilation has little effect on the flux and luminosity.

5. Conclusion

Based on the MMs catalytic proton decay, we present two MMs models of the energy resources in WDs. We discuss the luminosity to apply to 25 massive WDs and calculated the number of MMs captured by WDs. We also compare these luminosity of the two MMS model with the observations. We find that the luminosities increase as the temperature increases. The luminosities for most of massive WDs for model (II) are agree well with the observations and the difference is no more than one order of magnitude at relativistic high temperature (e.g., $T_6 = 1, 10$). However, the luminosities of the observations can be five orders of magnitude larger than those of model (I). According to our calculations and discussion, the monopole-catalyzed proton decay process may be an effective way, which can prevent WDs from cooling.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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