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A Model of Grand Unified Theory: Suggested Solution for CP-Violation Using Ideas of Phase Paths

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Abstract

This study demonstrates that beyond standard model (BSM) cosmic fundamental interactions—weak, strong, and electromagnetic forces—can be unified through a common basis of representation. This unification allows for the derivation of the fine structure constant with running points of $\alpha(t) \approx 1/(136.9038)$ at high energy scales, based on electroweak interactions. Through the application of the Ising model, the running point of the elementary charge e at high energy scales is determined, and Coulomb's law is actually derived from the Yukawa potential. Theoretically, based on S. Weinberg's electroweak interaction theory, this study unifies the strong and electromagnetic forces by representing them with r_{Yuka}, and further advances the reconstruction of the $SU(3)_C \times SU(1)_L \times U(1)_{EM}$ framework on the basis of electroweak interaction concepts. In fact, the cosmic fundamental forces can interchange at the mass gap, defined as the Yukawa turning phase at $r_{Yuka} \simeq 1.9404$ fm, with the $SU(3)_{Diag}$ structural constant f^{ijk} on glueballs calculated, estimating a spectrum mass gap of $\Delta_0 > 0$.

Keywords

BSM, Electroweak, Glueballs, Ising Model, Mass Gap, S. Weinberg

1. Introduction

It is well known that Einstein worked on the theory of unified fields (UFT) from the 1930s until 1955, during which he proposed several hypothetical field equations

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to describe the basic mathematical rules governing two fundamental interactions, though without success. Prior to the 1960s, only gravity and electromagnetic forces were recognized in the scientific community. There was no clear understanding of the third and fourth fundamental interactions in the universe at that time. In 1967, S. Weinberg published the theory of electroweak interactions [1], which introduced the concept of unifying forces through the Lie group by $SU(2) \times U(1)$, although he did not extend this unification to the nuclear force. Around the same time, the Yang-Mills theory provided a framework for understanding the representation of the Lie group by $SU(3)$, which governs $SU(3)$ as the rules for Quantum Chromodynamics (QCD) scales(Λ_{QCD}) [2]. The strong force, also known as the nuclear force, adheres to these rules, and QCD dictates that quarks must obey color confinement and color symmetry. This led scientists to establish a set of rules to construct models or physical frameworks to understand the behavior of elementary particles, a model now known as the Standard Model (SM). However, as advancements in high-energy physics continued, physicists increasingly encountered phenomena that could not be explained within the Standard Model, such as hypothetical supersymmetric (SUSY) particles, often referred to as "ghost particles." Although these particles have not been experimentally verified, they have posed significant challenges in scientific data analysis. It is widely believed that the development of a theory unifying the three cosmic interactions is essential, as it is seen as the most promising candidate among all Grand Unified Theories (GUTs), regardless of whether it includes color symmetry. In this context, data and results from CERN (2019) suggest the need for a theory that can provide a robust explanation for running coupling constants, which were precisely measured at values of 0.02 and 0.04 at high energy scales [3]. This raises critical questions: What is the significance of these findings? How do they relate to Einstein's UFT? These are sensitive and important issues (and if Dr. Pauli were alive today, he would likely pose these questions sharply). It is important to note that the discussion above is based on the behavior of fine structure constants, which also vary at high energy scales [4]-[8]. This paper is primarily based on the theory of electroweak interactions as proposed by S. Weinberg in 1967. For a long time, the widely known four fundamental cosmic interactions have been confined to the categories defined by this theory and the existing Grand Unified Theories (GUTs). Despite extensive research, there has been little progress in unifying these fields. Some GUT-related theories even predict proton decay, but no experimental evidence has yet confirmed this phenomenon. In this work, we aim to reconstruct electroweak interactions and calculate the running points of the fine structure constant, addressing a longstanding question in physics. Our study also responds to the famous physicist Richard Feynman's remarks on the fine structure constant, often referred to as "God's number." Furthermore, we introduce a new interpretation of the Yukawa potential turning phase, considering it as a spectral mass gap in interactions (electro-strong or strong nuclear force), which has long been hidden but can be explained within the framework of electroweak interactions.

For a significant period, the equations of Yang-Mills have remained unsolved at energy scales relevant to describing atomic nuclei. We demonstrate that the half-wavelength of glueballs can be denoted as $\lambda/2 = r_{Yuka}/2$, which constitutes plane waves of glueballs interacting with particles, and this finding is in complete agreement with experimental data [9] [10]. The motivations of studying unique connection of color charges (e.g., the $SU(5)$ model by SUSY, which is Beyond the Standard Model (BSM) and is behind widely-studied models) [11]. Therefore, we determine that the quark freedom in QCD scales is six, by using the method described in this paper and the well-known process of strong fine-tuning [12]. A key highlight of our work is the demonstration that the running points of coupling constants at high energy scales are actually connected to the vacuum light speed C, as derived in this paper. We conclude that, through the method of approximation, our findings are eventually linked to the work on Two- and Four-Point Functions by Sander Mooij and Mikhail Shaposhnikov [13]. Finally, this paper provides an answer to the outstanding problem concerning the value of the cutoff Λ.

2. Method

This work is grounded in the principles of S. Weinberg's theory of electroweak interactions (1967) and explores the transition from the Standard Model (SM) to Beyond Standard Model (BSM) physics at different energy scales. The method employed in this paper relies entirely on hand calculations, without the use of computers or analytical instruments. If one wonders about the exact relationship between strong and electromagnetic forces or how Yang-Mills theory explains the physics of nuclei and their constituents, this paper provides a simple yet profound answer, building consecutively on the work of S. Weinberg. By utilizing helicity, we can explore the spin-spin couplings of massless particles, that is $\hat{L} = \hat{r} \times \hat{p} = const$, $\hat{L} = \hat{r} \times \lim_{m\to\infty} \sum_{i=1}^{\infty} m_i \vec{v}_j = const$, in the early universe. Given that the cosmic mass was infinite during this period, we arrive at the second formula. It requires $\hat{r} \equiv \hat{x} + \hat{y} + \hat{z} = 0$, where $S' = 0$ is denoted as the scalar boson, while the angular momentum is projected onto the z-axis, implying the inflaton spin with $S' = 0$.

Massless particles, such as Sgluons with eight types of color charges and massless Binos[1], which are colorless, play a crucial role. The combination of Sgluons and Binos clearly violates color symmetry. However, the colorlessness of Binos serves as a "center" for the color symmetry of Sgluons. The combination relationships of colors are demonstrated with a deduction of colorlessness:

$$
\begin{aligned}
&g_{i,\tilde{B}} + g_{i,\tilde{g}} \overset{(L.E.)}{=} g_{i,\tilde{g}} \neq 0, \\
&\tilde{g} \xleftrightarrow{\Lambda_{GUT}} \tilde{B}, \forall r = r_{Yuka} = \infty, \tilde{m}_{\tilde{g}} = \tilde{m}_{\tilde{B}} = 0, \\
&g_{i,\tilde{g}} \overset{(loop)}{=} g_{i,\tilde{B}} = 0
\end{aligned}
\tag{1}
$$

where widely-known $\left[g_i, g_j\right] = if^{ijk} g_k, i = 1, 2, 3, \cdots, 8$.

In Quantum Chromodynamics (QCD), the concept of colorlessness is com-

[1]Using a capital "B" for "Binos" is a clear and effective way to distinguish it as a specific term of $\tilde{B}$.

monly represented by white, which is the color symmetry before mixing with higgsinos. This will be discussed further in the next section, where we refer to their common eigenstates. Note that r_{Yuka} as the "center" at high energy scales (in the context of Yang-Mills theory where particles are massless) is not arbitrary; it has a specific value that will be derived in later sections. This r_{Yuka} is positioned at the interacting top-point with $\cos\theta_w\big|_{\theta_w=\pi/4}$ in Feynman diagrams (see **Figure 1**). The corresponding values $T_a = \lambda_a/\sqrt{2}$ can be represented in Gell-Mann matrices. Similar to how neutralinos have a mixed state that produces four common eigenstates, this paper presents a compelling assumption. The symmetrical group of color confinement is represented by $SU(3)_G$, while $SU(3)_{Diag}$ denotes asymptotic freedoms without color attributes (R, G, B).

3. Results and Discussion

A) Tensors and the rotation matrix (with chirality versus helicity chart)

Given the cascade decays, assuming that

$$\left.\begin{array}{l} \tilde{g} \xrightarrow{A^0} \tilde{B}, \forall t > t_0 \\ \tilde{B} \xrightarrow{A^0} \tilde{g}, \forall t < t_0, CPT \\ \tilde{m}_{\tilde{g}} = \tilde{m}_{\tilde{B}} = 0 \\ \tilde{g} \xleftrightarrow{\Lambda_{GUT}} \tilde{B}, \forall r = r_{Yuka} = \infty \end{array}\right\} \tag{2}$$

where high energy scales is denoted by Λ_{GUT}. A^0 indicated as GUT-scale QCD axions. $\tilde{g}$ indicated as Sgluons. And $\tilde{B}$ indicated as Binos.

The term $\tilde{m}_{\tilde{g}} = \tilde{m}_{\tilde{B}} = 0$ refers to one of the common eigenstates of their masses.

The second reaction corresponds to the evolution of reversal time ($t < t_0$).

For the massless gaugino, denoted by $\tilde{m}_{\tilde{g}} = \tilde{m}_{\tilde{B}} = 0$, the relationships (rotation matrix) are shown below for an arbitrary matrix in physical mathematics. Specifically,

$$M = \exp(A) = \sum_{k=0}^{\infty} \frac{A^k}{k!} \tag{3}$$

where A represents the EM-tensor and is a Skew-symmetric matrix, shown as below. In $U(1)_{EM}$ group, let A be a diagonal matrix with $a_{ij} = a_{ji} = 0$:

$$A_{\alpha\beta} \equiv \begin{bmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & -B_z & B_y \\ -\frac{E_y}{c} & B_z & 0 & -B_x \\ -\frac{E_z}{c} & -B_y & B_x & 0 \end{bmatrix} \tag{4}$$

In this context, all components of the electric field (E-field) are forced to be zero. Due to the chaotic period during the early universe, this leads to the presence of a magnetic monopole, denoted by $\ln\hbar$, which ensures that all components of the magnetic field (B-field) are non-zero and must exist (indicating a non-zero matrix of $m \times n = 4 \times 4$). Particles during the inflation period possess supersymmetry (SUSY), which implies that they have magnetic monopoles associated with $\ln\hbar$. These monopoles can be incorporated into the electromagnetic tensor to ensure it is non-zero, thereby enabling their inclusion in subsequent calculations involving $M = \exp(A)$.

M represents the *strong-force tensor* within the *SU*(3) groups. Using this approach, we can also apply the electroweak tensor θ_w as shown below. The spontaneous breaking of SUSY leads to the combination of gauginos into two different bosons. Specifically:

$$M \equiv \begin{pmatrix} g \\ \gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix} \tag{5}$$

And

$$M = \exp(A) = \sum_{k=0}^{\infty} \frac{A^k}{k!} \tag{6}$$

Because of the zero charges associated with $(q_L = q_R = 0)$ in Weinberg's plane, and based on the pattern of weak isospin with $\theta_w = 0$, $\cos\theta_w = 1$, this can be indicated in progress as

$$Const \cdot \alpha = \frac{\pi e_L e_R}{|\vec{Y}'|} \overset{(Singul.)}{\equiv} \frac{e_L e_R}{\hbar c} \tag{7}$$

where $Const$ as slope, and α^{-1} as *x'-axis* on W-plane. It yields

$$\alpha = \left. \frac{|r|^2 (e_L e_R)}{Const \cdot \hbar c} \right|_{e_L = e_R} \propto \frac{e^2}{4\pi\hbar c} \tag{8}$$

The constant selected depends precisely on the Weinberg angle $\theta = 2\pi$, which determines $Const = 4\pi|r|^2$ as the surface area of the singular point, represented as a tight loop in high dimensions. In this following, $1/\varepsilon_0$ is chosen to ensure no units in the equation for α in the case of Equation (8). Consequently, this yields $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$.

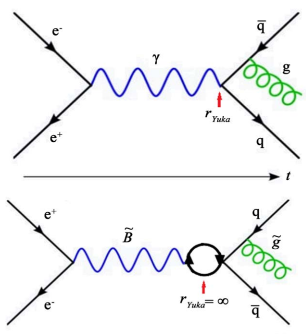

Figure 1. The Feymann diagrams.

Top (CP Conservation): $r_{Yuka} = finite$ **(localization)** is located at the interacting top-point and serves as a closure at low energy scales.

Bottom (CP Violation): $r_{Yuka} = \infty$ **(globalization)** forms a loop and diverges at high energy scales (e.g., in the early universe with $|v_i| = |v_j| = c, i \neq j$, where massless particles exist). To maintain translational invariance in quantum mechanics (ensuring energy conservation), additional terms are included in the diagram (e.g., antiparticles) with reversal in time ($t < t_0$). This diagram does not violate CPT symmetry; CP remains conserved while $t < t_0$, leading to the conservation of $U(1)$. Consequently, the photon's R-parity is +1, while the Bino's R-parity is −1, and similar considerations apply to gluons and sgluons. The loop contributes its energy to mix RGB colors to zero, thereby restoring color symmetry and ultimately returning to CP conservation.

Therefore the matrix is simplified, producing the eigenstate as a result of mixing states:

$$M \equiv \begin{pmatrix} g \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix}}_{eigenstate} \tag{9}$$

In the case of zero colors, which leads to a colorless state in Equation (9) at high energy scales, this further produces the common eigenstate (e.g., mass eigenstate) of the system. Consequently, their masses are equivalent to being massless:

$$m_{\tilde{g}} = 0 = \frac{m_{\tilde{B}}}{\cos\theta_w} \tag{10}$$

Given the high-energy state during the inflation era, A and M could be interchanged arbitrarily within an infinitesimally short time, as shown in Sect. III.A. However, the prohibition rules must still be adhered to.

By allowing $M \overset{(\Lambda_{GUT})}{\equiv} A \cdot Const$ to be interchanged, a satisfactory approach for unifying the two fundamental cosmic forces can be initiated:

$$\begin{aligned} A &= \exp(M) = \sum_{k=0}^{\infty} \frac{M^k}{k!}, \\ A &= \underbrace{\frac{M^0}{0!}}_{\substack{\equiv 0 \\ (flat.Space)}} + \underbrace{\frac{M^1}{1!} + \frac{M^2}{2!} + \frac{M^3}{3!} + \frac{M^4}{4!}}_{\substack{=0 \\ (Chiaral.Symmetry)}} + \underbrace{\frac{M^5}{5!} + \frac{M^6}{6!}}_{\substack{\neq 0 \\ (SUSY.Breakings)}} + \underbrace{H.O.T\cdots}_{\substack{=0 \\ (End.of.Inflation)}}, \\ A &\approx \frac{1}{144} M \end{aligned} \tag{11}$$

where

$$\begin{aligned} &\frac{M^1}{1!} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix} \equiv M, \\ &\left. \begin{aligned} \frac{M^5}{5!} &= \frac{1}{5!} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix} = M/120 \neq 0 \\ \frac{M^6}{6!} &= \frac{1}{6!} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{g} \\ \tilde{B} \end{pmatrix} = -M/720 \neq 0 \end{aligned} \right\} invisible \end{aligned} \tag{12}$$

The first term of the series corresponds to the flat space predicted by the Big Bang in the current Λ CDM model. The term following the second one will be discussed in the subsequent sections (refer to the three formulae below). Note that in Equation (12), the second and third formulae are concealed. At the instant of inflation, supersymmetry (SUSY) was spontaneously broken, forcing the eigenstates of mass, including $\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}^{(L.E.)} \neq 0$ (gluons ($\tilde{g} \to g$)) and graviphotons ($\tilde{B} \to$ LKPs with motional mass 10^8 eV and a lifetime 10^{-8} s), to be produced at 10^{-6} s. The numerous LKPs were scattered into pairs of photons and gravitons at a specific temperature level, traveling through the later universe at $t > 10^{-6}$ s. This type of exchange was restricted during the SUSY epoch, where high-dimensional considerations were allowed, and magnetic monopoles existed. By the end of inflation, all magnetic monopoles had degenerated (through an unknown "absorption mechanism") into SUSY particles, leading to the spontaneous breaking of SUSY at $t > 10^{-6}$ s. Despite this, the gauge conditions, including the hidden terms in Equation (12), remained intact.

Due to Equation (11) and (12), obviously all running points of fine structure constant is hidden in higher dimensions with $n = 5 \sim n = 6$ on expansions of matrix. Based on this, therefore assuming that $n \equiv 5.472$, hence $n! = 5.472! \approx 137 \cdot 2$. Such leads in

Due to Equations (11) and (12), it is evident that all running points of the fine structure constant are concealed within higher dimensions, particularly with $n = 5 \sim n = 6$ in the expansions of the matrix. Based on this assumption, if $n \equiv 5.472$, then $n! = 5.472! \approx 137 \cdot 2$. This, in turn, leads to

$$\frac{M^{5.472}}{5.472!} = \frac{1}{5.472!}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}\cos\theta_w & \sin\theta_w\\ -\sin\theta_w & \cos\theta_w\end{pmatrix}^{0.472}\right]\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix} \tag{13}$$

Thus, we obtain that the above expression is rotating on the plane of weak isospin, specifically considering the coterminal angles of the Weinberg angle:

$$\frac{M^{5.472}}{5.472!} = \frac{1}{5.472!}\left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}\cos\theta_w & \sin\theta_w\\ -\sin\theta_w & \cos\theta_w\end{pmatrix}\right]\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}\Bigg|_{\theta_w = (0.472)\cdot 2\pi} \tag{14}$$

Namely[2]

$$\frac{M^{5.472}}{5.472!} \approx \frac{1}{5.472!}\left[\underbrace{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}_{=1} + \underbrace{\begin{pmatrix}0.9985 & 0.0548\\ -0.0548 & 0.9985\end{pmatrix}}_{\equiv 1}\right]\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix} \tag{15}$$

Moreover,

$$\frac{M^{5.472}}{5.472!} \approx \frac{2}{5.472!}\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix} \overset{(\Lambda_{GUT})}{\approx} \frac{1}{136.9038}\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix} \approx \frac{1}{137}\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix} \tag{16}$$

[2]The term $\begin{pmatrix}0.9985 & 0.0548\\ -0.0548 & 0.9985\end{pmatrix} \equiv 1$ in Equation (15) could be rigorously defined as an integer, specifically 1, due to the nature of "gauge bosons." All the sections above are indicated as $SU(3)_{Diag}$ (*i.e.*, Beyond Standard Model, BSM).

Thus we obtain

$$A = \exp(M) = \frac{M^{5.472}}{5.472!} \overset{(L.E.)}{\approx} \frac{1}{137} M \tag{17}$$

Equation (17) is indicated as an observation at low energy. Thus, we establish a complete relationship between electromagnetic forces and the strong force through these deductions. It is important to note that this is not a coincidence; the primary rotation of the Weinberg angle was specifically performed to obtain the factor of 2. This factor, in conjunction with the Λ CDM model, *chiral symmetry* (as seen in Equation (11)), and supersymmetry (SUSY), was derived to accurately determine the factor in terms of $\frac{2}{5.472!} \approx \frac{1}{137}$. At this point, we can revisit the historical context with a clearer understanding. Additionally, the magnitude of $\alpha(t) \approx \frac{1}{136.9038}$ complies with the indicated data from the sets.

B) The Statement: The Ising model Phase Path r_{Yuka}

The famous Yukawa potential is expressed as:

$$V(r) = -g^2 \frac{e^{\kappa mr}}{r} \tag{18}$$

where g is the coupling constant, $\kappa \equiv c/\hbar$ represents a specific term in the interaction, m is the mass of the interaction particles, and r denotes the particle-waves, such as meson waves radiated from a nucleus or the core of a particle or boson (e.g., a photon, meson, or gluon). It is evident that the quantum system exhibits duality, possessing two or more classical limits, as represented by Equation (18) and Equation (19). According to the Ising model:

$$\langle \psi | \hat{F} | \psi \rangle = -\frac{\partial V(r)}{\partial r} = -g^2 \frac{\kappa mr e^{\kappa mr} - e^{\kappa mr}}{r^2} \tag{19}$$

The strong force must vanish at a unique critical phase (e.g., a phase path $r = r_{Yuka}$) in space, converting into the electromagnetic force governed by linear ordinary differential equations (ODEs). Based on this idea[3], therefore

$$\begin{aligned} &0 = -g^2 \frac{\kappa mr e^{\kappa mr} - e^{\kappa mr}}{r^2}, \\ &0 = g' \frac{-\kappa mr e^{\kappa mr} + e^{\kappa mr}}{r^2} = g' \frac{1}{r^2}\left[i\kappa mr\left(i e^{\kappa mr}\right) + e^{\kappa mr} \right], g' \neq g \end{aligned} \tag{20}$$

Note that $-\infty < r < \infty, r \neq 0$. The precise ranges will be provided later. Referencing Equation (5), it actually approaches zero for massless particles during the inflationary epoch. However, towards the end of this epoch, at the instant of 10^{-6} s, LKPs appear and acquire mass 10^8 eV due to SUSY breaking. Consequently, the second term $\tilde{g} \to g$ in the matrix of Equation (5) is forced to vanish, *i.e.*,

$$0 = g' \frac{1}{r^2}\left(\underbrace{i\kappa mr\left(i e^{\kappa mr}\right)}_{\substack{\neq 0 \\ (LKP, m = 10^8\,\mathrm{eV})}} + \underbrace{e^{\kappa mr}}_{\substack{\equiv 0 \\ (gluon)}} \right) = g' \frac{\kappa mr\left(i e^{\kappa mr}\right)}{r^2} = g' \frac{\kappa mr\left(\sin(\kappa mr)\right)}{r^2} \tag{21}$$

[3]This can be represented as a determinant by 2 × 2 with eigenvalue $\lambda = r$.

where we let the matrix element of $\sin\theta_w\big|_{\theta_w=0,2\pi}\equiv 0$ be defined. This allows the extracted $\kappa mr = 2\pi$ to hold significant importance. Furthermore[4],

$$r=\frac{h}{mc} \tag{22}$$

Considering that causality involves a "second-order constant perturbation" with factors of 99 (refer to **Appendix A** for the origin of 99),

$$r'=\frac{h}{\eta mc}\bigg|_{\eta=99} \tag{23}$$

Notably the starting point of r' is indicated from $r'=0$ to $r'=2r$, based on the principle of *causality*.

$$r=\frac{h}{2\cdot 99mc} \tag{24}$$

Substituting the cosmic physical constants by h,m,c, we obtain:

$$r_{Yuka}=\frac{1.05\times10^{-34}\ \text{J}\cdot\text{sec}}{(2)(99)\left(9.11\times10^{-31}\ \text{kg}\right)\cdot 3\times10^{8}\ \text{m}\cdot\text{sec}^{-1}} \tag{25}$$

Results with $r_{Yuka}\approx 1.9404\ \text{fm}$ were obtained for the phase path of interaction carriers (e.g., bosons such as hypothetical glueballs; see **Appendix B**). Additionally, by considering causality for electron matter-waves during electron collisions,

$$\overline{r}_{Yuka}\approx\frac{(1.9404+1.9560)\,\text{fm}}{2}\approx 1.950\ \text{fm} \tag{26}$$

In the case of electron-electron interactions (at the end of inflation[5], $m=m_e$, $10^{-36}\ \text{s}\le t\le 10^{-32}\ \text{s}$), the above expression $\overline{r}_{Yuka}$ aligns with Ting's experimental results from 1967 (*i.e.*, $\underbrace{\langle r\rangle_{\min}}_{\approx 1.954\ \text{fm}}-\overline{r}_{Yuka}=4\times10^{-18}\ \text{m}\sim 10^{-18}\ \text{m}$).

Notably, $r_{Yuka}\approx 1.9404\ \text{fm}$, as derived from Equation (25) to Equation (26), reveals that an estimated value for the spectrum mass gap has been missing from quantum Yang-Mills theory.

Remark.

See Equations (19) and (20), both of which are relevant to their corresponding matrices.

C) Representation by r_{Yuka} (a spectrum mass gap in *r*-basis)

This section presents a representation of r_{Yuka} in nuclear physics as part of the conclusions. **Deduction:** Following the establishment of Coulomb's law, the physical system of the universe evolved into a reactive equation at 10^{-6} s. During this period, graviphotons were produced in large quantities (with $\tau=10^{-8}\ \text{s}$ for LKPs), but they were unstable and quickly scattered to form electromagnetic fields (photons) and gravitational fields in free space, which then propagated throughout

[4]The expression in Equation (22) holds a different significance compared to the Compton wavelength.

[5]At the end of inflation, SUSY persisted momentarily, allowing the interchange of electron mass with photon mass due to the chaotic state of the universe and $m=m_e=0$, respectively. Notably, during the inflation epoch, as the universe expanded to a macroscopic scale approximating 1 meter, the size of $r_{Yuka},\overline{r}_{Yuka}$ is satisfactory, respectively.

the universe after inflation. As the uniform temperature decreased, the Weinberg angle continued to rotate until the particles reached a stable state. The rotation of the matrix indicates the constant interchange of roles among massless particles within the matrix.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{\pm\frac{\pi}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{27}$$

Therefore

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} g \\ \gamma_g \end{pmatrix} \xrightarrow{\pm\frac{\pi}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} g \\ \gamma_g \end{pmatrix} \tag{28}$$

This reveals that gluons participate in chromodynamics (QCD) within $|r_{Yuka}| \le 1.9404\ \text{fm}$, and graviphotons (γ_g) are instantaneously scattered into pairs of photons and gravitons, both of which have zero mass after scattering.

$$F(r) = -g'\frac{1}{r^2}\left[i\kappa mr\left(\mathrm{i}e^{\kappa mr}\right)\right]\Big|_{e^{\kappa mr}\equiv 1} = -g'i\frac{\kappa(im)r}{r^2}, im \ne 0 \tag{29}$$

At this moment, virtual photons mediate the electromagnetic force, resulting in a non-zero imaginary mass. Without this, the discussions would lose their significance. Mathematically arranging this, we obtain:

$$F(r) = g'\frac{\kappa mr\big|_{r=r_{Yuka}}}{r^2} \tag{30}$$

Note that r in the numerator holds significance as r_{Yuka}, as it is associated with the matrix from previous calculations. At $t \ge 10^{-6}\ \text{s}$, it is evident that electron-electron interactions are governed by Equation (26) as shown below:

$$\begin{aligned} \kappa mr_{Yuka} &= \frac{mcr_{Yuka}}{\hbar}, \\ \kappa mr_{Yuka} &= \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.997\times10^{8}\ \text{m}\cdot\text{sec}^{-1}\right)\left(1.94\times10^{-15}\ \text{m}\right)}{1.05456\times10^{-34}\ \text{J}\cdot\text{sec}}, \\ \kappa mr_{Yuka} &\approx 50.2268\times10^{-4}\ \frac{\text{kg}\cdot\text{m}^2}{\text{J}\cdot\text{sec}^2} \end{aligned} \tag{31}$$

The above is established in the context of high energy scales. (We will later see that this corresponds to the elementary charge of *e*.)

The strong force and electromagnetic forces undergo a linear transformation at point r_{Yuka} (*i.e.*, the unification of electrostrong interactions). Due to the nature of the opposing directions of their tensors, the calculations are performed within a repulsion field (where a negative sign for $-g'$ is assigned to electrons). Therefore, the real calculation should include the hidden terms[6].

$$-\frac{\left(\kappa mr_{Yuka}\right)^6}{10^5} \overset{(\Lambda_{GUT})}{\equiv} -e \approx -1.6055\times10^{-19}\ \text{C} \tag{32}$$

(LHS: at high-energy scales; RHS: at low-energy scales)

[6]The electric charge in an unstable universe was larger than its current value, analogous to the behavior of the fine structure constant; both variations are considered normal.

(Where $n=5, n=6$ originates from higher dimensions or general repulsion fields. The indication of 10^5 could be derived from the coupling constant of g'. The chaotic state of the early universe naturally normalized these units at the current low energy, a result confirmed by modern experiments (see **Table 1** for proof). Therefore, constant verification is unnecessary.) Substituting the above expression into Equation (30), we derive Coulomb's law:

$$F(r)=\xi g'\frac{-e}{r^2}\bigg|_{t<10^{-6}\,\mathrm{s}}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\bigg|_{t=10^{-6}\,\mathrm{s}} \tag{33}$$

Using cosmic extra dimensions to rigorously define

$$g'\equiv\frac{-e}{4\pi\varepsilon_0\xi},\xi\equiv10^{[\beta]},\left.\begin{matrix}5\le[\beta]\le11, repulsions\\ \beta=0, attractions\end{matrix}\right\} \tag{34}$$

The denominator parameter ξ represents the sum of diagonal elements after the matrix of the EM-tensor is diagonalized. The term β originates from the upper index of the four-vector potential in the Euclidean space metric. In this context, the numerator only includes $-e$ and cannot be expressed as e^2 mathematically; otherwise, e could not have been resolved in the previous discussions. In the specific case of physics based on the Yukawa potential, due to its mathematical nature, the parameter settings for the linear transformation to Coulomb's law (e.g., massless photon) are flawed: the Yukawa potential $V(r)$ has not been properly differentiated and substituting $m=0$ at that point is incorrect.

Based on the M-matrix conversion results presented in this paper, $m=0$ correctly refers to the *massless gluon* (not the massless photon) and corresponds to the definition of the EM-coupling constant g^2. However, this definition is incomplete in both physics and mathematics (particularly the latter), making it challenging to theoretically solve for the elementary charge, even though its value was experimentally determined by Millikan in the 1910s. According to our interpretation, by inversely solving for the charge and substituting it back into the correlation equations, Coulomb's law is derived theoretically.

D) Reconstruction for photon energy of $E=n\hbar\omega$ by Einstein

The representation by r_{Yuka} clearly indicates the point of SUSY breaking for massless particles (or spontaneous symmetry breaking (SSB) for gluons and photons at low energy scales). Consequently, r_{Yuka} can be fully incorporated into the matrix of Equation (4). Following this, in the context of the neutral current, as conceptualized by S. Weinberg, Sheldon Lee Glashow, and Abdus Salam (since photons are radiated by electrons), a dynamic framework for it, such as a Lagrangian, is required. Therefore,

$$L_N=T-V(r) \tag{35}$$

Given that r_{Yuka} corresponds to an electron, and $V(r)=0$ is associated with a free particle (such as a photon), after SUSY breaking or SSB,

$$L_N=T=\frac{m(\dot{r})^2}{2}\bigg|_{r=r_{Yuka}} \tag{36}$$

Table 1. Units normalization problem.

Events	Progress
High energy input is followed by low energy output in the observed physical system.	$units'\cdot\ln\left(H.E._{input}/L.E._{output}\right)=\infty,$ where $units'=arbitrary\overset{(H.E.)}{=}\{u_1',u_2',u_3',\cdots,\infty\},$ $units'\cdot\ln\left(\frac{(1/136.9038\cdots)\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}}{(1/137)\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}}\right)=\infty,$ $units\cdot\ln\left[\frac{\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}}{\begin{pmatrix}\tilde{g}\\ \tilde{B}\end{pmatrix}}\right]\overset{(L.E.)}{=}0,$ $units=arbitrary\overset{(L.E.)}{=}\{0,u_1,u_2,u_3,\cdots\}$
Normalization	$units\cap units'=\{u_1=u_1',u_2=u_2',u_3=u_3',\cdots,u_N=u_N'\}$ such leads in ***normalization*** produced by $1\equiv u_1/u_1'=u_2/u_2'=u_3/u_3'=u_N/u_N'$

The input event occurs at high energy scales and is subsequently output at low energy for all physical units. This process is based on the eigenstate properties of the rotation matrix in Equation (9).

Remark.

a) Equation (27) is clearly a representation.

b) Based on the previous discussions, we conclude that the range of the strong interaction is confined within $r\le 1.9404\,fm$, consistent with inherent physical principles. The typical effective range of electro strong interactions is situated at $-1.9404\ fm\le r<\infty, r\ne 0$.

c) Although the results of these equations were obtained with sufficiently small probabilities, they are still valid.

Regarding Equations (23) and (24), after rearranging Equation (23), we obtain:

$$2r=\frac{h}{\eta mc} \tag{37}$$

Using Compton wavelengths of $\lambda=h/mc$ for an electron with mass m, this leads to:

$$2r=\lambda/\eta \tag{38}$$

When a photon is instantaneously radiated at r_{Yuka} by an electron, there is no need to consider the perturbation term with η. Thus, we can directly set $\eta\equiv 1$. Therefore,

$$2r=\lambda \tag{39}$$

Leads in

$$\dot{r}\big|_{r=r_{Yuka}} = \dot{\lambda}/2 \tag{40}$$

Moreover[7],

$$L_N = \frac{1}{8} m\left(\dot{\lambda}\right)^2\Big|_{r=r_{Yuka}} \tag{41}$$

This complies with one of the eight components of the Lagrangian due to the factor of 1/8. At low energy scales, the Lagrangian is averaged in thermal equilibrium (*i.e.,* following the statistical Boltzmann distribution) when a photon is radiated. Considering the interactions between a fermion (an electron) and a gauge boson (a photon), we have[8]:

$$L_N = eJ_\mu^{em} A_\mu + \frac{g}{\cos\theta_W}\left(J_\mu^3 - \sin^2\theta_W J_\mu^{em} Z^\mu\right) \tag{42}$$

With the weak isospin of the right chiral state $T = 0$ for a stable electron (which does not participate in strong interactions due to its isospin $I = 0$), this leads to $J_\mu^3 = 0$. Additionally, with the weak hypercharge of the right chiral state $Y_W = -2$ for a stable electron, this yields $\theta_W = -\pi$. Associated with the above, therefore, Equation (42) becomes:

$$L_N = eJ_\mu^{em} A_\mu \tag{43}$$

where J_μ^{em} denotes the electromagnetic current and A_μ represents the electromagnetic tensor in four dimensions. Evidently, Equation (43) provides the Lagrangian, *i.e.*, the photon energy at r_{Yuka}. Based on the above, and further referencing Equation (37) with $\eta \equiv 1$ at r_{Yuka}, therefore

$$m = \frac{h}{2rc} \tag{44}$$

Using $E = mc^2$ with $2r = \lambda$ (Equation (39)) to calculate it, we obtain

$$\begin{gathered} E = \frac{hc}{2r} = \hbar\left(2\pi\frac{c}{2r}\right), \\ E = \hbar\left(2\pi c/\lambda\right) = \hbar\omega, n = 1 \end{gathered} \tag{45}$$

At this moment, we can finally understand why Einstein concluded the expression of photon energy as $E = n\hbar\omega$ in 1905.

<u>Remark.</u>

In the deductions where $\dot{\lambda}$ notably represents the meson wave function varying with time at r_{Yuka}, the expressions of the neutral current in this section align with $J_\mu = i\varepsilon\bar{\psi}\gamma_\mu\tau\psi$.

E) Importance: Light-speed of *C* in GUT scales (Factorial 2! given by loops)

The elementary charge *e* has been theoretically obtained in previous sections.

[7]Equation (41) one such example, shown as

$$L_{EW} = L_K + L_N + L_C + L_H + L_{HV} + L_{WWV} + L_{WWVV} + L_Y$$

Here L_N term is chosen for it.

[8]Note that this excludes the Higgs mechanism for massless photons.

Moving forward, we now require the mathematical-physical expression of the speed of light, C, which is particularly intriguing. Referencing Equation (32),

$$-\frac{\left(\kappa m r_{Yuka}\right)^6}{10^5} \overset{(\Lambda_{GUT})}{\equiv} -e \approx -1.6055\times10^{-19}\ \mathrm{C} \tag{46}$$

Moreover,

$$c^6 = \frac{10^5\left(1.6055\times10^{-19}\ \mathrm{C}\right)\hbar^6}{\left(m r_{Yuka}\right)^6} = \frac{10^5\left(1.6055\times10^{-19}\ \mathrm{C}\right)\left(1.05\times10^{-34}\ \mathrm{J\cdot sec}\right)^6}{\left(9.11\times10^{-31}\ \mathrm{kg}\right)^6\left(1.94\times10^{-15}\ \mathrm{m}\right)^6},$$

$$c = \left(7.06\times\left(10^{50}\right)\right)^{\frac{1}{6}} \approx 1.3850\times10^{8.\overline{33}} = 1.3850\times\left(\underbrace{10^{0.\overline{33}}}_{\approx 2.15443}\times10^8\right) \approx 2.9839\times10^8\left[\mathrm{m/sec}\right] \tag{47}$$

Similar to the measurement results in the history of physics (with an error ≤±1.31%), note that the factor of 1.38(5) precisely reveals the coefficients of the famous Boltzmann constant. Therefore, its illustration is as follows: With the fixed Boltzmann constant and the constant speed of light C, note that Equation (47) can be rewritten as:

$$c = \left(7.06\times\left(10^{50}\right)\right)^{\frac{1}{6}} = \left(Const\cdot\left(10^{50}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = Const \tag{48}$$

Obviously,

$$\left(10^{50}\right)^{\frac{1}{2}} = Const \tag{49}$$

So that

$$10^{25}\ \mathrm{eV} \overset{(\Lambda_{GUT})}{=} 10^{16}\ \mathrm{GeV} \tag{50}$$

Recall that the Grand Unified Theory (GUT) requires that three coupling constants precisely converge at point $\Lambda_{GUT} \approx 10^{16}\ \mathrm{GeV} \gg \Lambda_{QCD}$ (e.g., the points associated with the speed of light C within six-dimensional energy scales, which we currently hypothesize may be greater than 10^8 GeV in this "section"). It is evident that this convergence is not coincidental; it is directly related to the Supersymmetry (SUSY) discussed in this paper, much like the motivations behind widely-studied models (e.g., the $SU(5)$ model by SUSY, which is Beyond the Standard Model (BSM)).

<u>Remark.</u>

All running points converge at C, precisely aligning with the strong coupling constant of graviphotons (LKPs).

S-Duality: Here directly given that an approximation as below:

$$|\theta| \equiv \frac{1}{\sqrt{1+\sin^2 x_1+\cos^2 x_2}}\tan^{-1}\frac{\sqrt{1+\sin^2 x_3+\cos^2 x_4}}{\sqrt{1+\sin^2 x_1+\cos^2 x_2}} = 2\cdot\int_{x=0}^{x_4/2}\frac{\mathrm{d}x}{1+x^2} \overset{(\Lambda_{QCD})}{\approx} \int_{x=0}^{g_s=1.214}\frac{\mathrm{d}x}{2+\sin^2 x_1+\cos^2 x_2+\sin^2 x_3+\cos^2 x_4} \tag{51}$$

Here, $|\theta|$ represents the small angles between two fitted arc lengths, while $x_i, i=1,2,3,4$ (within QCD scaled ranges) denotes the coupling constants, also known as the Weinberg angle for force-charged carriers. These values are essentially approximated using the least squares method. The same slope starts from and is bounded at [10^{11}, 10^{18}] GeV, as assigned by the two fitted arc lengths (see the triangle in Figure 2). It is not difficult to observe that the slope is indicated as

$$\begin{aligned} \frac{\mathrm{d}x_i}{\mathrm{d}E} &\overset{(\Lambda_{GUT})}{=} \frac{1}{\frac{1}{2}mv^2+\frac{3}{8}m\frac{v^4}{c^2}+\cdots}\left.\frac{\mathrm{d}x_i}{\mathrm{d}v}\right|_{v=c} = \tan|\theta|, \\ |\theta| &\overset{(\Lambda_{GUT})}{\approx} \tan^{-1}\frac{\mathrm{d}x_i}{mc^2\mathrm{d}v} \end{aligned} \tag{52}$$

Evidently, $\dot{x}_i$ varies with the energy flow of $mc^2\mathrm{d}\Phi_{GUT}$ (in Λ_{GUT} scales). At such large energy scales, this directly results in $v=c$ and $\mathrm{d}x_i=0$, leading to $|\theta| \overset{(\Lambda_{GUT})}{\approx} \tan^{-1}(0/0)$, $\theta' \overset{(L.H.)}{=} \left(\tan^{-1}(0/0)\right)'$ intelligently. Equation (50) suggests that $v=c$ is located at the point of convergence of the coupling constants, meaning it runs to this point when using derivatives, as shown:

$$\begin{aligned} \theta' &\overset{(\Lambda_{GUT})}{\approx} \frac{1}{2+\sin^2 x_1+\cos^2 x_2+\sin^2 x_3+\cos^2 x_4+\cdots} \\ &= \left[\tan^{-1}\frac{\mathrm{d}x_i}{mc^2\mathrm{d}v}\right]'\Bigg|_{\substack{x_i=const\\ v=c}}, \theta\ge 0 \end{aligned} \tag{53}$$

Resulting in the following sequence:

$$\begin{aligned} \theta' &\overset{(\Lambda_{GUT})}{\approx} \frac{1}{2+\sin^2 x_1+\cos^2 x_2+\sin^2 x_3+\cos^2 x_4+\cdots} = \left[\tan^{-1}\frac{0}{0}\right]'\Bigg|_{x_i=const}, \theta\ge 0, \\ \theta' &\overset{(\Lambda_{GUT})}{\approx} \sum_{i=1}^{\infty}\frac{1}{1+\sin^2 x_i+\cos^2 x_i} = \left[\tan^{-1} Const\right]'\Big|_{x_i=const}, \\ \theta &\overset{(\Lambda_{GUT})}{\approx} \int_{\dot{x}}^{\dot{x}=c}\sum_{i=1}^{\infty}\frac{\mathrm{d}x}{1+\sin^2 x_i+\cos^2 x_i} = \tan^{-1} Const = Const, \\ Const &= \tan\theta = \frac{\mathrm{d}x_i}{mc^2\mathrm{d}v} \end{aligned} \tag{54}$$

i.e., the speed of light in a vacuum, expressed for convenience in terms of its square, is given by

$$c^2 = Const\cdot\left.\frac{\mathrm{d}x_i}{m\mathrm{d}v}\right|_{\substack{x_i=const\\ m=0}} = Const \tag{55}$$

Another approach could involve working within QCD scales, as outlined below. (Note: In Equation (55), $m=0$ pertains to massless particles at GUT scales.)

Let the coupling constants be represented as angles within QCD scales:

$$x_1\approx 1.214, x_2\approx 0.302822, x_3\approx 0.6295, x_4\approx 0.7180 \tag{56}$$

Based on smooth and continuous arc-lengths in the RGE scale μ (measured in GeV), the following two linear combinations are presented:

$$\begin{aligned} \theta'^{(\Lambda_{QCD})} &\approx \frac{1}{2+\sin^2(1.214)+\cos^2(0.302822)+\sin^2(0.6295)+\cos^2(0.7180)} \\ &= 0.249980 \approx 1/4, \\ \theta'^{(\Lambda_{QCD})} &\approx \frac{1}{2+\sin^2(0.302822)+\cos^2(1.214)+\sin^2(0.7180)+\cos^2(0.6295)} \\ &= 0.250024 \approx 1/4 \end{aligned} \tag{57}$$

i.e., all numbers would convergence onto 1/4:

$$\begin{aligned} &\frac{1}{2+\sin^2 x_1+\cos^2 x_2+\sin^2 x_3+\cos^2 x_4+\cdots} = 1/4, \\ &\sin^2 x_1+\cos^2 x_2+\sin^2 x_3+\cos^2 x_4+\cdots = 2 \equiv 2! \end{aligned} \tag{58}$$

Refer to the note below[9] for its corresponding QFT (*i.e.*, the equivalence of Equation (58) is denoted as Tree Level(s): $\Gamma^{(2)}_{tree} = i\left(k^2+m^2\right) > 0$). Specifically, all constants in Equation (54) are indicated as 1/3. Generalizing this into Equation (55), the calculated result of 1/3 becomes a hidden power component of the constant in both Equation (48) and Equation (55), respectively. By taking the logarithm and applying the concepts from Table I, the units of C are omitted at Λ_{GUT} scales. Therefore,

$$\log_{10} c = \frac{1}{3!}\cdot\log_{10}\left(Const\cdot\left(10^{50}\right)\right) \equiv Const \tag{59}$$

The origins from Equation (48) are self-consistent, and therefore, it was found that

$$\log_{10} c = \frac{1}{3}\cdot\log_{10} Const \equiv Const \tag{60}$$

Starting from Equation (55), we find that Equation (59) is physically equivalent to Equation (60) via instantons. Thus, by aligning the form of Equation (59) with Equation (60), we see that they both represent the same concept—extremely small sizes or distances, very close to Planck scales. Consequently, this alignment yields the precise fit points of the running coupling constants at Λ_{GUT} scales. The results are, indeed, remarkable[10]:

$$\begin{aligned} &\frac{1}{3}\cdot\log_{10}\left(3\times10^8\right)\cdot\frac{1}{10\cdot Const'}\bigg|_{Const'=7.06} \approx 0.04, \sim 10^{11}\ \text{GeV}, \\ &\frac{1}{3!}\cdot\log_{10}\left(3\times10^8\right)\cdot\frac{1}{10\cdot Const'}\bigg|_{Const'=7.06} \approx 0.02, \sim 10^{18}\ \text{GeV} \end{aligned} \tag{61}$$

[9]Equation (58) directly corresponds to the well-known Two-Point Function with a one-loop contribution; see Fig 1, 6, and 10 in Ref. [13], or **Figure 3** by this paper.

[10]The powers of 1/3 and 1/6, respectively, are derived from the hypercharge integers $Y = 0$ for an electron and then $I_3 = -\frac{1}{3}$ for electric charge, e.g., when an electron acquires $3\times\left(-\frac{1}{3}\right)$, leading to the power of 1/3 being obtained. This also relates to $SU(3)\times SU(2)\times U(1)$ in the context of some personal work in GUTs.

where the anti-logarithm: $\log_{10}^{-1}\left(Const\cdot\left(10^{50}\right)\right)\equiv\left.\frac{1}{10\cdot Const'}\right|_{Const'=7.06}$ suppresses four curves on p-103 of the PDF "Axion and ALP couplings—CERN Indico (2019) where α_6 could be regarded as being the fourth curve because of continuity: $\lim_{x_i\to x_j} f(x_i)=f(x_j)\equiv\alpha_6$), and the term of $(10\cdot Const')$ is assigned from Equation (48). Note that all numbers with constants could be merged ensemble. Above is not pointed out what ways calculating value of light-speed *C*, it is pointed out its vale is a constant regardless scenarios of running coupling constants, and in points of view of $\theta\equiv x_i$, it is exactly the convergence point of three coupling constants while all running in scales of Λ_{GUT}.

So far as, note that the indicated arc-lengths of picture are done by CERN (2019), one could preliminary refer to **Figure 2** where the hypotenuse of triangle fits four curves. This section is majority-contributed by the first author in this paper, and supervised by the second author. Where α_6 could be considered the fourth curve due to its continuity: $\lim_{x_i\to x_j} f(x_i)=f(x_j)\equiv\alpha_6$, the term C is derived from Equation (48). It's important to note that all numbers with constants can be merged. While the specific method for calculating the value of $(10\cdot Const')$ is not detailed here, it is established that C remains a constant, irrespective of the scenarios involving running coupling constants. From the perspective of $\theta\equiv x_i$, this constant precisely marks the convergence point of the three coupling constants as they run across Λ_{GUT} scales.

It is worthy to notice that, the arc-lengths cited here which are produced by CERN (2019), and one can preliminarily refer to **Figure 2**, where the hypotenuse of a triangle aligns with the four curves. *This section was primarily contributed by the first author of this paper, under the supervision of the second author.*

Remark

The derivation in the section aligns with the work of Kane, Gordon L. Specifically, the speed of light, denoted as C, is shown as the convergence point for all running points at high energy scales.

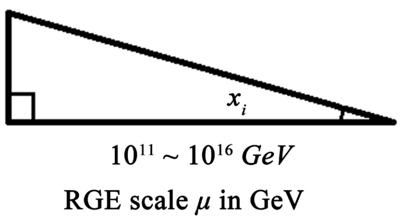

Figure 2. A simulation or approximation related to the problem of arc-lengths as curves, concerning the running couplings versus the RGE scale μ in GeV, is titled "Model II: Small Size Instanton Contribution by CERN." Notably, $x_i\equiv\theta\approx\tan\theta$ is considered because it is sufficiently small.

4. Conclusions

Beyond abstract equations, the unification of the three cosmic fundamental forces

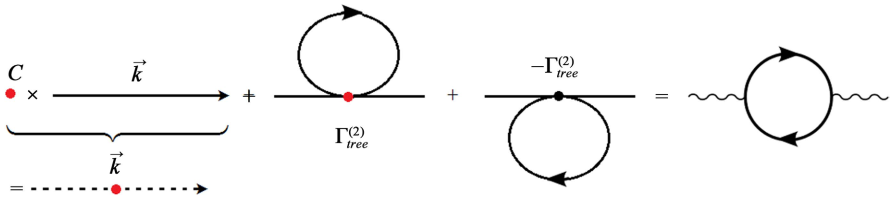

Figure 3. The tree levels of the two-point function are expressed in terms of $\Gamma_{tree}^{(2)}, -\Gamma_{tree}^{(2)}$. It is important to note that the amplitudes on the RHS originate from the one-loop level (*i.e.*, the point *C*) and the K-line in the scattered state on the extreme LHS. The point C is crucial in unfolding these two levels into one-loop structures, denoted as $\left(\Gamma_{tree}^{(2)}, -\Gamma_{tree}^{(2)}\right)$. This demonstrates the concept of a "factorial 2! loop". The resulting picture is truly miracle.

(interactions) into a single force has been achieved in this work. We have addressed and overcome the challenges associated with running points at high energy scales. The derived results for running points at high energy scales—including those of the fine structure constant $\alpha(t)$, the elementary charge e, the speed of light C, and Coulomb's law—have all been demonstrated in previous sections. Various difficulties in high-energy physics, such as running points at varying energy scales, massless particles, and the sources of photon energy, have largely been resolved through this work, which represents a significant scientific contribution. Aside from the Higgs mechanism, the representations for dynamics presented in this paper align with those of electroweak interaction theory, incorporating the novel concept of $r_{Yuka} \approx 1.9404 \text{ fm} \equiv k\Delta_0 > 0, k > 0$ in glueballs.

We found that the antiscreening colors with a trivial UV fixed point $\alpha_S(\mu^2) = 0$, therefore $G = SU(3)_{Diag} \times SU(2)_L \times U(1)_{EM} \subset SU(5)$ with $\Delta_0 > 0$, which can be completely derived by asymptotic freedoms, primarily introduced in this work (*i.e.*, providing an estimated value for the spectrum mass gap that has been elusive in Yang-Mills Theory). Consequently, this work also contributes to Grand Unified Theories (GUTs) without predicting proton decay. Additionally, the idea of a loop contributing its energy to maintaining color symmetry among massless particles at high energy scales is explored. Furthermore, we concluded our Beyond Standard Model (BSM) work with the formulation of $G = SU(3)_{Diag} \times SU(2)_L \times U(1)_{EM}$ (*i.e.*, Yang-Mills theory with a non-Abelian symmetry group), whose Lagrangian was derived for both Quantum Chromodynamics (QCD) and electromagnetic fields, in compliance with the representation of L_{YM}. We have provided verification for the six quark flavors: up (u), down (d), strange (s), charm (c), bottom (b), and top (t). Most notably, the running coupling constants of 0.02 and 0.04 at [10^{11}, 10^{18}] GeV, as reported by CERN in 2019, correspond to the speed of light c in a vacuum. Thus, we can conclude that the three fundamental cosmic forces are fully unified. This work could serve as a foundation for advancing the research to complete Einstein's unfinished Unified Field Theory (UFT) manuscript from 1955. Additionally, we have addressed the issue of strong CP-violation in **Appendix D**, where the significance of $\theta_{CP} \to 0^{\pm}$ is thoroughly explained.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendices

Appendix A. The Second-Order Constant Perturbation for Source with Number of 99

Perturbative QCD: Taking into account the quark confinement effect, we shall begin by deriving it within the intrinsic space framework, following the definition of the speed of light as established by special relativity. Namely,

$$C = \frac{\mathrm{d}|\vec{r}|}{\mathrm{d}t} \tag{A.1}$$

where $\mathrm{d}|\vec{r}|$ is the tiny average distance between quarks. Set the center of mass located at r_0 (*i.e.*, an eigen-constant), then

$$\mathrm{d}|\vec{r}| = \mathrm{d}(r - r_0) = \begin{cases} \mathrm{d}r, r_0 = const \\ \mathrm{d}r - \mathrm{d}(r_0 \lambda(t)), perturbation \end{cases} \tag{A.2}$$

Due to quark oscillations, it is evident that the center of mass is influenced by the time-dependent perturbation term $\lambda(t)$, causing the coordinate of the center of mass to be unfixed. Consequently, by omitting the first solutions in Equation (A.2), we obtain:

$$\mathrm{d}|\vec{r}| = \mathrm{d}r - \mathrm{d}(r_0 \lambda(t)) \tag{A.3}$$

Taking the derivative of Equation (A.1) with respect to time, thus we have

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}|\vec{r}|}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{d}r}{\mathrm{d}t} - \frac{\mathrm{d}(r_0 \lambda(t))}{\mathrm{d}t}\right] \tag{A.4}$$

Further organizing it, we obtain

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = r_0 \frac{\mathrm{d}^2(\lambda(t))}{\mathrm{d}t^2} \tag{A.5}$$

Given that the quark system within the electron undergoes simple harmonic oscillation, we shall introduce an arbitrary time-dependent perturbation term,

$$\lambda(t) \equiv \mathrm{e}^{i\lambda_0\omega_0 t} \tag{A.6}$$

where $0 < \lambda(t) < 1$. Substituting with Equation (A.6), therefore, Equation (A.5) becomes

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = r_0 \frac{\mathrm{d}^2\left(\mathrm{e}^{i\lambda_0\omega_0 t}\right)}{\mathrm{d}t^2} \tag{A.7}$$

After a calculation, thus we obtain

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = r_0 \cdot \lambda_0^2 \cdot (i\omega_0)^2 \left(\mathrm{e}^{i\lambda_0\omega_0 t}\right) \tag{A.8}$$

where λ_0^2 is referred to as the reduced perturbation term. In Equation (A.8), we observe that it is clearly a modification derived from the second-order perturbation theory in quantum mechanics.

Notably, according to the previous discourse, the common center of mass between quarks is not fixed due to the time-dependent perturbation (*i.e.*, the concept of delocalization), which is clearly a result of the external force, represented by the added perturbation. It is evident that in the intrinsic space between quarks, this external force corresponds to the strong force mediated by the gluon field. The mass-energy of the gluon field constitutes approximately 99% of the mass-energy in the intrinsic space. Therefore, the reduced perturbation term precisely represents this, meaning it is essentially the strong force itself, denoted as:

$$\lambda_0^2 = \left(\frac{99}{100}\right)^2 \equiv \frac{\tilde{\lambda}_0^2}{100^2} = 0.9801 < 1 \tag{A.9}$$

where $\tilde{\lambda}_0 \equiv \eta \equiv 99$ is extracted as the modification term of Equation (23), representing the significant "strong" component that is missing from QCD. It is important to note that the above discourse always assumes the state of high-energy perturbation, specifically, the strong perturbation, which is a relativistic quantum effect in QCD (*i.e.*, perturbative QCD). Notably, Equation (A.9) is effectively equivalent to the QCD coupling constants (*i.e.*, $\alpha_s < 1$). See Ref. [14].

Appendix B. Glueballs With $\Delta_0 > 0$ (One of Millennium Prize Problems)

Consider a plane-wave of a meson at $r = 1.954\ \text{fm}$ (denoted as $n = 0$) traveling and bounded at $r_{Yuka} \approx 1.9404\ \text{fm}$ (as the next lowest energy state $n' < n$) on glueballs. This interaction reveals the solution to the Yang-Mills existence and mass gap problem. The half-wavelength of glueballs, denoted as $\lambda/2 = r_{Yuka}/2$, constitutes the plane-waves of glueballs interacting with particles. This agrees with Equations. (25) and (26), and its mass 1278 MeV/c^2 aligns with $f_0(500)$ data in its 1000 MeV/c^2 or 1500 MeV/c^2 form.

Remark

One of the Millennium Prize Problems "Yang-Mills existence and mass gap" has been well-solved by this paper, marking the first resolution of this extremely difficult problem.

Appendix C. Distinguish: SM and BSM

Table A1. The differences of G groups between SM and BSM.

SM	BSM
$G = SU(3) \times SU(2)_L \times U(1)_{EM} \subset SU(5)$; $G = SU(3)_C \times SU(2)_L \times U(1)_{EM} \subset SU(5)$ (C.1)	$G = SU(3)_{Diag} \times SU(2)_L \times U(1)_{EM} \subset SU(5)$ (C.2)†
Color Symmetry (to sum R, G, B as to white)	Antiscreening Colors With a Trivial UV Fixed Point $\alpha_s(\mu^2) = 0$
Observable Color Confinement	Asymptotic Freedoms
Energy scales: Λ_{QCD} (C.3)	Energy scales: $\Lambda_{GUT} \gg \Lambda_{QCD}$ (C.4)

Remark

Equation (C.2) is recommended by this paper. Equation (C.3) and (C.4) are widely known.

Appendix D. Examination: Violations of Color Symmetry in $SU(3)_{Diag}$ Group

From Equations (9) to (10), at high energy scales, $r_{Yuka} = \infty$, $\tilde{m}_{\tilde{g}} = \tilde{m}_{\tilde{B}} = 0$ corresponds with the representations in Figure 1, such that

$$\gamma \xleftrightarrow{r_{Yuka} = finite} g \quad \text{(Top)} \tag{D.1}$$

Or

$$\tilde{B} \xleftrightarrow{r_{Yuka} = \infty} \tilde{g} \quad \text{(Bottom)} \tag{D.2}$$

Equation (D.1) represents particles under local gauge symmetry, while Equation (D.2) corresponds to higher energy (H.E.) contributions from loops. A clever approach can be employed here. †By dividing by r_{Yuka}, we obtain.

$$\begin{aligned} & g_{i,\tilde{B}} + g_{i,\tilde{g}} = g_{i,\tilde{g}} \neq 0, \\ & \lim_{r_{Yuka} \to \infty} \frac{g_{i,\tilde{B}}}{r_{Yuka}} + \frac{g_{i,\tilde{g}}}{r_{Yuka}} \overset{(\Lambda_{GUT})}{=} \lim_{r_{Yuka} \to \infty} \frac{g_{i,\tilde{g}}}{r_{Yuka}} = 0 \end{aligned} \tag{D.3}$$

This soon results in zero, indicating colorlessness (*i.e.*, color symmetry). The benefit of the bottom in Figure 1 is the invariance of translation due to the nature of quantum mechanics, which leads to Equation (D.1) transforming into

$$\begin{aligned} & g_{i,\gamma} + g_{i,g} = g_{i,g} \neq 0, \\ & \frac{g_{i,\gamma}}{r_{Yuka}} + \frac{g_{i,g}}{r_{Yuka}} = \frac{g_{i,g}}{r_{Yuka}} \quad \text{(Low Energy Scales)} \end{aligned} \tag{D.4}$$

Because of $\tilde{g} \to g$ after $\tilde{g}$ mixing with higgsinos and Equation (D.3) is translated into Equation (D.4) therefore

$$\lim_{r_{Yuka} \to \infty} \frac{g_{i,\tilde{g}}}{r_{Yuka}} \overset{(R,G,B)}{=} \frac{g_{i,g}}{r_{Yuka}} = 0 \quad \text{(One Loop Contribution)} \tag{D.5}$$

It can be considered that zero, representing colorlessness, arises from the divergence term in the Feynman diagram (refer to the loop in **Figure A1**). In the process of renormalization, we encounter couplings of colors (here, directly using colors as chromatic quantum numbers):

$$\begin{aligned} & I \equiv \int_B^R \frac{\mathrm{d}z}{z} = \int_0^R \frac{\mathrm{d}z}{z} - \int_0^B \frac{\mathrm{d}z}{z} = \ln R - \ln B + \ln 0 - \ln 0, \\ & I = \ln R - \ln B + \ln 0 - \ln 0, \\ & \varepsilon_R = \varepsilon_B \to 0, \\ & I = \ln \frac{R}{B} \end{aligned} \tag{D.6}$$

(R: $g = 1/3$ G: $g = 1/3$ B: $g = 1/3$)

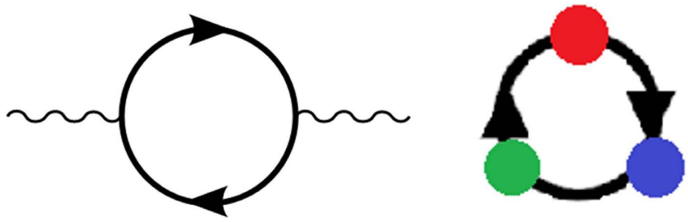

Figure A1. Antiscreening: The loop contributes its energy to achieve zero color for both gluons and sgluons, thereby facilitating color symmetry during their interactions with other particles.

Remark

Simply put, the loop plays a role in chromatic dynamics. Renormalization is utilized to connect different energy scales. The loop results in an $SU(3)_{Diag}$ group without color charges. Since there is no loop at the top of **Figure 1**, $SU(3)_C$ with color charges is required.

Note that the loops in **Figure 1** align with the following demonstrations: "Model II Small Size Instanton Contribution," by Belén Gavela, Univ. Autónoma de Madrid and IFT, H2020, Granada, June 3-7 (2019). Refer to the indicated α_{diag} 1 loop and/or α_{diag} 2 loop on p. 103 in the PDF "Axion and ALP couplings—CERN Indico."

As a result, the correctness of the deduction from Equations. (D.1) to Equation (D.5) can be considered as acceptance. The Lagrangian density is denoted as:

$$L_{QCD} = \bar{\psi}_i \left(i\gamma^{\mu}\partial_{\mu} - m \right)\psi_i - g_s A^a_{\mu}\bar{\psi}_i \gamma^{\mu} T^a_{ij}\psi_j - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \tag{D.7}$$

At $r = r_{Yuka} = \infty$, looping occurs due to antiscreening colors, resulting in $m = m_i = 0$ (massless quarks) and constant quark fields of $\psi_i = const$ (individual quarks or quark-gluon plasma, QGP), which subsequently leads to $\bar{\psi}_i \left(i\gamma^{\mu}\partial_{\mu} - m \right)\psi_i = i\bar{\psi}_i \gamma^{\mu}\partial_{\mu}\psi_i = 0$.

Subsequently, the coupling constant with $g_s = \sqrt{4\pi\alpha_s} = 0$ looped (*i.e.*, resulting in colorlessness). Thus, Equation (D.7) remains

$$L_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \tag{D.8}$$

The same mathematical form as that of EM fields, leading to the deduction that

$$L = \bar{\psi}\left(i\hbar c\gamma^{\alpha}D_{\alpha} - mc^2 \right)\psi - \frac{1}{4\mu_0}F_{\alpha\beta}F^{\alpha\beta} \tag{D.9}$$

At $r = r_{Yuka} = \infty$ looped we have $m = m_e = 0$ (massless fermions) and $\gamma^{\alpha} = 0$ when $\gamma^{\alpha} \neq \gamma^{\mu}$ (*i.e.*, $\alpha \neq \mu$). †Therefore **Equation (D.9)** becomes

$$L_{EM} = -\frac{1}{4\mu_0}F_{\alpha\beta}F^{\alpha\beta} \tag{D.10}$$

Comparing with **Equation (D.8)** and **Equation (D.10),** the mixed structure of group of $SU(3)_C \times U(1)_{EM}$ and $SU(3)_{Diag} \times U(1)_{EM}$ (or directly: $SU(3)_C \times SU(3)_{Diag} \subset SU(6)$) is obtained. To complie with $SU(3)$ in the SM because

$$L_{YM} = -\frac{1}{2}Tr\left(F^2\right) = -\frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu} \tag{D.11}$$

As a result, we define the angle of strong CP conservation as

$$\theta \equiv \tan^{-1}\frac{-4L_{YM}}{\left|F_{\mu\nu}^{a}F_{a}^{\mu\nu}\right|}, \theta = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \tag{D.12}$$

In the case of Equation (50), which operates within certain energy scales (denoted as Λ_{GUT}), as such, and later, as we shall see in Equation (E.1) and Equation (E.2), the scale approaches infinity ($\Lambda_{GUT} < \Lambda_{Landau} \to \infty$ and then $L_{YM} \to 0$).

Therefore,

$$\begin{aligned} &\theta = \lim_{\substack{L_{YM} \to 0 \\ \Lambda_{Landau} \to \infty}} \tan^{-1}\frac{-4L_{YM}}{\left|F_{\mu\nu}^{a}F_{a}^{\mu\nu}\right|} = 0, \\ &\theta_{CP} \to 0^{\pm} \end{aligned} \tag{D.13}$$

where $\theta_{CP} \to 0^{\pm}$ is indicated as the critical angle of CP-violation. (Note that $\theta = 0$ occurs in electroweak interactions at TeV-scales, which operate at much smaller energy scales than Λ_{GUT}).

Equation (D.13) implies that the breakthrough of symmetry in strong CP-violation occurs at GUT scales, leading to a simple Lie group or asymptotic freedom. As a result, we define $\theta_{CP} \equiv \beta_1(\alpha) \to 0^{-} < 0$ as asymptotic freedom in the Feynman diagram. The calculation of quark freedoms in the case of asymptotic freedom is performed using the physical Beta function:

$$\beta_1(\alpha) = \frac{\alpha^2}{\pi}\left(-\frac{11N}{6} + \frac{n_f}{3}\right) < 0 \tag{D.14}$$

For $SU(3)$ groups with $N = 3$, such gives

$$\frac{\pi\beta_1(\alpha)}{\alpha^2} = -\frac{33}{6} + \frac{n_f}{3} < 0 \tag{D.15}$$

where $\alpha^2 \approx \frac{g^2}{4\pi} \approx \frac{1}{\pi}, g \approx -2.002\cdots$ Substitute $\theta_{CP} \to 0^{-}$ into Equation (D.15) associated with Equation (D.13). Hence

$$\begin{aligned} &3\cdot\left(0^{-} + \frac{33}{6}\right) = n_f, \\ &n_f = 16.4\overline{9} < \frac{33}{2} \end{aligned} \tag{D.16}$$

Yields

$$\left[\!\left[n_f \right]\!\right] = 16 < \frac{33}{2} \tag{D.17}$$

The number 16 is clearly indicated as corresponding to the QCD $\left(n_f = n_{f'} = 6\right)$, QED $\left(n_f = 6\right)$, and Yang-Mills gauge particles $\left(n_f = 4\right)$ (with the exception of Higgs bosons $n_f = 0$, which are stationary) at Landau poles, within the current Standard Model (SM).

Another case: The negative sign of 0^{-} originates from $\bar{\psi}\psi \to -\psi\bar{\psi}$ (while considering the scenario in which $\Lambda_{GUT} \gg \Lambda_{QCD}$ is presented), which signifies the contribution from chiral condensed matter (e.g., axions as their candidates, coupled with massless quarks) at GUT scales, resulting in the production of quark freedoms. This concept is supported by the work done by K. Choi and J.E. Kim (1985). In the context of SM

$$n'_f = 6 < \frac{33}{2} \tag{D.18}$$

The six distinct flavors of quarks are precisely identified as up (u), down (d), strange (s), charm (c), bottom (b), and top (t). Each of these quark flavors has been observed and studied individually.

Appendix E. The Definite Answer for the Value of the Cutoff Λ

On p-3 of Ref. [13], Sander Mooij and Mikhail Shaposhnikov posed the question, "What is the value of the cutoff Λ?" Here, we provide the answer. By applying either Equation (58) or Equation (61), the solution for the cutoff is determined as

$$\Lambda \propto M_W e^{-\frac{c}{\alpha}}\Big|_{\alpha=c} = M_W e^{-1} \approx (0.3680)(80.39\ \text{GeV}) \approx 29.58\ \text{GeV} \tag{E.1}$$

where c means the speed of light. Hence

$$\begin{aligned} &\Lambda^4 \approx 10^{42}\,(\text{eV})^4 \to \infty, \\ &\Lambda_{Landau} \to \infty \end{aligned} \tag{E.2}$$

With a trivial solution (the UV fixed point):

$$\alpha_s\left(\mu^2\right) = \frac{1}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)} = 0 \tag{E.3}$$

where Λ_{Landau} is denoted as Landau poles.

Equation (E.3) describes the "coupling constant with $g_s = \sqrt{4\pi\alpha_s} = 0$ one-looped (*i.e.*, colorlessness)" as discussed around Equations. (D.7) to (D.8) in **Appendix D**. By comparing Equation (E.2) with the masses of Higgs bosons, we observe that

$$m_H = 125.09\ \text{GeV} \ll \Lambda_{Landau} \tag{E.4}$$

The actual presence of $m_H \ll \Lambda_{Landau}$ suggests that there is a necessity to fine-tune its related issues in the SM.