

# Multiverse/Hyperverses Models: (4 + 1)-Dimensional Landscape (Black Saturns, Bousso-Hawking Nucleation, Gogberashvili Multiverses, Schwarzschild-De Sitter Nurseries) and a (3 + 1)-Dimensional Model for Dark Energy

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## Abstract

We consider the *Hyperverses* as a collection of multiverses in a (4 + 1)-dimensional spacetime with gravitational constant  $G$ . Multiverses in our model are bouquets of thin shells (with synchronized intrinsic times). If  $g$  is the gravitational constant of a shell  $S$  and  $\varepsilon$  its thickness, then  $G \sim \varepsilon g$ . The physical universe is supposed to be one of those thin shells inside the local bouquet called *Local Multiverse*. Other remarkable objects of the Hyperverses are supposed to be black holes, black lenses, black rings and (generalized) Black Saturns. In addition, Schwarzschild-de Sitter multiversal nurseries can be hidden inside those Black Saturns, leading to their Bousso-Hawking nucleation. It also suggests that black holes in our physical universe might harbor embedded (2 + 1)-dimensional multiverses. This is compatible with outstanding ideas and results of Bekenstein, Hawking-Vaz and Corda about “black holes as atoms” and the condensation of matter on “apparent horizons”. It allows us to formulate conjecture 12.1 about the origin of the Local Multiverse. As an alternative model, we examine spacetime warping of our universe by external universes. It gives data for the accelerated expansion and the cosmological constant  $\Lambda$ , which are in agreement with observation, thus opening a possibility for verification of the multiverse model.

## Keywords

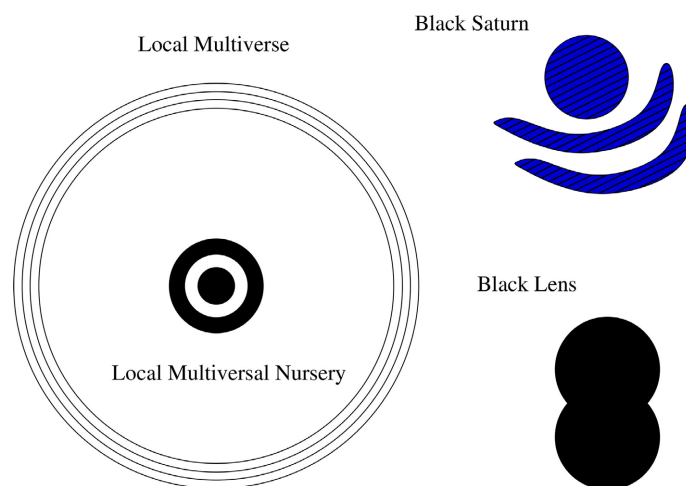
5-Dimensional Gravity, Black Hole, Black Saturn, Cosmological Constant,

## 1. Introduction

The multiverse as a collection of individual universes and the Hyperverse as a collection of multiverses have become a widely discussed and disputed area of cosmology [1] [2]. Whereas the universe is a region of space accessible to observers within that region, the strongest arguments against the multiverse/Hyperverse has been the notion that there is no possibility of observation or interaction from our universe with other universes. As a consequence—and as the major argument of decliners of the multiverse model—the verification or disproof of the existence of the multiverse so far has not been achieved.

A major step forward in the process of verification of the multiverse model was the observation of accelerated expansion of the universe and the prediction of the non-zero cosmological constant  $\Lambda$ . An additional argument in favor of the multiverse is the progress made in string theory, applying more than four spacetime dimensions for describing the multiverse as a landscape inhabited by our universe.

In the current paper, we use various approaches and scenarios in order to describe multiverses and the Hyperverse. First of all, we consider multiverses as bouquets of thin shells, embedded into a 5-dimensional spacetime (Hyperverse). The physical universe is supposed to be one of those thin shells inside the local bouquet called *Local Multiverse* (Figure 1).



**Figure 1.** Landscape of the 5-dimensional Hyperverse.

Nested Gogberashvili multiverses [3] seem to give us the most realistic scenario, compatible with the  $\Lambda$ CDM model.

The second approach focuses on a measurable interaction of universes via gravity. We assume that the accelerated expansion of the universe is caused by gravitational spacetime warping of our universe by other universes surrounding us.

Currently, the reason for the accelerated expansion of our universe is attributed

to an unknown force, called *Dark Energy*, which acts like a repulsive force (“antigravity”), when observed within our universe; or as “normal” gravity, when observed from outside of our universe [4]. If this is the case, it would also indicate a clear route towards verification of the multiverse model.

Values of the on-shell cosmological constant  $\Lambda$  are compared within these two models (Sections 6 and 7).

Other constituents of the Hyperverses, which are being addressed, are black holes, black rings and Black Saturns.

## 2. Schwarzschild-Tangherlini Black Holes

The Schwarzschild-type solution of Einstein field equations in higher dimensions was first derived by Tangherlini [5]. When the dimension  $D$  is equal to 5, one gets (putting  $c = 1$ ):

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_3^2 \tag{2.1}$$

where  $r$  is the radial coordinate and  $d\Omega_3^2$  is the line element of the unit 3-sphere  $S^3$ . The function  $f(r)$  is given by:

$$f(r) = 1 - \frac{16\pi GM}{3s_3 r^2}, \tag{2.2}$$

where  $M$  is the mass of the black hole and  $s_3 = 2\pi^2$  is the surface volume of  $S^3$ . Thus, the radius  $r_H$  of the event horizon is given by the formula

$$r_H = \sqrt{\frac{8GM}{3\pi}}. \tag{2.3}$$

(cf. ([6], ch.~6) for more details in any dimension  $D \geq 5$ ).

## 3. Multiversal Nurseries of de Sitter Bubbles

Frolov *et al.* [7] and Barabès-Frolov [8] suggested the formation of multiple de Sitter universes inside the event horizon of Schwarzschild black holes. Here we generalize it to the 5-dimensional case (cf. ([6], sect.~5.4.2) when  $D = 4$ ).

In static coordinates, the de Sitter metric is given by:

$$ds^2 = -\left(1 - \frac{r^2}{\alpha^2}\right)dt^2 + \left(1 - \frac{r^2}{\alpha^2}\right)^{-1}dr^2 + r^2d\Omega_3^2 \tag{3.1}$$

with cosmological horizon at  $r = \alpha$ . The simultaneous creation of multiple de Sitter shells at radius  $r_0$  corresponds to  $f(r_0) = 1 - r_0^2/\alpha^2$  that implies:

$$r_0^4 = \frac{8GM\alpha^2}{3\pi} \tag{3.2}$$

The intrinsic time of such de Sitter multiversal bubble corresponds to

$$d\tau = -\frac{3\pi r^2}{8GM}dr \tag{3.3}$$

with  $dr < 0$ .

Phase transitions from Schwarzschild to de Sitter metrics solve the problem of singularity at  $r = 0$ .

#### 4. Gogberashvili Multiverses in de Sitter-Schwarzschild Setting

The Gogberashvili solution (*cf.* [9]) of 5-dimensional Einstein field equations is given in terms of metrics of inner and outer regions:

$$ds_+^2 = -\left(1 - 2GM/r^2 + \Lambda_+ r^2\right) dt^2 \quad (4.1)$$

$$+ \left(1 - 2GM/r^2 + \Lambda_+ r^2\right)^{-1} dr^2 + r^2 d\Omega_3^2, \quad (4.2)$$

$$ds_-^2 = -\left(1 + \Lambda_- r^2\right) dt^2 + \left(1 + \Lambda_- r^2\right)^{-1} dr^2 + r^2 d\Omega_3^2, \quad (4.3)$$

separated by a timelike 4-dimensional spherical shell (brane, bubble) with FLRW metric:

$$ds^2 = -d\tau^2 + a^2(\tau) d\Omega_3^2, \quad (4.4)$$

where  $\tau$  is the intrinsic time of this spherical universe.

The outer metric looks like a 5-dimensional version of the *de-Sitter-Schwarzschild spacetime*.

The inner metric is, actually, de Sitter when  $\Lambda_- = -1/\alpha^2$  is negative. A nested multiversal version of this construction is described in [3]. The innermost region is supposed to be a *de Sitter multiversal nursery*.

Thus, the Gogberashvili solution is the insertion of inflating (3 + 1)-dimensional FLRW multiverses into a (4 + 1)-dimensional Schwarzschild-de Sitter spacetime (Hyperverses).

#### 5. Synchronization of Intrinsic Times

Intrinsic times of shells-universes in an authentic multiverse should be mutually synchronized.

For example, let's consider two FLRW universes with intrinsic times  $\tau_1$  and  $\tau_2$ . They are synchronized if there exists a strictly increasing bijective function  $\varphi$  such that  $\tau_2 = \varphi(\tau_1)$  and  $\varphi(0) = 0$ . In the simplest case, intrinsic times are just proportional:  $\tau_2 = k\tau_1$ .

In particular, it means that the whole Local Multiverse has the same timeline (up to appropriate synchronizations). Let's notice here that the problem of existence of cosmic time functions was studied by Hawking [10].

In such case, we can construct time-amalgamated products and coproducts of shells-universes. They are useful if we want to consider all strata of a multiverse simultaneously (*cf.* [11]).

#### 6. On-Shell Cosmological Constant in Gogberashvili Model

The expansion rate of the Universe in Gogberashvili model is given by the following formula:

$$\dot{a}^2 = -1 + \frac{\sigma^4 - 2(\Lambda_- + \Lambda_+) \sigma^2 + (\Lambda_- - \Lambda_+)^2}{4\sigma^2} a^2 \quad (6.1)$$

$$+ \frac{GM(\sigma^2 + \Lambda_- + \Lambda_+)}{\sigma^2 a^2} + \frac{G^2 M^2}{\sigma^2 a^6} \quad (6.2)$$

where  $\sigma$  is the intrinsic on-shell energy density.

We can compare it with the first Friedmann equation (with  $c = k = 1$ )

$$\dot{a}^2 = -1 + \frac{8\pi G\rho + \Lambda}{3} a^2, \quad (6.3)$$

where  $\rho$  is the volumetric mass density. It gives us the following formula for the on-shell cosmological constant  $\Lambda$ :

$$\Lambda = 3 \left[ \frac{\sigma^4 - 2(\Lambda_- + \Lambda_+) \sigma^2 + (\Lambda_- - \Lambda_+)^2}{4\sigma^2} \right] \quad (6.4)$$

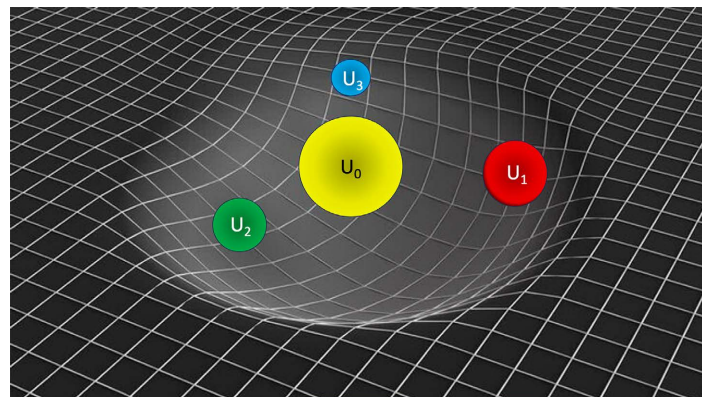
$$+ 3 \left[ \frac{GM(\sigma^2 + \Lambda_- + \Lambda_+)}{\sigma^2 a^4} + \frac{G^2 M^2}{\sigma^2 a^8} \right] - 8\pi G\rho. \quad (6.5)$$

Maybe we could neglect the middle term, but we don't know the values of  $\Lambda_-$  and  $\Lambda_+$  in the regions of 5-dimensional Hyperverses, surrounding our physical shell-universe.

## 7. Warping by External Universes, Dark Energy and Cosmological Constant

In this section we consider a different  $(3 + 1)$ -dimensional semiclassical model, in which universes are balls in Euclidian  $\mathbb{R}^3$ , separated by the void. External universes are considered as the source of dark energy.

Details of this model have been described in [4]. In short, we assumed that the multiverse is a collection of individual universes with identical physical laws and constants according to Tegmark's level 1 multiverse [1] [12]. We calculate the Newtonian gravitational force exerted on our universe by surrounding universes. In [4] identical masses were used for all universes (configuration A). Here we consider also configurations B and C described below (Figure 2).



**Figure 2.** Illustration of spacetime warping by three external universes.  $U_0$  is our Universe;  $U_1$ ,  $U_2$  and  $U_3$  are external universes. The positions and sizes of the external universes are illustrative, they are not identical to those used in Table 1.

The resulting force for the gravitational pull exerted by the external universes is provided in Table 1, together with other parameters such as acceleration  $a$ , velocity  $v$  and mass density  $\rho$ .

**Table 1.** Comparison of the parameters obtained for different multiverse models with three external universes or three black holes. The original configuration and details are described in [4].

Parameter	A	B	C
$F(N)$	$1.28 \times 10^{43}$	$4.03 \times 10^{43}$	$4.90 \times 10^{17}$
$a$ (m/s <sup>2</sup> )	$1.26 \times 10^{-11}$	$3.97 \times 10^{-11}$	$2.46 \times 10^{-24}$
$v$ (km/s)	5151	16267	$1.01 \times 10^{-9}$
$E_{\text{kin}}$ (J)	$1.35 \times 10^{67}$	$1.34 \times 10^{68}$	$1.01 \times 10^{29}$
$\rho$ (kg/m <sup>3</sup> )	$4.20 \times 10^{-31}$	$4.19 \times 10^{-30}$	$3.16 \times 10^{-69}$
$\Lambda$ (s <sup>-2</sup> )	$7.04 \times 10^{-40}$	$7.02 \times 10^{-39}$	$5.31 \times 10^{-78}$
$\Lambda$ (m <sup>-2</sup> )	$3.92 \times 10^{-57}$	$3.90 \times 10^{-56}$	$2.95 \times 10^{-95}$

A: Identical masses of all four universes.  $U_0$  is our Universe;  $U_1$ ,  $U_2$  and  $U_3$  are external universes. Their positions relative to  $U_0$  (0, 0, 0) are  $U_1$  (3, -1, 2),  $U_2$  (-4, -2, 1) and  $U_3$  (1, 2, -3). The numbers in parentheses indicate distances in radius of our universe. B:  $U_1$ : two-fold mass relatively to  $U_0$ ,  $U_2$ : threefold mass,  $U_3$ : half the mass. The positions are identical to A. C: Three hypermassive black holes with  $10^{11}$  solar masses each. The positions are identical to A.  $F(N)$ : gravitational pull (force) due to external universes according to Newton's Law of Gravitation.  $a$  (m/s<sup>2</sup>): acceleration of our universe due to the gravitational pull.  $v$  (km/s): resulting velocity at present time.  $E_{\text{kin}}$ : kinetic energy due to the gravitational pull = "dark energy" in this model.  $\rho$  (kg/m<sup>3</sup>): density of the dark energy.  $\Lambda$ : cosmological constant in s<sup>-2</sup> and m<sup>-2</sup>.

It allows us to calculate the corresponding  $\Lambda$  in the flat FLRW model. All parameters depend on the number, mass and position of external universes. Arranging these parameters in configurations A and B, we show that  $\Lambda$  can take values in the range from  $10^{-56}$  to  $10^{-55}$  m<sup>-2</sup>. Carfora and Familiari [13] published the value of  $\Lambda \sim 10^{-52}$  m<sup>-2</sup>.

Similar calculations can be done in the case of spherical shells in the (4 + 1)-dimensional Hyperverses.

## 8. Horizons, Seifert Fibrations and Black Saturns

(Cosmological and event) horizons in 5-dimensional gravity show a remarkable range of possibilities. Basically, they are Seifert fibrations or generalized Black Saturns. A higher-dimensional version of the rigidity theorem implies that a stationary black hole should be static or it should have a U(1)-symmetry. The following *refined topology theorem* (cf. [14], result 1) and ([15], theorem 4) describes possible horizons when  $D = 5$ .

**Theorem 8.1** *The topology of the horizon  $B$  in  $D = 5$  dimensions should be one of the following types:*

- If the U(1)-symmetry has a fixed point on  $B$ , then

$$B \cong \# j \cdot (S^1 \times S^2) \# L(p_1; q_1) \# \dots \# L(p_k; q_k) \quad (8.1)$$

where  $L(p_i; q_i)$  are lens spaces for  $1 \leq i \leq k$ .

- Otherwise,  $B \cong S^1 \times S^2$  or  $B \cong S^3/\Gamma$  where  $\Gamma$  is a certain finite subgroup of SO(4). All manifolds in this class are Seifert fibered spaces over  $S^2$  with positive

orbifold Euler characteristic.

By *Black Saturn* we understand the case when  $B \cong S^3 = L(1;0)$  surrounded by black rings of type  $S^1 \times S^2$ .

### 9. Fragmentation of Nariai Black Rings

We follow here some ideas of Bousso and Hawking [16]-[18]. Let's suppose that a black ring is, actually, a *Nariai spacetime* with the following metric in global coordinates:

$$\frac{ds^2}{l^2} = \frac{1}{3}(-d\tau^2 + \cosh^2(\tau)dx^2 + d\Omega_2^2) \tag{9.1}$$

Here  $\tau$  is the intrinsic time and  $l$  is the characteristic length.

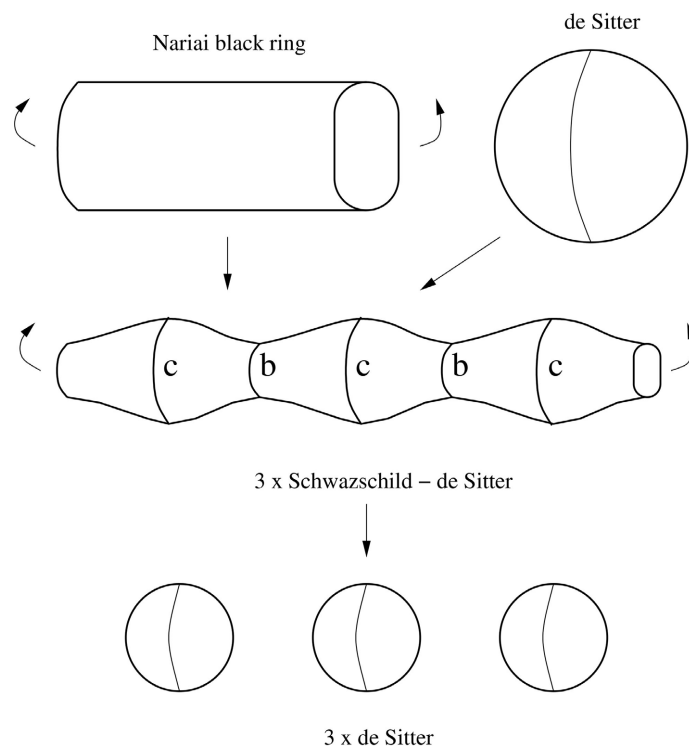
Nariai space is the product  $S^1 \times S^2$ . The  $S^2$  factor remains constant, while the  $S^1$  factor expands exponentially into the future and the past, forming a (1 + 1)-dimensional de Sitter spacetime.

Radii of cosmological and event horizons coincide, but they are always spatially separated. The metric for an observer, sandwiched between horizons, reduces to  $dS_2 \times S^2$  metric of the following form:

$$ds^2 = -(R^2 - z^2)d\tau^2 + \frac{dz^2}{R^2 - z^2} + d\Omega_2^2. \tag{9.2}$$

Two horizons are located at  $z = \pm R$  in these coordinates.

The Nariai spacetime is classically unstable. Consider the oscillation of 2-sphere as a function of the angular variable on  $S^1$ . The first-mode instability arises when the area of  $S^2$  is not constant, but given by  $l^2/3(1 + \varepsilon \cos(x))$  (**Figure 3**).



**Figure 3.** Fragmentation of black rings and spherical horizons.

It reverts the geometry into a nearly maximal de Sitter-Schwarzschild metric [19]. In this case, a black hole connects the opposite sides of a single asymptotic de Sitter region.

The  $n$ th mode instability corresponds to the area oscillation of type  $l^2/3(1 + \varepsilon \cos(nx))$ . It leads to the formation of  $n$  black hole interiors, connected by  $n$  asymptotically de Sitter regions.

Finally, since black holes evaporate, a Nariai black ring eventually splits into  $n$  disconnected components with  $S^3$  topology.

## 10. Spherical Horizons as Schwarzschild-De Sitter Black Holes

Suppose now that the intrinsic metric of the spherical horizon (in 5-dimensional Hyperverses) is itself a Schwarzschild-de Sitter black hole with metric:

$$ds^2 = -f(r)d\tau^2 + f(r)^{-1}dr^2 + r^2d\Omega_2^2 \quad (10.1)$$

where

$$f(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2. \quad (10.2)$$

Here  $\mu$  is the mass parameter and  $\Lambda$  is the on-shell cosmological constant. For  $0 < \mu < \frac{1}{3\sqrt{\Lambda}}$ ,  $f(r)$  has two positive roots  $r_c$  and  $r_b$ , corresponding to the cosmological and the black hole horizons respectively. For  $\mu = 0$  there will be no black hole horizon and one obtains the de Sitter solution.

Let's define the parameter  $\varepsilon$  by the formula:

$$\varepsilon = \sqrt{\frac{1 - 9\mu^2}{3}}. \quad (10.3)$$

The degenerate case, when the size of black hole horizon approaches the size of cosmological horizon, corresponds to  $\varepsilon \rightarrow 0$ .

It is convenient to introduce new time and radial coordinates,  $\psi$  and  $\chi$ , by the following formulas:

$$\tau = \frac{\psi}{\varepsilon\sqrt{\Lambda}}, \quad r = \frac{1}{\sqrt{\Lambda}} \left( 1 - \varepsilon \cos \chi - \frac{\varepsilon^2}{6} \right). \quad (10.4)$$

The black hole horizon lies at  $\chi = 0$  and the cosmological horizon at  $\chi = \pi$ . The new metric, in the first order approximation, is given by:

$$ds^2 = -\frac{1}{\Lambda} \left( 1 + \frac{2}{3} \varepsilon \cos \chi \right) \sin^2 \chi d\psi^2 \quad (10.5)$$

$$+ \frac{1}{\Lambda} \left( 1 - \frac{2}{3} \varepsilon \cos \chi \right) \chi^2 + \frac{1}{\Lambda} (1 - 2\varepsilon \cos \chi) d\Omega_2^2. \quad (10.6)$$

It describes Schwarzschild-de Sitter solutions of nearly maximal black hole size ([17], sect.~2).

In these coordinates, the topology of spacelike sections is  $S^1 \times S^2$ . So we can speak about handle creation in the originally spherical solution. The degenerate

case, when  $\varepsilon = 0$ , corresponds to the Nariai black ring.

## 11. Nucleation and Proliferation of Black Saturns

We have seen that both spherical horizons and black rings in the 5-dimensional Hyperverses can fragmentate into several spherical components. In addition, spherical components can evolve into black rings by the process of handle creation.

Thus, a (generalized) Black Saturn with several black rings can evolve into multiple new Saturns. The process could be infinitely reiterated. This is related to the proliferation of de Sitter spaces, originally discovered by Bousso and Hawking.

This is also related to multiversal nurseries of de Sitter bubbles and Gogberashvili multiverses, discussed in Sections 3 and 4.

In our current vision, the landscape of the 5-dimensional Hyperverses (or *Astral Cosmos*) represents a collection of multiverses, multiversal nurseries, black holes, black lenses, black rings and Black Saturns.

Our observable universe belongs to the Local Multiverse with multiple “parallel” shells-universes with synchronized time coordinates. If we restrict the Local Multiverse to 4-dimensional strata, then it is embedded into the Astral Cosmos described in this paper.

Higher-dimensional ( $D > 4$ ) strata of the Local Multiverse and higher-dimensional ( $D > 5$ ) strata of the Hyperverses require additional investigations.

## 12. Black Holes as Atoms and Hawking’s Apparent Horizons

This section was added to the article following reviewers’ suggestions. It compares our calculations with outstanding ideas and results of Bekenstein [20], Hawking-Vaz [21] [22], Corda and others ([23] and references therein).

Bekenstein [20] was the first, who developed the idea that black holes play the same role in gravitation as atoms in the quantum mechanics. It would imply that black hole masses have a discrete spectrum.

In 2014 Hawking [21] suggested that event horizons could not be the final result of the gravitational collapse. Instead of it, the matter should condense on so-called “apparent horizons”. The precise mechanism of such condensation was found by Vaz via entire solutions of Wheeler-DeWitt equation [22].

Recently, by solving black hole Schrödinger and Klein-Gordon equations, Corda showed that black holes are “self-interacting, highly excited, spherically symmetric, massive quantum shells generated by matter condensing on the apparent horizon” [23].

Clearly, nested Gogberashvili shells (sect. 4 and [3]) should play the role of such “apparent horizons” in our 5-dimensional case.

Corda’s solution gives “gravitational hydrogen atoms” with the condensation of matter on one layer. Our 5-dimensional results give also multi-layer solutions in Schwarzschild-de Sitter setting.

As a final remark, we would like to point out that the “harmony of transc cosmic spheres”, discovered in ([3], sect.~4), is related to eigenvalues of the Bekenstein’s

discrete mass spectrum mentioned above.

In view of all these results, the following “fantastic conjecture” starts to appear more realistic.

**Conjecture 12.1.** *The collection of 4-dimensional strata of the Local Multiverse, including the observable universe, originated in the Hyperverses as the multi-layer apparent horizon of a 5-dimensional black hole (or Black Saturn).*

### 13. Conclusions

The  $\Lambda$ CDM model works very well as long as there are no singularities involved. The avoidance of a singularity at time zero has been described in various modifications of the  $\Lambda$ CDM model and its fine-tuning is now moving into the range of higher than  $(3 + 1)$  spacetime dimensions. According to Barrabès and Hogan [6], spacetime models with dimensions 5 and greater are suitable targets for unifying gravity with other forces. More dimensions, on the other hand, require a multi-universe- or hyperverses-based model. The possibility of verification of this approach, however, has vigorously been disputed.

We initiated studies on various 5-dimensional hyperverses models on the basis of black holes with the aim to 1) describe these models and 2) to reduce the number of dimensions to  $3 + 1$  in order to be able to calculate the on-shell cosmological constant  $\Lambda$ . The objective was to compare the  $\Lambda$  obtained in these models with the one reported previously [4], which had been obtained in a  $(3 + 1)$  semiclassical multiverse model based on spacetime warping by external universes.

In the current investigation, this approach was expanded to contain universes of different masses, and, additionally to a model in which the universes were replaced by supermassive black holes. The results obtained for  $\Lambda$  using external universes are in agreement with those reported in the literature. Black holes, however, can be excluded, as an explanation for dark energy, since their masses, even in the case of supermassive black holes, are still too small to account for the necessary gravitational force.

We found that the cosmological constant  $\Lambda$  depends on the number, mass and position of the external universes. It must therefore be time dependent, since the positions of universes change over time. By fine-tuning parameters, number, mass, and positions of the external universes and by comparing values of  $\Lambda$  over a wider range of time, the multiverse model can be verified.

If similar results for  $\Lambda$  are obtained in the hyperverses models, described in this paper and in the semiclassical multiverse model, the door to verification of the multiverse/hyperverses approach will be open even wider. To this end, we studied various black hole models in five spacetime dimensions, starting with Schwarzschild-Tangherlini black holes. We continued with de Sitter universes inside the event horizon of Schwarzschild black holes (multiversal nurseries of de Sitter bubbles), which solve the problem of singularity at  $r = 0$  by phase transition. In addition, we considered Gogberashvili multiverses in Schwarzschild-de Sitter setting. This model describes a collection of inflating  $(3 + 1)$ -dimensional FLRW universes

with synchronized intrinsic times inside a  $(4 + 1)$ -dimensional Schwarzschild-de Sitter spacetime.

The cosmological constant was calculated in the Gogberashvili model. However, the formula (6.4)-(6.5), obtained for  $\Lambda$  could not be finally evaluated due to unknown off-shell components  $\Lambda_-$  and  $\Lambda_+$  of the surrounding Hyperverses.

We also considered horizon topologies in 5-dimensional black hole models, which are, basically, Seifert fibrations or generalized Black Saturns, according to refined topology theorem 8.1. Using Bousso-Hawking ideas, we have shown that Nariai black rings, are instable and finally split into components with  $S^3$  topology. Further transformations of black holes, which we investigated, include a complete landscape of multiverses, multiversal nurseries, black lenses, black rings and Black Saturns. The birth of new universes happens after condensations, nucleations and other transformations. It opens a huge open field for future researches.

## Acknowledgements

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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