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Cosmological Redshift Caused by Head-On Collisions with CMB Photons, Not by Expansion of Space

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Abstract

The Big Bang model was first proposed in 1931 by Georges Lemaitre. Lemaitre and Hubble discovered a linear correlation between distances to galaxies and their redshifts. The correlation between redshifts and distances arises in all expanding models of universe as the cosmological redshift is commonly attributed to stretching of wavelengths of photons propagating through the expanding space. Fritz Zwicky suggested that the cosmological redshift could be caused by the interaction of propagating light photons with certain inherent features of the cosmos to lose a fraction of their energy. However, Zwicky did not provide any physical mechanism to support his tired light hypothesis. In this paper, we have developed the mechanism of producing cosmological redshift through head-on collision between light and CMB photons. The process of repeated energy loss of visual photons through n head-on collisions with CMB photons, constitutes a primary mechanism for producing the Cosmological redshift z. While this process results in steady reduction in the energy of visual photons, it also results in continuous increase in the number of photons in the CMB. After a head-on collision with a CMB photon, the incoming light photon, with reduced energy, keeps moving on its original path without any deflection or scattering in any way. After propagation through very large distances in the intergalactic space, all light photons will tend to lose bulk of their energy and fall into the invisible region of the spectrum. Thus, this mechanism of producing cosmological redshift through gradual energy depletion, also explains the Olbers's paradox.

Keywords

Redshift, CMBR, Big Bang, Cosmology, Elastic Collisions, Tired Light

1. Introduction

Redshift in a photon of electromagnetic radiation is the increase in its wavelength during propagation from the source of emission to the observer. When this increase in wavelength is associated with a receding or separation velocity between the source and observer, the redshift is called Doppler redshift. When this increase in wavelength is associated with propagation through large gravitational potential difference, the redshift is called gravitational redshift. But when the increase in wavelength is associated with propagation through large distances in the intergalactic space, the redshift is called cosmological redshift. The mechanism of development of cosmological redshift is the focus of our study in this paper.

1.1. Big Bang Model and Expansion of Space

Under the currently accepted Standard Cosmological Model [1], the universe began with a Big Bang, went through a period of high inflation and has been expanding ever since. The Big Bang model was first proposed in 1931 by Georges Lemaitre and the Hubble-Lemaitre law states that the farthest galaxies are moving away fastest from Earth [2]. Hubble's law is considered a major observational support for the expansion of universe, as well as the Big Bang model. The famous equation describing the Hubble-Lemaitre law, where H_0 is constant of proportionality between the distance D to a galaxy and its speed of separation v_r, is given by,

$$v_r = H_0 D \tag{1}$$

The Hubble constant H_0 is given in (km/s)/Mpc, thus giving the speed in km/s of a galaxy 1 megaparsec away, and its value is about 70 (km/s)/Mpc.

In 1964, the Cosmic Microwave Background (CMB) radiation was discovered, which convinced many cosmologists to accept the Big Bang model in preference to the competing steady-state model of cosmic evolution. Among various non-standard cosmologies which attempted to explain Hubble's observations without invoking the expansion of space, Fritz Zwicky's tired light hypothesis was the most significant. Zwicky proposed that the cosmological redshift is caused by the interaction of the propagating light with certain inherent features of the cosmos [3]. Zwicky objected to the Doppler effect interpretation of cosmological redshift, but his tired light hypothesis did not provide any physical mechanism to support it.

1.2. Doppler and Cosmological Redshifts

The redshift of a photon is denoted by the letter z, given by the ratio of the fractional increase in its wavelength λ to its emission wavelength. Alternatively, the term $1 + z$ is also defined as the ratio of observed wavelength λ_{obsv} to the emitted wavelength λ_{emit}. For large z, a gamma ray may get redshifted to an X-ray, or initially visible light may get redshifted to a radio wave.

$$1 + z = \frac{\lambda_{obsv}}{\lambda_{emit}} \tag{2}$$

To calculate the redshift, we need to know the wavelength of the emitted light in the rest frame of the source. Redshifts cannot be calculated by looking at unidentified features of a spectrum.

When the measured redshift is correlated with the recessional or separation velocity of an astronomical source, it is called Doppler redshift. For recessional velocity v_r much smaller than speed of light c, the Doppler redshift z can be given by,

$$z = \frac{v_r}{c} \tag{3}$$

On the other hand, when the measured redshift is attributed to stretching of the wavelengths of photons propagating through an expanding space, the redshift is called cosmological redshift. Cosmological redshift is often interpreted as a Doppler shift due to the recessional velocity of distant galaxies. In that case, combining Equations (1) and (3), we get a relation between z and the Hubble constant H_0 as,

$$z = \frac{H_0 D}{c} \tag{4}$$

Correlation of redshift parameter z with Hubble constant H_0 as given by Equation (4) is intimately linked to the founding assumption of the Big Bang model regarding continuous expansion of the universe.

1.3. CMB and Extragalactic Background Light

The cosmic microwave background (CMB) is microwave radiation that fills all intergalactic and interstellar space in the universe. It is generally believed to be an important remnant of the Big Bang. The CMB is not very smooth and uniform. A faint anisotropy in this microwave background can be mapped by sensitive detectors. Detailed measurements of the CMB are important in cosmology, since any non-standard model of cosmology must explain this radiation. The CMB has a thermal black body spectrum at a temperature of 2.7 K. Most of the CMB photons have a frequency spectrum with average photon energy of 2.4×10^{-4} eV corresponding to an average frequency of about 58 GHz and average wavelength of about 5 mm. Most of the radiation energy in the universe is in the cosmic microwave background. Energy density of the CMB is about 0.260 eV/cm^3.

Extragalactic background light (EBL) is part of the diffuse extragalactic background radiation (DEBRA), which covers the entire electromagnetic spectrum [4] except the microwave. EBL is the accumulated radiation from star formation processes and contribution from active galactic nuclei (AGNs). After CMB, the EBL produces second-most energetic diffuse background. The understanding of EBL is fundamental for extragalactic very-high-energy (VHE) astronomy. VHE photons coming from cosmological distances are attenuated by pair production with EBL photons [5]. This interaction is dependent on the spectral energy distribution of the EBL.

2. Energy Attenuation of Cosmic Rays

Cosmic rays (CR) are charged particles like protons or atomic nuclei that move through intergalactic and interstellar space at nearly the speed of light. About 99% of the primary cosmic rays are atomic nuclei, and about 1% are electrons. Of the nuclei, about 90% are simple protons; 9% are alpha particles; and 1% are the nuclei of heavier elements [6]. The energy of most primary CR incident at the top of the atmosphere, lie in the range of 1 - 10 GeV. The total energy range is however much greater, covering 10^8 eV - 10^{20} eV range.

2.1. Origin of Cosmic Rays

Active galactic nuclei (AGN) with powerful jets have long been considered as possible sources of cosmic rays, including ultra-high energy cosmic rays (UHECRs). As per Ionic Gravitation model [7], the primary constituents of AGN are the highly Ionized Solid Iron Stellar Bodies (ISISB) which are the remnant cores of massive stars. Mass of such ISISB is estimated at about one solar mass with radius of about 50,000 km. Total positive charge accumulated in outer regions of these ionized solid bodies is estimated at about 10^{37} Coulombs. Assuming a small fraction of the circulating electrons get blown away in terminating explosions, the ionized solid core with a shell of circulating electrons, will be left with an excess of positive charge of the order of 10^{33} Coulombs. These ionized, solid core stellar remnants are known as Compact Stellar Objects.

Due to Newtonian and Ionic gravitation, inter-stellar matter in the surrounding space will get attracted to an ISISB and crash on to its ionized solid surface through the accretion process. Bombardment of the ionized solid surface of an ISISB by inter-stellar matter in the accretion process will produce all cosmic rays and gamma ray bursts. In AGN, under the influence of strong magnetic field associated with circulating electrons and the action of Lorentz forces, the accreting ionized matter will experience extreme compression along the ionized surface and get pushed towards the polar regions to produce strong Jets of ionized matter. These jets of ionized matter are also major sources of primary CR. The Coulomb force of the excess positive charge of an ISISB is the sole acceleration mechanism that imparts extremely high energies, even beyond 10^{20} eV to the positively charged primary cosmic rays.

2.2. CR Energy Decay during Propagation through EBL and CMB

Most of the cosmic rays that hit the Earth get blocked by Earth's atmosphere and magnetic field. Sometimes a few high energy CR will strike matter particles in the atmosphere and create a shower of secondary particles that travel to the ground. Attenuation of cosmic rays in earth's atmosphere have been extensively recorded and studied. However, attenuation of primary cosmic rays during their propagation through the intergalactic and interstellar space is currently a field of active research [6].

During the propagation through the intergalactic and interstellar space, a small fraction of CR energy is lost to various scattering and ionization processes in the prevailing dust, plasma, gases, and local magnetic field environment. Major fraction of the total energy of primary CR is lost during direct interaction between high energy charged particles of cosmic rays and the CMB and EBL photons. Another process that causes a small fraction of total energy loss from very high energy CR is the photo disintegration of CR particles triggered by the CMB and EBL photons.

The Greisen-Zatsepin-Kuzmin limit or GZK cutoff is a theoretical upper limit on the energy of CR protons traveling from other galaxies through the intergalactic medium to our galaxy [4]. The limit of 5×10^{19} eV (50 EeV) is set by the attenuation interactions of the protons with the EBL and CMB radiation over long distances (≈50 Mpc). As per the GZK limit, CR protons of energy greater than this threshold are predicted to get photo disintegrated under the inelastic collisions with CMB photons to produce nucleons and pions. Yet another mechanism of energy depletion from CR protons is the Bethe-Heitler pair production process. It is a key process in astro-particle physics wherein a particle and antiparticle pair like a positron and an electron are produced by a high energy photon on collision with a nucleus or another photon [8]. Extragalactic cosmic-ray protons at EeV energies are believed to undergo frequent Bethe-Heitler energy loss interactions with CMB photons, losing their energy through this process on Gyr timescales. In the next section we will analyze the energy loss of CR charged particles in their elastic collisions with EBL and CMB photons over long distances of CR propagation through intergalactic space.

3. Two Particle Head-On Elastic Collisions

In this section we compute the energy transfer during three simple cases of head-on collisions between 1) Large and small mass particles, 2) High energy charge particle and low energy photon, and 3) High energy photon and a low energy charge particle.

3.1. Head-On Elastic Collision between a Large and a Small Mass Particles

Let us consider two particles A and B of mass m_A and m_B, approaching each other at initial velocities U_a and $-U_b$ respectively, for a head-on collision. For a situation where mass m_A of particle A is much bigger than mass m_B of particle B, their final velocities V_a and V_b after an elastic collision (**Figure 1**) will be given by the momentum and energy conservation relations as,

$$m_A \cdot U_a - m_B \cdot U_b = m_A \cdot V_a + m_B \cdot V_b \tag{5}$$

$$\frac{1}{2} m_A \cdot U_a^2 + \frac{1}{2} m_B \cdot U_b^2 = \frac{1}{2} m_A \cdot V_a^2 + \frac{1}{2} m_B \cdot V_b^2 \tag{6}$$

Solving Equations (5) and (6) for V_a and V_b we get,

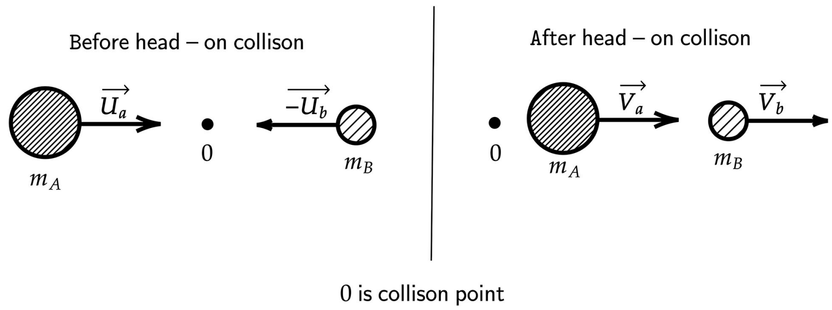

Figure 1. Head-on elastic collision between two unequal mass particles.

$$V_a = \frac{(m_A - m_B)U_a - 2m_B U_b}{m_A + m_B} \tag{7}$$

$$V_b = \frac{2m_A U_a + (m_A - m_B)U_b}{m_A + m_B} \tag{8}$$

Total energy transferred from A to B during the elastic collision is given by,

$$E_{tr} = \frac{1}{2}m_A\left(U_a^2 - V_a^2\right) = \frac{1}{2}m_A\left[\frac{4m_B\left(m_A U_a - m_B U_b\right)\left(U_a + U_b\right)}{\left(m_A + m_B\right)^2}\right] \tag{9}$$

These computations show that the magnitude of rebound or backscatter velocity V_b of smaller particle B is increased by nearly twice the initial velocity of particle A, for $m_A \gg m_B$. Further the kinetic energy of this backscattered smaller particle B is increased to nearly eight times its initial energy (for $U_a = U_b$), because of head-on collision with a much heavier particle A.

3.2. Head-On Elastic Collision between a High Energy Charged Particle and a Low Energy Photon

Commencing from the mass energy equivalence relation, $dm = dE/c^2$, the Equation for dynamic mass m of a particle with rest mass m_0 can be easily derived without invoking Special Theory of Relativity [9].

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{10}$$

With momentum p defined as $p = mv$ and total energy defined as $E = mc^2$, we get the following standard relations between E, p, m and kinetic energy T (**Appendix A**) as,

$$E^2 = m_0^2c^4 + p^2c^2 \quad \text{and} \quad E = m_0c^2 + T \tag{11}$$

$$\text{And} \quad pc = \sqrt{T^2 + 2Tm_0c^2} \tag{12}$$

Let us consider a case where a high energy particle A with initial energy E_{a1}, rest mass $m_0 = m_A$ and momentum p_1, makes head-on collision with a photon B of energy E_{b1}. It may be noted that initial total energy E_{a1} includes the rest mass energy m_0c^2 and the kinetic energy T_1 of the particle as per Equation (11). Kinetic energy of the particle in motion is physically stored in the motion induced electromagnetic field accompanying the particle. The interaction during head-on collision of a charged particle and a photon will be governed by the instantaneous superposed electromagnetic fields of the colliding particles. After collision, if the particle A continues to move forward with reduced energy E_{a2} and photon B rebounds back with energy E_{b2} (**Figure 2**) then the energy and momentum conservation equations will be written as,

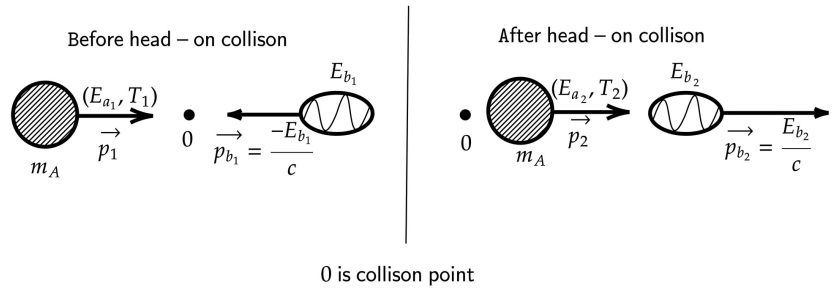

Figure 2. Head-on collision between a high energy charge particle and a low energy photon.

For energy conservation,

$$E_{a1} + E_{b1} = E_{a2} + E_{b2} \tag{13}$$

$$\text{Or } T_1 + m_0c^2 + E_{b1} = T_2 + m_0c^2 + E_{b2}$$

$$\text{Or } T_1 + E_{b1} = T_2 + E_{b2} \tag{14}$$

For momentum conservation,

$$p_1 - E_{b1}/c = p_2 + E_{b2}/c$$

$$\text{Or } p_1c - E_{b1} = p_2c + E_{b2} \tag{15}$$

On simplification, Equations (14) and (15) yield,

$$E_{b2} = \frac{(T_1 + p_1c) - (T_2 + p_2c)}{2} \tag{16}$$

And

$$(T_2 - p_2c) = (T_1 - p_1c) + 2E_{b1} \tag{17}$$

Substituting $p_2c = \sqrt{T_2^2 + 2T_2m_0c^2}$ and $p_1c = \sqrt{T_1^2 + 2T_1m_0c^2}$ in Equation (17) we get,

$$\left(T_2 - \sqrt{T_2^2 + 2T_2m_0c^2}\right) = \left(T_1 - \sqrt{T_1^2 + 2T_1m_0c^2}\right) + 2E_{b1} = x \tag{18}$$

Where x is computed from initial parameters T_1, E_{b1} and the rest mass m_0 of particle A.

Rearranging the terms in Equation (18) and squaring, we get,

$$T_2^2 + 2T_2m_0c^2 = T_2^2 + x^2 - 2xT_2$$

$$\text{Or } T_2 = \frac{x^2}{2\left(x + m_0c^2\right)} \tag{19}$$

And from Equation (16)

$$E_{b2} = \frac{\left(T_1 + \sqrt{T_1^2 + 2T_1 m_0 c^2}\right) - \left(T_2 + \sqrt{T_2^2 + 2T_2 m_0 c^2}\right)}{2} \tag{20}$$

From Equations (18), (19) and (20), for certain constant values of photon energy E_{b1}, we have plotted a curve of E_{b2}, the energy of backscattered photon [10] against various values of initial energy E_{a1} of electron as the high energy particle A, as shown in **Figure 3**.

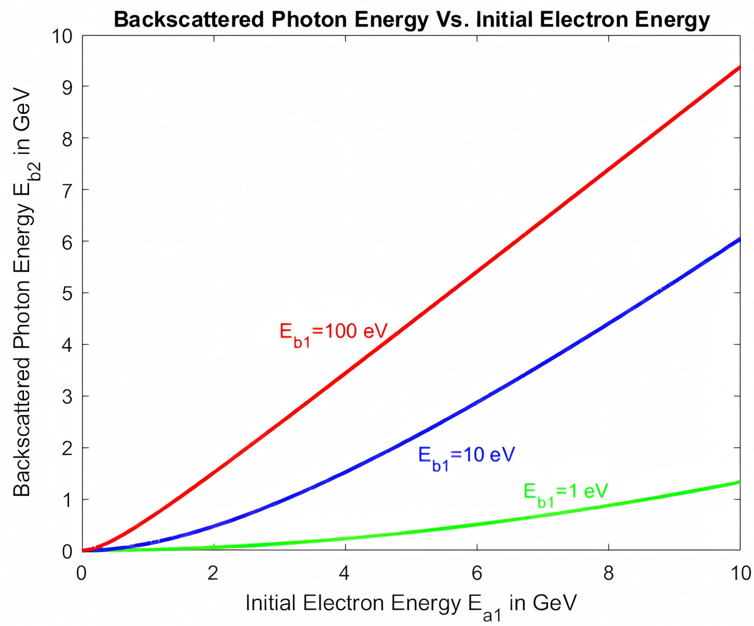

Figure 3. Backscattered photon energy after head-on collision with a high energy electron for three different initial photon energies.

Similarly, we have plotted a curve of E_{a2}, the reduced energy of electron as particle A, after the collision, as shown in **Figure 4**. Above equations are valid for computing backscattered photon energy from a head-on collision with a high energy proton by substituting proton rest mass for m_0. **Figure 5** shows the variation in backscattered photon energy E_{b2} against various values of initial energy E_{a1} of proton as the particle A.

3.3. Head-On Elastic Collision between a High Energy Photon and a Low Energy Charge Particle

Scattering phenomenon of a high energy photon from a nearly static electron, wherein the photon transfers a part of its energy to the recoiling electron and deflects from its original path, is commonly known as Compton scattering. Here we study a special case of head-on collision of a high energy photon with a low energy electron or another charge particle, which is quite uncommon in terrestrial environment. Let us consider a case where a high energy Photon A with initial energy E_{a1} makes head-on collision with a charge particle B of rest mass m_0, kinetic energy

T_1 and momentum $-p_1$. The interaction during head-on collision of a photon and the charge particle will be governed by the instantaneous superposed electromagnetic fields of the colliding particles. After the collision, if the photon A continues to move forward with reduced energy E_{a2} and particle B rebounds back with kinetic energy T_2 and momentum p_2 then the energy and momentum conservation equations will be written as,

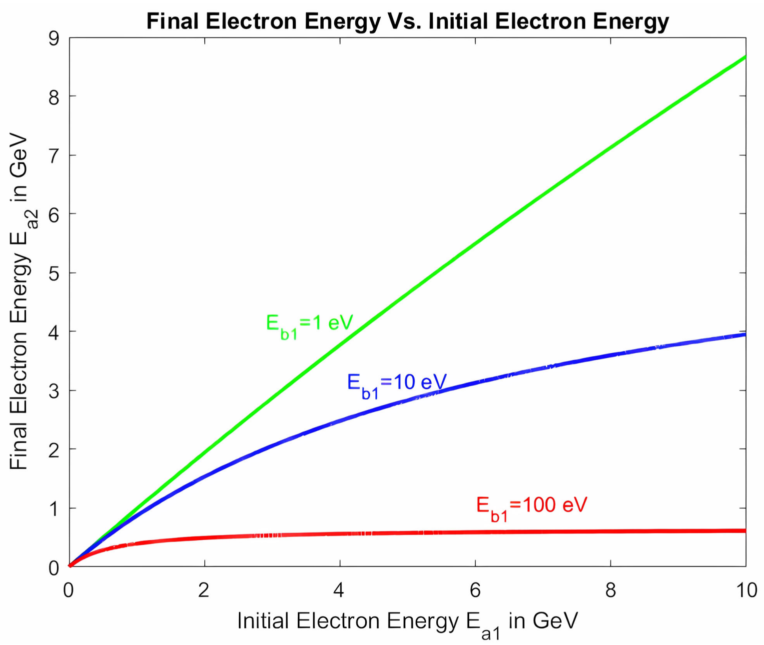

Figure 4. Residual electron energy after the head-on collision of a photon with a high energy electron for three different initial photon energies.

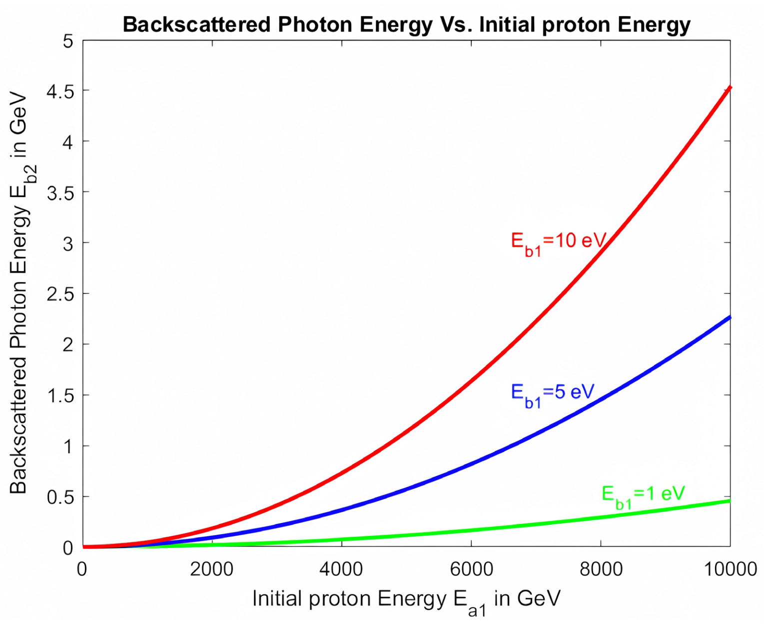

Figure 5. Backscattered photon energy after head-on collision with a high energy proton for three different initial photon energies.

For energy conservation,

$$E_{a1} + T_1 = E_{a2} + T_2 \tag{21}$$

$$\text{Or} \quad E_{a1} + \frac{1}{2} m_0 v_1^2 = E_{a2} + \frac{1}{2} m_0 v_2^2$$

$$\text{Or} \quad E_{a1} - E_{a2} = \frac{1}{2} m_0 \left(v_2^2 - v_1^2 \right) \tag{22}$$

For momentum conservation,

$$E_{a1}/c - p_1 = E_{a2}/c + p_2$$

$$\text{Or} \quad E_{a1} - E_{a2} = m_0 c \left(v_2 + v_1 \right) \tag{23}$$

On simplification, Equations (22) and (23) yield an absurd result ($v_2 - v_1 = 2c$) which shows that as a result of the head-on collision, both the photon and the rebound particle cannot continue to move in the same direction. That is because the momentum lost per unit energy transferred by a photon is much less than the momentum required per unit energy gained by a low energy charge particle. As such, during a head-on collision of a high energy photon with a low energy charge particle, the photon must transfer maximum momentum and minimum energy to the rebounding particle.

Therefore, let us consider a revised situation wherein a high energy Photon A with initial energy E_{a1} makes head-on elastic collision with a low energy charge particle B of rest mass $m_0 = m_B$, kinetic energy T_1 and momentum $-p_1$. After the collision, photon A transfers all its forward momentum to the particle B and splits itself into two equal parts C and D, with reduced energy E_{a2} left with each part. The two photons C and D get ejected in opposite directions in the transverse plane with equal and opposite momentum E_{a2}/c and $-E_{a2}/c$. The charge particle B rebounds back with kinetic energy T_2 and momentum p_2 (**Figure 6**). The energy and momentum conservation equations will now be written as,

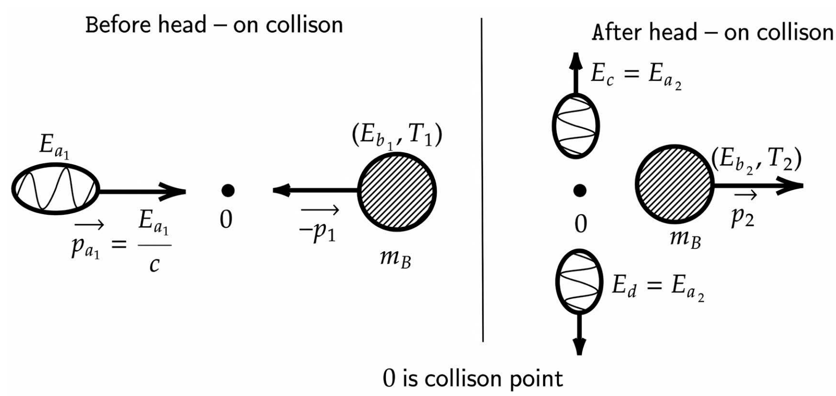

Figure 6. Head-on collision between a high energy photon and a low energy charge particle.

For energy conservation,

$$E_{a1} + T_1 = 2E_{a2} + T_2 \tag{24}$$

For momentum conservation,

$$E_{a1}/c - p_1 = p_2$$

$$\text{Or,}\quad E_{a1} - p_1 c = p_2 c$$

$$E_{a1} - \sqrt{T_1^2 + 2T_1 m_0 c^2} = \sqrt{T_2^2 + 2T_2 m_0 c^2} = y \tag{25}$$

Therefore,

$$T_2 = -m_0 c^2 + \sqrt{m_0^2 c^4 + y^2} \tag{26}$$

$$E_{a2} = \frac{E_{a1} - (T_2 - T_1)}{2} \tag{27}$$

From Equations (26) and (27) we can compute the values of T_2, the rebound kinetic energy of the particle B, and E_{a2} the total energy of each of the new pair of photons, C and D ejected in the transverse plane. For certain values of E_{a2}, photons C and D could be replaced by a newly produced particle antiparticle pair ejected in opposite directions in the transverse plane. When a positron and an electron are produced by a high energy photon on collision with a nucleus, it is known as Bethe-Heitler pair production, which is a key process in Astro particle physics. For various values of initial energy E_{a1} of photon A, we can plot the corresponding rebound kinetic energy T_2 of the low energy charge particle B for specific values of initial kinetic energy T_1. One such plot is shown in **Figure 7**. Similarly, energy E_{a2} of each transversal photon C or D can be plotted against various values of initial energy E_{a1} of photon A, for specific values of initial kinetic energy T_1, as shown in **Figure 8**. It can be clearly seen from **Figure 7** that rebound kinetic energy T_2 of the particle B keeps steadily increasing with initial energy E_{a1} of photon A, though at slightly reduced level.

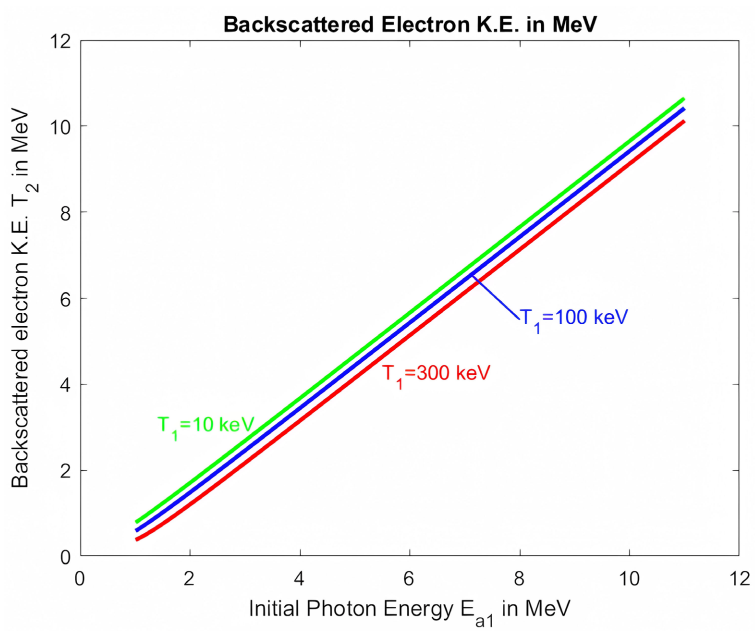

Figure 7. Backscattered electron kinetic energy after head-on collision with a high energy photon for different initial electron energies T_1.

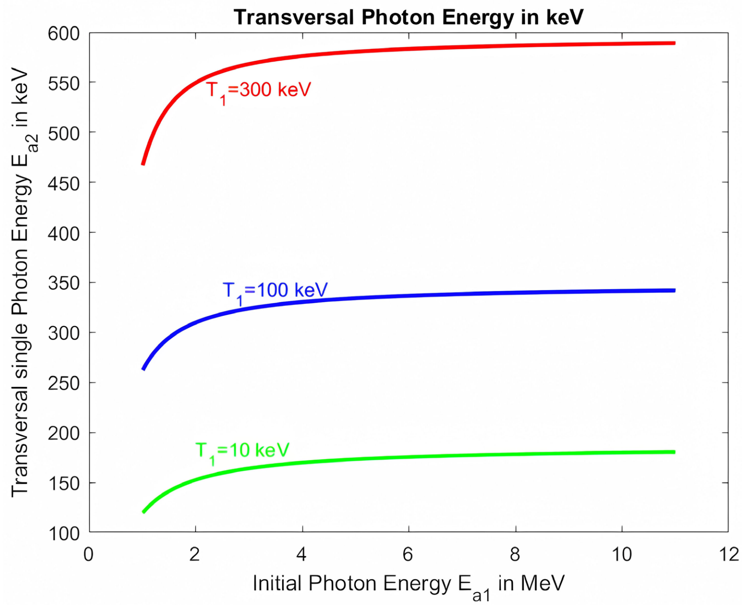

Figure 8. Single transversal photon energy after head-on collision of a high energy photon with an electron at different initial kinetic energies T_1.

Reduction in rebound kinetic energy of particle B is precisely by the amount of energy $2E_{a2}$ passed on to the two transversal photons C and D because of this head-on collision. It is clear from **Figure 8** that transversal photon energy mainly depends on the level of initial kinetic energy T_1 of the target electron and does not change with increasing initial energy E_{a1} of photon A. This is an important result which is also applicable for the pair production process by head-on collision of high energy gamma photons with low energy charge particles. In the case of Photon A making head-on elastic collision with a low energy charge particle B, when the initial kinetic energy T_1 of target particle is reduced to zero, we get back to the standard case of Compton scattering.

The foregoing analysis shows that the primary mechanism of energy attenuation from high energy CR protons and electrons is their head-on elastic collisions with EBL and CMB photons. Even though such head-on collisions are quite uncommon in terrestrial environment, this is a significant phenomenon in the inter-galactic and inter-stellar space. In this primary attenuation process EBL and CMB photons get backscattered with high energies up to Giga or even Tera electron volts (TeV). Apart from initial high energies of CR particles, the magnitude of backscattered photon energy also depends on the initial energies of EBL and CMB photons. The high energy electrons from primary CR can lose most of their energy to some higher energy EBL photons in this backscattering process. On the other hand, energy lost by high energy CR protons, under similar EBL environment is much less. However, the photons backscattered by very high energy CR protons can acquire up to TeV energies in this process. The famous Bethe-Heitler pair production process is not supported by this photon backscattering mechanism of CR energy attenuation.

Since the primary mechanism of CR energy attenuation leads to production of high energy gamma photons, there is a secondary mechanism of energy transfer from gamma photons to low energy electrons and ions through backscattering process. In this mechanism, high energy gamma photons make a head-on collision with low energy electrons or ions available in the intergalactic and interstellar space. As a result of this collision, most of the photon energy gets transferred to the backscattered electron or ion and the balance energy gets emitted as a pair of photons or electron-positron pair in the transverse plane. The Bethe-Heitler pair production process is supported by this mechanism of CR energy attenuation. However, the energy of the transversal photon pair does depend on the initial energy of the target electron or ion. The effectiveness of this secondary mechanism of CR energy attenuation gets limited by the availability of sufficient electrons and ions in the intergalactic and interstellar space.

4. Head-On Collision of a Higher Energy Photon with a Lower Energy Photon

There is yet another tertiary mechanism of energy attenuation from higher energy photons through their head-on collision with lower energy EBL and CMB photons by emitting a pair of lower energy photons in the transverse pane. We shall explore this mechanism in detail in this section and show its role in explaining the cause of cosmological redshift.

4.1. Electromagnetic Interaction during Head-On Collision of Two Photons

Photon is a short wavelet of propagating electromagnetic (EM) field. The EM photon consists of oscillating electric (E) and magnetic (B) fields, which are perpendicular to each other and also perpendicular to the direction of propagation as sketched in **Figure 9**. The E and B fields are governed by Maxwell's EM field equations in free space and are interrelated through magnetic vector potential A as

$$E = -\partial A/\partial t \quad \text{and} \quad B = \nabla \times A \quad \text{with} \quad \nabla \cdot A = 0$$

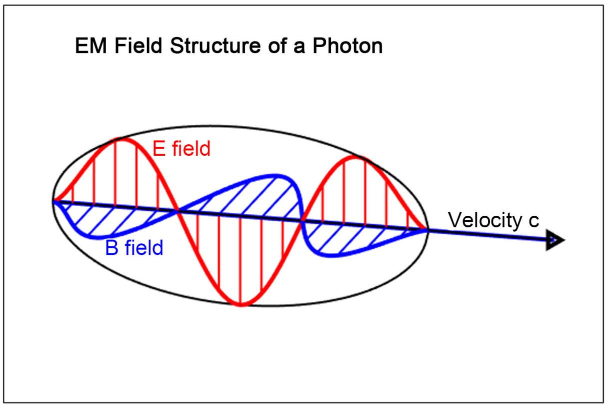

Figure 9. A photon is a short wavelet of oscillating electric (E) and magnetic (B) fields, which are perpendicular to each other as well as to the direction of propagation.

The total energy of a photon is proportional to frequency f of oscillations of E and B fields as per the Planck equation. However, the energy density of the EM wavelet is proportional to square of the amplitudes of E and B fields. If two identical photons are spatially superposed while their temporal oscillations are exactly in phase, then amplitudes of the E and B fields will get doubled and energy density will increase to four times the original one. Hence due to conservation of total energy, two identical photons can never get superposed in free space. As such, any tendency for in-phase superposition of two identical photons will be strongly resisted by the principle of total energy conservation. Such resistance is physically affected through EM interaction between the E and B field vectors of the two photons and leads to the scattering phenomenon. When two photons make a perfect head-on collision, with their field vectors spatially aligned and temporally in-phase at the point of their collision, the EM interaction between their E and B field vectors will govern the outcome of such a collision, in conjunction with total momentum and energy conservation.

While the total energy of a photon of frequency f is given by $h.f$, the linear momentum is given by $h.f/c$. The momentum of a particle in motion is intimately linked to its dynamic or kinetic energy, as against the rest mass energy. Since for a photon there is no rest mass, whole energy content of a photon is intimately linked to its momentum. If any obstruction on its propagation path could change the momentum of a photon, it would automatically lead to a corresponding change in its bound energy structure. During a head-on collision between two photons, if the momentum vectors are perfectly aligned in opposing directions and the EM field vectors are also spatially aligned at the point of collision, then their oppositely propagating E and B fields could electromagnetically interact to induce a change in the momentum of both photons. Even though such alignment between opposite momentum vectors and EM fields of two photons is quite feasible in principle, in practice it is a rare phenomenon with extremely low probability. As and when such head-on collisions between two photons occur, their outcome will be governed by the principle of conservation of total momentum and energy. Such head-on collisions between two gamma ray beams have already been experimentally tried out for pair production studies [11].

4.2. Backscattering Impossibility of Low Energy Photon

Let the initial energies of photon A be E_{a1} and that of B be E_{b1}. The initial momentum of these two photons will be $p_{a1} = E_{a1}/c$ and $p_{b1} = -E_{b1}/c$. The two colliding photons may interact due to instantaneous superposition of their electric and magnetic fields propagating in opposite directions. As a result of such interaction, let the photon B get rebound back or get backscattered. Let their energies after a head-on collision be E_{a2} and E_{b2} with corresponding momentum given by, $p_{a2} = E_{a2}/c$ and $p_{b2} = E_{b2}/c$. From the conservation of total energy and linear momentum we get the following two equations for computing the final energy and momentum in the direction of their original path.

For energy conservation

$$E_{a1} + E_{b1} = E_{a2} + E_{b2} \tag{28}$$

For momentum conservation

$$E_{a1}/c - E_{b1}/c = E_{a2}/c + E_{b2}/c \tag{29}$$

However, Equations (28) and (29) can be satisfied only if E_{b1} is zero. That means, there is no colliding photon B and hence no collision. That is, two photons can never make a head-on collision to produce backscattered photons. The fundamental reason for this impossibility is that the momentum of both colliding photons is $1/c$ times their total energies. While during head-on collision total momentum of the two photons gets reduced, their total energy remains constant. Therefore, the energy that corresponds to the reduction in their total momentum must no longer contribute to their linear momentum. That is, the energy that corresponds to the reduction in total momentum must get emitted in two equal parts in opposite directions, in the transverse plane.

4.3. Transverse Scattering of Low Energy Photons

Let us consider a situation where photon B of energy E_{b1} makes a head-on collision with high energy photon A of energy E_{a1}. After collision, photon B gets split into two photons C and D of energy E_{b2} each and get ejected in opposite directions in the transverse plane. Momentum of the new photons C and D will be equal and opposite in the transverse plane (**Figure 10**). In this case final momentum and energy conservation equations will be given as,

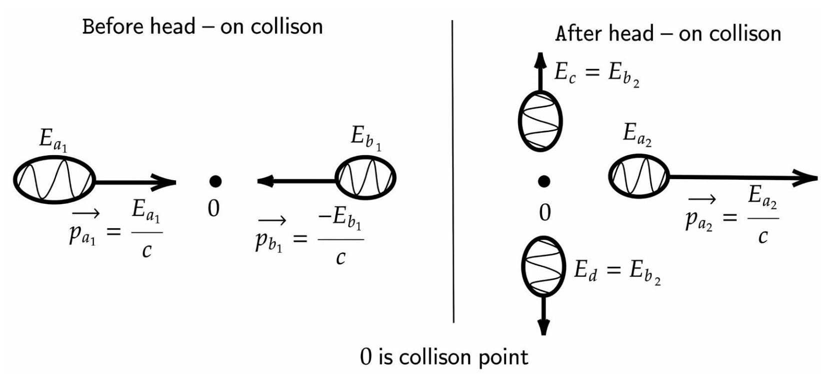

Figure 10. Head-on elastic collision between a higher energy photon and a lower energy photon.

Momentum conservation along original path,

$$E_{a1}/c - E_{b1}/c = E_{a2}/c \tag{30}$$

Momentum conservation in transverse plane,

$$0 = E_{b2}/c - E_{b2}/c \tag{31}$$

Total energy conservation,

$$E_{a1} + E_{b1} = E_{a2} + 2E_{b2} \tag{32}$$

Solving Equations (30) and (32) we get,

$$E_{a2} = E_{a1} - E_{b2} = E_{a1} - E_{b1} \quad \text{and} \quad E_{b2} = E_{b1} \tag{33}$$

Therefore, during head-on collision of photon B with a high energy photon A, two photons of energy E_{b1} each will get emitted in opposite directions in the transverse plane [11]. Energy of one of these photons will get extracted from the initial energy of photon A and the energy of the second photon will be carried over from the original photon B. Original high energy photon A, after losing energy E_{b1} in the collision, will continue on its original path as a modified photon A_m with reduced energy of (E_{a1} − E_{b1}). If the energy E_{a1} of the original high energy photon A is not greater than E_{b1} then such an interaction may not take place.

Let us examine the momentum conservation Equation (30) in some more detail. During the head-on elastic collision, high energy photon A loses its forward momentum by E_{b1}/c. Hence, photon A will also lose a part of its total energy that was associated with its lost momentum E_{b1}/c. That part of its total energy will be precisely E_{b1} which will get lost from photon A and get transferred to one of the transversal photons C emitted in the transverse plane. Similarly, during this head-on collision photon B too will lose its entire momentum of magnitude E_{b1}/c. Therefore, photon B will also lose its entire total energy E_{b1} during the collision, which will get transferred to the second transversal photon D emitted in the transverse plane.

5. Redshift Caused by Head-On Collision of Visible Photons with CMB Photons

5.1. Steady Energy Transfer from High Energy Photons to CMB Photons

The overall effect of a head-on collision between an incoming high energy photon A and a low energy or soft photon B is that the incoming photon A loses a part of its total energy equal to the total energy of the soft photon and continues on its original path with the reduced energy. If the reduced energy photon A, while continuing on its original path, makes another head-on collision with another soft photon then it will again lose a part of its energy equal to the total energy of the soft photon and continue on its original path with the further reduced energy. That is, if the incoming high energy photon A with initial energy E_{a1}, while traversing a large distance D in the intergalactic space, makes n such head-on collisions or hits with identical CMB soft photons of energy E_{cmb} and frequency f_{cmb} each, then the total energy lost by the high energy photon will be n times E_{cmb}. This process of repeated energy loss of gamma photons through n such head-on collisions or hits with EBL or CMB soft photons, constitutes a tertiary mechanism for CR energy attenuation. VHE photons coming from cosmological distances are attenuated by pair production with EBL photons. However, the process of repeated energy loss of visual photons through n such head-on collisions or hits with identical CMB soft photons, constitutes a primary mechanism for producing the Cosmological redshift z. While on the one hand this process of head-on collisions of higher energy photons with CMB photons results in steady reduction in the energy of higher energy photons, on the other hand it results in continuous increase or multiplication in the number of identical soft photons in the CMB.

5.2. Cosmological Redshift Caused by Head-On Collisions with CMB Photons

As introduced in section 1.2, the redshift z of a photon is generally given in terms of wave length λ, as the ratio of fractional increase in its wavelength during propagation to its original wavelength. The redshift can also be expressed in terms of photon frequency $f\,(=c/\lambda)$ or in terms of photon energy $E = hf$, where h is the Planck's constant. In terms of emitted frequency f_{em} and observed frequency f_{ob}, or emitted photon energy E_{em} and observed photon energy E_{ob}, Equation (2) for $1 + z$ can be re-written as,

$$1+z=\frac{f_{em}}{f_{ob}}=\frac{E_{em}}{E_{ob}} \tag{34}$$

However, the observed photon energy E_{ob} can be further expressed as the energy emitted E_{em} minus the energy lost E_{lost} by the photon during its propagation through the intergalactic space. As discussed in section 5.1 above, if the photon under consideration has made n head-on collisions with soft CMB photons of average energy E_{cmb} during propagation of distance D in the intergalactic space, then the energy lost E_{lost} can be given by,

$$E_{lost}=n\cdot E_{cmb} \tag{35}$$

Therefore, Equation (34) can now be written as,

$$1+z=\frac{E_{em}}{E_{em}-E_{lost}}=\frac{E_{em}}{E_{em}-n\cdot E_{cmb}}=\frac{f_{em}}{f_{em}-n\cdot f_{cmb}} \tag{36}$$

Let us consider the energy loss of visible light photons due to their head-on collisions with CMB photons during propagation through intergalactic space. Let us assume that, on average, visible photons make one head-on collision with CMB photon in every "d" distance covered in the space. If a particular photon, say a photon from Balmer β line, covers a total distance D from the source to the observer, then it is likely to make n = integer(D/d) head-on collisions with CMB photons during its entire journey. For various values of n we can compute the redshift parameter z or $1 + z$ from Equation (36) for the photon of given emission frequency. This way we can correlate the propagation distance D with the redshift parameter z. Similarly, by using Equation (4) we can correlate the propagation distance D with the redshift parameter z by making use of Hubble parameter H_0. Since, for low values of z, the Hubble relation (4) is matched with the observational empirical data, we can compare the two correlations to evaluate n for a given D. From this comparison we determine that a visual photon from Balmer β line with $f_\beta = 616.7$ THz, while propagating through the intergalactic space, makes one head-on collision with a CMB photon in approximately every $d = 0.42$ Mpc distance.

By making use of Equation (36) and computing number of hits n from D/d, we have plotted the redshift parameter $1 + z$ against propagation distance D, for Balmer β and γ photons as shown in Figure 11 by blue and green lines. Similarly, by using the Hubble redshift Equation (4), we have also plotted the space expansion redshift parameter $1 + z$ against distance D as shown in Figure 11 by the red line. It is interesting to note that while the Hubble redshift curve is a straight line, showing linear increase in z with distance D, the energy depletion redshift lines are curving upwards with increasing distance. That is because the denominator of Equation (36) tends to zero with depleting energy of the incoming photon or with increasing n or the propagation distance D. An important inference from this result is that higher values of cosmological redshift parameter z do not reflect proportionately higher distance D which is quite contrary to the Hubble prediction.

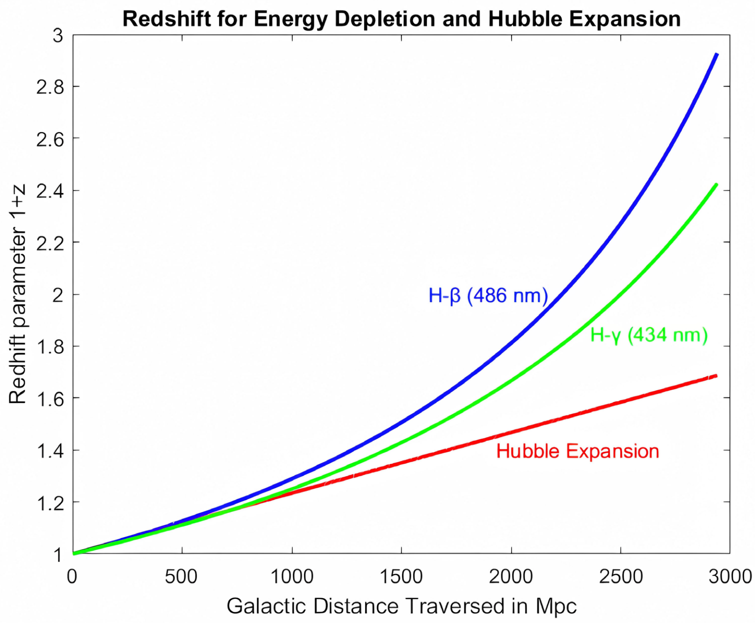

Figure 11. Redshift curves for Energy depletion of Balmer H-β & H-γ lines, and for Hubble Space Expansion.

Therefore, as noted above, the process of energy depletion of visual photons through repeated head-on collisions with CMB photons, constitutes a primary mechanism for producing the Cosmological redshift z. Following significant observations regarding this mechanism need to be highlighted.

1) Even though a head-on collision between a light photon and a CMB photon is quite feasible in principle, it is quite uncommon or rare in practice. Even in the vast expanses of the intergalactic space, it takes a light photon more than a million years to find a suitably oriented CMB photon on its way to make a head-on collision.

2) On a head-on collision with a CMB photon, the energy depleted from the incoming light photon gets pumped back into the CMB photons and the total

number of CMB photons keeps increasing with each hit. That is, this mechanism of pair production of CMB photons on each head-on collision with light photon helps to sustain the CMB photon population, preventing its natural attenuation with time.

3) After a head-on collision with a CMB photon, the incoming light photon, with reduced energy, keeps moving on its original path without any deflection or scattering in any way. This is a major plus point of this mechanism in comparison with photon energy attenuation by Compton scattering process.

4) After a large but finite number N of head-on collisions between a light photon and CMB photons, the light photon will lose most of its energy so that it no longer remains visible. The number N depends on the ratio of light photon frequency to the CMB photon frequency. Therefore, after propagation through very large distances ($D > N.d$) in the intergalactic space, all light photons will tend to lose bulk of their energy and fall into the invisible region of the spectrum. This explains the Olbers's paradox.

5.3. Impossibility of Expansion of Physical Space

Physical space is well known to support propagation of transverse waves. It is also well known from theory of elasticity that transverse waves can propagate only through an elastic solid or an elastic continuum and certainly not through any fluid medium. Whereas the metric scaling property or the metric tensor g_{ij} is only associated with coordinate space, the physical measurable properties of permittivity ε_0, permeability μ_0 and intrinsic impedance Z_0 are only associated with physical space. The popular notion of expansion of space is actually the metric expansion of coordinate space and not any real expansion of physical space. Propagation of transverse waves demands the physical space to be an elastic continuum and not a fluid medium. Hence any assumption of expansion of physical space like a fluid medium is invalid.

As per our current understanding about the physical nature of cosmological redshift, the wavelength λ of a visible photon is believed to get stretched due to the expansion of physical space. Since stretching of wavelength implies reduction in frequency and energy of the photon, there is no physical mechanism available for transferring a part of the photon energy to the expanding space or anything else. Further it is believed that stars, galaxies, and clusters do not expand with expansion of space due to their tight gravitational binding. Similarly, the contents of a photon are tightly bound together by the electromagnetic forces, which are far stronger than the gravitational binding, and hence cannot stretch along with assumed expansion of space. Hence, the assumed stretching of photons with hypothetical expansion of physical space is invalid and wrong.

6. Summary & Conclusions

When the increase in wavelength is associated with propagation through large distances in the intergalactic space, the redshift is called cosmological redshift.

Lemaître and Hubble, discovered a roughly linear correlation between distances to galaxies and their increasing redshifts. These observations were explained by Friedmann's solutions to Einstein's equations of general relativity. The correlation between redshifts and distances arises in all expanding models of universe as the cosmological redshift is commonly attributed to stretching of the wavelengths of photons propagating through the expanding space. Hubble's law is considered the first observational basis for the expansion of the universe, and today it serves as the main evidence in support of the Big Bang model.

Fritz Zwicky suggested that the cosmological redshift could be caused by the interaction of propagating light photons with certain inherent features of the Cosmos to lose a fraction of their energy. However, Zwicky's tired light hypothesis did not provide any physical mechanism to support it. In this paper, we have developed the mechanism of producing cosmological redshift through head-on collision between light and CMB photons, wherein the light photons gradually lose their energy as they propagate through large distances in the intergalactic space.

By analyzing the head-on collisions between high energy CR charged particles and low energy EBL photons, we have found that the soft photons get backscattered with very high energies even of the order of Giga or Tera electron volts. Thus, head-on collisions with EBL and CMB photons does provide an effective mechanism of energy attenuation of VHE cosmic ray particles. The famous Bethe-Heitler pair production process is not supported by this photon backscattering mechanism of CR energy attenuation. However, there is a secondary mechanism of energy transfer from gamma photons to low energy electrons and ions through backscattering process. In this mechanism, high energy gamma photons make a head-on collision with low energy electrons or ions available in the intergalactic and interstellar space. As a result of this collision, most of the gamma photon energy gets transferred to the backscattered electron or ion and some energy gets emitted as a pair of photons or electron-positron pair in the transverse plane. The Bethe-Heitler pair production process is supported by this mechanism.

Energy momentum analysis of head-on collision of a higher energy photon A with a lower energy photon B revealed that it is impossible for the photon B to get backscattered. Instead, we found that during such collision, the higher energy photon A loses energy exactly equal to the initial energy of the photon B and continues its motion on its original path. Two new photons C and D get emitted in opposite directions in the transverse plane with energy of each transversal photon equal to that of photon B. If the reduced energy photon A, while continuing on its original path, makes another head-on collision with another soft photon then it will again lose a part of its energy equal to the total energy of the soft photon and again continue on its original path with the further reduced energy. VHE photons coming from cosmological distances are attenuated by pair production with EBL photons through this mechanism.

The process of repeated energy loss of visual photons through n head-on collisions with CMB photons, constitutes a primary mechanism for producing the

Cosmological redshift z. While this process of head-on collisions of visual photons with CMB photons results in steady reduction in the energy of visual photons, it also results in continuous increase in the number of photons in the CMB. This way, for various values of repeated head-on collisions n, we can compute the red-shift parameter $1 + z$ from Equation (36) for the photon of given emission frequency. By comparing with the redshift parameter $1 + z$ obtained from empirical Hubble relation, we find that on an average a visual photon while propagating through the intergalactic space, makes one head-on collision with a CMB photon in approximately every 0.42 Mpc distance. On this basis we have plotted the red-shift parameter $1 + z$ against propagation distance D, for Balmer β and γ photons as shown in **Figure 11**. An important inference from this analysis is that higher values of cosmological redshift parameter z do not reflect proportionately higher distance D which is quite contrary to the Hubble prediction.

Finally, we conclude that the process of energy depletion of visual photons through repeated head-on collisions with CMB photons, constitutes a primary mechanism for producing the Cosmological redshift z. This mechanism of pair production of CMB photons on each head-on collision with light photon helps to sustain the CMB photon population, preventing its natural attenuation with time. After a head-on collision with a CMB photon, the incoming light photon, with reduced energy, keeps moving on its original path without any deflection or scattering in any way. After propagation through very large distances in the intergalactic space, all light photons will tend to lose bulk of their energy and fall into the invisible region of the spectrum, a phenomenon that explains the Olbers's paradox. Propagation of transverse waves demands the physical space to be an elastic continuum and not a fluid medium. Hence any assumption of expansion of physical space like a fluid medium is invalid.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix A

In physics, mass-energy equivalence is the concept that the mass of a body is a measure of its energy content. Mass is also a measure of Inertia of all forms of energy.

$$\mathrm{d}m = \frac{\mathrm{d}E}{c^2} \tag{A1}$$

From the inertial property of all forms of entrapped energy (Equation A1), we can derive the notion of dynamic mass m and develop its quantitative relationship with the rest mass m_0. Let a material particle P be at rest in some center of mass (CoM) fixed reference frame and let its rest mass in this frame be m_0. When at rest, the kinetic energy of this particle P will obviously be zero. Now let us assume that the particle P is set in motion through application of a constant force $\boldsymbol{F}$. Further, at an instant of time t, let the instantaneous velocity of P be $\boldsymbol{v}$, with corresponding kinetic energy content T. Since the energy content T will also exhibit the inertial property, let the quantitative measure of total inertia of P with rest mass m_0 and kinetic energy T, at the instant t, be given by m, the dynamic mass of the particle. If during a small interval of time $\mathrm{d}t$ the particle traverses a small distance $\mathrm{d}\boldsymbol{s}$ and gains a small amount of energy $\mathrm{d}E$ then the following relations will hold.

$$\boldsymbol{v} = \mathrm{d}\boldsymbol{s}/\mathrm{d}t \tag{A2}$$

And momentum,

$$\boldsymbol{p} = m\boldsymbol{v} \tag{A3}$$

Energy gain,

$$\mathrm{d}E = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} \tag{A4}$$

From Newton's second law of motion,

$$\boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t = \mathrm{d}(m\boldsymbol{v})/\mathrm{d}t = m \cdot \mathrm{d}\boldsymbol{v}/\mathrm{d}t + \boldsymbol{v} \cdot \mathrm{d}m/\mathrm{d}t \tag{A5}$$

From Equations A(4) and (A5),

$$\mathrm{d}E = m \cdot (\mathrm{d}\boldsymbol{v}/\mathrm{d}t) \cdot \mathrm{d}\boldsymbol{s} + \boldsymbol{v} \cdot (\mathrm{d}m/\mathrm{d}t) \cdot \mathrm{d}\boldsymbol{s} = m\boldsymbol{v} \cdot \mathrm{d}\boldsymbol{v} + v^2 \cdot \mathrm{d}m \tag{A6}$$

And from Equations (A1) and (A6) we get,

$$\mathrm{d}m = (mv/c^2) \cdot \mathrm{d}v + (v^2/c^2) \cdot \mathrm{d}m \tag{A7}$$

Let us make a substitution $x = v/c$ in Equation (A7) so that $\mathrm{d}x = \mathrm{d}v/c$ and

$$\mathrm{d}m = mx \cdot \mathrm{d}x + x^2 \cdot \mathrm{d}m \tag{A8}$$

$$\text{or,} \quad (1 - x^2)\mathrm{d}m = mx \cdot \mathrm{d}x$$

$$\text{or,} \quad \frac{\mathrm{d}m}{m} = \frac{x\mathrm{d}x}{1 - x^2} \tag{A9}$$

On integration this equation yields,

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - x^2}}$$

or, replacing x by v/c,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{A10}$$

This is a standard relation for the dynamic or relativistic mass of a particle in motion. Here, it is important to note that the derivation of dynamic mass m, in terms of rest mass m_0, did not involve special relativity. Instead, this derivation is entirely based on the inertial property of all forms of energy, including kinetic energy. Further, to deduce a separate relation for the kinetic energy T, in terms of m and m_0, we may rewrite Equations (A5) and (A6) as,

$$\mathrm{d}T = \frac{\mathrm{d}(mv)}{\mathrm{d}t} \cdot \mathrm{d}s = v \cdot \mathrm{d}(mv), \tag{A11}$$

and using Equation (10),

$$T = \int_0^v v \cdot \mathrm{d}\left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}} \right) \tag{A12}$$

or,

$$\begin{aligned} T &= m_0 \left[\int_0^v v \cdot \mathrm{d}\left(\frac{v}{\sqrt{1 - v^2/c^2}} \right) \right] \\ &= m_0 \left[\frac{v^2}{\sqrt{1 - v^2/c^2}} - \int_0^v \frac{v \cdot \mathrm{d}v}{\sqrt{1 - v^2/c^2}} \right] \\ &= m_0 \left[\frac{v^2}{\sqrt{1 - v^2/c^2}} + c^2 \sqrt{1 - v^2/c^2} - c^2 \right] \\ &= \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 = mc^2 - m_0 c^2 \end{aligned} \tag{A13}$$

Equation (A13) can be written in terms of total energy E as,

$$E = m_0 c^2 + T = mc^2 \tag{A14}$$

From Equations (A3), (A10) and (A14) we can derive the usual energy momentum relation as,

$$\begin{aligned} p^2 &= m^2 v^2 = \frac{m_0^2 v^2}{1 - v^2/c^2} = m_0^2 c^2 \left[\frac{v^2/c^2 + 1 - 1}{1 - v^2/c^2} \right] \\ &= \frac{m_0^2 c^2}{1 - v^2/c^2} - m_0^2 c^2 = m^2 c^2 - m_0^2 c^2 = \frac{E^2}{c^2} - m_0^2 c^2 \end{aligned} \tag{A15}$$

Rearranging the terms, Equation (A15) yields the standard momentum energy relation,

$$E^2 = p^2 c^2 + m_0^2 c^4 \tag{A16}$$

This shows that the kinetic energy of a body in motion is given by the difference between its dynamic mass and rest mass (m - m_0) times c^2. Similarly, all other dynamic or relativistic relations of SR can be shown to be resulting from the inertial property of all forms of energy represented by Equation (A1).