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Unruh Metric Tensor HUP via Planckian Space-Time Compared to HUP Based Complexity of Measured System Results to Obtain Inflaton Potential Magnitude

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Abstract

First of all, we restate a proof of a highly localized special case of a metric tensor uncertainty principle first written up by Unruh. Unruh did not use the Roberson-Walker geometry which we do, and it so happens that the dominant metric tensor we will be examining, is variation in δg_{tt}. The metric tensor variations given by δg_{rr}, $\delta g_{\theta\theta}$ and $\delta g_{\phi\phi}$ are negligible, as compared to the variation δg_{tt}. Afterwards, what is referred to by Barbour as emergent duration of time δt is from the Heisenberg Uncertainty principle (HUP) applied to δg_{tt} in such a way as to be compared with $\Delta x \Delta p \geq \frac{\hbar}{2} + \tilde{\gamma} \frac{\partial C}{\partial V}$ with V here a volume spatial term and $\tilde{\gamma}$ a complexification strength term and $\frac{\partial C}{\partial V}$ influence of complexity of physical system being measured in order to obtain a parameterized value for the initial value of an inflaton which we call V_0.

Keywords

Massive Gravitons, Heisenberg Uncertainty Principle (HUP)

1. Introduction

The first matter of business will be to introduce a framework of the speed of gravitons in "heavy gravity". Heavy Gravity is the situation where a graviton has a small rest mass and is not a zero mass particle, and this existence of "heavy gravity" is important since eventually, as illustrated by Will [1] [2] gravitons having a small mass could possibly be observed via their macroscopic effects upon astrophysical events. The second aspect of the inquiry of our manuscript will be to come up with a variant of the Heisenberg Uncertainty principle (HUP), in [3], with

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \tilde{\gamma} \frac{\partial C}{\partial V} \tag{1}$$

As opposed to

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad \text{Unless } \delta g_{tt} \sim O(1) \tag{2}$$

which we claim in the Planckian regime will de evolve, as being effectively as being equivalent to

$$\Delta x \Delta p \geq \frac{\hbar}{\delta g_{tt}} \tag{3}$$

We will be comparing Equation (1) and Equation (3) as well as writing

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \tag{4}$$

The second term in Equation (4) comes directly from a simplified inflaton expression which is [4] [5]

$$\begin{aligned} & a(t) = a_{\text{initial}} t^{\nu} \\ & \Rightarrow \phi = \ln\left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}} \\ & \Rightarrow \dot{\phi} = \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\ & \Rightarrow \frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{1.66^2 \cdot g_*}{m_P^2} \approx 10^{-5} \end{aligned} \tag{5}$$

In this we isolate out an expression for initial value of an inflaton which we call V_0 and that concludes our document once we link it to the issue of complexity which is generated as to black hole physics which is the final chapter of our study.

We reference what was done by Will in his living reviews of relativity article as to the "Confrontation between GR and experiment". Specifically, we make use of his experimentally based formula of [1] [2], with v_{graviton} the speed of a graviton, and m_{graviton} the rest mass of a graviton, and E_{graviton} in the inertial rest frame given as:

$$\left(\frac{v_{\text{graviton}}}{c}\right)^2 = 1 - \frac{m_{\text{graviton}}^2 c^4}{E_{\text{graviton}}^2} \tag{6}$$

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{\text{scale factor}} \sim 10^{-55}$, *i.e.* 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space-time singularity.

We will next discuss the implications of this point in the next section, of a non-zero smallest scale factor. Secondly, the fact we are working with a massive graviton, as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of modification of the values [6]

$$\left\langle (\delta g_{uv})^2 (\hat{T}_{uv})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2}$$
$$\xrightarrow[uv \to tt]{} \left\langle (\delta g_{tt})^2 (\hat{T}_{tt})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2} \tag{7}$$
$$\& \ \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$$

2. Nonzero Scale Factor, Initially and What This Is Telling Us Physically. Starting with a Configuration from Unruh

Begin with the starting point of [7] [8]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \tag{8}$$

We will be using the approximation given by Unruh, [7] [8]

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2}$$
$$(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A \tag{9}$$

If we use the following, from the Roberson-Walker metric [9].

$$g_{tt} = 1$$
$$g_{rr} = \frac{-a^2(t)}{1 - k \cdot r^2}$$
$$g_{\theta\theta} = -a^2(t) \cdot r^2 \tag{10}$$
$$g_{\phi\phi} = -a^2(t) \cdot \sin^2\theta \cdot d\phi^2$$

Following Unruh [7] [8], write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} \text{ meters} \tag{11}$$

Then, if $\Delta T_{tt} \sim \Delta\rho$

$$V^{(4)} = \delta t \cdot \Delta A \cdot r$$
$$\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \tag{12}$$
$$\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}$$

This Equation (11) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time for the stress energy tensor as given in Equation (12).

$$T_{ii} = diag(\rho, -p, -p, -p) \tag{13}$$

Then

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}} \tag{14}$$

Then,

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}$$
$$\text{Unless } \delta g_{tt} \sim O(1) \tag{15}$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \tag{16}$$

In fact, we have that from Giovannini [9], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if [9]

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \tag{17}$$

Then, there is no way that Equation (15) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$.

3. Obtaining a Bridge from Equation (2) to Equation (3). It Depends upon Using Equation (5) and Assuming Time Is for All Intensive Purposes Fixed at about Planck Time to Isolate V_0

Equation (17) is crucial here, and it depends upon the scalar term in Equation (17) have a time dependence only, which means it is for near Planck time, almost a constant term. *I.e.* for the sake of argument, in the near Planckian regime, we can figure that Equation (5) will have as far as evaluation of the argument the following configuration, *i.e.* [8]

$$a(t) \approx a_{\text{initial}} \cdot (t/t_P)^{\nu} \tag{18}$$

Given this we will be looking at, if we do the set up

$$\Delta x \Delta p \geq \frac{\hbar}{\delta g_{tt} = \left[a_{\text{initial}} \cdot (t/t_P)^{\nu}\right]^2 \left[\ln\left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}}\right]} \tag{19}$$

Comparing this Equation (19) with Equation (1), we obtain then if $\hbar = c = t_p = k_B = l_P = G = 1$ the following bound for V_0

$$V_0 \cong \left[\frac{\nu \cdot (3\nu - 1)}{8\pi}\right] \cdot \left[\exp\left(\frac{16\sqrt{\pi}}{\sqrt{\nu}} \cdot \frac{1}{a_{\min}^2 \cdot (t/t_p)^2}\right)\right] \cdot \left(\frac{1}{1 + 2\tilde{\gamma}\frac{\partial C}{\partial V}}\right)^2 \tag{20}$$

4. Evaluation of Equation (20) If We Are Near Planck Time. Two Limits

1st, What if we have expansion of the scale factor initially at greater than the speed of light?

Set $\nu \approx 10^{88}$ and then we can obtain if we are just starting off inflation say

$a_{\min}^2 \approx 10^{-44}$. Then

$$V_0 \cong \left[10^{176}\right] \cdot \left[\exp\left(16\sqrt{\pi}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\dfrac{\partial C}{\partial V}}\right)^2, \tag{21}$$

If we wish to have a Planck energy magnitude of the V_0 term, we will then be observing

$$V_0 \cong \left[10^{176}\right] \cdot \left[\exp\left(16\sqrt{\pi}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\dfrac{\partial C}{\partial V}}\right)^2, \xrightarrow[2\tilde{\gamma}\frac{\partial C}{\partial V} \approx \left[10^{88}\right]]{} o(1) \tag{22}$$

i.e. the system complexity will become effectively almost infinite, and this will be explained in the conclusion

$$2\tilde{\gamma}\frac{\partial C}{\partial V} \approx \left[10^{88}\right] \Rightarrow V_0 \cong o(1) \tag{23}$$

On the other hand, if there is a very small value for $2\tilde{\gamma}\dfrac{\partial C}{\partial V}$ we can see the following behavior for the Equation (21), namely

$$2\tilde{\gamma}\frac{\partial C}{\partial V} \approx o(1) \Rightarrow V_0 \cong \left[10^{176}\right] \tag{24}$$

i.e. low complexity and all that in the measurement process will then imply an enormous initial inflaton potential energy

Secondly, Now what if we have instead $\nu \approx 1$

$$V_0 \cong \left[\frac{1}{4\pi}\right] \cdot \left[\exp\left(\frac{16\sqrt{\pi}}{a_{\min}^2 \cdot \left(t/t_p\right)^2}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\dfrac{\partial C}{\partial V}}\right)^2 \tag{25}$$

The threshold if $2\tilde{\gamma}\dfrac{\partial C}{\partial V} \approx \left[10^{88}\right]$ *i.e.* a huge value for initial complexity would be effectively made insignificant in cutting down the initial inflaton lead to

$$\exp\left(\frac{16\sqrt{\pi}}{\sqrt{\nu}} \cdot \frac{1}{a_{\min}^2 \cdot \left(t/t_p\right)^2}\right) \xrightarrow[a_{\min}^2 \approx 10^{-88}]{} V_0 \cong \exp\left(10^{88}\right) \tag{26}$$

i.e. we come to the seemingly counter Intuitive expression that the initial inflaton potential would still be infinite if we used Equation (26) in Equation (21). Now let us consider how we can link this to the matter of complexity and the development of primordial black holes. This involves material from [10]-[17].

Table 1 from [10] assuming Penrose recycling of the Universe as stated in that document.

The limits in section four may give structural complexity data relevant to the following development. As given, see [10].

Table 1. A guide to cosmological structure formation involving black holes.

End of Prior Universe time frame	Mass (black hole): super massive end of time BH 1.989×10^{41} to about 10^{44} grams	Number (black holes) 10^6 to 10^9 of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10^{-5} to 10^{-4} grams (an order of magnitude of the Planck mass value)	Number (black holes) 10^{40} to about 10^{45}, assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Equation (1) to have say 10^{10} gravitons/second released per black hole	Mass (black hole) 10 grams to say 10^6 grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time. 10^{20} to at most 10^{25}

This increase in complexity can be with work tied into the following for black hole physics [11]

$$\begin{aligned}
& m \approx \frac{M_P}{\sqrt{N_{\text{gravitons}}}} \\
& M_{BH} \approx \sqrt{N_{\text{gravitons}}} \cdot M_P \\
& R_{BH} \approx \sqrt{N_{\text{gravitons}}} \cdot l_P \\
& S_{BH} \approx k_B \cdot N_{\text{gravitons}} \\
& T_{BH} \approx \frac{T_P}{\sqrt{N_{\text{gravitons}}}}
\end{aligned} \tag{27}$$

We will try to quantify all this in future research work to explain this in terms of the physics of phase transitions, in the universe and cyclic conformal cosmology. This means paying attention to the inputs of **Appendix A** and **Appendix B** as given below in future developments. Finally the physics of initial transformations as given in Table 1 should have some linkage eventually to [16] as to the idea of Gravity breath, as given by Dr. Corda.

5. First Major Implication to Investigate, *i.e.* Role of Complexity in Bridge from Different Black Hole Numbers as Given in Table 1

There are three regimes of black hole numbers given in Table 1. From Pre Planckian, to Planckian and then to post Planckian physics regimes. This is all assuming CCC cosmology. To start to make sense of this, we need to examine how one could achieve the complexity as indicated by Table 1 in the Planckian era.

To do this at a start, we will pay attention to a datum in reference [11], namely a Horizon, like a Schwarzschild black hole construction with

$$L_A = \sqrt{\frac{3}{\Lambda}} \tag{28}$$

In what [17] deems as a corpuscular gravity one would have a "kinetic energy term" per graviton

$$\in_G \cong \frac{M_p}{\sqrt{\tilde{N}}} \tag{29}$$

And the mass of a black hole, scaling as [17]

$$M_{\text{black hole}} \cong \sqrt{\tilde{N}} M_p \approx \tilde{N} \in_G \tag{30}$$

This in [11] has the exact same functional forms as is given in Equation (27) so then we have $\tilde{N} = N$ and furthermore [17] also has

$$\in_G \cong \frac{M_p}{\sqrt{\tilde{N}}} \cong \frac{\hbar}{L_A} \approx \frac{M_p}{\sqrt{N}} \tag{31}$$

If so for Black holes, we have the following relationship, *i.e.*

$$\sqrt{\Lambda} \cong \frac{\sqrt{3} M_p}{\hbar \sqrt{N}} \tag{32}$$

Now as to what is given in [18] as to Torsion, we have that as given in [18] that.

First look at numbers provided by [19] as to inputs, *i.e.* these are very revealing

$$\Lambda_{Pl} c^2 \approx 10^{87} \tag{33}$$

This is the number for the vacuum energy and this enormous value is 10^{122} times larger than the observed cosmological constant. Torsion physics, as given by [18] [19] is solely to remove this giant number.

In order to remove it, the reference [18] [19] proceeds to make the following identification, namely

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} = 0 \tag{34}$$

What we are arguing is that instead, one is seeing, instead [18] [19]

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda_{Pl} c^2}{3} \approx 10^{-122} \times \left(\frac{\Lambda_{Pl} c^2}{3}\right) \tag{35}$$

Our timing as to Equation (33) is to unleash a Planck time interval t about 10^{-43} seconds.

As to Equation (34) versus Equation (35) the creation of the torsion term is due to a presumed "graviton" particle density of

$$n_{Pl} \approx 10^{98}\ \text{cm}^{-3} \tag{36}$$

This Equation (36) is directly relevant to the basic assumption of how to have relevant Gravitons initially created as to obtain the huge increase in complexity alluded to, in order to obtain the number of micro black holes in the Planckian era [18] [19].

i.e. assume that there are, then say initially up to 10^{98} gravitons, initially, and then from there, go to Table 1 to assume what number of micro sized black holes are available.

i.e. Table 1 has said a figure of 10^{45} to at most 10^{50} micro sized black holes, presumably for 10^{98} gravitons being released, and this is meaning we have say 10^{50} black holes of say of Planck mass, to work with.

If say we have 10^{50} Black holes of Planck mass we will then be examining that as to the magnitude of the Inflaton potential in our final chapter and how this pertains to black holes, in the creation of complexity by our modified HUP argument. *i.e.* summing up.

6. Comparing the Given Inflaton Magnitude as Due to the HUP Argument, with the Dramatic Increase in Complexity as Indicated

In order to do this we will be making correlations between say 10^{50} micro sized black holes, as proportional to V_0 in some fashion, with V_0. Looking at Equation (21) to (26) it is most likely that the magnitude of expansion of the scale factor would have to be greater than the speed of light, dictating a preference as to $2\tilde{\gamma}\frac{\partial C}{\partial V}$ being very large, *i.e.* the complexity would be gained way up and the magnitude of the magnitude of the coefficient of the scale factor would be less than $\nu \approx 10^{88}$ but significantly larger than 1.

In a future study we will detail different scenarios as to what the coefficient ν could be as also linked to corresponding complexity factors of $2\tilde{\gamma}\frac{\partial C}{\partial V}$ would be, as to give a range of options. As to what to expect. Doing this though we still need to justify how we can have a nonzero graviton mass. Which is our final section.

7. The Role of Barbour Emergent Time and Our Evolution of Black Holes as Seen in Table 1. *i.e.* How We Can Justify Writing Very Small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^{+}$ Values with Still a Non-Zero Graviton Mass. *i.e.* This Justifies Our BEC Condensate Treatment of Gravitons

To begin this process, we will break it down into the following coordinates as to why only the variation in $g(tt)$ survives which is essential to our HUP to begin with Equation (12).

In the rr, $\theta\theta$ and $\phi\phi$ coordinates, we will use the Fluid approximation, $T_{ii} = diag(\rho, -p, -p, -p)$ [20] with

$$\begin{aligned}
&\delta g_{rr} T_{rr} \geq -\left|\frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}}\right| \xrightarrow[a \to 0]{} 0 \\
&\delta g_{\theta\theta} T_{\theta\theta} \geq -\left|\frac{\hbar \cdot a^2(t)}{V^{(4)}\left(1 - k \cdot r^2\right)}\right| \xrightarrow[a \to 0]{} 0 \\
&\delta g_{\phi\phi} T_{\phi\phi} \geq -\left|\frac{\hbar \cdot a^2(t) \cdot \sin^2\theta \cdot d\phi^2}{V^{(4)}}\right| \xrightarrow[a \to 0]{} 0
\end{aligned} \tag{37}$$

If as an example, we have negative pressure, with T_{rr}, $T_{\theta\theta}$ and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^{+}$, since there is no way that $p = -\rho$ is zero valued.

Having said this, the value of δg_{tt} being nonzero, will be part of how we will be looking at a lower bound to the graviton mass which is not zero. To do this though we will have inflation created by the switching of the initially enormous potential energy to a very high level of kinetic energy which is tied into Barbor emergent time, *i.e.* the emergent time concept is used in our lower nonzero bound to a massive graviton, which is important.

8. Lower Bound to the Graviton Mass Using Barbour's Emergent Time

In order to start this approximation, we will be using Barbour's value of emergent time [21] [22] restricted to the Plank spatial interval and massive gravitons, with a massive graviton [23]

$$(\delta t)^2_{\text{emergent}} = \frac{\sum_i m_i l_i \cdot l_i}{2\cdot(E-V)} \to \frac{m_{\text{gravtion}} l_P \cdot l_P}{2\cdot(E-V)} \tag{38}$$

Initially, as postulated by Babour [21] [22], this set of masses, given in the emergent time structure could be for say the planetary masses of each contribution of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton. Recall that we can write.

If $(\delta t)^2_{\text{emergent}} = \delta t^2$ in Equation (12), using Equation (12) and Equation (38).

We can arrive at the identification of

$$m_{\text{gravtion}} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{E-V}{\Delta T_{tt}^2} \tag{39}$$

Key to Equation (39) will be identification of the kinetic energy which is written as $E-V$. This identification will be the key point raised in this manuscript. Note that [23] raises the distinct possibility of an initial state, just before the "big bang" of a kinetic energy dominated "pre inflationary" universe. *I.e.* in terms of an inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [23]. The key finding which is in [23] is, that, if the kinetic energy is dominated by the "inflaton" that

$$K.E. \sim (E-V) \sim \dot{\phi}^2 \propto a^{-6} \tag{40}$$

This is done with the proviso that $w < -1$, in effect, what we are saying is that during the period of the "Planckian regime" we can seriously consider an initial density proportional to Kinetic energy, and call this *K.E.* as proportional to [20] w ranging in value of −1 to 1

$$\rho_w \propto a^{-3(1-w)} \tag{41}$$

This will allow for us to switch from the enormous inflaton Potential energy we identified to the initial enormous kinetic energy which starts the jump off, of the dynamics seen in **Table 1** as well as the information given in [24].

9. Future Work as to the Complexity Factor Used in the HUP Document. *i.e.* the Author of [3] Has Graciously Sent Updates Involving Electromagnetism and Other Items

[25] and [26] have Nye's generalization of [3] and this will be important due to the following, *i.e.* [27] has this as to a cosmology with initially strong E and B fields to contend with that there would be a minimum scale factor influenced by the treatment of the early universe having E and B fields or their early universe analogue, as given by

$$\begin{aligned} &\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0 \\ &\hat{\lambda}(\text{defined}) = \Lambda c^2 / 3 \\ &a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\hat{\lambda}(\text{defined})} \left(\sqrt{\alpha_0^2 + 32\hat{\lambda}(\text{defined}) \cdot \mu_0 \omega \cdot B_0^2} - \alpha_0 \right) \right]^{1/4} \end{aligned} \tag{42}$$

where the following is possibly linkable to minimum frequencies linked to *E* and *M* fields [28], and possibly relic Gravitons

$$B > \frac{1}{2 \cdot \sqrt{10\mu_0 \cdot \omega}} \tag{43}$$

Finally is the question of applicability of the Riemann Penrose inequality which is [29], p. 431, which is stated as:

Riemann Penrose Inequality: Let (***M***, ***g***) be a complete, asymptotically flat 3-manifold with Non negative-scalar curvature, and total mass m, whose outermost horizon Σ has total surface area A. Then

$$m_{\text{total mass}} \geq \sqrt{\frac{A_{\text{surface Area}}}{16\pi}} \tag{44}$$

And the equality holds, iff (***M***, ***g***) is isometric to the spatial isometric spatial Schwartzshield manifold ***M*** of mass ***m*** outside their respective horizons.

Assume that the frequency, say using the frequency of Equation (42), and $A \approx A_{\min}$ of Equation (43) is employed. So then say we have by dimensional analysis from a wave in a medium the following, assuming it is traveling at light speed. Then

$$\begin{aligned} &(v = \text{velocity} \equiv c) = f(\text{frequency}) \times \lambda(\text{wavelength}) \\ &\Rightarrow \omega \approx \omega_{\text{initial}} \sim \frac{c}{d_{\min}} \sim \frac{1}{d_{\min}}\bigg|_{c \equiv 1} \; \& \; d_{\min} \sim A^{1/3} \propto a_{\min} \end{aligned} \tag{45}$$

Assume that we also set the input frequency as to Equation (43) as according to $10 < \zeta \leq 37$ *i.e.* does

$$\begin{aligned} &\left(m_{\text{total mass}} \sim 10^{\zeta} \cdot m_{\text{graviton}}\right)^2 \propto a_{\min}^3 / 16\pi \\ &\Leftrightarrow \omega \approx \omega_{\text{initial}} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta} \cdot m_{\text{graviton}}\right)^{-2/3} \end{aligned} \tag{46}$$

This will heavily influence future work in delineating the HUP, complexity and other factors.

Finally consider the following, namely we wish to incorporate the following as far as graviton production from relic black holes, in Table 1 and in our analysis

We claim that if this is an initial frequency and that it is connected with relic graviton production, that the minimum frequency would be relevant to Equation (42), and may play a part as to admissible B fields. Furthermore, if say $N = N_{\text{graviton}} \approx 10^{\zeta}; 10 < \zeta \leq 37$, then [30] with

$$N = N_{\text{graviton}}\Big|_{r_H} = \frac{c^3}{G \cdot \hbar} \cdot \frac{1}{\Lambda} \approx \frac{1}{\Lambda} \tag{47}$$

which in turn would lead to [31]

$$m_{\text{graviton}} = \frac{\hbar}{c} \cdot \sqrt{\frac{2\Lambda}{3}} \approx \sqrt{\frac{2\Lambda}{3}} \tag{48}$$

which is different from the De Sitter version of graviton mass given in [32]

$$m_g^2 = -(2/3) \cdot \Lambda \tag{49}$$

10. Comment Included as to Why DE (and DM) Is Likely Still Necessary, Even If We Consider Topological Defects, as Brought up by [33]

In [33], Lieu has the postulation that one could still obtain the Galaxy rotation curves in his MNRAS article by topological defects, rather than by DM (and DE). While the article is interesting, I wish to go to a part of the manuscript which is most intriguing to me as to what it purports the following

Go to its Equation (9), (10) and (11) which we will re write as

$$\mathrm{d}S^2 = c^2 g^2(r)\mathrm{d}t^2 - f^2(r)\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{50}$$

This line element leads to the following expressions as to the Einstein field tensors

$$r^2 G_{00} = 1 - \frac{1}{f^2(r)} + \frac{2rf'(r)}{f^3(r)} \tag{51}$$

$$r^2 G_{11} = 1 - \frac{1}{f^{-2}(r)} + \frac{2g'(r)}{g(r)} \tag{52}$$

In the case of a weak field approximation, which is given in the manuscript, [33] looks at the case of

$$g(r) = 1 + \Phi(r) \tag{53}$$

While in a multi shell (matter?) approximation is given by

$$\Phi(r) = \alpha \cdot s \cdot c^2 \sum_{n=n_{s,1}}^{n_r} \frac{\Theta(r-R)}{nR} \tag{54}$$

Whereas we then have

$$\frac{c^2 g'(r)}{g(r)} = \frac{\alpha \cdot s \cdot c^2}{r} \delta(r-R) \tag{55}$$

Here, Θ is a step function whereas we also have Equation (55) as having a delta function.

Meanwhile in doing this, for galaxies, *i.e.* not in the regime of analysis of our problem

$$f(r) = 1 + \alpha \cdot s \cdot \delta(r - R) \quad (56)$$

and

$$g(r) = 1 + \alpha \cdot s \cdot \frac{\Theta(r - R)}{r} \quad (57)$$

Then, to our surprise

$$\begin{aligned} &G_{00} \to G_{00} = \frac{8\pi G\rho(r)}{c^2} \\ &G_{11} = 0 \end{aligned} \quad (58)$$

Here we have that the first equation in Equation (58) is the density.

Also,

If we do not do the truncation specified in Equation (56) and in Equation (57) we still have

$$\begin{aligned} &\theta_{\text{light ray angle bent}} \approx 2^{-1/3}\pi^{-2/3}s^{2/3}a^{1/3} \\ &s \ll R \le \Delta \ll a \\ &nR - a \approx \Delta \end{aligned} \quad (59)$$

While this is indeed very clever, we do not have any line metric like Equation (50) in our analysis, in fact in the uncertainty principle we worked with, we only have functionally the time component of a modified Schwartzshield metric to work with, in fact then what this [33] is doing is using a geometry of multi sphered topological defects ensheathed about each other plus the limit of weak field approximation in order to obtain its results.

We are not assuming in our analysis a weak field approximation! In fact in what we are working with we are assuming due to the enormity of the initial inflaton potential a very strong field, implied by enormous graviton numbers at the start of expansion of the universe.

Again let me highlight this. The entire [33] is using by its construction a metric explicitly mixing time and space as well as multi sphered topological "spheres" whereas we are close to, but not embracing a near singularity (not exactly). *i.e.* it is a small region of space, not a point singularity, but it is no where near the size of what would be for light bending let alone gravitational rotation curves.

The geometry of the two analysis are completely different. The scale of the spatial analysis are wildly different

In short, as far as macro scale, [33] may be in large scale a "proof" in the late regime of spatial expansion of the universe that the DM (and possibly DE) models do need a re do. It in no way is commensurate with the geometry of analysis which this paper is based upon.

Fund

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: The Generalized HUP Term in Operators

Finally, as far as Equation (15) is concerned, there is one serious linkage issue to classical and quantum mechanics, which should be the bridge between classical and quantum regimes, as far as space time applicable. Namely, from Wald [12], if we look at first of all arbitrary operators, A and B

$$(\Delta A)^2 \cdot (\Delta B)^2 \geq \frac{1}{2i}\langle [A, B] \rangle \tag{A1}$$

Appendix B: Scenarios as to the Value of Entropy in the Beginning of Space-Time Nucleation

We will be looking at inputs so if we look at [13] so that if
$E \sim M \sim \Delta T_{tt} \cdot \delta t_{\text{time}} \cdot \Delta A \cdot l_P$

$$S(\text{entropy}) = \ln Z + \frac{(E \sim \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot l_P)}{k_B T_{\text{temperature}}} \tag{B1}$$

And using Ng's infinite quantum statistics, we have to first approximation [14] [15]

$$\begin{aligned} S(\text{entropy}) &\sim \ln Z + \frac{((E \sim \Delta T_{tt}) \cdot \delta t \cdot \Delta A \cdot l_P)}{k_B T_{\text{temperature}}} \\ &\sim \ln Z + \frac{\hbar}{k_B T_{\text{temperature}} \delta g_{tt}} \\ &\xrightarrow[T_{\text{temperature}} \to \#\text{anything}]{} [S(\text{entropy}) \sim n_{\text{count}}] \end{aligned} \tag{B2}$$

This is due to a very small but non vanishing δg_{tt} with the partition functions covered by [25], and also due to [14] [15] with n_{count} a non-zero number of initial "particle" or information states, about the Planck regime of space-time, so that the initial entropy is nonzero.