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Fermions: Spin, Hidden Variables, Violation of Bell's Inequality and Quantum Entanglement

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Abstract

Using real fields instead of complex ones, it was recently claimed, that all fermions are made of pairs of coupled fields (strings) with an internal tension related to mutual attraction forces, related to Planck's constant. Quantum mechanics is described with real fields and real operators. Schrodinger and Dirac equations then are solved. The solution to Dirac equation gives four, real, 2-vectors solutions $\psi_1 = \begin{pmatrix} U_1 \\ D_1 \end{pmatrix}$ $\psi_2 = \begin{pmatrix} U_2 \\ D_2 \end{pmatrix}$ $\psi_3 = \begin{pmatrix} U_3 \\ D_3 \end{pmatrix}$ $\psi_4 = \begin{pmatrix} U_4 \\ D_4 \end{pmatrix}$ where (ψ_1, ψ_4) are coupled via linear combinations to yield spin-up and spin-down fermions. Likewise, (ψ_2, ψ_3) are coupled via linear combinations to represent spin-up and spin-down anti-fermions. For an incoming entangled pair of fermions, the combined solution is $\Psi_{in} = c_1\psi_1 + c_4\psi_4$ where c_1 and c_4 are some hidden variables. By applying a magnetic field in +Z and +x the theoretical results of a triple Stern-Gerlach experiment are predicted correctly. Then, by repeating Bell's and Mermin Gedanken experiment with three magnetic filters σ_θ, at three different inclination angles θ, the violation of Bell's inequality is proven. It is shown that all fermions are in a mixed state of spins and the ratio between spin-up to spin-down depends on the hidden variables.

Keywords

Fermions, Spin, Hidden Variables, Bell's Inequality Violation, Spin Entanglement

1. Introduction

One of the most intriguing phenomena in Quantum mechanics is Quantum entanglement. It originates in the debate about the EPR paradox [1] starting in 1935, when Einstein have refused to accept the concept of "spooky action". Einstein claimed that there must be some hidden variables to explain the results. However, this explanation was disproved in 1964 by Bell [2] [3], and later by others [4]-[8]. It is quantum entanglement phenomena that is used to strongly declare that quantum mechanics must use complex fields, since otherwise one cannot explain entanglement.

Based on the formulation of Quantum mechanics in terms of real fields [9], it is possible to explain the Bell's inequality violation in terms of hidden variables. In fact, other fundamental quantum phenomena such as interference are explained as well. Moreover, a new interpretation of fermions internal structure emerges and provides an explanation to Planck's constant.

The result of this new approach does not change from those of the standard complex description of quantum mechanics. But it helps to better understand the internal structure of fermions and their spin characteristics.

2. Real Formulation of Schrödinger and Dirac Equations

As described already [9], for a time-independent classical Hamiltonian of a free particle, with mass m, the Schrödinger Equation:

$$-i\hbar\frac{\partial}{\partial t}\psi\left(x,t\right)=\mathcal{H}\psi\left(x,t\right) \tag{1}$$

can be solved by separating it into real and imaginary components

$$\psi\left(x,t\right)=\Psi=\varphi_1+i\varphi_2 \tag{2}$$

the Schrödinger equation becomes:

$$+\hbar\frac{\partial}{\partial t}\varphi_2=\mathcal{H}_r\varphi_1-\mathcal{H}_i\varphi_2 \tag{3}$$

$$-\hbar\frac{\partial}{\partial t}\varphi_1=\mathcal{H}_i\varphi_1+\mathcal{H}_r\varphi_2 \tag{4}$$

In other words, the traditional Schrödinger Equation describes two coupled real fields, with real operators acting in real 3-dimensional space. Needless to say, any Hermitian operator can be split into a sum of two real operators

It will be an assumption herewith, that the quantum description and characteristics of a single fermion are the result of a coupling interaction between two real components (entities) which composes the single fermion.

In the following, we show that the solution to Dirac's equation $\left(i\hbar\gamma^{\mu}\partial_{\mu}-mc\right)\Psi=0$ with real fields are four 2-vector fields $\psi_1=\begin{pmatrix}U_1\\D_1\end{pmatrix}$ $\psi_2=\begin{pmatrix}U_2\\D_2\end{pmatrix}$ $\psi_3=\begin{pmatrix}U_3\\D_3\end{pmatrix}$ $\psi_4=\begin{pmatrix}U_4\\D_4\end{pmatrix}$ that represent fermion and anti-fermion with spin-up u^{UP} and spin-down u^{DN} mixtures. Using the Pauli σ_z and σ_x operators, we show how the spin states are created by linear combinations of ψ_1 and ψ_4 in case of the fermions (and similarly by ψ_2 and ψ_3 in the anti-fermion). Moreover, the Stern-Gerlach 3-steps experiment is exactly according to this real formalism and the results are made obvious. Further, because of some hidden variables and the coupled solution, Bell's inequality violation is proven to hold due to equal mixture of u^{UP} and u^{DN} states in the incoming entangled particles.

3. Dirac Equation with Real Wave Functions

The relativistic Dirac Equation, describing a free Fermion of mass m is given by:

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\right)\Psi = 0 \tag{5}$$

One may separate the Dirac operator $i\hbar\gamma^{\mu}\partial_{\mu} - mc$ and the complex wave function Ψ into their **real and imaginary parts** [9].

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

With $\psi_1 = \begin{pmatrix} U_1 \\ D_1 \end{pmatrix}$ $\psi_2 = \begin{pmatrix} U_2 \\ D_2 \end{pmatrix}$ $\psi_3 = \begin{pmatrix} U_3 \\ D_3 \end{pmatrix}$ $\psi_4 = \begin{pmatrix} U_4 \\ D_4 \end{pmatrix}$ all real components.

As will be shown, these 4 components represent two fermions and two anti-fermions. Each pair is the source for two opposing spin states.

After some work and **boosting** to a system moving with the particle along the +x axis ($p_y, p_z = 0 \rightarrow \partial_y, \partial_z = 0$)

the Dirac equations take the form:

$$-\frac{mc^2}{\hbar}\Psi_1 = +\partial_t\Psi_4 - c\sigma_x\partial_x\Psi_4 \tag{6}$$

$$-\frac{mc^2}{\hbar}\Psi_4 = -\partial_t\Psi_1 - c\sigma_x\partial_x\Psi_1 \tag{7}$$

$$-\frac{mc^2}{\hbar}\Psi_2 = +\partial_t\Psi_3 + c\sigma_x\partial_x\Psi_3 \tag{8}$$

$$-\frac{mc^2}{\hbar}\Psi_3 = -\partial_t\Psi_2 + c\sigma_x\partial_x\Psi_2 \tag{9}$$

This shows, that, Ψ_1 is coupled with Ψ_4 and Ψ_2 is coupled with Ψ_3. As will be shown later, linear combinations of these represent spin-up and spin-down fermion and anti-fermion.

4. Eight Real Components

Each Ψ_i is a 2-vector with real components. Thus, the Dirac Equation is actually 8 equations of real components with coupled pairs $\left(\Psi_1, \Psi_4\right)$, and $\left(\Psi_2, \Psi_3\right)$.

Applying a time derivative to the first equation of each pair and using the second component of each pair, leads to:

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)U_1 + c\partial_x\partial_t U_1 = -\frac{mc^3}{\hbar}\partial_x U_4 \tag{10}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2 + \partial_t^2\right)D_1 - c\partial_x\partial_t D_1 = +\frac{mc^3}{\hbar}\partial_x D_4 \tag{11}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)U_4-c\partial_x\partial_t U_4=+\frac{mc^3}{\hbar}\partial_x U_1 \tag{12}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)D_4+c\partial_x\partial_t D_4=-\frac{mc^3}{\hbar}\partial_x D_1 \tag{13}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)U_2+c\partial_x\partial_t U_2=-\frac{mc^3}{\hbar}\partial_x U_3 \tag{14}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)D_2-c\partial_x\partial_t D_2=+\frac{mc^3}{\hbar}\partial_x D_3 \tag{15}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)U_3+c\partial_x\partial_t U_3=-\frac{mc^3}{\hbar}\partial_x U_2 \tag{16}$$

$$\left(\left(\frac{mc^2}{\hbar}\right)^2+\partial_t^2\right)D_3-c\partial_x\partial_t D_3=+\frac{mc^3}{\hbar}\partial_x D_2 \tag{17}$$

These 8 real components equations demonstrate the existence of coupled pairs: ($U_1 \Leftrightarrow U_4$), ($D_1 \Leftrightarrow D_4$), ($U_2 \Leftrightarrow U_3$) and ($D_2 \Leftrightarrow D_3$).

These equations show, that every fermion is composed of 4 real fields which are coupled in a yet to be explored manner.

5. Solution

The solutions are described in the following:

$$\Psi_1=\begin{pmatrix}U_1\\D_1\end{pmatrix}=\begin{pmatrix}\cos\left(px-(\omega_0+cp)t\right)\\ \sin\left(px-(\omega_0-cp)t\right)\end{pmatrix}$$

$$\Psi_4=\begin{pmatrix}U_4\\D_4\end{pmatrix}=\begin{pmatrix}\sin\left(px+(\omega_0+cp)t\right)\\ \cos\left(px+(\omega_0-cp)t\right)\end{pmatrix}$$

$$\Psi_2=\begin{pmatrix}U_2\\D_2\end{pmatrix}=\begin{pmatrix}\cos\left(px-(\omega_0+cp)t\right)\\ \sin\left(px-(\omega_0-cp)t\right)\end{pmatrix}$$

$$\Psi_3=\begin{pmatrix}U_3\\D_3\end{pmatrix}=\begin{pmatrix}\sin\left(px+(\omega_0-cp)t\right)\\ \cos\left(px+(\omega_0+cp)t\right)\end{pmatrix}$$

Here $p\equiv\frac{p_x}{\hbar}$, where p_x is the x component of the momentum. When boosted to a rest system (where $\partial_x=0$) we obtain for all components

$$\left[\partial_t^2+\left(\frac{mc^2}{\hbar}\right)^2\right]\Psi_i=0 \tag{18}$$

Solving this equation by setting $\Psi=\cos(\omega t)$ or $\Psi=\sin(\omega t)$ shows that all components of this fermion at the rest frame, are oscillating at a rate given by $\omega_0=\frac{mc^2}{\hbar}$. For an electron $\omega_0\approx 7.7\times 10^{11}\ \text{GHz}(\approx 0.512\ \text{MeV})$.

There are two particle states involved. One, denoted by (+) precessing around the X axis in a positive right direction, and the other, denoted by (−), precessing around the X axis in the opposite (left) direction:

$\theta_+(t)=\omega_0 t$ a right-rotation and $\theta_-(t)=-\omega_0 t$ a left rotation, with $\theta_+(t)=-\theta_-(t)$

At a boosted system, where $p=0$: $\Psi_1=\begin{bmatrix}\cos(\theta_+(t))\\ \sin(\theta_-(t))\end{bmatrix}$ and $\Psi_4=\begin{bmatrix}\sin(\theta_+(t))\\ \cos(\theta_+(t))\end{bmatrix}$

As will be shown next, a fermion is actually a mixture of both Ψ_1 and Ψ_4. We denote this as (Ψ_1,Ψ_4).

Likewise, an anti-fermion is a mixture of both Ψ_2 and Ψ_3, denoted as (Ψ_2,Ψ_3), where $\Psi_2=\begin{bmatrix}\cos(\theta_+(t))\\ \sin(\theta_-(t))\end{bmatrix}$ and

$$\Psi_3=\begin{bmatrix}\sin(\theta_+(t))\\ \cos(\theta_+(t))\end{bmatrix}$$

6. Spin up and Spin down

We will use now the above results for a boosted system, namely, one that moves with the massive particle.

Define u^{UP}, u^{DN}, and v^{UP}, v^{DN} 2-vectors as follows:

$$u^{UP}=\frac{1}{2}(\Psi_1+\sigma_x\Psi_4)=\begin{bmatrix}\cos(\theta_+(t))\\ 0\end{bmatrix} \tag{19}$$

$$u^{DN}=\frac{1}{2}(\Psi_1-\sigma_x\Psi_4)=\begin{bmatrix}0\\ \sin(\theta_-(t))\end{bmatrix} \tag{20}$$

$$v^{UP}=\frac{1}{2}(\Psi_2+\sigma_x\Psi_3)=\begin{bmatrix}\cos(\theta_+(t))\\ 0\end{bmatrix} \tag{21}$$

$$v^{DN}=\frac{1}{2}(\Psi_2-\sigma_x\Psi_3)=\begin{bmatrix}0\\ \sin(\theta_-(t))\end{bmatrix} \tag{22}$$

To find their spin, we apply σ_z upon each of the states, to find ($\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, $\sigma_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$):,

$$\sigma_z u^{UP}=+1u^{UP} \tag{23}$$

$$\sigma_z u^{DN}=-1u^{DN} \tag{24}$$

$$\sigma_z v^{UP}=+1v^{UP} \tag{25}$$

$$\sigma_z v^{DN}=-1v^{DN} \tag{26}$$

These represent two eigenfunctions of spin-up and spin-down for the fermion u and two eigenfunctions of spin up and spin down for the anti-fermion v solution.

So far we have shown the following:

Every fermion is composed of 2 generic real fields Ψ_1 and Ψ_4 while every anti-fermion is similarly composed of 2 generic real fields Ψ_2 and Ψ_3

Each 2-component fields (Ψ_1, Ψ_4) and (Ψ_2, Ψ_3)) are coupled via the operator σ_x in such a manner, that two spin-states are created:

$u^{UP} = \frac{1}{2}(\Psi_1 + \sigma_x \Psi_4)$ representing a spin-up fermion

$u^{DN} = \frac{1}{2}(\Psi_1 - \sigma_x \Psi_4)$ representing a spin-down fermion

$v^{UP} = \frac{1}{2}(\Psi_2 + \sigma_x \Psi_3)$ representing a spin-up anti-fermion

$v^{DN} = \frac{1}{2}(\Psi_2 - \sigma_x \Psi_3)$ representing a spin-down anti-fermion

Equivalently,

$$\Psi_1 = u^{UP} + u^{DN}$$

$$\Psi_4 = \sigma_x \left(u^{UP} - u^{DN}\right)$$

$$\Psi_2 = v^{UP} + v^{DN}$$

$$\Psi_3 = \sigma_x \left(v^{UP} - v^{DN}\right)$$

The interaction energy between the fermion and the magnetic field $\vec{B}$ is given by $\Delta H = \vec{\mu} \cdot \vec{B} = \frac{1}{2}\hbar g_f B_z$ where g_f is the gyromagnetic factor of the fermion.

Under a magnetic field $\vec{B}_0 = \hat{k} B_z$ the change in energies of the above states is given by

$$\Delta H u^{UP} = +\frac{1}{2}\hbar g_f B_z u^{UP} \tag{27}$$

$$\Delta H u^{DN} = -\frac{1}{2}\hbar g_f B_z u^{DN} \tag{28}$$

$$\Delta H v^{UP} = +\frac{1}{2}\hbar g_f B_z v^{UP} \tag{29}$$

$$\Delta H v^{DN} = -\frac{1}{2}\hbar g_f B_z v^{DN} \tag{30}$$

This demonstrates, that in the presence of a magnetic field, there exist two spin states (up and down) for u and two spin states (up and down) for v. It is the energy difference between the two spin states that appears in the form of deflection in their paths and the split in the beam of incoming particles in proportion with the square of their incoming speed, the strength of the applied magnetic field and the constants $\hbar$ and g_f.

The energy difference between two states is given by

$$\tilde{u}^{UP} \Delta H u^{UP} - \tilde{u}^{DN} \Delta H u^{DN} = +\frac{1}{2}\hbar g_f B_z \cos^2(\theta) + \frac{1}{2}\hbar g_f B_z \sin^2(\theta) = \hbar g_f B_z \tag{31}$$

The energy difference between the two states is independent of time and of location along the x-axis.

The two states are time-dependent and position-dependent, yet their spin does not change with time and in space:

$$u^{UP} = \begin{bmatrix} \cos\left(\theta_+(t)\right) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\left((\omega_0 - pc)t\right) \\ 0 \end{bmatrix} \quad (32)$$

$$u^{DN} = \begin{bmatrix} 0 \\ \sin\left(\theta_-(t)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\left((\omega_0 - pc)t\right) \end{bmatrix} \quad (33)$$

$$v^{UP} = \begin{bmatrix} \cos\left(\theta_+(t)\right) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\left((\omega_0 - pc)t\right) \\ 0 \end{bmatrix} \quad (34)$$

$$v^{DN} = \begin{bmatrix} 0 \\ \sin\left(\theta_-(t)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\left((\omega_0 - pc)t\right) \end{bmatrix} \quad (35)$$

One can see again that

$$\Psi_1 = \begin{bmatrix} \cos\left(\theta_+(t)\right) \\ \sin\left(\theta_-(t)\right) \end{bmatrix} = u^{UP} + u^{DN}$$

$$\Psi_4 = \begin{bmatrix} \sin\left(\theta_+(t)\right) \\ \cos\left(\theta_+(t)\right) \end{bmatrix} = \sigma_x\left(u^{UP} - u^{DN}\right)$$

Also,

$$\sigma_z \Psi_1 = u^{UP} - u^{DN}$$

$$\sigma_z \Psi_4 = -\sigma_x\left(u^{UP} + u^{DN}\right)$$

$$\sigma_x \Psi_4 = u^{UP} - u^{DN}$$

$$\sigma_x \Psi_4 = \sigma_z \Psi_1$$

$$\sigma_z \Psi_{1`} = \sigma_z\left(u^{UP} + u^{DN}\right) = u^{UP} - u^{DN} = \sigma_x \Psi_4$$

These relations lead to:

$$\sigma_z \Psi_1 = \sigma_x \Psi_4$$

$$\sigma_z \Psi_4 = -\sigma_x \Psi_1$$

$$\sigma_x \Psi_1 = \Psi_4$$

We notice, that $\sigma_x u^{UP}$ behaves like a spin-**down** state, while $\sigma_x u^{DN}$ behaves like a spin-**up** state. Indeed, $\sigma_z\left(\sigma_x u^{UP}\right) = -\sigma_x\sigma_z u^{UP} = -\left(\sigma_x u^{UP}\right)$ and $\sigma_z\left(\sigma_x u^{DN}\right) = -\sigma_x\sigma_z u^{DN} = +\left(\sigma_x u^{DN}\right)$. (Notice though that $\sigma_x u^{DN}$ and u^{UP} may differ by a phase change between sin and cos, and so do $\sigma_x u^{UP}$ and u^{DN}). Thus, $\sigma_x u^{DN} = u'^{UP}$ and $\sigma_x u^{UP} = u'^{DN}$.

7. A Gedanken Stern-Gerlach Experiment

In view of the above relations, we evaluate now the Stern-Gerlach triple slit results.

In a Stern-Gerlach experiment (Figure 1), a beam of fermions enters the system. Any entering beam of fermions Ψ_{in} is considered to be aa mixture of spin-up and spin down, determined by two hidden variables (constants) $c_1 \geq 0$ and

$c_4 \geq 0$ and with the probabilistic demand $c_1^2 + c_4^2 = 1$).

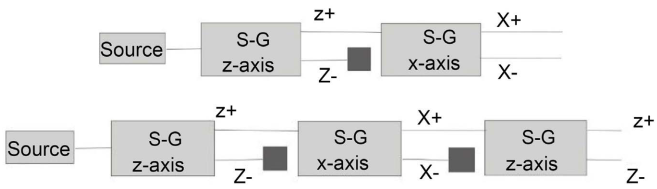

Figure 1. A Stern-Gerlach (S-G) experiment with deflection in the X direction. Spin-up splits into spin-up and spin-down in the X direction. The spin-up in the X direction then splits again into spin-up and spin-down in the Z direction. This shows that a spin-up electron will always split by an external magnetic field into two beams according to the magnetic field direction. So, the applied magnetic field treats a spin-up as if it is made of entangled spin-up and spin-down particles u^{UP} and u^{DN}.

$$\Psi_{in} = c_1\Psi_1 + c_4\Psi_4$$

The source emits fermions, that is a mixture of spin-up and spin-down particles. One cannot know the state of each of the incoming beam of particles. They are in a mixed state: a mixture of up and down spins. Only a measurement by application of an external interacting magnetic fields will be able to differentiate between the states.

The output of the third apparatus which measures the deflection on the z axis again shows an output of z− as well as z+. Given that the input to the second S-G apparatus consisted only of z+, it can be inferred that a S-G apparatus must be altering the states of the particles that pass through it.

If we write

$$\Psi_1 = \begin{bmatrix} \cos(\theta_+(t)) \\ \sin(\theta_-(t)) \end{bmatrix} = u^{UP} + u^{DN}$$

$$\Psi_4 = \begin{bmatrix} \sin(\theta_+(t)) \\ \cos(\theta_+(t)) \end{bmatrix} = \sigma_x\left(u^{UP} - u^{DN}\right)$$

One can then describe the entering fermion Ψ_{in} as a mixture of both Ψ_1 and Ψ_4.

$$\Psi_{in} = c_1\Psi_1 + c_4\Psi_4 = \left(c_1 + c_4\sigma_x\right)u^{UP} + \left(c_1 - c_4\sigma_x\right)u^{DN}$$

Applying a magnetic field in the +z gives:

$$\sigma_z\Psi_{in} = \left(c_1 - c_4\sigma_x\right)u^{UP} - \left(c_1 + c_4\sigma_x\right)u^{DN}$$

In case we block the down spins beam, and then apply the B_x field we obtain

$$\sigma_x\Psi_{bloked} = \sigma_x\left(c_1 - c_4\sigma_x\right)u^{UP} = c_1\sigma_x u^{UP} - c_4 u^{UP}$$

And since $\sigma_x u^{UP}$ behaves like a spin-**down** state, we replace $\sigma_x u^{UP}$ by u'^{DN} to obtain

$$\sigma_x\Psi_{bloked} = c_1 u'^{DN} - c_4 u^{UP}$$

which is, surprisingly, again a mixture of spin-up and spin-down particles.

Thus, even though we blocked the spin-down particles, the new interaction with B_x, **recreates a mixed beam** of spin-up and spin-down particles.

If the incoming beam is made of fermions with spin-up only (no spin-down fermions, $c_4 = 0$, $c_1 = 1$)

$$\Psi_{fermions} = c_1 \Psi_1 = u^{UP} + u^{DN}$$

$$\sigma_z \Psi_{fermions} = u^{UP} - u^{DN}$$

And after blocking the down beam and applying the B_x field we obtain

$$\sigma_x \sigma_z \Psi_{fermions} = \sigma_x u^{UP}$$

This time there is no mixture of states, just an inversion from up to down.

This suggests a method to test whether an incoming beam is pure or a mixture of states.

As seen above, the concept of a fermion of given spin is wrong. A fermion is neither a Ψ_1 nor a Ψ_4 particle. Rather, it is a coupled pair (Ψ_1, Ψ_4) that makes a spin-up and spin-down mixture.

The solutions to Dirac equation for a free fermion provide Ψ_1 or Ψ_4 as free fermions (Ψ_2 or Ψ_3 as free anti-fermions).

In fact

$$\Psi_1 = u^{UP} + u^{DN}$$

$$\Psi_4 = \sigma_x \left(u^{UP} - u^{DN} \right)$$

with

$u^{UP} = \frac{1}{2}(\Psi_1 + \sigma_x \Psi_4)$ representing **a spin-up fermion**.

$u^{DN} = \frac{1}{2}(\Psi_1 - \sigma_x \Psi_4)$ representing **a spin-down fermion**.

Similarly for anti-fermions:

$$\Psi_2 = v^{UP} + v^{DN}$$

$$\Psi_3 = \sigma_x \left(v^{UP} - v^{DN} \right)$$

with

$v^{UP} = \frac{1}{2}(\Psi_2 + \sigma_x \Psi_3)$ **a spin-up anti-fermion**.

$v^{DN} = \frac{1}{2}(\Psi_2 - \sigma_x \Psi_3)$ **a spin-down anti-fermion**.

One cannot discuss fermions with spin-up or spin-down. They exist as a mixture, where their spins appear only under interaction with an external magnetic field.

8. Quantum Entanglement

Quantum entanglement [10] is a phenomenon where a group of particles being generated, interacting, or sharing spatial proximity in such a way that the quantum state of each particle of the group cannot be described independently of the state of the others, including when the particles are separated by a large distance. There are therefore two beams to consider A and B (Alice and Bob) which emerge from a single source simultaneously.

In the following, we will show, that entanglement is a result of the internal structure of the fermions and two unknown **hidden variables**.

Fermions, according to this description are made of two stringlike particles, strongly connected to each other in a non-linear form.

It will be shown, that all fermions may be viewed as made of two coupled fields.

The Dirac equation has 4 real solutions. Two fermions and two anti-fermions. Each of them is made of two real fields. The fields are coupled together in a non-linear form creating two possible states, one with spin-up and one with spin-down.

9. Incoming Beam

What is the ratio between up and down spins in the incoming beam?

The incoming beam (A or B) is written as

$$\Psi_{in} = c_1\Psi_1 + c_4\Psi_4 = (c_1 + c_4\sigma_x)u^{UP} + (c_1 - c_4\sigma_x)u^{DN}$$
$$= (c_1 - c_4)u^{UP} + (c_1 + c_4)u^{DN}$$

The square of its components represents the average number of particles. Obviously then the ratio is given by (recall that $c_1^2 + c_4^2 = 1$):

$$\frac{N_{UP}}{N_{DN}} = \frac{1 - 2c_1\sqrt{1 - c_1^2}}{1 + 2c_1\sqrt{1 - c_1^2}}$$ which is 1 for $c_1 = 0,1$ only.

Therefore, for either $c_1 = 0$. Or $c_4 = 0$ the beams of the entangled particles have equal amounts (50%) of spin-up and (50%) of spin-down particles.The two hidden parameters c_1 and c_4, force the correlation between spin-up and spin down in the two entangled beams. So, if at A the spin measured is up, then the other beam must have a similar particle but down. Otherwise, the numbers will not be equal.

At these two cases, the incoming beams will be either $\Psi_{in} = \Psi_1$, or, $\Psi_{in} = \Psi_4$

However, if $\Psi_{in} = \Psi_1$, then $\sigma_z\Psi_{in} = \sigma_z\Psi_1 = \sigma_x\Psi_4 = \sigma_x\sigma_x\Psi_1 = \Psi_1$. This means $\sigma_z\Psi_1 = \Psi_1$.

If $\Psi_{in} = \Psi_4$, then $\sigma_z\Psi_{in} = \sigma_z\Psi_4 = -\sigma_x\Psi_1 = -\Psi_4$. Namely, $\sigma_z\Psi_4 = -\Psi_4$.

Therefore,

$$\sigma_z\Psi_{in} = \mp\Psi_{in}$$

Under these special circumstances, where either $c_1 = 0$ or $c_4 = 0$, Ψ_{in} behaves like either spin-up, or, spin-down.

In other words, when the beams of the entangled particles is made of equal amounts (50%) of spin-up and (50%) of spin-down particles, the combined entangled beam behaves like either a spin-up or a spin down particle. Obviously, if one is measured to be a spin-up, the other must be a spin-down. As a matter of fact, the actual process of creation of entangled pairs assures this equal distribution of spins.

10. Violation of Bell's Inequality

Local hidden variable theories fail, when measurements of the spin of entangled particles along different axes are considered. If a large number of pairs of such measurements are made (on a large number of pairs of entangled particles), then statistically, if the local realist or hidden variables view were correct, the results would always satisfy Bell's inequality [2] [3]. A number of experiments have shown in practice that Bell's inequality is not satisfied. However, prior to 2015, all of these experiments had loophole problems that were considered the most important by the community of physicists. When measurements of the entangled particles are made in moving relativistic reference frames, in which each measurement (in its own relativistic time frame) occurs before the other, the measurement results remain correlated. More and more experimental results [8] [9] demonstrate the violation of Bell's inequality.

The fundamental issue about measuring spin along different axes is that these measurements cannot have definite values at the same time—they are incompatible in the sense that these measurements maximum simultaneous precision is constrained by the uncertainty principle. This is contrary to what is found in classical physics, where any number of properties can be measured simultaneously with arbitrary accuracy. It has been proven mathematically that compatible measurements cannot show Bell-inequality-violating correlations, and thus entanglement is a fundamentally non-classical phenomenon.

In the following, we will test such a Gedanken experiment, where a large number of pairs of entangled particles are measured statistically. We will test whether the results satisfy Bell's inequality. (e.g. Bell [2] [3] and Mirmin [6]).

Applying a magnetic field in an arbitrary angle θ with respect to the Z axis, we define the following σ_θ operator

$$\sigma_\theta = \cos\theta\sigma_z + \sin\theta\sigma_x$$

One can use the σ_θ operator, repeatedly at different angles to obtain

$$\sigma_\theta\Psi_{in} = \sigma_x\left[\left(c_1\cos\theta + c_4\sin\theta\right)\Psi_4 + \left(c_1\sin\theta - c_4\cos\theta\right)\Psi_1\right]$$

The Bell Mermin suggestion is the repeated application of the filter at 0˚, 135˚ and 225˚ degrees with respect to the X-axi

$$\sigma_0\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[c_1\sqrt{2}\Psi_4 - c_4\sqrt{2}\Psi_1\right]$$

$$\sigma_{180-45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[\left(c_4 - c_1\right)\Psi_4 + \left(c_4 + c_1\right)\Psi_1\right]$$

$$\sigma_{180+45}\Psi_{in} = \sigma_x\frac{\sqrt{2}}{2}\left[\left(-c_1 - c_4\right)\Psi_4 + \left(c_4 - c_1\right)\Psi_1\right]$$

This can be rewritten as:

$$\phi_1 = A\Psi_1 + a\Psi_4$$

$$\phi_2 = B\Psi_1 + b\Psi_4$$

$$\phi_3 = C\Psi_1 + c\Psi_4$$

And the statistical combination over all incoming filtered beams will result in summing over the 27 possible combinations of A B and C. There are 27 possible combinations of triads such as AAA ABC etc. for the Ψ_1 part, and likewise 27 triads for the Ψ_4 part (such as aaa, aab etc.).

Counting all the possible combinations and summing, results in

$$27A + 27B + 27C = 27(A + B + C) = 27\frac{\sqrt{2}}{2}\left(c_1\sqrt{2} + 2c_4 - 2c_1\right)$$

So, in 1 out of 27^2 experiments, the average filtered beam $\langle\phi\rangle$ will be

$$\frac{1}{27}\sigma_x\frac{\sqrt{2}}{2}\left[\left(c_1\sqrt{2} + 2c_4 - 2c_1\right)\Psi_1 + \left(2 - \sqrt{2}\right)c_4\Psi_4\right]$$

Since $\sigma_x\Psi_1 = \Psi_4$ and $\sigma_x\Psi_4 = \Psi_1$ this can be written as

$$\langle\phi\rangle = \frac{\sqrt{2}}{54}\left[\left(c_1\sqrt{2} + 2c_4 - 2c_1\right)\Psi_4 + \left(2 - \sqrt{2}\right)c_4\Psi_1\right]$$

Since we are interested in counting the average number of spin-up and spin down in the beam we substitute Ψ_1 Ψ_4 with u^{UP} and u^{DN} using

$$\Psi_1 = u^{UP} + u^{DN}$$

$$\Psi_4 = \sigma_x\left(u^{UP} - u^{DN}\right)$$

And after rearranging terms

$$\langle\phi\rangle = \frac{\sqrt{2}}{54}\left\{\left(c_1\left(\sqrt{2} - 2\right) + \left(4 - \sqrt{2}\right)c_4\right)u^{DN} - \left(c_1\left(\sqrt{2} - 2\right) + \sqrt{2}c_4\right)u^{UP}\right\}$$

The expected ratio between spin-up to spin-down is then

$$\frac{N_{UP}}{N_{DN}} = \frac{c_1\left(\sqrt{2} - 2\right) + \sqrt{2}c_4}{c_1\left(\sqrt{2} - 2\right) + \left(4 - \sqrt{2}\right)c_4}$$

For c_1 between 0 to 1:

$$55\% \le \frac{N_{UP}}{N_{DN}} \le 100\%$$

Therefore, the number of spin-ups will never exceed the number of spin-downs, and it will occur at least 55% of the times. They will be equal only when $c_1 = 1$, namely, the incoming beam is made of equal amounts of spin-ups and spin downs.

If $c_1 = 1$ and $c_4 = 0$ then $\frac{N_{UP}}{N_{DN}} = 1$.

If $c_1 = 0$ and $c_4 = 1$ then $\frac{N_{UP}}{N_{DN}} = 0.55$.

11. A Comment about the Violation of Heisenberg's Uncertainty

What happens in the situation when Bob measures a given spin in the +Z say spin-up) and Alice measures in the +X?

Alice will be notified by Bob of his spin up outcome in the +Z and will then know her result in the +Z (spin-down) together with measured spin in the +X direction. This supposedly creates a violation of the Heisenberg's uncertainty rule.

However, one must be careful with the defining restriction of Heisenberg inequality. It prohibits the simultaneous measurement of two conjugate pairs, such as spin in the z and x directions.

It states that there is a limit to the precision to which conjugate pairs of physical properties, such as spin in two conjugate directions, can be simultaneously measured.

Notice however, that **information≠ measurement**. Therefore, there is no violation of Heisenberg's uncertainty principle and locality is not violated.

12. A 2-String Analog to the Schrödinger Equation

To further emphasize the justification of the two strings description of fermions, we repeat here the already described theoretical explanation of two strings coupling [9] of two real coupled fields.

Let a single particle of mass m be described by two classical real strings, $\varphi_1(x,t)$ and $\varphi_2(x,t)$. Here, the functions, $\varphi_1(x,t)$ and $\varphi_2(x,t)$ represent the amplitudes of the perturbation of the strings from the x-axis as a function of time and position.

Assume coupling between these two strings, given by a constant k_s, and described by the following coupled differential equations:

$$\frac{\partial \varphi_1}{\partial t} = +k_s \frac{\partial^2 \varphi_2}{\partial x^2} \tag{36}$$

$$\frac{\partial \varphi_2}{\partial t} = -k_s \frac{\partial^2 \varphi_1}{\partial x^2} \tag{37}$$

It seems to have a peculiar behavior, where the spatial curvature in one field, affects the change in time of the second field and vice a versa.

A physical interpretation to these two equations is the following interaction model. Consider two strings φ_1 and φ_2. Let $\varphi_1(x,t)$ represent the amplitude of string 1 at time *t* and at position *x*. Let τ_s be some tension force acting in the string.

Assuming next:

$$\frac{1}{k_{0s}} \frac{\partial \tau_s}{\partial t} = -\frac{\hbar}{2} \tag{38}$$

The above coupled equations now read

$$\frac{\partial \varphi_1}{\partial t} = +\frac{\hbar}{2m} \frac{\partial^2 \varphi_2}{\partial x^2} \tag{39}$$

$$\frac{\partial \varphi_2}{\partial t} = -\frac{\hbar}{2m} \frac{\partial^2 \varphi_1}{\partial x^2} \tag{40}$$

which is the coupled real presentation of Schrödinger Equation.

This gives a physical meaning to the Planck constant, namely, independent of a particle's mass, the Planck constant $\hbar$ is derived from the internal features of the strings (real fields). It represents somehow the reaction of the internal tensions in the strings to perturbations. Up to a proportionality constant, one can write $\hbar \approx -\frac{1}{k_{0s}(t)}\frac{\partial \tau_s}{\partial t}$ (In order for this equation to make sense, k_{0s} must be a time-dependent variable).

This leads to the conclusion:

$$\tau_s(t) = -\hbar \int k_{0s}(t)\,\mathrm{d}t \tag{41}$$

k_{0s} has the dimensions of 1/sec, so we may assign it the meaning of the number of exchanges (interacting particles) per second, between the two strings.

In other words, $\tau_s(t)$ is the total number of exchanged particles and it is proportional to $\hbar$. The minimal possible tension in a string will be $\hbar$ (a single exchange).

So, the tension in the strings is proportional to the Planck constant $\hbar$, and to the basic coupling between the two strings.

Based on the following assumptions:

1) A Classical Fermion is made up of two interacting string-like entities (hypergluons).

2) Tension in the strings is proportional to the coupling strength between the two strings.

3) The coupling force between the two strings is assumed to be the result of preons exchange between these massless bosons.

4) The force is proportional to the duration of the exchange (the actual number of exchanged preons per second).

One is lead to conclude, that Planck's constant $\hbar$, is the proportionality constant, between the total exchange between the two strings, and the tension in these strings.

This interpretation of a particle as made up of two real coupled strings, which tensions and interaction are connected, is equivalent to a single particle complex wave function, described by Schrödinger Equation.

When the original Schrödinger Equation is used with complex wave function, this internal string-like characteristic is not showing because we treat the real and imaginary parts as a single entity. However, when the Equation is separated into two parts, these two parts can be treated as independent strings interacting with each other where both tension and interaction fall off abruptly inversely proportional to time. This may be a result of weakening due to increased distance between the two strings, together with tension drop inside the strings.

13. Conclusions

By making all quantum mechanics formulation real, we show that both Schrödinger and Dirac equations can be considered as representing real coupled entities. These entities represent two interacting strings (non-relativistic Schrödinger equation) or as 4 interacting double strings (relativistic Dirac Equation).

The above analysis shows, that there are hidden variables involved in the internal structure of any fermion. Any fermion will have its mass being the result of the internal interaction force between its components and the Planck constant.

These hidden variables play an important role in quantum entanglement.

The picture successfully describes interference. It suggests an interaction mechanism between the two strings which relates to Planck's constant. Moreover, it describes the spins of fermions as a result of two hidden parameters which dictates the outcome of Stern-Gerlach experiments. It also explains the violation of Bell's inequality in all quantum entanglement experiments.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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