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Classical Cosmology III. Modified Tired Light and Distance Modulus for Supernovae

Lorenzo Zaninetti

Physics Department, University of Turin, Turin, Italy
Email: l.zaninetti@alice.it

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Abstract

We analyze a simple model for tired light in a cosmological environment, a generalized model, and a spectroscopic model. The three models are tested on different compilations for the distance modulus of supernovae. The tests are negative for the simple tired light and the spectroscopic models, but positive for the generalized tired light model. The percentage error of the distance modulus for the generalized tired light model compared with the distance modulus of standard cosmology is less than one percent over the considered ranges in redshift.

Keywords

Galaxy Groups, Large Scale Structure of the Universe Cosmology

1. Introduction

The increase of the redshift with distance for the galaxies is usually explained by Hubble's law, stated in 1929 [1]:

$$v = H_0 x \frac{\mathrm{km}}{\mathrm{Mpc}}, \tag{1}$$

where v is the velocity of the expansion of the universe, H_0 the Hubble constant, and x the distance in Mpc. In the same year, a gravitational explanation for the redshift was also introduced, by Zwicky [2] and the formula for the change of frequency of light in a gravitational framework is

$$\frac{\Delta\nu}{\nu} = \frac{1.4\pi G\rho DL}{c^2}. \tag{2}$$

Here, ν is the frequency, G is the Newtonian gravitational constant, ρ is the density in g/cm^3, D is the distance after which the perturbing effect begins to fade out, L is the distance, and c is the speed of light. The above formula can be considered the first alternative to the Doppler effect and the reader interested in the evolution of the tired light hypothesis over the years 1929-1939 can read a review by Krag in 2017 [3]. We now report some recent approaches to the tired light hypothesis. A comparison of the Hubble diagram calculated from the observed redshift data of 280 Supernovae (SNs) in the range of $z = 0.0104$ to 8.1 with Hubble diagrams derived from the exponential tired light and the ΛCDM cosmology was made by Marosi in 2014 [4]. A standard model of tired light was assumed by LaViolette in 2021 [5] in which the photon loses energy during its travel according to

$$E(r) = E_0 \exp(-\beta r), \tag{3}$$

where β is a coefficient of energy attenuation and r is the distance.

The New Tired Light (NTL) hypothesis was developed by Ashmore in 2022 and has equations

$$H_0 = \frac{2n_e h r_e}{m_e}, \tag{4}$$

$$z = \exp\left(\frac{H_0 * d}{c}\right) - 1, \tag{5}$$

where z is the redshift, n_e is the number density of matter, h is Planck's constant, r_e is the classical radius of the electron, m_e is the mass of the electron, c is the speed of light, d is the distance and H_0 is the Hubble constant; see Equations (13) and (14) in [6]. The dichotomy between standard cosmology and tired light was analyzed in 2022 by Premović [7]. Some questions are still to be solved.

1) Have all the options for tired light really been explored?

2) What are the differences in terms of percentage error for the distance modulus of SNs in the case of tired light versus standard cosmology?

In order to answer the above questions, Section 2 analyses three models of tired light and Section 3 reviews the standard cosmology and the adopted statistics. Section 4 contains the results for the three new formulae applied to three compilations for the distance modulus of supernovae.

2. Tired Light

In this section, we present the simple tired light model, the generalized tired light model, and a model from spectroscopy.

2.1. The Simple Tired Light Model

We assume that the frequency ν of a photon which travels through intergalactic space decreases according to the following ordinary differential equation (ODE), called the Beer-Lambert law, after [3] [8] [9],

$$\frac{\mathrm{d}}{\mathrm{d}x}\nu(x) = -k n_e \nu(x), \tag{6}$$

where n_e is the number density of matter in $1/\mathrm{m}^3$ and k an attenuation coefficient in m². This ODE is solved while assuming the initial condition $\nu(0)=\nu_0$

$$\nu(x)=\nu_0 e^{-kn_e x}. \tag{7}$$

We now define the redshift as a function of the wavelength λ and the initial wavelength λ_0 by

$$z=\frac{\lambda-\lambda_0}{\lambda_0}. \tag{8}$$

Making use of

$$\nu\lambda=c, \tag{9}$$

where c is the speed of light, we obtain

$$z=e^{kn_e x}-1, \tag{10}$$

or

$$\ln(z+1)=kn_e x. \tag{11}$$

A Taylor expansion around $z=0$ gives

$$z=kn_e x, \tag{12}$$

and the expansion velocity, v, in the radial Doppler framework can be obtained by multiplying both sides by c,

$$v=kn_e xc. \tag{13}$$

The above equation is expressed in terms of Hubble's Law [1]

$$v=H_0 x\frac{\text{km}}{\text{Mpc}}, \tag{14}$$

which gives

$$H_0=kn_e c. \tag{15}$$

The distance modulus for tired light is

$$(m-M)=25+\frac{5\ln\left(\frac{\ln(z+1)c}{H_0}\right)}{\ln(10)}. \tag{16}$$

2.2. The Generalized Tired Light Model

We assume that the frequency ν of a photon decreases according to the following non-linear law

$$\frac{\mathrm{d}}{\mathrm{d}x}\nu(x)=-an_e\nu(x)^{\alpha}, \tag{17}$$

where n_e is the number density of matter in 1/m³ and a the attenuation coefficient in $\text{Hz}^{1-\alpha}\cdot\text{m}^2$. We call the above ODE the generalized tired light (GTL) model. Imposing the initial condition $\nu(0)=\nu_0$, the solution is

$$\nu(x)=\frac{1}{\left(an_e x\alpha-an_e x+e^{-\alpha\ln(\nu_0)}\nu_0\right)^{\frac{1}{-1+\alpha}}}. \tag{18}$$

We now continue inserting

$$a = \frac{\nu_0^{1-\alpha} H_0}{c n_e}, \tag{19}$$

and as a consequence the redshift is

$$z = c^{-\frac{1}{-1+\alpha}} \left(c + \left(-1 + \alpha \right) x H_0 \right)^{\frac{1}{-1+\alpha}} - 1. \tag{20}$$

The inversion of the above formula gives

$$x = -\frac{\left(-\left(z+1 \right)^{\alpha} + z + 1 \right) c}{H_0 \left(-1 + \alpha \right) \left(z+1 \right)}, \tag{21}$$

and the distance modulus in GTL is

$$\left(m - M \right) = 25 + \frac{5 \ln \left(-\frac{\left(-\left(z+1 \right)^{\alpha} + z + 1 \right) c}{H_0 \left(-1 + \alpha \right) \left(z+1 \right)} \right)}{\ln \left(10 \right)}. \tag{22}$$

2.3. The Model from Spectroscopy

We now introduce a modification to the Bouguer-Lambert-Beer as suggested in Equation (1) of [10]

$$A = \log \left(\frac{I_{in}}{I_{out}} \right) = \epsilon C^{\alpha} x^{\beta}, \tag{23}$$

where A is the apparent absorbance, I_{in} the input light intensity, I_{out} the output light intensity transmitted through the sample, ϵ the effective specific absorbance, C the sample concentration, α a positive correction coefficient of the concentration, x the path length and β a positive correction coefficient of the path length. The frequency decreases in accordance with the following law

$$\nu = \nu_0 \mathrm{e}^{-\epsilon C^{\alpha} x^{\beta}}. \tag{24}$$

We now continue posing

$$\epsilon = \frac{H_0}{C^{\alpha} c}, \tag{25}$$

and the equation which defines the redshift is

$$\ln \left(z+1 \right) = \frac{H_0 x^{\beta}}{c}. \tag{26}$$

The above equation can be inverted in order to give the distance

$$x = \ln \left(z+1 \right)^{\frac{1}{\beta}} c^{\frac{1}{\beta}} H_0^{-\frac{1}{\beta}}, \tag{27}$$

and therefore the distance modulus for the spectroscopic model is

$$\left(m - M \right) = \ln \left(z+1 \right)^{\frac{1}{\beta}} c^{\frac{1}{\beta}} H_0^{-\frac{1}{\beta}}. \tag{28}$$

3. Preliminaries

In this section we review the standard cosmology and the adopted statistics.

3.1. The Standard Cosmology

In ΛCDM cosmology the *Hubble distance* D_{H} is defined as

$$D_{\mathrm{H}} \equiv \frac{c}{H_0}, \tag{29}$$

where c is the speed of light and H_0 is the Hubble constant. We then introduce the parameter Ω_{M}

$$\Omega_{\mathrm{M}} = \frac{8\pi G \rho_0}{3H_0^2}, \tag{30}$$

where G is the Newtonian gravitational constant and ρ_0 is the mass density at the present time. We now introduce another parameter, Ω_Λ:

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}, \tag{31}$$

where Λ is the cosmological constant, see [11]. Once Ω_Λ and H_0 are found the numerical value of the cosmological constant is derived, $\Lambda \approx 1.2 \frac{1}{\mathrm{m}^2}$.

The two previous parameters are connected with the curvature Ω_K by

$$\Omega_{\mathrm{M}} + \Omega_\Lambda + \Omega_K = 1. \tag{32}$$

The comoving distance, D_{C}, is

$$D_{\mathrm{C}} = D_{\mathrm{H}} \int_0^z \frac{\mathrm{d}z'}{E(z')}, \tag{33}$$

where $E(z)$ is the "Hubble function"

$$E(z) = \sqrt{\Omega_{\mathrm{M}}(1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda}. \tag{34}$$

Details on how to derive the distance modulus in ΛCDM cosmology through the Padé approximant can be found in formula (11) in [12].

3.2. The Adopted Statistics

In the case of the distance modulus, the merit function χ^2 is

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{(m-M)_i - (m-M)(z_i)_{th}}{\sigma_i} \right]^2, \tag{35}$$

where N is the number of SNs, $(m-M)_i$ is the observed distance modulus evaluated at a redshift of z_i, σ_i is the error in the observed distance modulus evaluated at z_i, and $(m-M)(z_i)_{th}$ is the theoretical distance modulus evaluated at z_i, see formula (15.5.5) in [13]. The reduced merit function χ^2_{red} is

$$\chi^2_{red} = \chi^2 / NF, \tag{36}$$

where $NF = N - k$ is the number of degrees of freedom, N is the number of SNs, and k is the number of free parameters. Another useful statistical parameter is the associated Q-value, which has to be understood as the maximum probability of obtaining a better fitting, see formula (15.2.12) in [13]:

$$Q = 1 - \mathrm{GAMMQ}\left(\frac{N-k}{2}, \frac{\chi^2}{2}\right), \tag{37}$$

where GAMMQ is a subroutine for the incomplete gamma function. The Akaike information criterion (AIC), see [14], is defined by

$$\mathrm{AIC} = 2k - 2\ln(L), \tag{38}$$

where L is the likelihood function. We assume a Gaussian distribution for the errors; then the likelihood function can be derived from the χ^2 statistic $L \propto \exp\left(-\frac{\chi^2}{2}\right)$ where χ^2 has been computed by Equation (35), see [15] [16]. Now the AIC becomes

$$\mathrm{AIC} = 2k + \chi^2. \tag{39}$$

The goodness of the approximation in evaluating a physical variable p is evaluated by the percentage error δ

$$\delta = \frac{\left|p - p_{approx}\right|}{p} \times 100, \tag{40}$$

where p_{approx} is an approximation of p.

4. Astrophysical Results

In this section we test the new formulae for the distance modulus of SNs on the three different compilations.

4.1. Union 2.1 Compilation

The first astronomical test we perform is on the 580 SNs of the Union 2.1 compilation, see [17], which is available at http://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt: in this compilation a calibrated distance versus redshift is provided. The cosmological parameters are given in **Table 1**.

Table 1. Numerical values of χ^2, χ^2_{red}, Q and AIC of the Hubble diagram for the Union 2.1 compilation: k stands for the number of parameters, H_0 is expressed in km·s^{-1}·Mpc^{-1}; 580 SNs.

cosmology	Equation	k	parameters	χ^2	χ^2_{red}	Q	AIC
ΛCDM	(11) [12]	3	$H_0 = (69.56 \pm 0.1)$; $\Omega_M = (0.238 \pm 0.01)$; $\Omega_\Lambda = (0.661 \pm 0.01)$	562.59	0.975	0.658	569.39
Tired light	(16)	1	$H_0 = (53.68 \pm 0.17)$	5187	8.95	0	5189
Generalized tired light	(22)	2	$H_0 = (68.54 \pm 0.32)$; $\alpha = 3.122$	585	1.01	0.41	589
Spectroscopic model	(28)	2	$H_0 = (150.52 \pm 2.17)$; $\beta = 0.84$	1419	2.45	0	1423

Figure 1 gives the best fit in the ΛCDM cosmology for the Union 2.1 compilation.

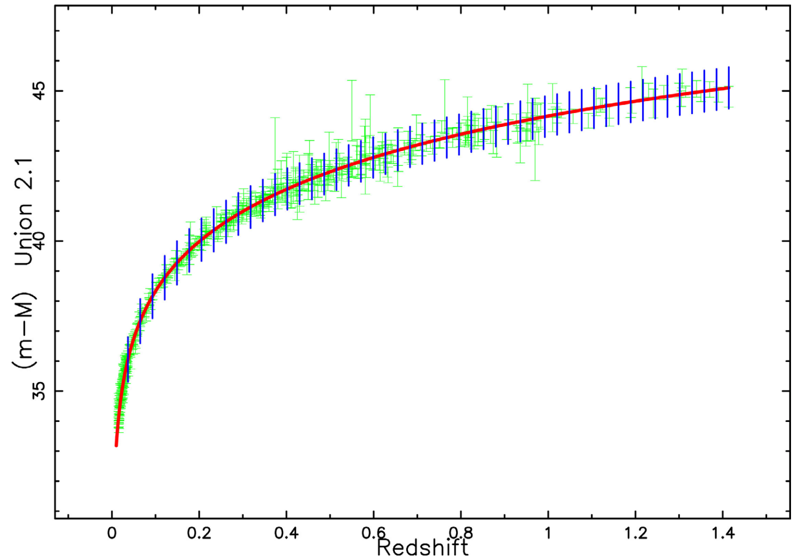

Figure 1. Hubble diagram for the Union 2.1 compilation, green points with error bars. The solid red line represents the best fit for the distance modulus in ΛCDM cosmology. The theoretical uncertainties are represented through green vertical lines by applying the law of errors of Gauss with the uncertainties and parameters as in the first line of Table 1.

Figure 2 compares the ΛCDM cosmology, GTL, and the spectroscopic model for the Union 2.1 compilation. The goodness of the distance modulus for the generalized tired light versus ΛCDM cosmology as function of the redshift can be evaluated with formula (40), see Figure 3.

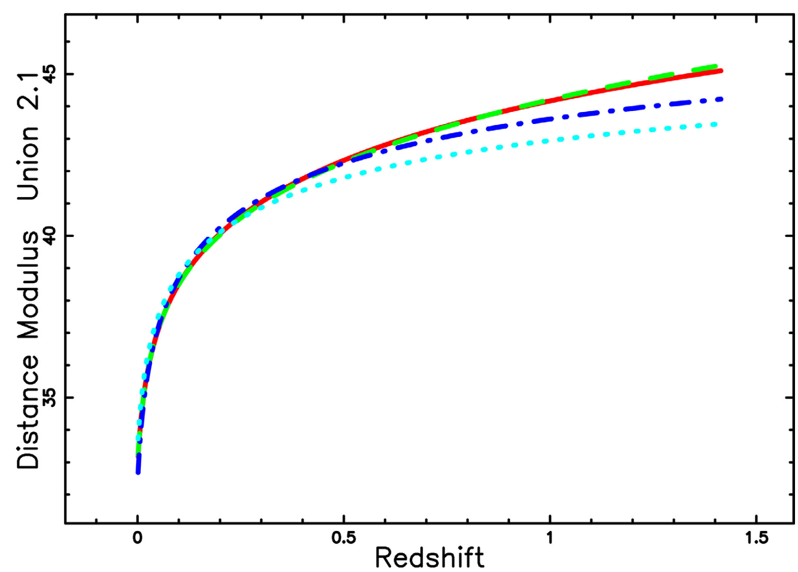

Figure 2. Best fit relative to the Union 2.1 compilation in ΛCDM cosmology (red full line), STL (cyan dotted line), GTL (green dashed line), and spectroscopic model (dash-point-dash line).

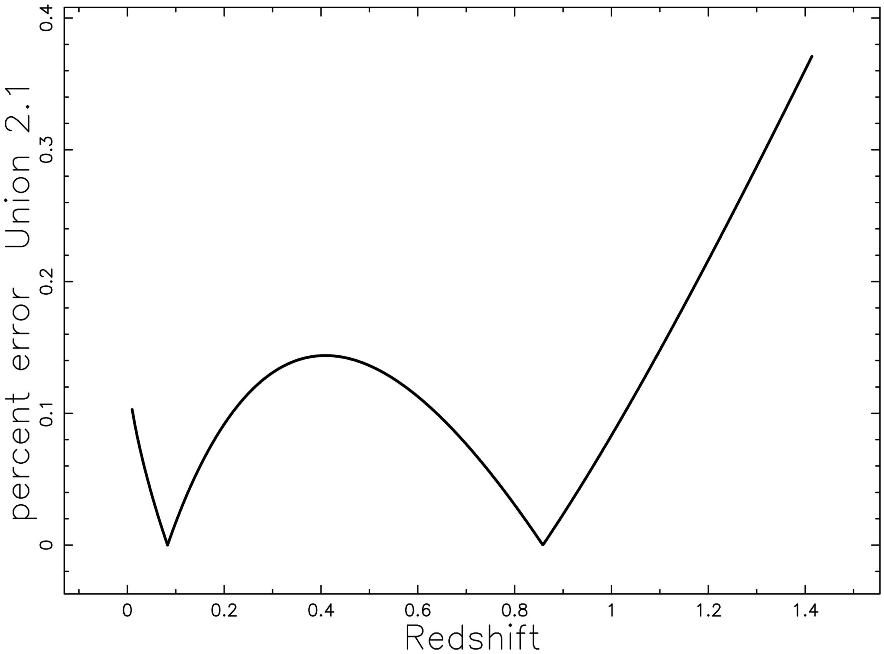

Figure 3. The percentage error between ΛCDM cosmology and GTL as a function of the redshift relative to the Union 2.1 compilation.

4.2. JLA Compilation

The second test we perform is on the the joint light-curve analysis (JLA), which contains 740 SNs [18] with data available on CDS at http://cdsweb.u-strasbg.fr/. The cosmological parameters with the JLA compilation are given in **Table 2**.

Table 2. Numerical values of χ^2, χ^2_{red}, Q and AIC of the Hubble diagram for the JLA compilation, k stands for the number of parameters, H_0 is expressed in km·s^{-1}·Mpc^{-1}; 740 SNs.

cosmology	Equation	k	parameters	χ^2	χ^2_{red}	Q	AIC
ΛCDM	(11) [12]	3	$H_0 = (70.71 \pm 0.1)$; $\Omega_M = (0.238 \pm 0.01)$; $\Omega_\Lambda = (0.621 \pm 0.01)$	626.53	0.85	0.998	632.53
Tired light	(16)	1	$H_0 = (52.49 \pm 0.14)$	5923	8.01	0	5925
GTL	(22)	2	$H_0 = (68.29 \pm 0.3)$; $\alpha = 3.12$	634.45	0.85	0.99	638.45
Spectroscopic model	(28)	2	$H_0 = (149.89 \pm 2.1)$; $\beta = 0.84$	1812	2.45	0	1816

Figure 4 gives the best fit in the ΛCDM cosmology for the JLA compilation.

Figure 5 compares the ΛCDM cosmology, GTL, and the spectroscopic model for the JLA compilation. The goodness of the distance modulus for GTL versus the ΛCDM cosmology as a function of the redshift is presented in **Figure 6**.

4.3. Pantheon Compilation

The third test is performed on the Pantheon sample of 1048 SN Ia [19] [20] with calibrated data available at https://archive.stsci.edu/prepds/ps1cosmo/jones_datatable.html, see **Table 3**.

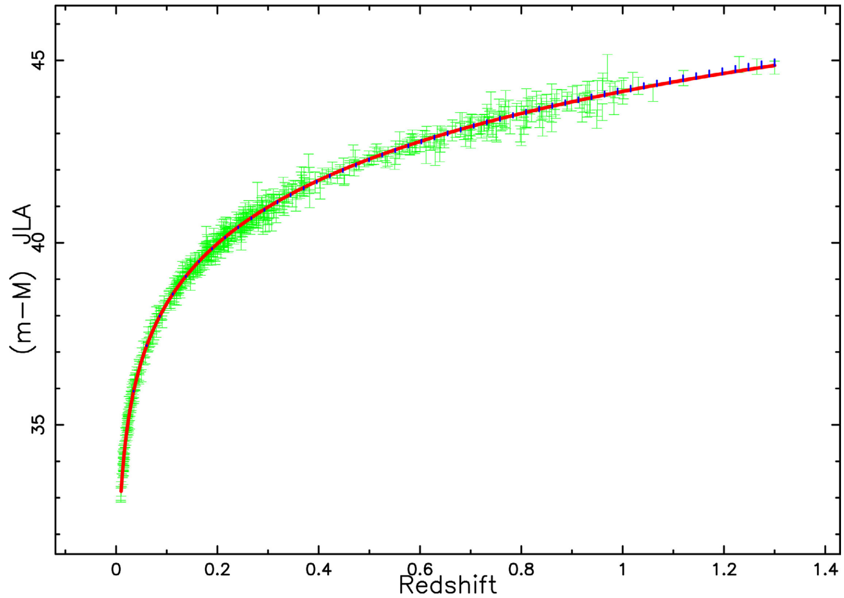

Figure 4. Hubble diagram for the JLA compilation, green points with error bars. The solid red line represents the best fit for the distance modulus in the ΛCDM cosmology. The theoretical uncertainties are represented through green vertical lines by applying the law of errors of Gauss with the uncertainties and parameters as in the first line of Table 2.

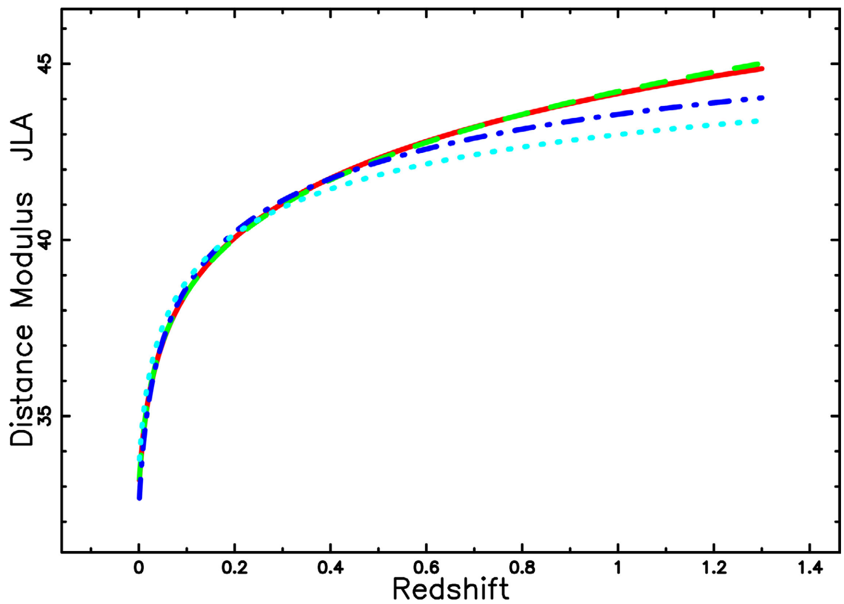

Figure 5. Best fit relative to the JLA compilation in ΛCDM cosmology (red full line), STL (cyan dotted line), GTL (green dashed line) and spectroscopic model (dash-point-dash line).

Figure 7 reports the best fit in the ΛCDM cosmology for the Pantheon compilation.

Figure 8 compares the ΛCDM cosmology, GTL, and the spectroscopic model for the Pantheon compilation. The goodness of the distance modulus for GTL versus the ΛCDM cosmology as a function of the redshift is given in Figure 9.

Table 3. Numerical values of χ^2, χ^2_{red}, Q and AIC of the Hubble diagram for the Pantheon sample, k stands for the number of parameters, H_0 is expressed in km·s⁻¹·Mpc⁻¹; 1048 SN Ia.

cosmology	Equation	k	parameters	χ^2	χ^2_{red}	Q	AIC
ΛCDM	[12]	3	$H_0=(68.209\pm0.2)$; $\Omega_M=(0.278\pm0.02)$; $\Omega_\Lambda=(0.651\pm0.02)$	1054.71	1.01	0.41	1060.71
Tired light	(16)	1	$H_0=(52.55\pm0.1)$	12,004	11.46	0	12,006
GTL	(22)	2	$H_0=(68.04\pm0.2)$; $\alpha=3.07$	1115	1.06	0.93	1119
Spectroscopic model	(28)	2	$H_0=(154.34\pm1.52)$; $\beta=0.84$	3365	3.21	0	3369

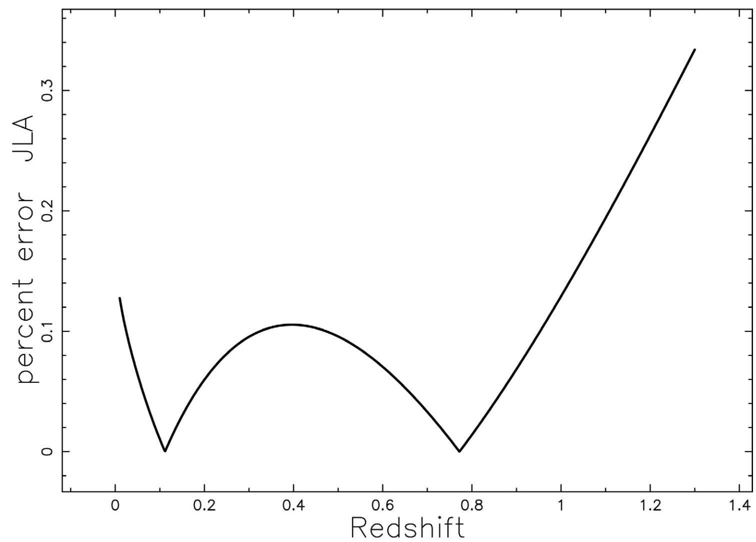

Figure 6. The percentage error between ΛCDM cosmology and GTL as function of the redshift relative to the JLA compilation.

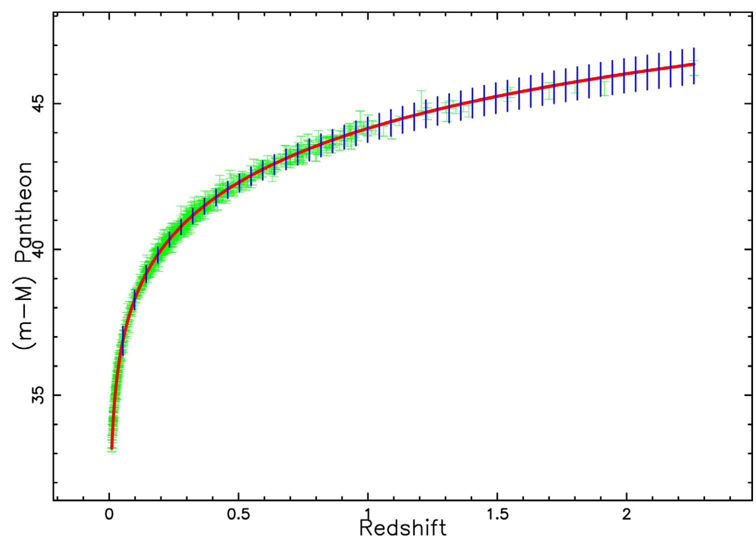

Figure 7. Hubble diagram for the Pantheon compilation, green points with error bars. The solid red line represents the best fit for the distance modulus in ΛCDM cosmology. The theoretical uncertainties are represented through green vertical lines by applying the law of errors of Gauss with the uncertainties and parameters as in the first line of Table 3.

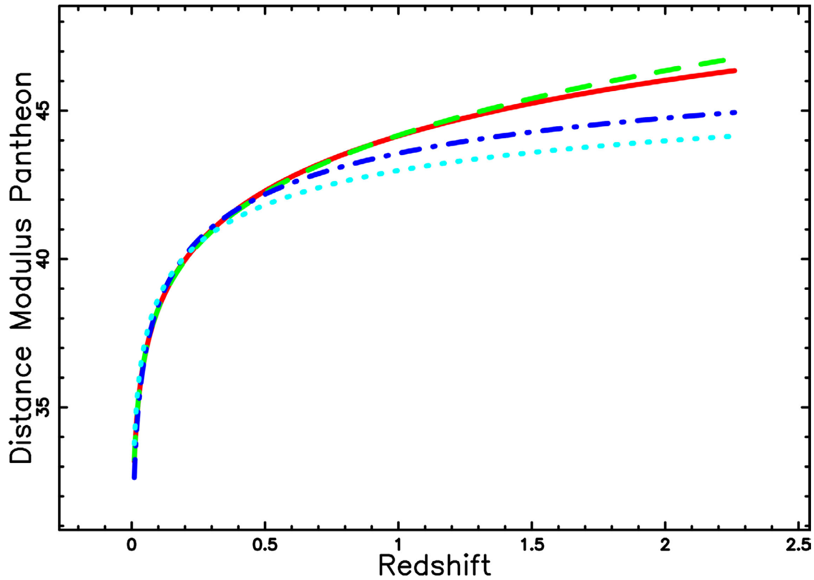

Figure 8. Best fit relative to the Pantheon compilation in ΛCDM cosmology (red full line), STL (cyan dotted line), GTL (green dashed line) and spectroscopic model (dash-point-dash line).

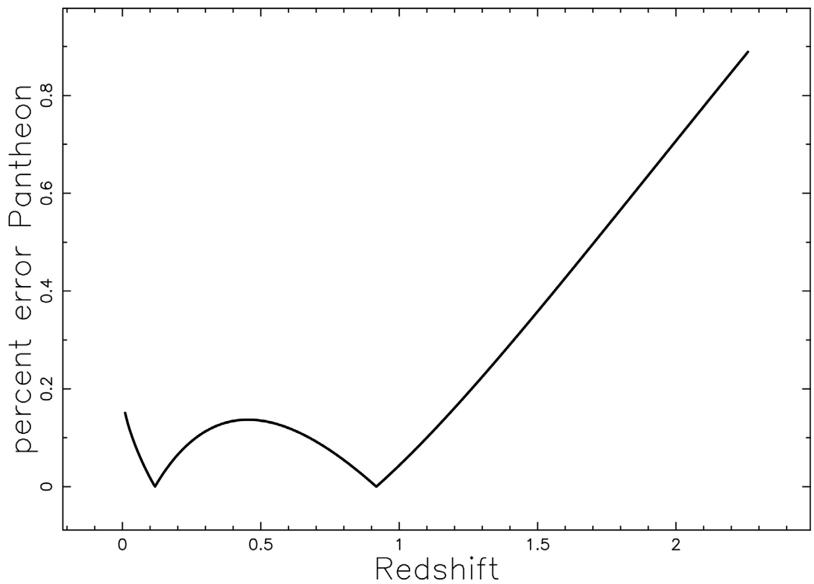

Figure 9. The percentage error between ΛCDM cosmology and GTL as function of the redshift relative to the Pantheon compilation.

5. Conclusions

We have analyzed three models for the distance modulus in the framework of the tired light hypothesis: the simple tired light model, see formula (16), the generalized tired light model, see formula (22), and a spectroscopic model, see formula (28).

A careful analysis of **Tables 1-3** allows rejecting the simple tired light model and the spectroscopic model because they have a large χ^2.

But the generalized tired light model fits well the data of the distance modulus and the percentage error with respect to the ΛCDM cosmology is less than 1% over the range in redshift, [0 - 2.3], for the three astronomical compilations here considered, see **Figure 3**, **Figure 6** and **Figure 9**. The advantages of the generalized tired light model are:

1) The relation between distance and redshift is analytic and invertible.
2) The distance modulus has an analytic expression.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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