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On a Cosmological Model with Variable Time Flow

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Abstract

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric is an exact solution of the Einstein field equations and it describes a homogeneous, isotropic and expanding universe. The FLRW metric and the Friedmann equations form the basis of the ΛCDM model. In this article, a metric which is based on the FLRW metric and that includes a space scale factor and a newly introduced time scale factor $\mathcal{T}(t)$ is elaborated. The assumption is that the expansion or contraction of the dimensions of space and time in a homogeneous and isotropic universe depend on the energy density. The Christoffel symbols, Ricci tensor and Ricci scalar are derived. By evaluating the results using Einstein's field equations and the energy momentum tensor, a hypothetical modified cosmological model is obtained. This theoretical model provides for a cosmic inflation, the accelerated expansion of spacetime as well avoids the flatness and fine-tuning problems.

Keywords

Cosmology, Dark Energy, Cosmic Inflation, Flatness Problem, Hubble Tension

1. Introduction

According to the Friedmann-Lemaître-Robertson-Walker (FLRW) metric and the Friedmann equations, the development of spacetime is represented by the cosmological scale factor, which is usually denoted by $a(t)$ or $R(t)$. The FLRW-universe is spatially homogeneous and isotropic and evolves over time. However, time in the FLRW metric is of Newtonian nature as it "flows equably without regard to anything external". The question arises as to whether the dimension of time in a metric describing an isotropic and homogeneous universe is also affected

[1] by the gravitational effect of energy and mass[1]. Let us therefore make the following assumptions and accept the listed principles and postulates:

1) The cosmological principle: The universe is homogeneous and isotropic when viewed on a sufficiently large scale (hereinafter referred to as the "Cosmological Principle").

2) The constant light speed postulate: The light speed c is constant when measured in any inertial frame of reference.

3) The assumption of the variability of time flow: The time flow in a homogeneous and isotropic universe depends on energy density and pressure.

By adopting the constant light speed postulate, the proposal contemplated herein is not based on variable speed of light (VSL) theories (see for VSL theories e.g. [2]-[4]). The assumption of variable time flow is a more fundamental approach.

2. Spacetime Scale Factors and Spacetime Parameters

Let us consider a sufficiently short time interval $\Delta t_e := t(e_2) - t(e_1)$, for example a photon with a period amounting to $T_e = \Delta t_e$ measured at the time of its emission $t(e_2)$ and accordingly a sufficiently short time interval $\Delta t_0 := t(a_2) - t(a_1)$, for example said photon with a period amounting to $T_0 = \Delta t_0$ measured today at the time of its absorption $t(a_2)$. Let us consider a variable time flow between the time of photon emission $t(e_2)$ and its absorption at $t(a_2)$. A dimensionless time scale factor $\mathcal{T}(t)$ is defined as the ratio between the proper time interval Δt_e and comoving time interval of today Δt_0:

$$\mathcal{T}(t) := \frac{\Delta t_0}{\Delta t_e}, \text{ with } \mathcal{T}(t) \neq 0 \wedge \mathcal{T}(t_0) := 1, \tag{1}$$

The notation $\mathcal{T}_e$ for $\mathcal{T}(t_e)$ (etc.) will be used. In a universe with a variable time flow but without an expansion or contraction of space, the photon is red- or blue-shifted when emitted ($c(\tau)T_e$) in the reference frame e and measured today on the basis of $c(\tau)T_0$. If the period of the photon increases, that is $T_0 > T_e$, then the photon is redshifted.

The space scale factor $\mathcal{S}(t)$ is introduced as the ratio between the proper distance $\Delta x_e = x_{e_2} - x_{e_1}$ at time t_e and the comoving distance $\Delta x_0 = x_{a_2} - x_{a_1}$ measured today (t_0) in an expanding or contracting space. It is defined as follows:

$$\mathcal{S}(t) := \frac{\Delta x_e}{\Delta x_0}, \text{ with } \mathcal{S}(t) \neq 0 \wedge \mathcal{S}(t_0) := 1. \tag{2}$$

In the last step, the spacetime parameters represented by the time parameter $\mathcal{G}(t)$ and the space parameter $\mathcal{J}(t)$ are defined as follows:

$$\mathcal{G}(t) := \frac{\dot{\mathcal{T}}}{\mathcal{T}}, \tag{3}$$

$$\mathcal{J}(t) := \frac{\dot{\mathcal{S}}}{\mathcal{S}}. \tag{4}$$

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[1] In [1] a metric with a variable time flow as an alternative to dark energy was proposed. Unfortunately, the paper was withdrawn.

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The "over-dot" is used as a notation for the first time derivative (e.g. $\dot{\mathcal{T}}$), and correspondingly for the second time derivative (e.g. $\ddot{\mathcal{T}}$). The space scale factor $\mathcal{S}$ and space parameter $\mathcal{J}$ will be used in the context of the proposed metric and cosmological model with variable time flow, and, for clarity, the cosmological scale factor denoted as R or a and the Hubble parameter H will be used in the context of the FLRW metric and Friedmann equations.

3. The Dynamic Metric

The FLRW metric $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + R^2\left(\frac{1}{1-kr^2}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right)$ (see e.g. [5]) shall be extended by the time scale factor $\mathcal{T}(t)$ and the cosmological scale factor a shall be replaced by the space scale factor $\mathcal{S}(t)$. It is expected (see for the FLRW metric e.g. [6], p. 382) that the spacetime is of the form $T \times \Sigma$, where T represents the time direction and Σ is a maximally symmetric three-manifold. It seems to be necessary that the partial derivatives of a position vector have to be determined. Based on these results, the covariant basis vectors and co- and contravariant metric tensors can be derived. If spacetime is of the form $T \times \Sigma$, the transformation of the metric to pseudo-spherical coordinates is straightforward.

Let us consider a four-dimensional manifold with a signature of the form $(-,+,+,+)$ and an event in spacetime $E(ct,x,y,z)$. Let us further consider the scale factors $\mathcal{T}_e$ and $\mathcal{S}_e$ in the reference frame e as well as a sufficiently short time interval $\mathrm{d}t_e$ and a distance $\mathrm{d}x_e$. With Equations (1) and (2), the transformation rules $\mathrm{d}t = \mathcal{T}_e\mathrm{d}t_e$ for the time dimension and $\mathrm{d}x_{i_a} = 1/\mathcal{S}_e\,\mathrm{d}x_{i_e}$ for the space dimension $i = x,y,z$ are obtained. To obtain the basis vectors, the partial derivatives of the position vector $\boldsymbol{E}$ can be calculated. For the partial time-derivative in direction of e the notation $\partial\tilde{t}$ is used. Further, we apply the chain rule, consider the Cosmological Principle and obtain:

$$\frac{\partial}{\partial\tilde{t}} = \frac{1}{\mathcal{T}_e}\frac{\partial}{\partial t},\ \frac{\partial}{\partial\tilde{x}} = \mathcal{S}_e\frac{\partial}{\partial x},\ \frac{\partial}{\partial\tilde{y}} = \mathcal{S}_e\frac{\partial}{\partial y},\ \frac{\partial}{\partial\tilde{z}} = \mathcal{S}_e\frac{\partial}{\partial z}. \tag{5}$$

Based on these results, the covariant basis vectors and the covariant metric tensor of the signature $(-,+,+,+)$ can be calculated using the dot product $g_{\mu\nu} = \boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu$. If the indices in $\mathcal{S}_e$ and $\mathcal{T}_e$ are omitted, the covariant metric tensor yields $g_{\mu\nu} = \mathrm{diag}\left(\mathcal{T}^{-2},\mathcal{S}^2,\mathcal{S}^2,\mathcal{S}^2\right)$. This further leads to the contravariant metric tensor as the inverse of the covariant form $g_{\mu\nu}g^{\mu\sigma} = \delta_\nu^\sigma$ yielding $g^{\mu\nu} = \mathrm{diag}\left(\mathcal{T}^2,\mathcal{S}^{-2},\mathcal{S}^{-2},\mathcal{S}^{-2}\right)$. Indeed, the expectation according to which the spacetime is of the form $T \times \Sigma$ is valid, so the metric can be transformed using the usual transformation rules (see e.g. [5] p. 7), which then reads

$$\mathrm{d}s^2 = -\frac{c^2}{\mathcal{T}^2}\mathrm{d}t^2 + \mathcal{S}^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right). \tag{6}$$

Accordingly, if a parameter for a constant curvature k is introduced (see for details [6], p. 329-332), the following dynamic metric yields:

$$\boxed{\mathrm{d}s^2 = -\frac{c^2}{\mathcal{T}^2}\mathrm{d}t^2 + \mathcal{S}^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right)}. \tag{7}$$

An extended FLRW metric was obtained from these calculations. The only difference in relation to the FLRW metric is that the time direction is scaled by the time scale factor. This metric describes a homogeneous, isotropic, expanding (or otherwise, contracting) universe. I am of the opinion that the variable time flow reflected by this metric does not contradict the Cosmological Principle. In the FLRW metric, the spatial dimensions of the universe's spacetime may be contracted or expanded in scale, in the metric derived above, the dimensions of space and/or time may be contracted or expanded.

4. The Christoffel Symbols (of the Second Kind)

The relation between the metric tensor and the Christoffel symbols (of the second kind) is given ([5], see Section 1.3 on p. 2) by:

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}g^{\mu\rho}\left[\partial_{\lambda}g_{\nu\rho} + \partial_{\nu}g_{\rho\lambda} - \partial_{\rho}g_{\nu\lambda}\right], \text{ whereas } \Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\lambda\nu} \tag{8}$$

With the metric tensor (Equation (7)), the non-zero components of the contra- and covariant metric tensors are

$$g_{00} = -\frac{1}{\mathcal{T}^2},\ g_{11} = \frac{\mathcal{S}^2}{1-kr^2},\ g_{22} = \mathcal{S}^2r^2,\ g_{33} = \mathcal{S}^2r^2\sin^2\theta, \tag{9}$$

and

$$g^{00} = -\mathcal{T}^2,\ g^{11} = \frac{1-kr^2}{\mathcal{S}^2},\ g^{22} = \frac{1}{\mathcal{S}^2r^2},\ g^{33} = \frac{1}{\mathcal{S}^2r^2\sin^2\theta}. \tag{10}$$

The detailed step-by-step calculation of all Christoffel symbols is set out in the supplemental material[2]. The index 0 is used equivalently to the index t, and consequently 1 for r, 2 for θ and 3 for φ.

$$\Gamma^{0}_{00} = \Gamma^{t}_{tt} = -\frac{\dot{\mathcal{T}}}{c\mathcal{T}}, \tag{11}$$

$$\Gamma^{0}_{11} = \Gamma^{t}_{rr} = \frac{\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}}{c\left(1-kr^2\right)}, \tag{12}$$

$$\Gamma^{0}_{22} = \Gamma^{t}_{\theta\theta} = \frac{\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}r^2}{c}, \tag{13}$$

$$\Gamma^{0}_{33} = \Gamma^{t}_{\varphi\varphi} = \frac{\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}r^2\sin^2\theta}{c}, \tag{14}$$

$$\Gamma^{1}_{01} = \Gamma^{1}_{10} = \Gamma^{r}_{tr} = \Gamma^{r}_{rt} = \frac{\dot{\mathcal{S}}}{c\mathcal{S}}, \tag{15}$$

$$\Gamma^{1}_{11} = \Gamma^{r}_{rr} = \frac{kr}{1-kr^2}, \tag{16}$$

$$\Gamma^{1}_{22} = \Gamma^{r}_{\theta\theta} = r\left(kr^2-1\right), \tag{17}$$

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[2] The supplemental material is made available by the author on request.

$$\Gamma^1_{33} = \Gamma^r_{\varphi\varphi} = r\left(kr^2 - 1\right)\sin^2\theta, \tag{18}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \Gamma^\theta_{t\theta} = \Gamma^\theta_{\theta t} = \frac{\dot{\mathcal{S}}}{c\,\mathcal{S}}, \tag{19}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r}, \tag{20}$$

$$\Gamma^2_{33} = \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta, \tag{21}$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \Gamma^\varphi_{t\varphi} = \Gamma^\varphi_{\varphi t} = \frac{\dot{\mathcal{S}}}{c\,\mathcal{S}}, \tag{22}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \Gamma^\varphi_{r\varphi} = \Gamma^\varphi_{\varphi r} = \frac{1}{r}, \tag{23}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \Gamma^\varphi_{\theta\varphi} = \Gamma^\varphi_{\varphi\theta} = \cot\theta. \tag{24}$$

The Christoffel symbols of the dynamic metric proposed herein have, in comparison with the FLRW metric, an additional term Γ^t_{tt}. The Christoffel symbols Γ^t_{rr}, $\Gamma^t_{\theta\theta}$ as well $\Gamma^t_{\varphi\varphi}$ also contain a factor of $\mathcal{T}^2$. All other Christoffel symbols remain unchanged compared to the FLRW metric. If it is assumed that the time scale factor is constant ($\mathcal{T} = 1$), and consequently the derivative of the time scale factor becomes zero ($\dot{\mathcal{T}} = 0$), all the Christoffel symbols listed above will become the Christoffel symbols of the FLRW metric. This is true for most of the equations in this paper.

5. Ricci Tensor and Ricci Scalar

Detailed step-by-step calculations of the Ricci tensor and Ricci scalar are presented in the supplemental material. The relation between the Riemann and the Ricci tensor is in accordance with ([5], Equations 1.2.2, 1.3.5 and 1.3.9 on p. 2):

$$R_{\mu\nu} = R^\rho_{\mu\rho\nu} = \partial_\rho\Gamma^\rho_{\mu\nu} - \partial_\nu\Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{\mu\nu} - \Gamma^\rho_{\lambda\nu}\Gamma^\lambda_{\rho\mu}. \tag{25}$$

The non-zero components of the Ricci tensor are as follows:

$$R_{tt} = -\frac{3}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}} + \frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}\right), \tag{26}$$

$$R_{rr} = \frac{\mathcal{T}^2\mathcal{S}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}} + \frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}} + 2\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2} + \frac{2c^2k}{\mathcal{T}^2\mathcal{S}^2}\right)\frac{1}{1-kr^2}, \tag{27}$$

$$R_{\theta\theta} = \frac{\mathcal{T}^2\mathcal{S}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}} + \frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}} + 2\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2} + \frac{2c^2k}{\mathcal{T}^2\mathcal{S}^2}\right)r^2, \tag{28}$$

$$R_{\varphi\varphi} = \frac{\mathcal{T}^2\mathcal{S}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}} + \frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}} + 2\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2} + \frac{2c^2k}{\mathcal{T}^2\mathcal{S}^2}\right)r^2\sin^2\theta. \tag{29}$$

The Ricci scalar (or scalar curvature) is the contraction of the Ricci tensor (see [5], Equations 1.3.10 on p. 2):

$$R = g^{\mu\nu}R_{\mu\nu}, \tag{30}$$

which results in:

$$R=6\frac{\mathcal{T}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}+\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2}+\frac{kc^2}{\mathcal{T}^2\mathcal{S}^2}\right). \tag{31}$$

6. Energy Momentum Tensor and Energy Density

In order to be able to solve the field equations, we will have to take a closer look at the energy-momentum tensor. The approach is the same as that for the FLRW metric where a perfect and continuous fluid is considered (see [6], p. 35, 117-119).

$$T^{\mu\nu}=(\rho+p)U^{\mu}U^{\nu}+pg^{\mu\nu}, \tag{32}$$

where $\rho(t)$ is the energy density, $p(t)$ is the pressure and U^{μ} is the four-velocity of the perfect fluid. With the inverse of the Minkowski metric, the energy-momentum tensor becomes $T^{\mu\nu}=\mathrm{diag}(\rho,p,p,p)$.

By applying the contravariant (Equation (9)) and covariant (Equation (10)) metric tensors, the components of the tensor yield

$$T^{\mu\nu}=\begin{bmatrix}\mathcal{T}^2\rho c^2 & 0 & 0 & 0\\ 0 & p\mathcal{S}^{-2}\left(1-kr^2\right) & 0 & 0\\ 0 & 0 & p(\mathcal{S}r)^{-2} & 0\\ 0 & 0 & 0 & p(\mathcal{S}r\sin\theta)^{-2}\end{bmatrix}. \tag{33}$$

The trace is accordingly $g_{\mu\nu}T^{\mu\nu}=-\rho+3p$. Now, the divergence of the energy-momentum tensor can be examined. The starting point is the conservation equation ([6], p. 118):

$$\nabla_{\mu}T^{\mu\nu}=\partial_{\mu}T^{\mu\nu}+\Gamma^{\mu}_{\mu\lambda}T^{\lambda\nu}+\Gamma^{\nu}_{\mu\lambda}T^{\mu\lambda}\overset{!}{=}0. \tag{34}$$

In a first step, the components of the terms $\nu=0$ are calculated:

$$\partial_{\mu}T^{\mu 0}=\partial_0 T^{00}=\frac{\mathcal{T}^2}{c}\dot{\rho}+\frac{2}{c}\mathcal{T}\dot{\mathcal{T}}\rho. \tag{35}$$

For the second term, the Christoffel symbols obtained in Section 4 can be applied:

$$\begin{aligned}\Gamma^{\mu}_{\mu\lambda}T^{\lambda 0}&=\Gamma^{\mu}_{\mu 0}T^{00}\\&=\left(\Gamma^{0}_{00}+\Gamma^{1}_{01}+\Gamma^{2}_{02}+\Gamma^{3}_{03}\right)T^{00}\\&=\frac{\mathcal{T}^2}{c}\left(-\frac{\dot{\mathcal{T}}}{\mathcal{T}}+3\frac{\dot{\mathcal{S}}}{\mathcal{S}}\right)\rho,\end{aligned} \tag{36}$$

and the third term results in

$$\begin{aligned}\Gamma^{0}_{\mu\lambda}T^{\mu\lambda}&=\Gamma^{0}_{00}T^{00}+\Gamma^{0}_{11}T^{11}+\Gamma^{0}_{22}T^{22}+\Gamma^{0}_{33}T^{33}\\&=-\frac{\dot{\mathcal{T}}}{\mathcal{T}}\frac{\mathcal{T}^2}{c}\rho+\frac{\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}}{c\left(1-kr^2\right)}p\frac{1-kr^2}{\mathcal{S}^2}+\frac{1}{c}\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}r^2\frac{1}{\mathcal{S}^2r^2}\\&\quad+\frac{1}{c}\mathcal{T}^2\dot{\mathcal{S}}\mathcal{S}r^2\sin^2\theta\, p\frac{1}{\mathcal{S}^2r^2\sin^2\theta}\\&=\frac{\mathcal{T}^2}{c}\left(-\frac{\dot{\mathcal{T}}}{\mathcal{T}}\rho+3\frac{\dot{\mathcal{S}}}{\mathcal{S}}p\right).\end{aligned} \tag{37}$$

The calculation of the spatial components of the energy-momentum tensor is provided in the supplemental material. Partial derivatives are given as follows:

$$\partial_i p = 0. \tag{38}$$

We now can derive the overall result:

$$\nabla_\mu T^{\mu\nu} = \frac{\mathcal{T}^2}{c}\dot{\rho} + \frac{2}{c}\mathcal{T}\dot{\mathcal{T}}\rho + \frac{\mathcal{T}^2}{c}\left(-\frac{\dot{\mathcal{T}}}{\mathcal{T}} + 3\frac{\dot{\mathcal{S}}}{\mathcal{S}}\right)\rho + \frac{\mathcal{T}^2}{c}\left(-\frac{\dot{\mathcal{T}}}{\mathcal{T}}\rho + 3\frac{\dot{\mathcal{S}}}{\mathcal{S}}p\right) \overset{!}{=} 0, \tag{39}$$

which amounts to

$$\dot{\rho} = -3\frac{\dot{\mathcal{S}}}{\mathcal{S}}(\rho + p). \tag{40}$$

The next step is to introduce the equation of state to express the relationship between the energy density and pressure for radiation and mass ([6], p. 334):

$$p_i = \omega_i \rho_i, \text{ with } i = \{rad, m\} \tag{41}$$

where *rad* stands for radiation and *m* for mass (**Table 1**).

Table 1. Energy density—pressure relationship.

i	ω_i	ρ_{ω_i}
rad	1/3	ρ_{rad}
m	0	ρ_m

Finally, we obtain from Equation (40), considering Equation (41) and the definition 4:

$$\boxed{\frac{\dot{\rho}}{\rho} = -3\mathcal{J}(\omega+1)}. \tag{42}$$

To solve this equation, it is required that ω must be constant (see [6], p. 334). By integrating Equation (41) we obtain $\ln\rho + \ln(const.) \propto (-3(\omega+1)\ln\mathcal{S})$ and therefore $\rho(\mathcal{S}) \propto const.\left(\mathcal{S}^{-3(\omega+1)}\right)$. If we normalize the scale factor to the value for today $\mathcal{S}_0 = 1$, the constant becomes ρ_0^{-1}, which finally results in

$$\rho(\mathcal{S}) \propto \rho_0\left(\mathcal{S}^{-3(\omega+1)}\right). \tag{43}$$

Finally, considering the values for ω, we obtain

$$\rho_{rad} = \frac{\rho_{rad,0}}{\mathcal{S}^4}, \quad \rho_m = \frac{\rho_{m,0}}{\mathcal{S}^3}. \tag{44}$$

7. Einstein's Field Equations and the Modified Cosmological Model

Now, we can solve Einstein's field equations in their general forms.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{45}$$

In a first step, the equation will be solved for $T_{00} = \frac{\rho c^2}{\mathcal{T}^2}$:

$$R_{00}-\frac{1}{2}Rg_{00}=\frac{8\pi G}{c^4}T_{00}, \tag{46}$$

$$\Rightarrow -\frac{3}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}\right)-\frac{1}{2}6\frac{\mathcal{T}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}+\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2}+\frac{kc^2}{\mathcal{T}^2\mathcal{S}^2}\right)\left(-\frac{1}{\mathcal{T}^2}\right)=\frac{8\pi G}{c^4}\frac{\rho c^2}{\mathcal{T}^2}, \tag{47}$$

$$\Rightarrow \boxed{\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2}=\frac{8\pi G}{3}\frac{\rho}{\mathcal{T}^2}-\frac{kc^2}{\mathcal{S}^2\mathcal{T}^2}}. \tag{48}$$

This is the modified first Friedmann equation. It contains an additional term $\mathcal{T}^{-2}$ on the right-hand side. If the time scale factor is set to 1, it is reduced to the original Friedmann equation. Now, the field equations for $T_{11}=\frac{\mathcal{S}^2}{1-kr^2}p$ are assessed:

$$R_{11}-\frac{1}{2}Rg_{00}=\frac{8\pi G}{c^4}T_{11}, \tag{49}$$

$$\begin{aligned}\Rightarrow &\frac{\mathcal{T}^2\mathcal{S}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}+2\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2}+\frac{2c^2k}{\mathcal{T}^2\mathcal{S}^2}\right)\frac{1}{1-kr^2}\\ &-\frac{1}{2}6\frac{\mathcal{T}^2}{c^2}\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}+\frac{\dot{\mathcal{S}}^2}{\mathcal{S}^2}+\frac{kc^2}{\mathcal{T}^2\mathcal{S}^2}\right)\frac{\mathcal{S}^2}{1-kr^2}=\frac{8\pi G}{c^4}p\frac{\mathcal{S}^2}{1-kr^2},\end{aligned} \tag{50}$$

$$\Rightarrow -2\frac{\ddot{\mathcal{S}}}{\mathcal{S}}-2\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}-\left(\frac{\dot{\mathcal{S}}}{\mathcal{S}}\right)^2=\frac{8\pi G}{c^2}\frac{p}{\mathcal{T}^2}+\frac{kc^2}{\mathcal{S}^2\mathcal{T}^2}. \tag{51}$$

These two results constitute the conditions for the development of the spacetime parameters. The results for R_{22} and R_{33} are equal to R_{11}. If we plug the result obtained into Equation (48) and eliminate $\mathcal{J}^2$, we obtain

$$\boxed{-\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}\right)=\frac{4\pi G}{3c^2}\frac{\rho c^2+3p}{\mathcal{T}^2}}. \tag{52}$$

Equation (52) is the modified second Friedmann equation. It contains an additional term $\mathcal{T}^{-2}$ on the right-hand side and the product of $\mathcal{G}$ and $\mathcal{J}$ on the left-hand side. If the time scale factor is set to constant 1 and, consequently, $\mathcal{G}$ becomes zero, it reduces to the original Friedmann equation.

8. Redshift

To derive the redshift, the geodesic equation for a light wave travelling radially ($\mathrm{d}\theta=\mathrm{d}\varphi=0$) is used, which is according to Equation (7):

$$0=\mathrm{d}s^2=-\frac{1}{\mathcal{T}^2}c^2\mathrm{d}t^2+\mathcal{S}^2\left(\frac{\mathrm{d}r^2}{1-kr^2}\right). \tag{53}$$

The crest of the light wave, observed at position $r=r_a$ and time $t=t_a$, was emitted at time $t=t_e$ in the past and at a distant position $r=r_e$. Integrating over the path in both space and time that the light wave travels results in:

$$c\int_{t_e}^{t_a}\frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}}=-\int_{r_e}^{r_a}\frac{\mathrm{d}r}{\sqrt{1-kr^2}}. \tag{54}$$

The next crest is emitted at $t = t_e + \frac{\lambda_e}{c}$ and observed at $t = t_a + \frac{\lambda_a}{c}$. Integrating over path and time yields:

$$c\int_{t_e+\frac{\lambda_e}{c}}^{t_a+\frac{\lambda_a}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} = -\int_{r_e}^{r_a} \frac{\mathrm{d}r}{\sqrt{1-kr^2}}. \tag{55}$$

Equations (54) and (55) can be combined to:

$$0 = c\int_{t_e}^{t_a} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} - c\int_{t_e+\frac{\lambda_e}{c}}^{t_a+\frac{\lambda_a}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} \tag{56}$$

$$= c\int_{t_e}^{t_e+\frac{\lambda_e}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} + c\int_{t_e+\frac{\lambda_e}{c}}^{t_a} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} - c\int_{t_e+\frac{\lambda_e}{c}}^{t_a+\frac{\lambda_a}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} \tag{57}$$

$$= c\int_{t_e}^{t_e+\frac{\lambda_e}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} - c\int_{t_a}^{t_e+\frac{\lambda_e}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} - c\int_{t_e+\frac{\lambda_e}{c}}^{t_a+\frac{\lambda_a}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} \tag{58}$$

$$= c\int_{t_e}^{t_e+\frac{\lambda_e}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} - c\int_{t_a}^{t_a+\frac{\lambda_a}{c}} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}}. \tag{59}$$

Let us assume that $\frac{\lambda_e}{c}$ and $\frac{\lambda_a}{c}$ are sufficiently small enough, thus the above result is

$$\frac{t_a + \lambda_a c}{\mathcal{T}_a\mathcal{S}_a} - \frac{t_a}{\mathcal{T}_a\mathcal{S}_a} = \frac{t_e + \lambda_e c}{\mathcal{T}_e\mathcal{S}_e} - \frac{t_e}{\mathcal{T}_e\mathcal{S}_e} \tag{60}$$

$$\Rightarrow \frac{\lambda_a}{\lambda_e} = \frac{\mathcal{T}_a\mathcal{S}_a}{\mathcal{T}_e\mathcal{S}_e} \tag{61}$$

$$\Rightarrow z = \frac{1}{\mathcal{T}_e\mathcal{S}_e} - 1, \text{ with } \mathcal{T}_a\mathcal{S}_a = 1. \tag{62}$$

Let us define a component of the redshift related to the time scale factor

$$z_{\mathcal{T}} := \frac{1}{\mathcal{T}} - 1,\ \mathcal{T} = \frac{1}{z_{\mathcal{T}} + 1}, \tag{63}$$

and accordingly the component of the redshift related to the space scale factor:

$$z_{\mathcal{S}} := \frac{1}{\mathcal{S}} - 1,\ \mathcal{S} = \frac{1}{z_{\mathcal{S}} + 1}. \tag{64}$$

Without any further restriction, let us consider a photon with a wavelength λ_0 at the beginning, moving through the dynamic spacetime and experiencing two causes for a redshift. The first cause results in a redshift z_1 and λ_1 and the second cause results in z_2 and λ_2. For the resulting redshift z_3 we conclude that:

$$z_1 = \frac{\lambda_1}{\lambda_0} - 1,\ z_2 = \frac{\lambda_2}{\lambda_1} - 1,\ z_3 = \frac{\lambda_2}{\lambda_0} - 1. \tag{65}$$

The second equation can be rearranged to obtain the result for λ_1 which is then plugged into the first equation:

$$z_1 = \frac{\lambda_2}{(z_2 + 1)\lambda_0} - 1 \Rightarrow \lambda_2 = \lambda_0 (z_2 + 1)(z_1 + 1). \tag{66}$$

The result for λ_2 from Equation (66) is now used to calculate z_3 given in Equation (65):

$$z_3 = (z_1 + 1)(z_2 + 1) - 1, \tag{67}$$

which then can be combined with Equations (65) and (66):

$$z = \frac{1}{\mathcal{S}\mathcal{T}} - 1. \tag{68}$$

This is the result of Equation (62). An observed redshift z_{obs} composed of one component $z_{\mathcal{S}}$ related to the space scale factor and another component $z_{\mathcal{T}}$ related to the time scale factor can be denoted as:

$$z_{obs} + 1 = (z_{\mathcal{S}} + 1)(z_{\mathcal{T}} + 1). \tag{69}$$

This result is also, with the exception of the redshifting by the expansion of space, in line with the outcome of the Schwarzschild-metric, where the energy of a photon emitted at time t_e and received today can be expected to yield

$$E_{ph_0} = h\nu_0 = h\nu_e\sqrt{-g^{tt}}, \tag{70}$$

where the frequency is denoted as ν and h is the Planck constant.

9. Distances and Revised Hubble Law

In the FLRW metric, the Hubble law can be derived by calculating the proper distance and assuming a flat geometry with $k = 0$ (see [7], p. 571-576; [8]). In the dynamic metric (Equation (7)), we start with Equation (54):

$$c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{\mathcal{T}\mathcal{S}} = \int_0^d \frac{\mathrm{d}r}{\sqrt{1-kr^2}}, \tag{71}$$

Let $u(t) := \mathcal{S}(t)\mathcal{T}(t)$. With

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \dot{\mathcal{S}}\mathcal{T} + \dot{\mathcal{T}}\mathcal{S}, \tag{72}$$

Equation (71) becomes

$$c\int_{u_e}^{u_0} \frac{1}{\mathcal{T}\mathcal{S}\left(\dot{\mathcal{S}}\mathcal{T} + \dot{\mathcal{T}}\mathcal{S}\right)}\mathrm{d}u = \int_0^d \frac{\mathrm{d}r}{\sqrt{1-kr^2}}. \tag{73}$$

Finally, with

$$\frac{\mathrm{d}u}{\mathrm{d}z} = -\frac{1}{\mathcal{T}^2\mathcal{S}^2}, \tag{74}$$

we obtain

$$c\int_0^z \frac{1}{\mathcal{J}(z) + \mathcal{G}(z)}\mathrm{d}z = \int_0^d \frac{\mathrm{d}r}{\sqrt{1-kr^2}}. \tag{75}$$

In a flat universe ($k = 0$) and assuming $\mathcal{J}(z) + \mathcal{G}(z) = \mathcal{J}_0 + \mathcal{G}_0$ are constant for $z \ll 0.1$ and can therefore be moved outside the integral, this yields

$$\boxed{d = \frac{cz}{\mathcal{J}_0 + \mathcal{G}_0}}. \tag{76}$$

This result is in line with the expectations based on Section 1, according to which a change in the time scale factor leads to a change in the wavelength of a photon. Consequently, if the distance is calculated based on an observed redshift z, the time scale factor must be considered because the space parameter and the time parameter interfere with each other. It could even result that no redshift is observed, although space expands, particularly if the time dimension was or were contracted to the same extent as space expands. In a time-like static universe, where the time "flows equably without regard to anything external", the redshift depends solely on the development of the space scale factor since the emission of the photon. In the proposed model, the values based on redshift will be diluted; however, in a slightly different way than in the Friedmann universe, as can be seen from Equations (48) and (52). Conservation of the energy of photons means that all of the photons emitted by the source will eventually pass through a sphere at comoving distance x from the emitter ([6], p. 346). The flux is diluted by the following additional effects: the individual photons redshift by a factor $(\mathcal{S}\mathcal{T})^{-1}$, and the photons hit the sphere less frequently, because two photons emitted a time δt apart will be measured at a time $(\mathcal{S}\mathcal{T})^{-1}\delta t$ apart. If the calculation of a distance involves the observed redshift, only the portion of the redshift caused by the expansion of space is to be considered, that is, the observed redshift will have to be corrected by the following factor:

$$z_{\mathcal{S}} = \frac{z_{obs}+1}{z_{\mathcal{T}}+1} - 1. \tag{77}$$

The distance in the model discussed herein is larger than the distance in the FLRW universe if the observed redshift is to be corrected by the blueshift portion caused by the contraction of the time dimension.

10. On the Dynamic of the Model

In order to further analyze the dynamic of the model discussed herein, let us rewrite Equation (48) by defining the critical densities as

$$\rho_c := \frac{3\mathcal{J}_0^2}{8\pi G},\ \rho_k := -\frac{3k}{8\pi G\mathcal{S}^2}, \tag{78}$$

and, considering Equation (43), the density parameters for mass, radiation and curvature as

$$\Omega_{rad,0} := \frac{\rho_{rad}}{\rho_c},\ \Omega_{m,0} := \frac{\rho_m}{\rho_c},\ \Omega_{k,0} := -\frac{k}{\mathcal{J}_0^2\mathcal{S}^2}. \tag{79}$$

Equation (48) can now be written to yield

$$\mathcal{J}^2 = \mathcal{J}_0^2\mathcal{T}^{-2}\left(\Omega_{rad,0}\mathcal{S}^{-4} + \Omega_{m,0}\mathcal{S}^{-3} + \Omega_{k,0}\mathcal{S}^{-2}\right), \tag{80}$$

which reveals a different dynamic than the ΛCDM model (see for the latter [6], p. 337):

$$H^2 = H_0^2\left(\Omega_{rad,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{k,0}a^{-2} + \Omega_{\Lambda,0}\right),\ \text{with } \rho_\Lambda := \frac{\Lambda}{8\pi G}, \Omega_{\Lambda,0} := \frac{\rho_\Lambda}{\rho_c}. \tag{81}$$

For the further analysis, let us now assume a matter-dominated universe and rewrite Equation (52) using (44):

$$-\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}\right)=\frac{4\pi G}{3}\frac{\rho_{0,m}}{\mathcal{S}^3\mathcal{T}^2}, \tag{82}$$

which can be written as

$$-\mathcal{S}^2\mathcal{T}\left(\ddot{\mathcal{S}}\mathcal{T}+\dot{\mathcal{T}}\dot{\mathcal{S}}\right)=\frac{4\pi G}{3}\rho_{0,m}. \tag{83}$$

The right-hand side of the equation is constant with $\rho_0 \geq 0$, which can be chosen to represent today's energy density or the density at any other point in time. As it can be seen from Equation (83), $\dot{\mathcal{T}}$ is negative, when $\ddot{\mathcal{S}}$ is positive and $\ddot{\mathcal{S}}$ is negative, when $\dot{\mathcal{T}}$ is positive. In this paper, only the cases of $\mathcal{S} > 0 \wedge \mathcal{T} > 0$ of the four possible configurations given by the combinations of $\mathcal{S} \gtrless 0$ and $\mathcal{T} \gtrless 0$ will be examined. Consequently, there are three sub-cases ($\ddot{\mathcal{S}}\mathcal{T} > 0$, $\ddot{\mathcal{S}}\mathcal{T} = 0$ and $\ddot{\mathcal{S}}\mathcal{T} = 0$):

Table 2. Cases of configuration *I*: $\mathcal{S} > 0 \wedge \mathcal{T} > 0$.

$\ddot{\mathcal{S}}\mathcal{T} > 0 \Rightarrow \ddot{\mathcal{S}} > 0 \wedge \dot{\mathcal{T}}\dot{\mathcal{S}} < -\ddot{\mathcal{S}}\mathcal{T} < 0$	if $\dot{\mathcal{S}} > 0 \Rightarrow \dot{\mathcal{T}} < 0$ A: accelerated expansion of space if $\dot{\mathcal{S}} < 0 \Rightarrow \dot{\mathcal{T}} > 0$ B: accelerated contraction of space
$\ddot{\mathcal{S}}\mathcal{T} = 0 \Rightarrow \ddot{\mathcal{S}} = 0 \wedge \dot{\mathcal{T}}\dot{\mathcal{S}} < -\ddot{\mathcal{S}}\mathcal{T} = 0$	if $\dot{\mathcal{S}} > 0 \Rightarrow \dot{\mathcal{T}} < 0$ C: continuous expansion of space if $\dot{\mathcal{S}} < 0 \Rightarrow \dot{\mathcal{T}} > 0$ D: continuous contraction of space
$\ddot{\mathcal{S}}\mathcal{T} < 0 \Rightarrow \ddot{\mathcal{S}} < 0 \wedge \dot{\mathcal{T}}\dot{\mathcal{S}} < -\ddot{\mathcal{S}}\mathcal{T}$	$\dot{\mathcal{S}} > 0$ E: decelerated expansion of space $\dot{\mathcal{S}} < 0$ F: decelerated contraction of space

An accelerated expansion of the universe requires that the derivative of the time scale factor and, consequently, the time parameter is negative (case A in Table 2). As can be seen from the table, the model permits all cases of accelerated, continuous, and decelerated expansion or contraction of space. However, it does not allow a steady state of space, because $\ddot{\mathcal{S}} \neq 0$, if $\dot{\mathcal{S}} = 0$.

11. The Inflationary Universe

The second Friedmann equation implies that the acceleration of the expansion is always negative, unless the cosmological constant with an energy density—pressure relationship of $\omega_\Lambda = -1$ is introduced. Let us now assume a completely radiation-dominated universe and rewrite Equation (52) using 44:

$$-\left(\frac{\ddot{\mathcal{S}}}{\mathcal{S}}+\frac{\dot{\mathcal{T}}\dot{\mathcal{S}}}{\mathcal{T}\mathcal{S}}\right)=\frac{8\pi G}{3}\frac{\rho_{0,rad}}{\mathcal{S}^4\mathcal{T}^2},\tag{84}$$

The right-hand side of the equation is constant with $\rho_0 \geq 0$, which can be chosen to represent today's energy density or the density at any other point in time. Considering configuration I with $\mathcal{S}>0 \wedge \mathcal{T}>0$ and the three sub-cases ($\ddot{\mathcal{S}}\mathcal{T}>0$, $\ddot{\mathcal{S}}\mathcal{T}=0$ and $\ddot{\mathcal{S}}\mathcal{T}=0$) depicted in Section 1, it can be concluded that $\dot{\mathcal{T}}$ must be negative and $\dot{\mathcal{T}}\dot{\mathcal{S}}$ must be less than $-\ddot{\mathcal{S}}\mathcal{T}$ (see **Table 2**, case A), if $\ddot{\mathcal{S}}$ shall cause an accelerated expansion of space (hence $\ddot{\mathcal{S}}>0 \wedge \dot{\mathcal{S}}>0$). Equation (84) can be rewritten as follows:

$$\ddot{\mathcal{S}}=-\mathcal{J}\mathcal{G}\mathcal{S}-\frac{8\pi G}{3}\frac{\rho_{0,rad}}{\mathcal{S}^3\mathcal{T}^2}.\tag{85}$$

As long as the term on the right side of the equation is greater than zero, the (space-like) expansion is accelerated:

$$\ddot{\mathcal{S}}\begin{cases}>0 & \text{for } -\mathcal{G}\mathcal{J}>\dfrac{8\pi G}{3}\dfrac{\rho_{0,rad}}{\mathcal{S}^4\mathcal{T}^2}\\ =0 & \text{for } -\mathcal{G}\mathcal{J}=\dfrac{8\pi G}{3}\dfrac{\rho_{0,rad}}{\mathcal{S}^4\mathcal{T}^2}\\ <0 & \text{for } -\mathcal{G}\mathcal{J}<\dfrac{8\pi G}{3}\dfrac{\rho_{0,rad}}{\mathcal{S}^4\mathcal{T}^2}\end{cases}\tag{86}$$

If the term becomes equal to or less than zero, the inflation epoch ends. A negative time parameter ($\mathcal{G}<0$) implies a blueshift by a factor $\mathcal{T}$. An accelerated expansion of space, such as inflation, therefore has the consequence that 1) the energy density dilutes by a factor of $\mathcal{S}^4$, 2) but at the same time, that the density increases by a factor of $\mathcal{T}$. If e.g. the time parameter $\mathcal{T}$ is "starting" at a value of the inverse of Planck time and is observed at the end of the inflationary epoch with $\mathcal{T}=1$, the energy density increased by a factor of 1.85×10^{43}. This considerable energy burst was diluted by the expansion of space ($\mathcal{S}^{-4}$); as a result, in our example, an expansion of space by a factor of 6.56×10^{10} would conserve the initial energy density. Nothing prevents us from assuming an arbitrarily large $\mathcal{T}$, which implies that the vehemence of inflation depends not only on the initial energy density, but mainly on the initial value of the time scale factor $\mathcal{T}$. However, depending on the initial conditions and the development of the space and time parameters, the blueshift caused by the contraction of the time dimension either increased the energy density at the beginning of the inflationary epoch, or, at least, delayed the dilution of the energy density and the end of the inflation.

12. A Solution to the Flatness and Fine-Tuning Problems

Evaluating the Friedmann equations, the flatness or fine-tuning problems arise because the universe has an observed density parameter that is very close to 1. In other words, the universe is close to the critical density. The problem is that for the universe to be very close to the critical density after its expansion and evolution, it must have been even closer at earlier times according to the Friedmann equations. There is no known reason for the density of the universe to be close to the critical density, and this appears to be a strange coincidence. Many attempts have been made to explain the flatness problem, and modern theories now include the idea of inflation (see e.g. [9] [10]) which predicts the observed flatness of the Universe. In view of the modified cosmological model, we start with Equation (48) and define the critical density and the density parameter as follows:

$$\rho_c := \frac{3\mathcal{J}^2}{8\pi G}. \tag{87}$$

$$\Omega := \frac{\rho}{\rho_c}. \tag{88}$$

The critical density and its parameters may change over time. Equation 48 can be rearranged as follows:

$$\mathcal{J}^2 - \frac{8\pi G\rho}{3\mathcal{T}^2} = -\frac{kc^2}{\mathcal{S}^2\mathcal{T}^2}. \tag{89}$$

$$\Rightarrow \mathcal{J}^2\left(1 - \frac{8\pi G\rho}{3\mathcal{J}^2\mathcal{T}^2}\right) = -\frac{kc^2}{\mathcal{S}^2\mathcal{T}^2}. \tag{90}$$

$$\Rightarrow \mathcal{J}^2\left(1 - \frac{\rho}{\rho_c\mathcal{T}^2}\right) = -\frac{kc^2}{\mathcal{S}^2\mathcal{T}^2}. \tag{91}$$

$$\Rightarrow \boxed{\mathcal{T}^2 - \Omega = -\frac{kc^2}{\dot{\mathcal{S}}^2}}. \tag{92}$$

It can be seen that the time scale factor compensates any deviation of the energy density parameter from 1. In other words, if the current curvature is near zero, it is not necessary that the critical density at earlier times (when $\dot{\mathcal{S}} \gg 0$) must have been closer to 0 than today, if or because it was compensated by the time scale factor. The outcome is also in line with what was derived for the inflationary universe; it was expected that inflation would require that $\dot{\mathcal{T}} < 0$ and therefore the time scale factor would decrease.

13. Discussion

In the ΛCDM model, the observeation of distant supernovae beeing fainter than expected [11]-[13], led to the conclsuion that the expansion of the universe is accelerating with time [14]. The driving force behind the acceleration is unknown. Some suggestions attribute the observations to vacuum energy. In view of the herein discussed theoretical model, the difference between the expected distance calculated on the basis of the flux density and the redshift can be traced back to two effects: the first effect results from the redshift respectively blueshift $z_{\mathcal{T}}$ caused by the contracted, respectively, the expanded time dimension at time of emission of the light and the second of a decreased flux caused by the accelerated expansion of space. As it could be seen from Equation (83), $\dot{\mathcal{T}}$ is negative, when $\ddot{\mathcal{S}}$ is positive and vice-versa. Given the observation, the model allows the conclusion that the time scale factor $\mathcal{T}$ at the time of emission of the light of the observed supernovae was greater than 1 and dropped to today's value of 1. Further, it was shown in section 2 that the expansion of space requires a contraction of the time dimension. However, it is not yet possible to conclude whether the the time scale factor dropped below 1 at the end of the inflationary epoch and increased to became greater than 1 at $z = 1.5$ to $z = 0.1$. This brought together, leads to the following predictions:

1) The Hubble parameter—as determined on the basis of geometrical or other measures that do not involve the redshift—will differ from the Hubble parameter if determined on the basis of redshit. In the first case, the value of Hubble parameter corresponds to the space parameter; in the second case, the Hubble parameter includes the portion of the redshift, respectively of the blueshift, caused by the time parameter (see Equation (75)).

2) A small difference between the expectation values of the distances 1) based on geometrical or other measures that do not involve the redshift and, 2) based on the redshift, should also be observable for redshifts $z < 0.1$.

3) It can be expected that this difference of the distances is either smaller or greater in relation of ultrahigh-redshift supernovae than it is predicted by the ΛCDM model. In the case of a slighter difference of the expectation values at higher redshift would reveal an acceleration of the expansion of space increasing with time. If ultrahigh-redshift supernovae are brighter than expected, it could be conluded that the time scale factor was below 1 at this time.

With the cosmological model discussed herein, it seems possible to explain the accelerated expansion of spacetime, the inflationary period and the problems of flatness and fine-tuning. A (time-) dynamic development of the time dimension is postulated at the metric level. As has already been shown [15], it does not seem possible at present to realise a consistent quantum theory of gravity that leads to the unification of gravity with the other forces. With a view to the Extended Theories of Gravity (see [15]), quantum and also post-quantum theories (see [16] [17]), it does not seem unreasonable (and not only for symmetry considerations) to introduce—a priori—a dynamisation, *i.e.* a variability of the dimension of time on the one hand and to allow necessary a-posteriori-adaptations in this respect on the other. The metrics for describing the gravitational fields of rotating or non-rotating, charged or uncharged masses, *i.e.* the Schwarschild, Kerr, Reissner-Nordström and Kerr-Newman metrics, already presuppose a spatially dynamic time dimension.

14. Conclusion

A dimensionless time scale factor $\mathcal{T}(t)$ was introduced and a dynamic metric and hypothecial modified cosmological model were elaborated which describe the expansion and contraction of the space and time dimensions in a homogeneous and isotropic model of the universe. It was shown that a variable flow of time may provide answers to some cosmological problems. The modified cosmological model includes an alternative to the standard inflationary theory, and further resolves classical cosmological constant and flatness questions. At a technical level, the theoretical model is not yet as well developed as the standard ΛCDM model and many topics of high importance have not yet been addressed.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Magain, P. and Hauret, C. (2016) Variable Time Flow as an Alternative to Dark Energy. arXiv: 1606.06169. https://doi.org/10.48550/ARXIV.1606.06169

[2] Albrecht, A. and Magueijo, J. (1999) Time Varying Speed of Light as a Solution to Cosmological Puzzles. *Physical Review D*, **59**, Article ID: 043516. https://doi.org/10.1103/physrevd.59.043516

[3] Barrow, J.D. (1999) Cosmologies with Varying Light Speed. *Physical Review D*, **59**, Article ID: 043515. https://doi.org/10.1103/physrevd.59.043515

[4] Magueijo, J. (2003) New Varying Speed of Light Theories. *Reports on Progress in Physics*, **66**, 2025-2068. https://doi.org/10.1088/0034-4885/66/11/r04

[5] Mueller, T. and Grave, F. (2009) Catalogue of Spacetimes. arXiv: 0904.4184v3. https://doi.org/10.48550/ARXIV.0904.4184

[6] Carroll, S.M. (2019) Spacetime and Geometry: An Introduction to General Relativity. Cambridge University Press. https://doi.org/10.1017/9781108770385

[7] Spatschek, K.H. (2021) Astrophysik. Springer. https://doi.org/10.1007/978-3-662-63626-8

[8] Hogg, D.W. (1999) Distance Measures in Cosmology. arXiv: as-tro-ph/9905116. https://doi.org/10.48550/ARXIV.ASTRO-PH/9905116

[9] Guth, A.H. (1981) Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Physical Review D*, **23**, 347-356. https://doi.org/10.1103/physrevd.23.347

[10] Linde, A.D. (1982) A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Physics Letters B*, **108**, 389-393. https://doi.org/10.1016/0370-2693(82)91219-9

[11] Riess, A.G., Filippenko, A.V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P.M., *et al.* (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, **116**, 1009-1038. https://doi.org/10.1086/300499

[12] Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A., Nugent, P., Castro, P.G., *et al.* (1999) Measurements of Ω and Λ from 42 High-Redshift Supernovae. *The Astrophysical Journal*, **517**, 565-586. https://doi.org/10.1086/307221

[13] Riess, A.G., Strolger, L., Tonry, J., Casertano, S., Ferguson, H.C., Mobasher, B., *et al.* (2004) Type IA Supernova Discoveries at $z > 1$ from the *Hubble Space Telescope*: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. *The Astrophysical Journal*, **607**, 665-687. https://doi.org/10.1086/383612

[14] Filippenko, A.V. (2001) Einstein's Biggest Blunder? High-Redshift Supernovae and the Accelerating Universe. *Publications of the Astronomical Society of the Pacific*, **113**, 1441-1448. https://doi.org/10.1086/324512

[15] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive

Test for General Relativity. *International Journal of Modern Physics D*, **18**, 2275-2282. https://doi.org/10.1142/s0218271809015904

[16] Oppenheim, J. (2023) A Postquantum Theory of Classical Gravity? *Physical Review X*, **13**, Article ID: 041040. https://doi.org/10.1103/physrevx.13.041040

[17] Vankov, A.A. (2007) Proper Mass Variation under Gravitational and Coulomb Force Action in Relativistic Mechanics of Point Particle. arXiv: 0710.1119. https://arxiv.org/abs/0710.1119