

The Maxwell-Heaviside Equations Explained by the Theory of Informatons

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Abstract

In the articles “*Newtons Law of Universal Gravitation Explained by the Theory of Informatons*” and “*The Gravitational Interaction between Moving Mass Particles Explained by the Theory of Informatons*” the gravitational interaction has been explained by the hypothesis that *information carried by informatons* is the substance of gravitational fields, i.e. the medium that the interaction in question makes possible. From the idea that “information carried by informatons” is its substance, it has been deduced that—on the macroscopic level—a gravitational field manifests itself as a dual entity, always having a field- and an induction component (E_g and B_g) simultaneously created by their common sources. In this article we will mathematically deduce the Maxwell-Heaviside equations from the kinematics of the informatons. These relations describe on the macroscopic level how a gravitational field (E_g , B_g) is generated by whether or not moving masses and how spatial and temporal changes of E_g and B_g are related. We show that there is no causal link between E_g and B_g .

Keywords

Gravity, Gravitational Field, Maxwell Equations, Informatons

1. Introduction

Generally, the gravitational field is set up [1]-[4] by a given distribution of—whether or not moving—objects and it is completely defined by a vector field with two components: the “*g-field*” characterized by the vector E_g and the “*g-induction*” characterized by the vector B_g . These components each have a value defined at every point of space and time and are thus, relative to an IRF O , regarded as functions of the space and time coordinates.

Let us focus on the contribution to a gravitational field of one of its sources: a certain moving mass particle with rest mass m_0 . More specifically we focus on the contribution of that particle to the flow of g-information at an arbitrary point P in the field. That flow is made up of informatons that pass near P in a specific direction with velocity \mathbf{c} and it is characterized by N , the rate per unit area at which these informatons cross an elementary surface perpendicular to the direction in which they move. The cloud of these informatons in the vicinity of P is characterized by its density n : n is the number of informatons per unit volume. N and n are linked by the relationship:

$$n = \frac{N}{c} \quad (1)$$

The definition of an informaton implies [1] [2] that every informaton that passes near P is characterized by two attributes that refer to its emitter: its g-index s_g and its β -index s_β . s_g , the magnitude of the g-index, is the elementary quantity of g-information. It is a fundamental physical constant. s_β refers to the state of motion of the source of the informaton and is defined by the relationship

$$s_\beta = \frac{\mathbf{c} \times \mathbf{s}_g}{c} \quad (2)$$

The informatons emitted by m_0 that pass near P with velocity \mathbf{c} contribute there to the *density of the g-information flow* with an amount $(N \cdot s_g)$. That quantity is the rate per unit area at which g-information emitted by m_0 crosses an elementary surface that, at P , is perpendicular to the direction of the g-indices of the involved informatons. It is the contribution of m_0 to the g-field at P . We refer to it as

$$\mathbf{E}_g = N \cdot \mathbf{s}_g$$

And the same informatons contribute there to the *density of the g-information cloud* with an amount $(n \cdot s_\beta)$. That quantity determines at P the amount of β -information per unit volume. It is the contribution of m_0 to the g-induction at P and we refer to it as:

$$\mathbf{B}_g = n \cdot \mathbf{s}_\beta$$

In **Figure 1**, we consider an informaton that is part of the flow of informatons that—at the moment t —passes at P with velocity \mathbf{c} . It is completely defined by its attributes s_g and s_β . $\Delta\theta$ is its characteristic angle: the angle between the lines carrying s_g and \mathbf{c} that is characteristic for the movement of the emitter.

The infinitesimal changes of N and of n at Q between the moments t and $(t + dt)$ are determined by the spatial change of the density of the flow and the density of the cloud of informations that during that elementary period travels from P to Q . This also applies to the infinitesimal changes of s_g and s_β at Q between the moments t and $(t + dt)$: they are determined by the spatial changes of the g-index and of the β -index of an informaton that during that elementary period travels from P to Q .

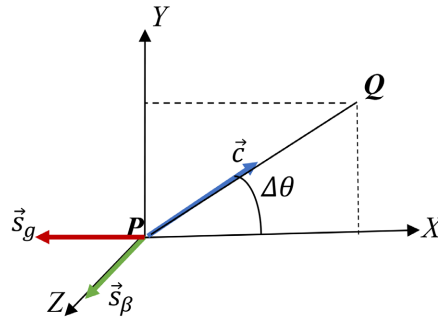


Figure 1. An informaton moving from P to Q

On the macroscopic level, this implies that there must be a relationship between the change in time of the gravitational field $(\mathbf{E}_g, \mathbf{B}_g)$ at an arbitrary point P and the spatial variation of that field in the vicinity of P .

The intensity of the spatial variation of the components of the gravitational field at P is characterized by $div \mathbf{E}_g$, $div \mathbf{B}_g$, $rot \mathbf{E}_g$ and by $rot \mathbf{B}_g$ and the rate at which these components change in time by $\frac{\partial \mathbf{E}_g}{\partial t}$ and $\frac{\partial \mathbf{B}_g}{\partial t}$.

From the above it can be concluded that it makes sense to investigate the relationships between the quantities that characterize the spatial variations of $(\mathbf{E}_g, \mathbf{B}_g)$ and the rate's at which they change in time.

2. $div \mathbf{E}_g$ —The First Equation in Free Space

The physical fact that the rate at which g-information flows inward a closed empty space must be equal to the rate at which it flows outward, can be expressed as [1] [2]: $\oiint_S \mathbf{E}_g \cdot d\mathbf{S} = 0$

So (theorem of Ostrogradsky) [5]:

$$div \mathbf{E}_g = 0$$

Thus, in vacuum the law of conservation of g-information can be expressed as followed:

1) At a matter free point P of a gravitational field, the spatial variation of \mathbf{E}_g obeys the law: $div \mathbf{E}_g = 0$.

This is the first equation of Maxwell-Heaviside in vacuum.

Corollary: At a matter free point P of a gravitational field

$$\frac{\partial}{\partial t} [N \cdot \cos(\Delta\theta)] = 0.$$

Because [5]

$$div \mathbf{E}_g = div(N \cdot \mathbf{s}_g) = grad(N) \cdot \mathbf{s}_g + N \cdot div(\mathbf{s}_g) \tag{3}$$

It follows from the first equation of Maxwell-Heaviside that:

$$grad(N) \cdot \mathbf{s}_g + N \cdot div(\mathbf{s}_g) = 0$$

1. First we calculate: $grad(N) \cdot \mathbf{s}_g$.

Referring to **Figure 1**:

$$\text{grad}(N) = \frac{N_Q - N_P}{PQ} \cdot \mathbf{e}_c = \frac{N_Q - N_P}{c \cdot dt} \cdot \mathbf{e}_c$$

Because an informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q , (with $PQ = c \cdot dt$).

$$\frac{N_Q - N_P}{dt} = \frac{N(t+dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

So:

$$\text{grad}(N) = -\frac{1}{c} \cdot \frac{\partial N}{\partial t} \cdot \mathbf{e}_c = -\frac{1}{c} \cdot \frac{\partial N}{\partial t} \cdot \frac{\mathbf{c}}{c}$$

And:

$$\text{grad}(N) \cdot \mathbf{s}_g = -\frac{1}{c^2} \cdot \frac{\partial N}{\partial t} \cdot \mathbf{c} \cdot \mathbf{s}_g = \frac{1}{c} \cdot \frac{\partial N}{\partial t} \cdot \mathbf{s}_g \cdot \cos(\Delta\theta) \tag{4}$$

2. Next, we calculate: $N \cdot \text{div}(\mathbf{s}_g)$.

$$\text{div}(\mathbf{s}_g) = \frac{\oiint \mathbf{s}_g \cdot d\mathbf{S}}{dV}$$

For that purpose, we calculate the double integral over the closed surface S formed by the infinitesimal surfaces dS that are at P and Q perpendicular to the flow of informatons (perpendicular to \mathbf{c}) and by the tube that connects the edges of these surfaces (and that is parallel to \mathbf{c}). $dV = c \cdot dt \cdot dS$ is the infinitesimal volume enclosed by S :

$$\text{div}(\mathbf{s}_g) = \frac{\oiint \mathbf{s}_g \cdot d\mathbf{S}}{dV} = \frac{\mathbf{s}_g \cdot d\mathbf{S} \cdot \cos(\Delta\theta_P) - \mathbf{s}_g \cdot d\mathbf{S} \cdot \cos(\Delta\theta_Q)}{dS \cdot c \cdot dt}$$

Because an informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q , (with $PQ = c \cdot dt$) is:

$$\frac{\cos(\Delta\theta_P) - \cos(\Delta\theta_Q)}{dt} = \frac{\cos[\Delta\theta(t)] - \cos[\Delta\theta(t+dt)]}{dt} = \frac{\partial[\cos(\Delta\theta)]}{\partial t}$$

So

$$\text{div}(\mathbf{s}_g) = \frac{1}{c} \cdot \mathbf{s}_g \cdot \frac{\partial\{\cos(\Delta\theta)\}}{\partial t}$$

And:

$$N \cdot \text{div}(\mathbf{s}_g) = \frac{N}{c} \cdot \mathbf{s}_g \cdot \frac{\partial\{\cos(\Delta\theta)\}}{\partial t} \tag{5}$$

3. Substitution of (4) and (5) in (3) gives:

$$\frac{1}{c} \cdot \frac{\partial N}{\partial t} \cdot \mathbf{s}_g \cdot \cos(\Delta\theta) + \frac{N}{c} \cdot \mathbf{s}_g \cdot \frac{\partial\{\cos(\Delta\theta)\}}{\partial t} = 0$$

Or:

$$\frac{\partial}{\partial t} [N \cdot \cos(\Delta\theta)] = 0 \quad (6)$$

3. $div \mathbf{B}_g$ —The Second Equation in Free Space

We refer to **Figure 1** and notice that:

$$\mathbf{s}_g = -s_g \cdot \mathbf{e}_x \quad \text{and} \quad \mathbf{s}_\beta = \frac{\mathbf{c} \times \mathbf{s}_g}{c} = s_g \cdot \sin(\Delta\theta) \cdot \mathbf{e}_z$$

From mathematics [5] we know:

$$div \mathbf{B}_g = div(n \cdot \mathbf{s}_\beta) = grad(n) \cdot \mathbf{s}_\beta + n \cdot div(\mathbf{s}_\beta) \quad (7)$$

1. First we calculate: $grad(n) \cdot \mathbf{s}_\beta$

$grad(n) \cdot \mathbf{s}_\beta = 0$ because $grad(n)$ is perpendicular to \mathbf{s}_β . Indeed n changes only in the direction of the flow of informatons, so $grad(n)$ has the same orientation as \mathbf{c} .

2. Next we calculate: $n \cdot div(\mathbf{s}_\beta)$

$$div(\mathbf{s}_\beta) = \frac{\oiint \mathbf{s}_\beta \cdot d\mathbf{S}}{dV}$$

We calculate the double integral over the closed surface S formed by the infinitesimal surfaces $dS = dzdy$ that are at P and at Q perpendicular to the X-axis and by the tube that connects the edges of these surfaces.

Because \mathbf{s}_β is oriented along the Z-axis the flux of \mathbf{s}_β through the planes $dzdy$ and $dx \cdot dz$ is zero, while the fluxes through the planes $dx \cdot dy$ are equal and opposite. So we can conclude that:

$$div(\mathbf{s}_\beta) = \frac{\oiint \mathbf{s}_\beta \cdot d\mathbf{S}}{dV} = 0$$

3. Both terms of the expression (7) of $div \mathbf{B}_g$ are zero, so $div \mathbf{B}_g = 0$, what implies (theorem of Ostrogradsky) that at a matter free point for every closed surface S in a gravitational field:

$$\iint_S \mathbf{B}_g \cdot d\mathbf{S} = 0$$

We conclude:

2) *At a matter free point P of a gravitational field, the spatial variation of \mathbf{B}_g obeys the law: $div \mathbf{B}_g = 0$.*

This is the second equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that the β -index of an informaton is always perpendicular to both its g-index \mathbf{s}_g and its velocity \mathbf{c} .

4. $rot \mathbf{E}_g$ —The Third Equation in Free Space

The density of the flow of g-informaton carried by informatons that—at the moment t —passes near P with velocity \mathbf{c} (**Figure 1**) is defined as:

$$\mathbf{E}_g = N \cdot \mathbf{s}_g = -N \cdot s_g \cdot \mathbf{e}_x$$

We know that [5]

$$\text{rot } \mathbf{E}_g = \{ \text{grad}(N) \times \mathbf{s}_g \} + N \cdot \text{rot}(\mathbf{s}_g) \tag{8}$$

1. First we calculate: $\{ \text{grad}(N) \times \mathbf{s}_g \}$

This expression describes the component of $\text{rot } \mathbf{E}_g$ caused by the spatial variation of N in the vicinity of P when $\Delta\theta$ remains constant. N has the same value at all points of the infinitesimal surface that, at P , is perpendicular to the flow of informatons. So $\text{grad}(N)$ is parallel to \mathbf{c} and its magnitude is the increase of the magnitude of N per unit length. Thus, with $PQ = c \cdot dt$, $\text{grad}(N)$ is determined by:

$$\text{grad}(N) = \frac{N_Q - N_P}{PQ} \cdot \frac{\mathbf{c}}{c} = \frac{N_Q - N_P}{c \cdot dt} \cdot \frac{\mathbf{c}}{c}$$

And:

$$\text{grad}(N) \times \mathbf{s}_g = \frac{N_Q - N_P}{c \cdot dt} \cdot \frac{\mathbf{c}}{c} \times \mathbf{s}_g = \frac{N_Q - N_P}{c \cdot dt} \cdot \mathbf{s}_\beta$$

The density of the flow of informatons at Q at the moment t is equal to the density of that flow at P at the moment $(t - dt)$, so:

$$\frac{N_Q - N_P}{dt} = \frac{N(t-dt) - N(t)}{dt} = - \frac{\partial N}{\partial t}$$

And taking into account that:

$$\frac{N}{c} = n$$

we obtain:

$$\text{grad}(N) \times \mathbf{s}_g = - \frac{\partial n}{\partial t} \cdot \mathbf{s}_\beta \tag{9}$$

2. Next we calculate: $\{ N \cdot \text{rot}(\mathbf{s}_g) \}$

This expression describes the component of $\text{rot } \mathbf{E}_g$ caused by the spatial variation of $\Delta\theta$ —the orientation of the g -index—in the vicinity of P —when N remains constant. $(\Delta\theta)_P$ is the characteristic angle of the informatons that pass near P and $(\Delta\theta)_Q$ is the characteristic angle of the informatons that at the same moment pass near Q (Figure 2).

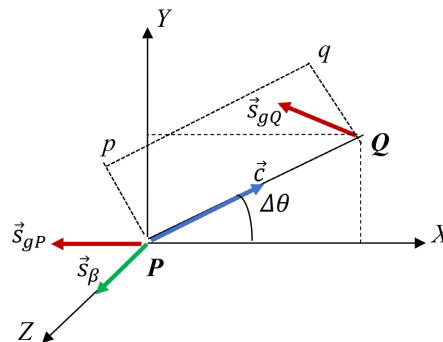


Figure 2. Calculation of $\text{rot}(\bar{\mathbf{s}}_g)$

For the calculation of $rot(s_g) = \frac{\oint s_g \cdot dl}{dS}$

we calculate $\oint s_g \cdot dl$ along the closed path $PQqpP$ that encircles dS :
 $dS = PQ \cdot Pp = c \cdot dt \cdot Pp$ (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it)¹

$$N \cdot rot(s_g) = N \cdot \frac{s_g \cdot \sin[(\Delta\theta)_Q] \cdot Qq - s_g \cdot \sin[(\Delta\theta)_P] \cdot pP}{c \cdot dt \cdot Pp} \cdot e_z$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, so:

$$N \cdot rot(s_g) = N \cdot \frac{s_g \cdot \sin[\Delta\theta(t - dt)] \cdot Qq - s_g \cdot \sin[\Delta\theta(t)] \cdot pP}{c \cdot dt \cdot Pp} \cdot e_z$$

The rate at which $\sin(\Delta\theta)$ in P changes at the moment t , is:

$$\frac{\partial\{\sin(\Delta\theta)\}}{\partial t} = \frac{\sin\{\Delta\theta(t)\} - \sin\{\Delta\theta(t - dt)\}}{dt}$$

And taking into account that

$$n = \frac{N}{c}$$

we obtain:

$$N \cdot rot(s_g) = -n \cdot s_g \cdot \frac{\partial\{\sin(\Delta\theta)\}}{\partial t} \cdot e_z = -n \cdot \frac{\partial}{\partial t} \{s_g \cdot \sin(\Delta\theta)\} \cdot e_z$$

or

$$N \cdot rot(s_g) = -n \cdot \frac{\partial s_\beta}{\partial t} \quad (10)$$

3. Combining the results (9) and (10), we obtain:

$$\begin{aligned} rot E_g &= grad(N) \times s_g + N \cdot rot(s_g) \\ &= - \left(\frac{\partial n}{\partial t} \cdot s_\beta + n \cdot \frac{\partial s_\beta}{\partial t} \right) \\ &= - \frac{\partial(n \cdot s_\beta)}{\partial t} = - \frac{\partial B_g}{\partial t} \end{aligned} \quad (11)$$

We conclude:

3) *At a matter free point P of a gravitational field, the spatial variation of E_g and the rate at which B_g is changing are connected by the relation:*

$$rot E_g = - \frac{\partial B_g}{\partial t}$$

This is the third equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of $(n \cdot s_\beta)$ —the density of the cloud of β -information—at a point of a gravitational field goes together with a spatial variation of $(N \cdot s_g)$ —the density of the flow of g -information—in the vicinity of that

¹The contributions along PQ and qp are equal and opposite. They cancel each other.

point.

The mentioned relation implies (theorem of Stokes [4]):

$$\oint \mathbf{E}_g \cdot d\mathbf{l} = -\iint_S \frac{\partial \mathbf{B}_g}{\partial t} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iint_S \mathbf{B}_g \cdot d\mathbf{S} = -\frac{\partial \Phi_B}{\partial t}$$

The orientation of the surface vector $d\mathbf{S}$ is linked to the orientation of the path L by the “rule of the corkscrew”. $\Phi_B = \iint_S \mathbf{B}_g \cdot d\mathbf{S}$ is called the “ β -information-flux through S ”.

So, in a gravitational field, the rate at which the surface integral of \mathbf{B}_g over a surface S changes is equal and opposite to the line integral of \mathbf{E}_g over the boundary L of that surface.

5. $rot \mathbf{B}_g$ —The Fourth Equation in Free Space

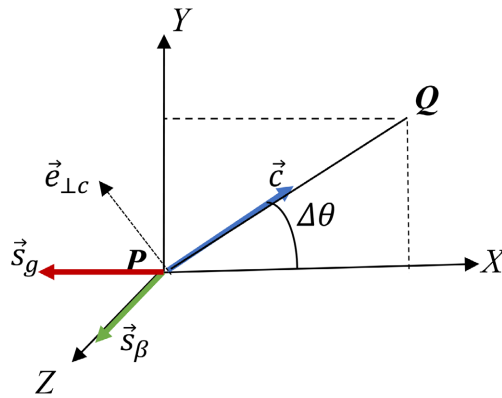


Figure 3. An informaton moving from P to Q

We consider again \mathbf{E}_g and \mathbf{B}_g , the contributions of the informatons that—at the moment t —pass with velocity c near P , to the g -field and to the g -induction at that point. Referring to Figure 3:

$$\mathbf{E}_g = N \cdot \mathbf{s}_g = -N \cdot s_g \cdot \mathbf{e}_x$$

and

$$\mathbf{B}_g = n \cdot \mathbf{s}_\beta = n \cdot \frac{\mathbf{c} \times \mathbf{s}_g}{c} = n \cdot s_g \cdot \sin(\Delta\theta) \cdot \mathbf{e}_z$$

A. Let us calculate $rot \mathbf{B}_g$. We know that [5]

$$rot \mathbf{B}_g = \{grad(n) \times \mathbf{s}_\beta\} + n \cdot rot(\mathbf{s}_\beta) \tag{12}$$

1. First we calculate: $\{grad(n) \times \mathbf{s}_\beta\}$

This expression describes the component of $rot \mathbf{B}_g$ caused by the spatial variation of n in the vicinity of P when $\Delta\theta$ remains constant. n has the same value at all points of the infinitesimal surface that, at P , is perpendicular to the flow of informatons. So $grad(n)$ is parallel to \mathbf{c} and its magnitude is the increase of the magnitude of n per unit length.

With $PQ = c \cdot dt$, $grad(n)$ is determined by:

$$\text{grad}(n) = \frac{n_Q - n_P}{PQ} \cdot \frac{\mathbf{c}}{c} = \frac{n_Q - n_P}{c \cdot dt} \cdot \frac{\mathbf{c}}{c}$$

The density of the cloud of informatons at Q at the moment t is equal to the density of that flow at P at the moment $(t - dt)$, so:

$$\frac{n_Q - n_P}{dt} = \frac{n(t - dt) - n(t)}{dt} = -\frac{\partial n}{\partial t}$$

And

$$\text{grad}(n) = -\frac{1}{c} \cdot \frac{\partial n}{\partial t} \cdot \frac{\mathbf{c}}{c} = -\frac{1}{c} \cdot \frac{\partial n}{\partial t} \cdot \mathbf{e}_c$$

The vector $\{\text{grad}(n) \times \mathbf{s}_\beta\}$ is perpendicular to the plane determined by \mathbf{c} and \mathbf{s}_β . So, it lies in the XY -plane and is there perpendicular to \mathbf{c} forming an angle $\Delta\theta$ with the axis OY . Taking into account the definition of vectoral product we obtain:

$$\text{grad}(n) \times \mathbf{s}_\beta = -\frac{1}{c} \cdot \frac{\partial n}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) \cdot (\mathbf{e}_c \times \mathbf{e}_z)$$

With $\mathbf{e}_c \times \mathbf{e}_z = -\mathbf{e}_{\perp c}$:

$$\text{grad}(n) \times \mathbf{s}_\beta = \frac{1}{c} \cdot \frac{\partial n}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) \cdot \mathbf{e}_{\perp c}$$

And, taking into account that $n = \frac{N}{c}$, we obtain:

$$\text{grad}(n) \times \mathbf{s}_\beta = \frac{1}{c^2} \cdot \frac{\partial N}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) \cdot \mathbf{e}_{\perp c} \tag{13}$$

2. Next we calculate $\{n \cdot \text{rot}(\mathbf{s}_\beta)\}$

This expression is the component of $\text{rot} \mathbf{B}_g$ caused by the spatial variation of \mathbf{s}_β in the vicinity of P when n remains constant. For the calculation of

$$\text{rot}(\mathbf{s}_\beta) = \frac{\oint \mathbf{s}_\beta \cdot d\mathbf{l}}{dS} \cdot \mathbf{e}_{\perp c}$$

With dS the encircled area, we calculate $\oint \mathbf{s}_\beta \cdot d\mathbf{l}$ along the closed path $PpQqP$ that encircles dS : $dS = PQ \cdot Pp = c \cdot dt \cdot Pp$ (Figure 4). (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it).

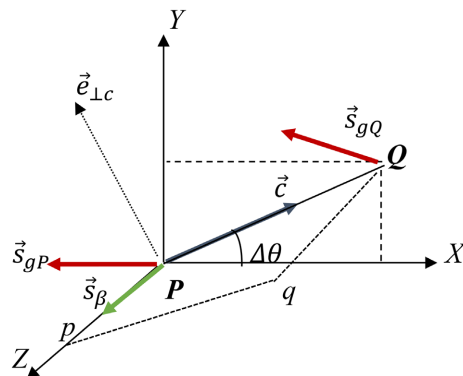


Figure 4. Calculation of $\text{rot}(\mathbf{s}_\beta)$

$$rot(s_\beta) = \frac{\oint s_\beta \cdot d\mathbf{l}}{dS} \cdot \mathbf{e}_{\perp c} = \frac{s_g \cdot \sin[(\Delta\theta)_P] \cdot Pp - s_g \cdot \sin[(\Delta\theta)_Q] \cdot qQ}{c \cdot dt \cdot Pp} \cdot \mathbf{e}_{\perp c}$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, so:

$$rot(s_\beta) = \frac{\oint s_\beta \cdot d\mathbf{l}}{dS} \cdot \mathbf{e}_{\perp c} = \frac{s_g \cdot \{\sin[\Delta\theta(t)] \cdot Pp - s_g \cdot \sin[\Delta\theta(t-dt)]\} \cdot qQ}{c \cdot dt \cdot Pp} \cdot \mathbf{e}_{\perp c}$$

The rate at which $\sin(\Delta\theta)$ at P changes at the moment t , is:

$$\frac{\partial\{\sin(\Delta\theta)\}}{\partial t} = \frac{\sin\{(\Delta\theta)[t]\} - \sin\{(\Delta\theta)[t-dt]\}}{dt}$$

So:

$$rot(s_\beta) = s_g \cdot \frac{1}{c} \cdot \frac{\partial[\sin(\Delta\theta)]}{\partial t} \cdot \mathbf{e}_{\perp c}$$

And with $n = \frac{N}{c}$, we finally obtain:

$$n \cdot rot(s_\beta) = s_g \cdot \frac{1}{c^2} \cdot N \cdot \frac{\partial[\sin(\Delta\theta)]}{\partial t} \cdot \mathbf{e}_{\perp c} \tag{14}$$

3. Substituting the results (13) and (14) in (12) gives:

$$\begin{aligned} rot \mathbf{B}_g &= \frac{1}{c^2} \cdot s_g \cdot \left\{ \frac{\partial N}{\partial t} \cdot \sin(\Delta\theta) + N \cdot \frac{\partial[\sin(\Delta\theta)]}{\partial t} \right\} \cdot \mathbf{e}_{\perp c} \\ &= \frac{1}{c^2} \cdot s_g \cdot \frac{\partial}{\partial t} [N \cdot \sin(\Delta\theta)] \cdot \mathbf{e}_{\perp c} \end{aligned} \tag{15}$$

B. Next we calculate $\frac{\partial \mathbf{E}_g}{\partial t}$ starting from [5]

$$\frac{\partial \mathbf{E}_g}{\partial t} = \frac{\partial N}{\partial t} \cdot \mathbf{s}_g + N \cdot \frac{\partial \mathbf{s}_g}{\partial t}$$

and from:

$$\mathbf{s}_g = -s_g \cdot \mathbf{e}_x \quad \text{and} \quad \frac{\partial \mathbf{s}_g}{\partial t} = s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \mathbf{e}_y$$

We obtain:

$$\frac{\partial \mathbf{E}_g}{\partial t} = -\frac{\partial N}{\partial t} \cdot s_g \cdot \mathbf{e}_x + N \cdot s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \mathbf{e}_y$$

Taking into account:

$$\mathbf{e}_x = \cos(\Delta\theta) \cdot \mathbf{e}_c - \sin(\Delta\theta) \cdot \mathbf{e}_{\perp c} \quad \text{and} \quad \mathbf{e}_y = \sin(\Delta\theta) \cdot \mathbf{e}_c + \cos(\Delta\theta) \cdot \mathbf{e}_{\perp c}$$

we obtain:

$$\begin{aligned} \frac{\partial \mathbf{E}_g}{\partial t} &= \left[-\frac{\partial N}{\partial t} \cdot s_g \cdot \cos(\Delta\theta) + N \cdot s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \sin(\Delta\theta) \right] \cdot \mathbf{e}_c \\ &+ \left[\frac{\partial N}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) + N \cdot s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \cos(\Delta\theta) \right] \cdot \mathbf{e}_{\perp c} \end{aligned}$$

or:

$$\frac{\partial \mathbf{E}_g}{\partial t} = s_g \cdot \left\{ -\frac{\partial}{\partial t} [N \cdot \cos(\Delta\theta)] \cdot \mathbf{e}_c + \frac{\partial}{\partial t} [N \cdot \sin(\Delta\theta)] \cdot \mathbf{e}_{\perp c} \right\}$$

Taking into account (6), we find:

$$\frac{\partial \mathbf{E}_g}{\partial t} = s_g \cdot \frac{\partial}{\partial t} [N \cdot \sin(\Delta\theta)] \cdot \mathbf{e}_{\perp c} \quad (16)$$

C. From (15) and (16), we conclude:

$$\text{rot } \mathbf{B}_g = \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t}$$

4) *At a matter free point P of a gravitational field, the spatial variation of \mathbf{B}_g and the rate at which \mathbf{E}_g is changing are connected by the relation:*

$$\text{rot } \mathbf{B}_g = \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t}$$

This is the fourth equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of $(N \cdot s_g)$ —the density of the flow of g-information—at a point of a gravitational field goes together with a spatial variation of $(n \cdot s_\beta)$ —the density of the cloud of β -information—in the vicinity of that point.

This relation implies (theorem of Stokes [5]): *In a gravitational field, the rate at which the surface integral of \mathbf{E}_g over a surface S changes is proportional to the line integral of \mathbf{B}_g over the boundary L of that surface:*

$$\oint \mathbf{B}_g \cdot d\mathbf{l} = \frac{1}{c^2} \iint_S \frac{\partial \mathbf{E}_g}{\partial t} \cdot d\mathbf{S} = \frac{1}{c^2} \frac{\partial}{\partial t} \iint_S \mathbf{E}_g \cdot d\mathbf{S} = \frac{1}{c^2} \frac{\partial \Phi_G}{\partial t}$$

The orientation of the surface vector $d\mathbf{S}$ is linked to the orientation of the path on L by the “rule of the corkscrew”. $\Phi_G = \iint_S \mathbf{E}_g \cdot d\mathbf{S}$ is called the “g-information-flux through S ”.

6. The Maxwell-Heaviside Equations

The volume-element at a point P inside a mass continuum is in any case an emitter of g-information and, if the mass is moving, it is also a source of β -information. In [1] it is shown that the instantaneous value of ρ_G —the mass density at P —contributes to the instantaneous value of $\text{div } \mathbf{E}_g$ at that point with an amount $-\frac{\rho_G}{\eta_0}$; and in [2] it is shown that the instantaneous value of \mathbf{J}_G —the mass flow density—contributes to the instantaneous value of $\text{rot } \mathbf{B}_g$ at P with an amount $-\nu_0 \cdot \mathbf{J}_G$.

It is evident that at a point of a gravitational field—linked to an IRF \mathcal{O} —one must take into account the contributions of the local values of $\rho_G(x, y, z; t)$ and of $\mathbf{J}_G(x, y, z; t)$. This results in the generalization and expansion of the laws at a mass free point. By superposition we obtain:

6.1. Equation 1

At a point P of a gravitational field, the spatial variation of \mathbf{E}_g obeys the law:

$$\operatorname{div} \mathbf{E}_g = -\frac{\rho_G}{\eta_0}$$

In integral form:

$$\Phi_G = \iint_S \mathbf{E}_g \cdot d\mathbf{S} = -\frac{1}{\eta_0} \cdot \iiint_G \rho_G \cdot dV$$

6.2. Equation 2

At a point P of a gravitational field, the spatial variation of \mathbf{B}_g obeys the law:

$$\operatorname{div} \mathbf{B}_g = 0$$

In integral form:

$$\Phi_B = \iint_S \mathbf{B}_g \cdot d\mathbf{S} = 0$$

6.3. Equation 3

At a point P of a gravitational field, the spatial variation of \mathbf{E}_g and the rate at which \mathbf{B}_g is changing are connected by the relation:

$$\operatorname{rot} \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$$

In integral form:

$$\oint \mathbf{E}_g \cdot d\mathbf{l} = -\iint_S \frac{\partial \mathbf{B}_g}{\partial t} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iint_S \mathbf{B}_g \cdot d\mathbf{S} = -\frac{\partial \Phi_B}{\partial t}$$

6.4. Equation 4

At a point P of a gravitational field, the spatial variation of \mathbf{B}_g and the rate at which \mathbf{E}_g is changing are connected by the relation:

$$\operatorname{rot} \mathbf{B}_g = \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t} - \nu_0 \cdot \mathbf{J}_G$$

In integral form:

$$\oint \mathbf{B}_g \cdot d\mathbf{l} = \frac{1}{c^2} \iint_S \frac{\partial \mathbf{E}_g}{\partial t} \cdot d\mathbf{S} - \nu_0 \cdot \iint_S \mathbf{J}_G \cdot d\mathbf{S} = \frac{1}{c^2} \cdot \frac{\partial}{\partial t} \iint_S \mathbf{E}_g \cdot d\mathbf{S} - \nu_0 \cdot \iint_S \mathbf{J}_G \cdot d\mathbf{S}$$

These are the laws of Heaviside-Maxwell.

In the frame of the gravitoelectromagnetic description of the gravitational phenomena and laws the Maxwell-Heaviside equations describe on the macroscopic level how a gravitational field (\mathbf{E}_g , \mathbf{B}_g) is generated by whether or not moving masses and how spatial and temporal changes of \mathbf{E}_g and \mathbf{B}_g are related. Gravitoelectromagnetism (GEM) was developed by Oliver Heaviside [6] and Oleg Jefimenko [7] as a separate theory expanding Newton's law referring to the kinetic effects of gravity. They started from the idea that the gravitational field generated by moving mass particles must be analogue to the electromag-

netic field generated by moving charges.

7. Conclusion

The mathematical deductions of the laws of Maxwell-Heaviside from the kinematics of the informatons, confirm that these equations indicate that there is no causal link between \mathbf{E}_g and \mathbf{B}_g . Therefore, *we must conclude that a gravitational field is a dual entity always having a “field-” and an “induction-” component simultaneously created by their common sources: time-variable masses and mass flows*². The Heaviside-Maxwell equations are consistent with special relativity. Indeed they are analogue to Maxwell’s equations in EM and it is proved [8] that these are consistent with special relativity.

8. Epilogue

The theory of informatons unifies gravitation with electromagnetism. Indeed, with the theory of informatons it is not only possible to explain the phenomena and the laws of gravitation but also those of electromagnetism [3] [4]. It is sufficient to add the following rule to the postulate of the emission of informatons:

C. *Informatons emitted by an electrically charged particle (a “point charge” q) at rest in an IRF, carry an attribute referring to the charge of the emitter, namely the e-index. e-indices are represented as \vec{s}_e and defined by:*

1. *The e-indices are radial relative to the position of the emitter. They are centrifugal when the emitter carries a positive charge ($q = +Q$) and centripetal when the charge of the emitter is negative ($q = -Q$).*

2. *s_e , the magnitude of an e-index depends on Q/m_0 , the charge per unit of mass of the emitter. It is defined by:*

$$s_e = \frac{1}{K \cdot \varepsilon_0} \cdot \frac{Q}{m_0} = 8.32 \times 10^{-40} \cdot \frac{Q}{m_0} \text{ kg} \cdot \text{m}^3 \cdot \text{s}^{-1} \cdot \text{C}^{-1}$$

where $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is the permittivity constant.

1) It follows that a point charge at rest in an IRF \mathcal{O} is surrounded by a cloud of informatons carrying, besides g-, “e-information”. Focusing on the e-information, that cloud can be identified as the “electric field” of q , that at the macroscopic level, is completely characterized by a vector field \mathbf{E} . In the definition of \mathbf{E} , the factor $(\frac{q}{\varepsilon_0})$ takes the role that is played by the factor $(-\frac{m_0}{\eta_0})$ in the definition of \mathbf{E}_g . The laws of conservation of e-information (Gauss’s law) and of the interaction between point charges at rest (Coulombs law) are the analogues respectively of the law of conservation of g-information and of Newton’s law of universal gravitation.

2) It also follows that a “point charge” q moving relative to IRF \mathcal{O} is the source of informatons with two attributes: the e-index \vec{s}_e and the b-index \vec{s}_b :

²On the understanding that the induction-component equals zero if the source of the field is time independent.

$$s_b = \frac{\mathbf{c} \times \mathbf{s}_e}{c}$$

Thus a moving point charge is surrounded by a cloud of informatons carrying besides e-information also “*b-information*” or “*magnetic information*”. That cloud can be identified as the “*electromagnetic field*” of q . On the macroscopic level it is completely characterized as a vector field always having a field- and an induction component (\mathbf{E} and \mathbf{B}) simultaneously created by their common sources. In the definition of \mathbf{B} the factor $(\mu_0 \cdot q)$ takes the role that is played by the factor $(-v_0 \cdot m_0)$ in the definition of \mathbf{B}_g . Lorentz law regarding the interaction between moving point charges is the analogue of the law regarding the interaction between moving mass particles.

3) The mathematical deductions that in the frame of this article lead to the Maxwell-Heaviside equations that govern the gravitational field, remain valid in the case of the electromagnetic field where they lead to Maxwells equations for the EM field. The role played by the factor $(-\frac{\rho_G}{\eta_0})$ in the case of a mass continuum with mass density ρ_G is played by the factor $(\frac{\rho_E}{\epsilon_0})$ in the case of a charge continuum with charge density ρ_E . And the role played by the factor $(-v_0 \cdot \mathbf{J}_G)$ in the case of a mass flow with flow density \mathbf{J}_G is taken over by the factor $(\mu_0 \cdot \mathbf{J}_E)$ in the case of a charge flow with flow density \mathbf{J}_E .

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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