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A New Fuzzy Number Similarity Measure Based on Quadratic-Mean Operator for Handling Fuzzy Recommendation Problems

Shijay Chen

Department of Information Management, National United University, Taipei City
Email: sjchen@nuu.edu.tw

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Abstract

This study presents a new approach that advances the algorithm of similarity measures between generalized fuzzy numbers. Following a brief introduction to some properties of the proposed method, a comparative analysis based on 36 sets of generalized fuzzy numbers was performed, in which the degree of similarity of the fuzzy numbers was calculated with the proposed method and seven methods established by previous studies in the literature. The results of the analytical comparison show that the proposed similarity outperforms the existing methods by overcoming their drawbacks and yielding accurate outcomes in all calculations of similarity measures under consideration. Finally, in a numerical example that involves recommending cars to customers based on a nine-member linguistic term set, the proposed similarity measure proves to be competent in addressing fuzzy number recommendation problems.

Keywords

Quadratic-Mean Operator, Generalized Fuzzy Numbers, Similarity Measures, Fuzzy Number Recommendation

1. Introduction

Numerous approaches for determining the degree of similarity between fuzzy numbers have been proposed to assist fuzzy decision-making [1]-[3] and fuzzy risk analysis [4]-[7]. Many of these approaches, however, have limitations in one way or another. For example, some of them fail to yield results of similarity degree accurately, especially when the fuzzy numbers are generalized. To address this issue, this study presents a novel technique based on quadratic-mean operators that measures similarity between generalized fuzzy numbers (GFN). Using 36 sets of

GFNs, we compare the proposed approach against seven existing methods presented in [3]-[5], [8]-[10] and [11]. The results show that the proposed method competently overcomes the drawbacks inherent in the extant methods, and can be used to solve fuzzy number information retrieval problems.

The remainder of this study is structured as follows. Section II first provides a definition for quadratic mean and GFN, and subsequently reviews seven existing measures of similarity between GFNs. Based on analyses of the drawbacks of these similarity measures, Section 3 presents a new method for calculating the degrees of similarity between GFNs, and demonstrates some properties of the method. Section 4 compares the proposed method with existing measures by considering an example. Section 5 applies the proposed measure of similarity to fuzzy number recommendation problems. Conclusions are drawn in Section 6.

2. Preliminaries

This section briefly describes the concepts of the quadratic mean of positive real numbers [12] and GFN [1], [13], as well as seven existing similarity measures of fuzzy numbers [3]-[5], [8]-[11].

2.1. Quadratic Mean

Quadratic mean, also known as root mean square, is the square root of the average of the squares of a set of numbers. The quadratic mean of positive numbers a_1, a_2, … and a_n is defined as

$$\sqrt{\frac{\sum_{i=1}^{n} a_i^2}{n}}, \tag{1}$$

where $1 \le i \le n$ [12].

2.2. Generalized Trapezoidal Fuzzy Numbers

Chen [1] [13] denoted a generalized trapezoidal fuzzy number by $\tilde{A} = (a, b, c, d; w)$, where $0 < w \le 1$, and a, b, c and d are real numbers. The GFN $\tilde{A}$ represents a fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

(1) $\mu_{\tilde{A}}$ is a continuous mapping from R onto the closed interval [0, 1],

(2) $\mu_{\tilde{A}}(x) = 0$, where $-\infty < x \le a$,

(3) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$,

(4) $\mu_{\tilde{A}}(x) = w$, where $b \le x \le c$,

(5) $\mu_{\tilde{A}}(x)$ is strictly decreasing on $[c, d]$,

(6) $\mu_{\tilde{A}}(x) = 0$, where $d \le x < \infty$.

If $w = 1$, then the GFN $\tilde{A}$ is a normal trapezoidal fuzzy number that can be denoted as $\tilde{A} = (a, b, c, d)$. If $a = b$ and $c = d$, then $\tilde{A}$ is a crisp interval. If $b = c$, then $\tilde{A}$ is a generalized triangular fuzzy number. If $a = b = c = d$ and $w = 1$, then $\tilde{A}$ is a real number. **Figure 1** presents a generalized trapezoidal fuzzy number $\tilde{A}$, which may represents a decision-maker's linguistic opinions denoted by a series of ordinal values (e.g., values ranging from *very good* to *bad*). The value w is the degree of confidence in the decision-maker's linguistic opinion, where $0 < w \le 1$.

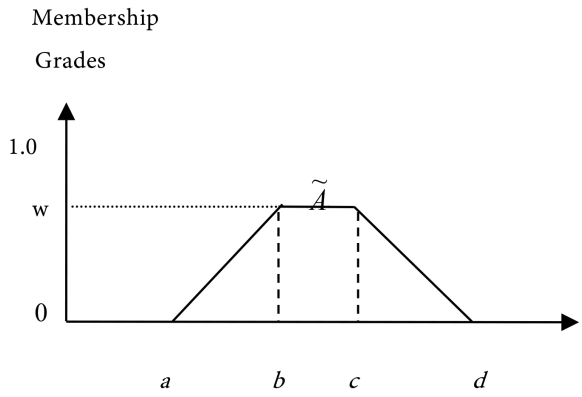

Figure 1. Generalized trapezoidal fuzzy number $\tilde{A}$.

2.3. Review of the Existing Similarity Measures between Fuzzy Numbers

1) Chen [4] presented a distance-based similarity measure between trapezoidal fuzzy numbers by considering two trapezoidal fuzzy numbers and $\tilde{B}$, where $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$. The degree of similarity $S(\tilde{A}, \tilde{B})$ between $\tilde{A}$ and $\tilde{B}$ is calculated as follows:

$$S(\tilde{A}, \tilde{B}) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}, \tag{2}$$

where $S(\tilde{A}, \tilde{B}) \in [0,1]$, if $\tilde{A}$ and $\tilde{B}$ are triangular fuzzy numbers, where $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, then the degree of similarity $S(\tilde{A}, \tilde{B})$ between $\tilde{A}$ and $\tilde{B}$ can be calculated as follows [4]:

$$S(\tilde{A}, \tilde{B}) = 1 - \frac{\sum_{i=1}^{3} |a_i - b_i|}{3}, \tag{3}$$

A larger value of the $S(\tilde{A}, \tilde{B})$ indicates a greater similarity between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

2) Lee [3] presented a similarity measure of trapezoidal fuzzy numbers that can be used to manipulate fuzzy opinions in group decision-making, which involves calculating the degree of similarity $S(\tilde{A}, \tilde{B})$ between trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ using the following formula:

$$S(\tilde{A}, \tilde{B}) = 1 - \frac{\left\| \tilde{A} - \tilde{B} \right\|_{l_p}}{\left\| U \right\|} \times 4^{-\frac{1}{p}}, \tag{4}$$

where U is the universe of discourse:

$$\left\| \tilde{A} - \tilde{B} \right\|_{l_p} = \left(\sum_{i=1}^{4} \left(|a_i - b_i| \right)^p \right)^{\frac{1}{p}}, \tag{5}$$

And

$$\|U\| = \max(U) - \min(U). \tag{6}$$

The larger the value $S(\tilde{A}, \tilde{B})$, the greater the similarity between the trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

3) Hsieh and Chen [9] presented a similarity measure that was based on the graded mean integration-representation distance, which calculates the degree of similarity $S(\tilde{A}, \tilde{B})$ between fuzzy numbers $\tilde{A}$ $\tilde{B}$ as follows:

$$S(\tilde{A}, \tilde{B}) = \frac{1}{1 + d(\tilde{A}, \tilde{B})}, \tag{7}$$

where $d(\tilde{A}, \tilde{B}) = \left|P(\tilde{A}) - P(\tilde{B})\right|$, and $P(\tilde{A})$ and $P(\tilde{B})$ are the graded mean integration representations of $\tilde{A}$ and $\tilde{B}$, respectively. If $\tilde{A}$ and $\tilde{B}$ represent triangular fuzzy numbers, then the graded mean integration representations $P(\tilde{A})$ and $P(\tilde{B})$ of $\tilde{A}$ and $\tilde{B}$ are respectively defined as follows [9]:

$$P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}, \tag{8}$$

$$P(\tilde{B}) = \frac{b_1 + 4b_2 + b_3}{6}, \tag{9}$$

If $\tilde{A}$ and $\tilde{B}$ denote trapezoidal fuzzy numbers, then the graded mean integration representations $P(\tilde{A})$ and $P(\tilde{B})$ of $\tilde{A}$ and $\tilde{B}$, respectively, are defined as follows [9]:

$$P(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}, \tag{10}$$

$$P(\tilde{B}) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}. \tag{11}$$

The larger the value of $S(\tilde{A}, \tilde{B})$, the greater the similarity between the GFNs $\tilde{A}$ and $\tilde{B}$.

4) Chen and Chen [5] presented an approach for measuring the degree of similarity between GFNs using a technique known as the Simple Center of Gravity Method (SCGM). It first calculates the center of gravity of trapezoidal or triangular GFNs, and then the degree of similarity between GFNs. The degree of similarity $S(\tilde{A}, \tilde{B})$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_{\tilde{B}})$, is calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4}|a_i - b_i|}{4}\right] \times \left(1 - \left|x^*_{\tilde{A}} - x^*_{\tilde{B}}\right|\right)^{B(S_{\tilde{A}}, S_{\tilde{B}})} \times \frac{\min\left(y^*_{\tilde{A}}, y^*_{\tilde{B}}\right)}{\max\left(y^*_{\tilde{A}}, y^*_{\tilde{B}}\right)}, \tag{12}$$

where $S(\tilde{A}, \tilde{B}) \in [0,1]$ and $B(S_{\tilde{A}}, S_{\tilde{B}})$ is defined as follows:

$$B(S_{\tilde{A}}, S_{\tilde{B}}) = \begin{cases} 1, & \text{if } S_{\tilde{A}} + S_{\tilde{B}} > 0, \\ 0, & \text{if } S_{\tilde{A}} + S_{\tilde{B}} = 0, \end{cases} \tag{13}$$

where $S_{\tilde{A}}$ and $S_{\tilde{B}}$ denote the lengths of the bases of the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, respectively, and are defined as follows:

$$S_{\tilde{A}} = a_4 - a_1, \tag{14}$$

$$S_{\tilde{B}} = b_4 - b_1. \tag{15}$$

The two values $y^*_{\tilde{A}}$ and $x^*_{\tilde{A}}$ are calculated as follows:

$$y^*_{\tilde{A}} = \begin{cases} \dfrac{w_{\tilde{A}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2 \right)}{6}, & \text{if } a_1 \neq a_4 \text{ and } 0 < w_{\tilde{A}} \leq 1, \\ \dfrac{w_{\tilde{A}}}{2}, & \text{if } a_1 = a_4 \text{ and } 0 < w_{\tilde{A}} \leq 1, \end{cases} \tag{16}$$

$$x^*_{\tilde{A}} = \frac{y^*_{\tilde{A}}(a_3 + a_2) + (a_4 + a_1)(w_{\tilde{A}} - y^*_{\tilde{A}})}{2w_{\tilde{A}}} \tag{17}$$

The values of $y^*_{\tilde{B}}$ and $x^*_{\tilde{B}}$ can similarly be calculated using formulae (16) and (17). The larger the value of $S(\tilde{A}, \tilde{B})$, the greater the similarity between the GFNs $\tilde{A}$ and $\tilde{B}$.

5) Yong *et al.* [11] introduced an approach for measuring the degree of similarity between GFNs based on the radius of gyration (ROG) to overcome the shortcomings of Chen and Chen's similarity measure [5]. According to Yong *et al.*'s method, the degree of similarity $S(\tilde{A}, \tilde{B})$ between generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_{\tilde{B}})$ can be calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4} \right] \times \left(1 - \left| r_y^{\tilde{A}} - r_y^{\tilde{B}} \right| \right)^{B(S_{\tilde{A}}, S_{\tilde{B}})} \times \frac{\min\left(r_x^{\tilde{A}}, r_x^{\tilde{B}} \right)}{\max\left(r_x^{\tilde{A}}, r_x^{\tilde{B}} \right)}, \tag{18}$$

where $S_{\tilde{A}}$ and $S_{\tilde{B}}$ denote the lengths of the bases of the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, respectively, and are defined as follows:

$$r_x^{\tilde{A}} = \sqrt{\frac{(I_x)_1 + (I_x)_2 + (I_x)_3}{\dfrac{((a_3 - a_2) + (a_4 - a_1)) \times w_{\tilde{A}}}{2}}} \tag{19}$$

$$r_y^{\tilde{A}} = \sqrt{\frac{(I_y)_1 + (I_y)_2 + (I_y)_3}{\dfrac{((a_3 - a_2) + (a_4 - a_1)) \times w_{\tilde{A}}}{2}}} \tag{20}$$

where

$$(I_x)_1 = \frac{(a_2 - a_1) w^3_{\tilde{A}}}{12} \tag{21}$$

$$(I_x)_2 = \frac{(a_3 - a_2) w^3_{\tilde{A}}}{3} \tag{22}$$

$$(I_x)_3 = \frac{(a_4 - a_3) w^3_{\tilde{A}}}{12} \tag{23}$$

$$\left(I_y\right)_1 = \frac{\left(a_2-a_1\right)^3 w_{\tilde{A}}}{4} + \frac{\left(a_2-a_1\right)a_1^2 w_{\tilde{A}}}{2} + \frac{2\left(a_2-a_1\right)^2 a_1 w_{\tilde{A}}}{3} \tag{24}$$

$$\left(I_y\right)_2 = \frac{\left(a_3-a_2\right)^3 w_{\tilde{A}}}{3} + \left(a_3-a_2\right)a_2^2 w_{\tilde{A}} + \left(a_3-a_2\right)^2 a_2 w_{\tilde{A}} \tag{25}$$

$$\left(I_y\right)_3 = \frac{\left(a_4-a_3\right)^3 w_{\tilde{A}}}{12} + \frac{\left(a_4-a_3\right)a_3^2 w_{\tilde{A}}}{2} + \frac{\left(a_4-a_3\right)^2 a_3 w_{\tilde{A}}}{3} \tag{26}$$

Similarly, $r_x^{\tilde{B}}$ and $r_y^{\tilde{B}}$ is calculated using formulae (19) and (20). Again, a larger $S\left(\tilde{A},\tilde{B}\right)$ value indicates a greater similarity between the fuzzy numbers.

6) Chen [8] reported a case that involved using three criteria (*i.e.*, the similarity of the shapes, spreads of their membership functions, and the relative distance between the two GFNs) to determine the degree of similarity between two GFNs $\tilde{A}$ and $\tilde{B}$. These criteria are also applicable for fuzzy number ranking problems [14]. Furthermore, a method of measuring the degree of similarity between GFNs through a geometric-mean averaging operator [8] was also proposed to overcome the drawbacks of the similarity measure found in the study of Yong *et al.* [11]. The degree of similarity $S\left(\tilde{A},\tilde{B}\right)$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, $\tilde{A}=\left(a_1,a_2,a_3,a_4;w_{\tilde{A}}\right)$ and $\tilde{B}=\left(b_1,b_2,b_3,b_4;w_{\tilde{B}}\right)$, is calculated as follows:

$$S\left(\tilde{A},\tilde{B}\right) = \left[\sqrt[4]{\prod_{i=1}^{4}\left(2-\left|a_i-b_i\right|\right)}-1\right] \times \frac{\min\left(y_{\tilde{A}}^*,y_{\tilde{B}}^*\right)}{\max\left(y_{\tilde{A}}^*,y_{\tilde{B}}^*\right)}, \tag{27}$$

where $y_{\tilde{A}}^*$ and $y_{\tilde{B}}^*$ can be calculated using formula (16). The larger the value of $S\left(\tilde{A},\tilde{B}\right)$, the greater the similarity between the GFNs $\tilde{A}$ and $\tilde{B}$.

7) In an effort to circumvent the pitfall of Chen and Chen's similarity measure [5], Wei and Chen [10] used arithmetic-mean averaging operator and perimeter of the generalized trapezoidal fuzzy number to measure the degree of similarity between GFNs, in which, the degree of similarity $S\left(\tilde{A},\tilde{B}\right)$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, $\tilde{A}=\left(a_1,a_2,a_3,a_4;w_{\tilde{A}}\right)$ and $\tilde{B}=\left(b_1,b_2,b_3,b_4;w_{\tilde{B}}\right)$ can be calculated by the following formula:

$$S\left(\tilde{A},\tilde{B}\right) = \left[1-\frac{\sum_{i=1}^{4}\left|a_i-b_i\right|}{4}\right] \times \frac{\min\left(P\left(\tilde{A}\right),P\left(\tilde{B}\right)\right)+\min\left(w_{\tilde{A}},w_{\tilde{B}}\right)}{\max\left(P\left(\tilde{A}\right),P\left(\tilde{B}\right)\right)+\max\left(w_{\tilde{A}},w_{\tilde{B}}\right)}, \tag{28}$$

where $P\left(\tilde{A}\right)$ and $P\left(\tilde{B}\right)$ represent the perimeters of the two generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, respectively, as follows:

$$P\left(\tilde{A}\right) = \sqrt{\left(a_1-a_2\right)^2+w_{\tilde{A}}^2} + \sqrt{\left(a_3-a_4\right)^2+w_{\tilde{A}}^2} + \left(a_3-a_2\right) + \left(a_4-a_1\right), \tag{29}$$

$$P\left(\tilde{B}\right) = \sqrt{\left(b_1-b_2\right)^2+w_{\tilde{B}}^2} + \sqrt{\left(b_3-b_4\right)^2+w_{\tilde{B}}^2} + \left(b_3-b_2\right) + \left(b_4-b_1\right), \tag{30}$$

The larger the value of $S\left(\tilde{A},\tilde{B}\right)$, the greater the similarity between the GFNs $\tilde{A}$ and $\tilde{B}$.

3. New Similarity Measure Between GFNS Based on Quadratic-Mean Operator

Although all of the seven methods described in the precedent section could calculate the degree of similarity between fuzzy numbers, they share a common weakness in that they may fail to obtain the similarity measure accurately in certain situations. The pitfall of these similarity measures could be elaborated by further examining the criteria provided in Chen's [8] method. Consider two sets of GFNs as follows.

The degree of similarity $S\left(\tilde{A}_1, \tilde{A}_2\right) = 0.85$ and the degree of similarity $S\left(\tilde{B}_1, \tilde{B}_2\right) = 0.85$ can be obtained from the Wei and Chen's similarity measure (*i.e.*, formulae (29) - (30)). However, $S\left(\tilde{A}_1, \tilde{A}_2\right)$ and $S\left(\tilde{B}_1, \tilde{B}_2\right)$ should not be identical. The GFNs $\tilde{B}_1$ and $\tilde{B}_2$ are more similar than the GFNs $\tilde{A}_1$ and $\tilde{A}_2$ because the shapes of the GFNs in $\tilde{B}_1$ and $\tilde{B}_2$ are identical, but those in $\tilde{A}_1$ and $\tilde{A}_2$ are not. Consequently, a new method for handling the GFN similarity measure is needed to overcome these drawbacks. Here we introduce a new similarity measure based on the quadratic-mean operator to calculate the degree of similarity between GFNs, and explicate some properties of the proposed similarity measure.

Unlike the arithmetic mean, which might oversimplify variance, and geometric mean, which is less sensitive to outlier values, the quadratic mean uniquely amplifies differences in the spread and shape of fuzzy numbers. This property aligns with the core requirement of accurately measuring similarity in GFNs. Consider two generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, where $\tilde{A} = \left(a_1, a_2, a_3, a_4; w_{\tilde{A}}\right)$, $\tilde{B} = \left(b_1, b_2, b_3, b_4; w_{\tilde{B}}\right)$, $0 \le w_{\tilde{A}} \le 1$, $0 \le w_{\tilde{B}} \le 1$, $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, and $0 \le b_1 \le b_2 \le b_3 \le b_4 \le 1$. The degree of similarity $S\left(\tilde{A}, \tilde{B}\right)$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$S\left(\tilde{A}, \tilde{B}\right) = \left[1 - \sqrt{\frac{\sum_{i=1}^{4}\left(a_i - b_i\right)^2}{4}}\right] \times \frac{\min\left(w_{\tilde{A}}, w_{\tilde{B}}\right)}{\max\left(w_{\tilde{A}}, w_{\tilde{B}}\right)}, \tag{31}$$

The larger the value of $S\left(\tilde{A}, \tilde{B}\right)$, the greater the similarity between the GFNs $\tilde{A}$ and $\tilde{B}$.

Chen and Chen [5] described three properties of GFN similarity measures that are denoted here as $\tilde{A}$ and $\tilde{B}$, as follows:

Property 1: Two GFNs $\tilde{A}$ and $\tilde{B}$ are identical if and only if $S\left(\tilde{A}, \tilde{B}\right) = 1$.

Property 2: $S\left(\tilde{A}, \tilde{B}\right) = S\left(\tilde{B}, \tilde{A}\right)$.

Property 3: If $\tilde{A} = (a, a, a, a; 1.0)$ and $\tilde{B} = (b, b, b, b; 1.0)$ denote two real numbers, then $S\left(\tilde{A}, \tilde{B}\right) = 1 - |a - b|$.

The following proofs show that the proposed similarity measure based on the quadratic-mean operator satisfies these properties.

Property 1: Two generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical if and only if $S\left(\tilde{A}, \tilde{B}\right) = 1$.

Proof:

(i) If $\tilde{A}$ and $\tilde{B}$ are identical, then $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_4 = b_4$, and $w_{\tilde{A}} = w_{\tilde{B}}$. The degree of similarity between $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$S\left(\tilde{A},\tilde{B}\right)=\left[1-\sqrt{\frac{\sum_{i=1}^{4}\left(a_i-b_i\right)^2}{4}}\right]\times\frac{\min\left(w_{\tilde{A}},w_{\tilde{B}}\right)}{\max\left(w_{\tilde{A}},w_{\tilde{B}}\right)}$$
$$=\left[1-\sqrt{\frac{0}{4}}\times 1\right]$$
$$=1.$$

(ii) If $S\left(\tilde{A},\tilde{B}\right)=1$, then:

$$S\left(\tilde{A},\tilde{B}\right)=\left[1-\sqrt{\frac{\sum_{i=1}^{4}\left(a_i-b_i\right)^2}{4}}\right]\times\frac{\min\left(w_{\tilde{A}},w_{\tilde{B}}\right)}{\max\left(w_{\tilde{A}},w_{\tilde{B}}\right)}$$
$$=1.$$

This implies that the calculating result of square root function in formula (31) must be zero, *i.e.*, $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_4 = b_4$, and $w_{\tilde{A}}$ and $w_{\tilde{B}}$ are equal. Therefore, the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical.

Property 2: $S\left(\tilde{A},\tilde{B}\right)=S\left(\tilde{B},\tilde{A}\right)$.

Proof: Since

$$S\left(\tilde{A},\tilde{B}\right)=\left[1-\sqrt{\frac{\sum_{i=1}^{4}\left(a_i-b_i\right)^2}{4}}\right]\times\frac{\min\left(w_{\tilde{A}},w_{\tilde{B}}\right)}{\max\left(w_{\tilde{A}},w_{\tilde{B}}\right)},$$

$$S\left(\tilde{B},\tilde{A}\right)=\left[1-\sqrt{\frac{\sum_{i=1}^{4}\left(b_i-a_i\right)^2}{4}}\right]\times\frac{\min\left(w_{\tilde{B}},w_{\tilde{A}}\right)}{\max\left(w_{\tilde{B}},w_{\tilde{A}}\right)},$$

where $\sum_{i=1}^{4}\left(a_i-b_i\right)^2=\sum_{i=1}^{4}\left(b_i-a_i\right)^2$ and $\frac{\min\left(w_{\tilde{A}},w_{\tilde{B}}\right)}{\max\left(w_{\tilde{A}},w_{\tilde{B}}\right)}=\frac{\min\left(w_{\tilde{B}},w_{\tilde{A}}\right)}{\max\left(w_{\tilde{B}},w_{\tilde{A}}\right)}$.

Thus, $S\left(\tilde{A},\tilde{B}\right)=S\left(\tilde{B},\tilde{A}\right)$.

Property 3: If $\tilde{A}=\left(a,a,a,a;1.0\right)$ and $\tilde{B}=\left(b,b,b,b;1.0\right)$ are two real numbers, then $S\left(\tilde{A},\tilde{B}\right)=1-\left|a-b\right|$.

Proof: The following is derived from formula (31):

$$S\left(\tilde{A},\tilde{B}\right)=\left[1-\sqrt{\frac{\sum_{i=1}^{4}\left(a-b\right)^2}{4}}\right]\times\frac{\min\left(1,1\right)}{\max\left(1,1\right)}$$
$$=\left[1-\sqrt{\frac{4\left(a-b\right)^2}{4}}\right]\times 1 \tag{32}$$
$$=1-\left|a-b\right|.$$

Consider the two GFNs $\tilde{A}=(0.1,0.2,0.3,0.4;1)$ and $\tilde{B}=(0.3,0.4,0.5;1)$. The generalized triangular fuzzy number $\tilde{B}$ can be represented by the generalized trapezoidal fuzzy number $\tilde{B}=(0.3,0.4,0.4,0.5;1)$. The degree of similarity between the two GFNs can be computed from formula (31) as follows:

$$S(\tilde{A},\tilde{B})=\left[1-\sqrt{\sum_{i=1}^{4}(a_i-b_i)^2/4}\right]\times\frac{\min(w_{\tilde{A}},w_{\tilde{B}})}{\max(w_{\tilde{A}},w_{\tilde{B}})}$$

$$=\left[1-\sqrt{(0.1-0.3)^2+(0.2-0.4)^2+(0.3-0.4)^2+(0.4-0.5)^2/4}\right]\times 1$$

$$=0.8419.$$

4. Comparing the Proposed Similarity Measure with Existing Methods

This section compares the proposed similarity measure with seven existing similarity measures [3]-[5], [8]-[10] and [11] using 36 sets of GFNs as listed in **Figure 2**. **Table 1** compares the calculation results of all eight similarity measures. Some of the 36 sets of GFNs are sourced from [5], [8], [10], [11], and some are extended from **Figure 1** in Section 2. All of these GFNs conform to the three criteria [8] described in Section 2. Inaccuracies or incongruences resulting from calculations using existing similarity measures [3]-[5], [8]-[10] and [11] are highlighted in **Table 1**, and the underlying pitfalls are elaborated along with an analytical review of the results.

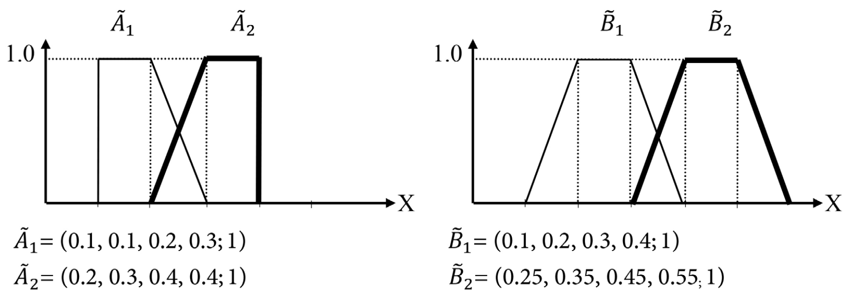

Figure 2. Two sets of GFNs.

Table 1. Calculation results were obtained through various similarity measures.

	Lee's Method [3]	Hsieh-and-Chen's Method [9]	Chen's Method [4]	Chen-and-Chen's Method [5]	Yong *et al.*'s Method [11]	Chen's Method [8]	Wei and Chen's Method [10]	The Proposed Method
Set 1	0.9617	1	0.975	0.8357	0.7954	0.8356	0.95	0.9646
Set 2	1	1	1	1	1	1	1	1
Set 3	0.5	0.7692	0.7	0.42	0.4028	0.5997	0.68	0.6979
Set 4	0.5	0.7692	0.7	0.49	0.4931	0.7	0.7	0.7
Set 5	1	1	1	0.8	0.8	0.8	0.8248	0.8
Set 6	N/A	1	1	1	1	1	1	1
Set 7	0	0.909	0.9	0.9	0.81	0.9	0.9	0.9

Continued

Set 8	0.5	0.909	0.9	0.54	0.5754	0.5991	0.8411	0.8775
Set 9	0.6667	0.909	0.9	0.81	0.8112	0.9	0.9	0.9
Set 10	0.8333	1	0.9	0.9	0.8854	0.8974	0.7833	0.8586
Set 11	0.75	1	0.9	0.72	0.6914	0.72	0.8003	0.9
Set 12	0.8	0.9375	0.9	0.78	0.7744	0.8959	0.8289	0.8419
Set 13	0.4	0.7692	0.7	0.49	0.8961	0.6971	0.6222	0.6838
Set 14	0.25	0.7692	0.7	0.49	0.7781	0.7	0.7	0.7
Set 15	0.5	0.7692	0.7	0.49	0.4931	0.7	0.7	0.7
Set 16	0.6407	0.7692	0.7	0.49	0.5622	0.6915	0.5014	0.5617
Set 17	0.5	0.6897	0.55	0.3025	0.309	0.55	0.55	0.55
Set 18	0.3333	0.6897	0.55	0.3025	0.1302	0.5418	0.2464	0.3485
Set 19	0.6	0.8333	0.8	0.5486	0.5905	0.6854	0.7794	0.7969
Set 20	0.6	0.8333	0.8	0.5486	0.5899	0.6854	0.7794	0.7969
Set 21	0.9	0.9091	0.9	0.81	0.8568	0.9	0.9	0.9
Set 22	0.9	0.9091	0.9	0.81	0.8551	0.9	0.9	0.9
Set 23	0.9	1	0.9	0.8077	0.8255	0.8077	0.8055	0.9
Set 24	0.9	1	0.9	0.8028	0.8255	0.8028	0.8012	0.8945
Set 25	1	1	1	0.7	0.7	0.7	0.7209	0.7
Set 26	0.75	1	0.95	0.9048	0.4328	0.9042	0.6215	0.6505
Set 27	0.3	0.7317	0.65	0.4279	0.5394	0.6492	0.65	0.6464
Set 28	0.5333	0.7407	0.65	0.4225	0.5394	0.65	0.65	0.65
Set 29	0.5	0.8571	0.85	0.7296	0.7898	0.8493	0.85	0.8419
Set 30	0.6667	0.87	0.85	0.7225	0.7876	0.85	0.85	0.85
Set 31	0.8571	1	0.9	0.8077	0.8275	0.8077	0.8055	0.9
Set 32	0.8571	0.909	0.9	0.7269	0.7979	0.8053	0.8055	0.8586
Set 33	0.7143	0.8333	0.8	0.5744	0.6725	0.7155	0.716	0.7764
Set 34	0.5714	0.7692	0.7	0.4397	0.5562	0.6256	0.6265	0.6838
Set 35	0.5	0.9375	0.95	0.9183	0.9346	0.9494	0.95	0.9293
Set 36	0.6667	0.9524	0.95	0.9025	0.9264	0.95	0.95	0.95

Note: " N/A " means that the degree of similarity between GFNs is unattainable with the similarity measure in question; " " means incorrectly yielded results.

1) Set 1 of **Figure 3** evidently indicates that the two sets of GFNs $\tilde{A}$ and $\tilde{B}$ are not equal because the shapes of the GFNs are different. However, using Hsieh and Chen's [9] similarity measure, a value $S\left(\tilde{A},\tilde{B}\right)=1$ is incorrectly yielded nevertheless, as depicted in **Table 1**.

2) The GFNs depicted in Set 3 are clearly different from those depicted in Set 4, and the two sets of GFNs in Set 4 are more similar to each other in shapes and spreads than those in Set 3, despite the same relative distance between $\tilde{A}$ and

$\tilde{B}$ in both Set 3 and 4. However, according to **Table 1**, the methods of Chen [4], Hsieh and Chen [9], and Lee [3] yield the same degree of similarity for Sets 3 and 4, indicating a potential flaw in these methods.

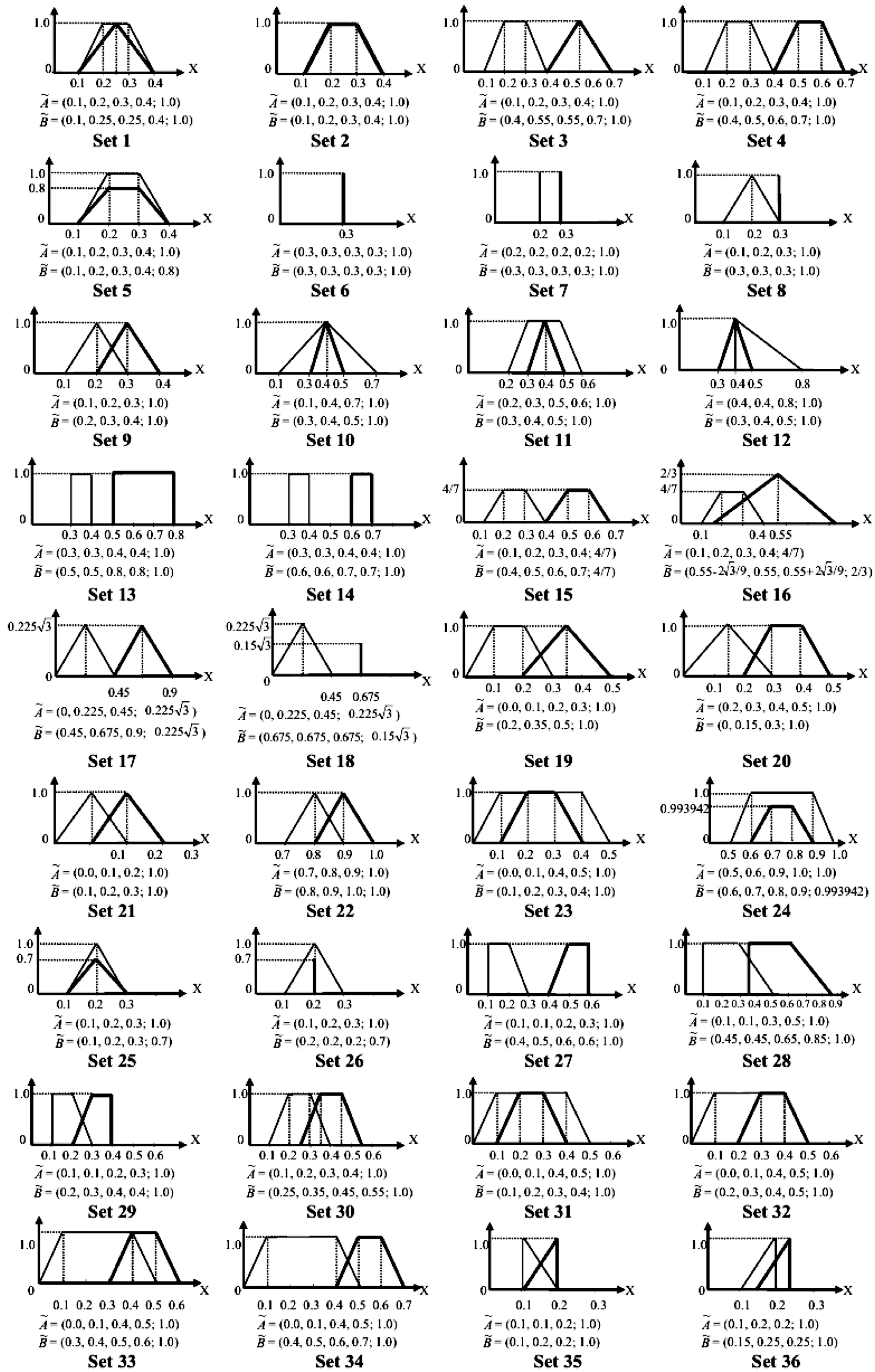

Figure 3. The 36 sets of GFNs.

3) As evidenced by **Table 1**, the same result, $S\left(\tilde{A},\tilde{B}\right)=1$, is yielded with the methods of Chen [4], Hsieh and Chen [9], and Lee [3], suggesting $\tilde{A}$ and $\tilde{B}$ of Set 5 are identical. This result is, however, incorrectly yielded because, according to **Figure 3**, $\tilde{A}$ and $\tilde{B}$ are different, as can be observed from their distinctive shapes.

4) It can be observed from Set 6 that Lee's method [3] is inapplicable to calculate the degree of similarity between two identical real values because the denominator U of formula (4) would become zero, yielding the incorrect result $S\left(\tilde{A},\tilde{B}\right)=\infty$. Moreover, while **Table 1** shows that Lee's method [3] yields a result $S\left(\tilde{A},\tilde{B}\right)=0$ on Set 7, indicating no similarity exists between the two real numbers contained in the set, it is clearly observable from **Figure 3** that distinguishable similarity $S\left(\tilde{A},\tilde{B}\right)$ does exist.

5) **Table 1** indicates that the methods of Chen [4] and Hsieh and Chen [9] yield the same degree of similarity for Sets 8 and 9, which are apparently different from each other based on observations of the geometry shown in **Figure 3**.

6) Sets 10, 11 and 12 are clearly different sets of GFNs. Corresponding results in **Table 1**, however, indicate that the same degree of similarity could be yielded by Chen's method for each of the GFNs in these sets.

7) **Table 1** indicates that Hsieh and Chen's method [9] yields a result $S\left(\tilde{A},\tilde{B}\right)=1$, indicating the two numbers concerned are identical, which is incorrect because $\tilde{A}$ and $\tilde{B}$ of Set 10 are clearly different, as could be seen from their geometric shapes in **Figure 2**. Likewise, Set 11 and **Table 1** show opposing results when Hsieh and Chen's method is applied [9].

8) Set 14 geometrically exhibits a higher degree of similarity than Set 13, whereas the results calculated using Chen's [4], Chen and Chen's [5] and Hsieh and Chen's [9]methods are the same, according to **Table 1**. Additionally, the methods of Lee [3] and Yong *et al.* [11] yield an incorrect result, showing Set 13 is more similar than Set 14.

9) Likewise, the numbers of Set 15 are geometrically more similar than those of Set 16 as far as the shapes of the GFNs are concerned. Yet, according to **Table 1**, the methods of Chen [4], Chen and Chen [5], and Hsieh and Chen [9] yield the same degree of similarity on both sets. Furthermore, the methods of Lee [3] and Yong *et al.* [11] yield an incorrect result showing Set 16 is more similar than Set 15, opposite to their geometric shapes.

10) **Table 1** indicates that the methods of Chen [4], Chen and Chen [5], and Hsieh and Chen [9] yield the same degrees of similarity on Sets 17 and 18, despite distinguishable differences between Set 17 and Set 18.

11) Set 19 is highly homogenous with Set 20, except a minor difference in the placement of the two numbers within the sets. However, **Table 1** indicates that by Yong *et al.*'s method [11], a result is yielded that falsely demonstrates a larger degree of similarity in Set 19 than that of Set 20.

12) Set 21 is highly similar to Set 22 in terms of relative distance between the GFNs in both Sets 21 and 22. Additionally, the shapes of the four GFNs in Sets 21

and 22 are identical. Opposed to this observation, however, the method of Yong *et al.* [11] yields a result that shows a larger degree of similarity in Set 21 larger than in Set 22, as shown in **Table 1**.

13) Sets 23 and 24 maintain a same relative distance between the GFNs. The shapes of the GFN $\tilde{A}$ in both sets are the same, while the GFN $\tilde{B}$ in Set 24 is proportionally of a smaller height than that in Set 23. Thus, intuitively, the degree of similarity in Set 23 is larger than that in Set 24. However, **Table 1** indicates that the methods of Chen [4], Hsieh and Chen [9], Lee [3], and Yong *et al.* [11] yield the same degree of similarity on both sets.

14) Sets 25 and 26 are distinguishably different in their geometric shapes, with the GFNs in Set 25 of a higher similarity than those in Set 26 because both GFNs in Set 25 are of a triangular shape, whereas the GFNs in Set 26 are of mixed shapes. However, the methods of Chen [8] and Chen and Chen [5] yield results that are incongruent with the geometry, as **Table 1** indicates.

15) In Set 25 of **Figure 2**, $\tilde{A}$ and $\tilde{B}$ are clearly different. However, the methods of Chen [4], Hsieh and Chen [9], and Lee [3] yield a result $S\left(\tilde{A},\tilde{B}\right)=1$, suggesting $\tilde{A}$ and $\tilde{B}$ are identical. Similarly, $\tilde{A}$ and $\tilde{B}$ in Set 26 of **Figure 2** are distinctive numbers. However, Hsieh and Chen's method [9] yields an incorrect result $S\left(\tilde{A},\tilde{B}\right)=1$.

16) Sets 27 and 28 each contains two distinctive sets of GFNs, with Set 28 of a higher similarity than Set 27. The shapes of the GFNs in Set 28 are identical, but those in Set 27 are not. Additionally, the relative distance between the GFNs in Set 28 is shorter than that between the GFNs in Set 27. However, **Table 1** indicates that the methods of Chen [4], Yong *et al.* [11], and Wei and Chen [10] yield the same degree of similarity for Sets 27 and 28. Furthermore, Chen and Chen's method [5] incorrectly suggests that Set 27 has greater similarity than Set 28.

17) Sets 29 and 30 each contains two distinctive sets of GFNs, with Set 30 of a higher similarity than Set 29 because the two sets of GFNs in Set 30 are of an identical geometric shape, while those in Set 29 are of a mirrored shape. However, **Table 1** indicates that the methods of Chen [4], and Wei and Chen [10] yield the same degree of similarity for Sets 29 and 30. Furthermore, the methods of Chen and Chen [5] and Yong *et al.* [11] yield an incorrect result that Set 29 has stronger similarity than Set 30.

18) Sets 31, 32, 33 and 34 each contains two different sets of GFNs. The GFNs in Set 31 are of a higher degree of similarity than those in Sets 32, 33 and 34 because of varying relative distances between the GFNs $\tilde{A}$ and $\tilde{B}$. In particular, the relative distance in Set 31 is shorter than those in the other sets. Despite these differences, the methods of Lee [3], Chen [4], and Wei and Chen [10] yield the same degree of similarity for Sets 31 and 32, as could be observed from **Table 1**.

19) Both Sets 35 and 36 contain two different sets of GFNs. The GFNs in Set 36 are more similar than those in Set 35 because the shapes in Set 36 are identical, while those in Set 35 are mirrored. However, **Table 1** indicates that the methods of Chen [4] and Wei and Chen [10] yield the same degree of similarity for Sets 29 and 30.

In sum, all of the seven approaches established in previous studies fall short in providing accurate or congruent results of similarity measures for GFNs in one way or another, as is evidenced by **Table 1** and **Figure 2**. In contrast, it is noteworthy that none of the inaccuracies or incongruences summarized above can be found in the results calculated by the method proposed in this study, indicating that the proposed similarity measure could competently overcomes the drawbacks inherent to the existing methods.

5. A Numerical Example of Fuzzy Recommendation Process

Martinez *et al.* [15] noted that a recommendation process comprises the following steps: (a) fusion of human linguistic evaluating values, (b) calculation of the similarity between user profile and recommended items, and (c) providing a recommendation for user. We assume n different recommended items exist, such that B = $\{b_1, b_2, \ldots, b_n\}$, which is described by a set of m features C = $\{c_1, c_2, \ldots, c_m\}$, which in return can be described by a nine-member linguistic term set as shown in **Table 2**.

Table 2. A nine-member linguistic term set [5].

Linguistic Terms	GFNs
Absolutely-low	(0.0, 0.0, 0.0, 0.0; 1.0)
Very low	(0.0, 0.0, 0.02, 0.07; 1.0)
Low	(0.04, 0.1, 0.18, 0.23; 1.0)
Fairly low	(0.17, 0.22, 0.36, 0.42; 1.0)
Medium	(0.32, 0.41, 0.58, 0.65; 1.0)
Fairly high	(0.58, 0.63, 0.80, 0.86; 1.0)
High	(0.72, 0.78, 0.92, 0.97; 1.0)
Very high	(0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely-high	(1.0, 1.0, 1.0, 1.0; 1.0)

The algorithm for recommendation process is detailed as follows.

Step 1: Use the weighted mean method and the GFN arithmetic operations to fuse the evaluating linguistic values $\tilde{R}_{ij} = \left(r_{1ij}, r_{2ij}, r_{3ij}, r_{4ij}; w_{\tilde{R}_{ij}}\right)$ and the real numbers w_i, where $\tilde{R}_{ij}$ denote the degree of strength and weight of feature c_i in item b_j, $0 \le w_i \le 1$, $1 \le i \le m$ and $1 \le j \le n$, to get the evaluating value $\tilde{R}_j$ of the item b_j, shown as follows:

$$\tilde{R}_j = \frac{\sum_{i=1}^{n}\left(1+w_i\right) \otimes \tilde{R}_{ij}}{\sum_{i=1}^{n}\left(1+w_i\right)} \tag{33}$$

where

$$\begin{aligned}\left(1+w_i\right) \otimes \tilde{R}_{ij} &= \left(1+w_i\right) \otimes \left(r_{1ij}, r_{2ij}, r_{3ij}, r_{4ij}; w_{\tilde{R}_{ij}}\right) \\ &= \left[\left(1+w_i\right) \times r_{1ij}, \left(1+w_i\right) \times r_{2ij}, \left(1+w_i\right) \times r_{3ij}, \left(1+w_i\right) \times r_{4ij}; \left(1+w_i\right) \times w_{\tilde{R}_{ij}}\right],\end{aligned} \tag{34}$$

and $\tilde{R}_j$ is an GFN.

Step 2: Use the proposed similarity measure to calculate the degree of similarity between the GFN $\tilde{R}_j$ and each linguistic term shown in Table 2. Translate the GFN $\tilde{R}_j$ into a linguistic term shown in Table 2, which has the largest degree of similarity with respect to $\tilde{R}_j$.

Step 3: The closest item to the user necessities will be recommended.

Assume that a customer is looking for a new car from four different cars available for selection B = {b_1, b_2, b_3, b_4}, which is described by a set of three features C = {c_1, c_2, c_3}, where c_1 is *Price*, c_2 is *Safety*, and c_3 is *Comfort*. There are some evaluating values represented by GFNs as shown in Table 3, where $\tilde{R}_{ij}$ denotes the degree of strength of feature c_i in car b_j, w_i denotes the weight of feature c_i for the customer, $0 \le w_i \le 1$, $1 \le i \le 3$ and $1 \le j \le 4$.

Table 3. Evaluating the values of cars b_1, b_2, b_3 and b_4.

	$w_1 = 0.3$	$w_2 = 0.8$	$w_3 = 0.6$
	c_1	c_2	c_3
b_1	$\tilde{R}_{11}$ = (Low; 0.8) = (0.04, 0.1, 0.18, 0.23; 0.8)	$\tilde{R}_{21}$ = (High; 0.9) = (0.72, 0.78, 0.92, 0.97; 0.9)	$\tilde{R}_{31}$ = (Low; 1.0) = (0.04, 0.1, 0.18, 0.23; 1.0)
b_2	$\tilde{R}_{12}$ = (Fairly-high; 0.9) = (0.58, 0.63, 0.80, 0.86; 0.9)	$\tilde{R}_{22}$ = (Low; 1.0) = (0.04, 0.1, 0.18, 0.23; 1.0)	$\tilde{R}_{32}$ = (Medium; 1.0) = (0.32, 0.41, 0.58, 0.65; 1.0)
b_3	$\tilde{R}_{13}$ = (Very-low; 1.0) = (0.0, 0.0, 0.02, 0.07; 1.0)	$\tilde{R}_{23}$ = (Very-high; 0.8) = (0.93, 0.98, 1.0, 1.0; 0.8)	$\tilde{R}_{33}$ = (High; 0.9) = (0.72, 0.78, 0.92, 0.97; 0.9)
b_4	$\tilde{R}_{14}$ = (Medium; 1.0) = (0.32, 0.41, 0.58, 0.65; 1.0)	$\tilde{R}_{24}$ (High; 1.0) = (0.72, 0.78, 0.92, 0.97; 1.0)	$\tilde{R}_{34}$ = (Low; 0.9) = (0.04, 0.1, 0.18, 0.23; 0.9)

The proposed algorithm is used to address the fuzzy recommendation process problem:

[Step 1] Based on formulas (33), (34) and Table 3, the evaluating value $\tilde{R}_1$ of car b_1 is calculated as follows:

$$\tilde{R}_1 = \frac{\sum_{i=1}^{3}(1+w_i)\otimes\tilde{R}_{i1}}{\sum_{i=1}^{3}(1+w_i)}$$
$$=(0.3004, 0.3604, 0.4634, 0.5134; 0.9064).$$

Similarly, other evaluating values are obtained: $\tilde{R}_2$ = (0.2845, 0.3521, 0.4877, 0.5472; 0.9723), $\tilde{R}_3$ = (0.6013, 0.6409, 0.7017, 0.7326; 0.8894) , $\tilde{R}_4$ = (0.3779, 0.4462, 0574, 0.6296; 0.966).

[Step 2] Based on formulas (29) ~ (32) and Table 2, the degrees of similarity between the evaluating value $\tilde{\tilde{R}}_1$ of car b_1 and the linguistic terms shown in Table 2 are obtained as follows:

S($\tilde{R}_1$, Absolutely-low) = 0.5276

S($\tilde{R}_1$, Very low) = 0.5515

$S(\tilde{R}_1$, Low) = 0.6597
$S(\tilde{R}_1$, Fairly low) = 0.799
$S(\tilde{R}_1$, Medium) = 0.8215
$S(\tilde{R}_1$, Fairly high) = 0.6255
$S(\tilde{R}_1$, High) = 0.509
$S(\tilde{R}_1$, Very high) = 0.3887
$S(\tilde{R}_1$, Absolutely-high) = 0.3657

Because $S(\tilde{R}_1$, Medium) = 0.8215 has the largest value, the evaluating value $\tilde{R}_1$ of car b_1 is translated into the linguistic term "Medium". Similarly, because the degree of similarity $S(\tilde{R}_2$, Medium) = 0.8975 is lager than the degrees of similarity between the evaluating value $\tilde{R}_2$ and any other of the linguistic terms shown in Table 2, the evaluating value $\tilde{R}_2$ of car b_2 is translated into the linguistic term "Medium". Likewise, because the degree of similarity $S(\tilde{R}_3$, Fairly high) = 0.817 is lager than the degrees of similarity between the evaluating value $\tilde{R}_3$ and any other of the linguistic terms shown in Table 2, the evaluating value $\tilde{R}_3$ of car b_3 is translated into the linguistic term "Fairly high". Following a same procedure, the evaluating value $\tilde{R}_4$ of car b_4 is translated into the linguistic term "Medium".

[Step 3] According to the results of Step 2, car b_3 is deemed the closest item to the user necessities, and will be the car model to be recommended.

6. Conclusions

Based on an analytical review of numerous methods in the literature pertaining to the calculation of similarity between fuzzy numbers, this study presents a new approach that advances the algorithm of similarity measure between GFNs. Following a brief introduction to some properties of the proposed method, a comparative analysis based on 36 sets of GFNs was performed, in which the degree of similarity of the GFN pairs was calculated with the proposed method and each of the seven methods existing in the literature, respectively. The results of the comparison show that the proposed similarity outperforms the existing similarity measures by overcoming their drawbacks and yielding accurate outcomes in all calculations for the degree of similarity between the GFNs under consideration. Finally, the proposed measure of similarity was adopted to advance an algorithm for handling fuzzy number recommendation problems. A numerical example is supplied that involves applying the algorithm to recommend cars to customers. The result indicates that the proposed algorithm is competent in determining the most appropriate car models to be recommended to satisfy customers' needs.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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