

Optimization Algorithms for Sidelobes SSL Reduction: A Comparative Study

Mohsen Denden¹, Aymen Alhamdan²

¹Department of Computer Science, Higher Institute of Applied Sciences of Sousse, Sousse University, Sousse, Tunisia

²Department of Electrical Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia
Email: mohsen.denden@isi.rnu.tn, Aalhamdan@cti.edu.sa

How to cite this paper: Denden, M. and Alhamdan, A. (2024) Optimization Algorithms for Sidelobes SSL Reduction: A Comparative Study. *Journal of Computer and Communications*, 12, 120-132.
<https://doi.org/10.4236/jcc.2024.127009>

Received: June 21, 2024

Accepted: July 26, 2024

Published: July 29, 2024

Copyright © 2024 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

The development of new technologies in smart cities is often hailed as it becomes a necessity to solve many problems like energy consumption and transportation. Wireless networks are part of these technologies but implementation of several antennas, using different frequency bandwidths for many applications might introduce a negative effect on human health security. In wireless networks, most antennas generate sidelobes SSL. SSL causes interference and can be an additional resource for RF power that can affect human being health. This paper aims to study algorithms that can reduce SSL. The study concerns typical uniform linear antenna arrays. Different optimum side lobe level reduction algorithms are presented. Genetic algorithm GA, Chebyshev, and Particle Swarm Optimization algorithm are used in the optimization process. A comparative study between the indicated algorithms in terms of stability, precision, and running time is shown. Results show that using these algorithms in optimizing antenna parameters can reduce SSL. A comparison of these algorithms is carried out and results show the difference between them in terms of running time and SSL reduction Level.

Keywords

SSL, Radio Wave Interference, SAR, Linear Antenna Arrays, Genetic Algorithm, Chebyshev, Particle Swarm Optimization

1. Introduction

In the world, many countries follow the World Health Organization recommendations relating to public exposure to electromagnetic fields. The safety measures chosen by a majority of states consist of taking the necessary precautions against possible health risks linked to exposure to electromagnetic fields

[1]. Furthermore, the Council of the European Union announced Decree No. 1999/519/EC of July 12, 1999, [2] concerning general exposure to electromagnetic fields. When researchers [3]-[6], introduced new smart city technologies, they rarely discussed if they could affect the traditional urban process or the negative effect on human health [7] [8]. Some antenna propagation is characterized by main radiation named also the main lobe and another radiation named SSL or sidelobes. SSL is sometimes used to cover particular regions, but in most cases, they cause problems of interference and unused radiation that contribute to negative radio-frequency effects. In Wireless networks, linear antennas can provide excellent radiation direction-changing capability and provide high gain by adjusting the characteristics of each element of the antenna array. Compared to a single antenna element [9] [10], the antenna array provides more performance and flexibility for new array applications because it is desirable to monitor the main beam, increase directivity, and minimize side lobe level in some cases. In literature, many developed algorithms are used to reduce the level of side lobes, such as genetic algorithm, Chebyshev, and particle swarm. The factors mentioned previously determine the array factor, which is used in the calculation of the directivity (and consequently the gain). The total field of an array can be formed by multiplying the field of a single element at a selected reference point (usually the origin) and the array factor. Note that this multiplication property is only valid when all the elements of the array have the same element pattern; hence, in this paper, all elements are assumed to be identical and installed in the same orientation. The gain is equal to the directivity multiplied by a loss coefficient depending on the antenna type. When losses are negligible, the gain is equal to the directivity. The most common antenna arrays are the linear array and the most common types of linear and uniformly spaced antenna arrays are Uniform and Chebyshev arrays. Many algorithms are used to reduce the side Lobe Level, most of them are enhanced versions of Genetic and Particle Swarm algorithms. In the literature, results regarding which algorithm is suitable to reduce the SSL radiation level are diversified. This work compares the effectiveness of the most treated algorithms and contributes to improving the final decision regarding the best algorithm.

2. Related Work

Due to their high performance, antenna arrays have been the subject of many research topics [11]-[13]. The research focused on developing optimization algorithms, designing new reduced shapes, and concept Ultra Wideband frequency cases. Developing this type of antenna responds to an urgent need for new technologies such as the Internet of Things, mobile networks, etc. Recently the authors in [14] developed an improved genetic algorithm to suppress the peak side lobe level (PSSL). The authors considered that N array element selection is a non-deterministic polynomial hard problem. For this, they proposed a genetic algorithm to reduce sub-optimal reconfigurable element patterns. In the paper

[15] the authors used the TLBO algorithm to synthesize the radiation pattern of a non-uniform array antenna. The concept of non-uniform elements permits a space gain of 78% compared to the traditional uniform array antenna. In the work of [16], the authors proposed two combined techniques to reduce the side lobe in an array antenna. Both techniques are based on a genetic algorithm. The first developed technique determines which element excitation should operate and the second determines the space between elements. The authors in [17] used the fruit fly algorithm for the synthesis of array antennas. To improve the performance of their algorithms, the authors introduced generation mechanisms, orthogonal crossing operations, and quantum selection. The proposed algorithm is tested on cases of circular antenna arrays and compared to other algorithms and showed a superiority. A similar idea has been presented in the work of [18]. The authors summarized the use of antenna arrays in the wireless environment. A description of new antenna arrays used in radio links was introduced. They therefore analyzed the techniques used to improve the operation of this antenna type. A comparative analysis of different antenna array simulation and design techniques was conducted. This study did not take into account the antenna arrays used in 6G networks. The propagation loss research will need to be conducted. The work of [19] consisted of the synthesis of new algorithms for the analysis of adaptive and reconfigurable antenna arrays. The effect of artificial intelligence on the design of these antennas is studied. The paper showed AI's importance in configuring and controlling the antenna arrays, especially for fixing the desired digital beams. The authors in [20] studied several antenna array architectures in the mmw case. A description of multiple forms of antenna was presented to meet the requirements imposed by the development of mobile technologies. The circular antenna array is studied as a particular case that showed a flat gain, unlike other regular shapes. According to the same study, this characteristic makes the antennas more stable and resists angle variations due to external phenomena linked to the deployment of these antennas. The work in [21] considered a comparison of several BFN-type circuits to control the beamforming of an antenna array. Different BFN circuits are discussed and compared. This work proposed also innovative ideas and developments for future research in this area. In the article [22], the authors applied an evolutionary algorithm to optimize two antenna arrays. The presented algorithm showed an improvement and proved its effectiveness. Nevertheless, the presented algorithms need to be generalized by applying them to various examples to generalize their usefulness. In the article [23], the authors applied PSO, ABC, JS, and MA algorithms on a linear array antenna. A comparison study between these algorithms is presented. The results showed the superiority of the MA and JS algorithms. This work must take into account the design of the antenna diversity. The authors in [24] developed approaches for tracking a moving object in the near field environment (Fresnel zone). The objective was to determine the position and speed of the considered object using an ad-hoc observation model that takes into account the phase profile of a large receiving array. For this, the au-

thors derived the posterior Cramer-Rao Lower Bound (P-CRLB) and proved that the loss of positioning information outside Fresnel comes from an increase in the ranging error rather than from inaccuracies of angular estimation. New technologies and applications require urgently the development of wireless communications [25] [26]. Technologies such as AI, collaborative robotics, digital twins, and additive manufacturing enable better planning, improved safety, and better human-machine interaction [27]-[29]. Compared to other domains, AI antenna application is not widely exploited, so many additional efforts should be considered.

3. Problem Description

A broadside linear antenna array of $2M$ isotropic patch radiators has been considered as shown in **Figure 1**, in which each element is excited with non-uniform current excitation. All elements are assumed to be identical. The array elements are assumed uncoupled and equally spaced along the z-axis with its center at the origin (**Figure 1**). The array is symmetric in both geometry and excitation. Array factor, in general, is a function of several elements, their geometrical arrangements, relative magnitudes, relative phases, and their relative spacing.

Since the array factor does not depend on the directional characteristics of the radiating elements, it can be formulated by replacing the actual elements with isotropic (point) sources. Referring to **Figure 1**, the array factor $AF(\theta, I, \phi)$ in azimuth plane (x-y plane) with symmetric amplitude distributions may be written as [30].

$$AF(\theta, I, \phi) = 2 \sum_m^M I_m \cos\left(\frac{2m-1}{2} kd \cos(\theta + \phi)\right)$$

where θ denotes the zenith angle measured from the broadside direction of the array. I_m and ϕ are respectively, the current excitation amplitude, the excitation phase of m^{th} array element, and d is the inter-elements spacing between two consecutive elements. In this paper ϕ is kept as zero, $k = \frac{2\pi}{\lambda}$ is the wave numbers and λ is the signal wavelength. The array elements are numbered 1 to M from the origin in a symmetric array where the total number of elements is $2M$.

4. Optimization Algorithms

The main purpose of this work is to study the low-side lobe radiation pattern for linear antenna arrays with the constraint of a fixed beam width. For this purpose, we propose, to use Genetic Algorithm, PSO, and Chebyshev. A comparative study of the efficiency of each other will be considered.

4.1. Genetic Algorithm

John Holland developed a Genetic Algorithm (GA) at the University of Michigan. It is a meta-heuristic that searches the feasible region of an optimization

problem. GA has been applied to many problems in various domains such as engineering, finance, and economics [31]. GA consists of a data structure of individuals called Population. Individuals are also called chromosomes. Each chromosome is evaluated by an equation known as the fitness function or cost function, which is usually the objective function of the corresponding optimization problem.

The important parameters of GA are:

- Selection: this is based on the fitness criterion to choose which chromosome from a population will go on to reproduce.
- Reproduction: the propagation of individuals from one generation to the next.
- Crossover: this operator exchanges genetic material which is the feature of an optimization problem. Single point crossover is used here.
- Mutation: the modification of chromosomes for single individuals.
- Stopping criteria - the iteration stops when the maximum number of cycles is reached. The grand minimum fitness and its corresponding chromosome string or the desired solution are finally obtained.

The basic steps involved in implementing GA are simple. These are listed as follows:

- 1) genes, chromosomes, and coding a parameter set;
- 2) create an initial population;
- 3) evaluate the fitness of each population member;
- 4) invoke natural selection;
- 5) select population members for mating;
- 6) generate off-springs; Mutate selected members of the population;
- 7) terminate or go to step b.

A simple flow graph for the Genetic Algorithm is shown in **Figure 2**.

4.2. Particle Swarm Optimization PSO

Particle Swarm Optimization (PSO) was invented by Russell Eberhart and James Kennedy in 1995. PSO is a computational method that optimizes a problem by iteratively trying to improve a candidate solution concerning a given measure of quality. PSO optimizes a problem by having a population of candidate solutions, here dubbed particles, and moving these particles around in the search space according to simple mathematical formulae over the particle's position and velocity [32]. From some of the information available, a particle must decide his next move, it is necessary to decide its new speed.

To do this, it linearly combines three informations:

- Its current speed.
- Its best performance.
- The best performance of its neighbors (his informants).

Using three parameters sometimes called confidence factors that weigh three trends:

- Trends to follow its path.
- Conservative (retrace his steps).
- Trend “panurgian” (follow the best neighbor).

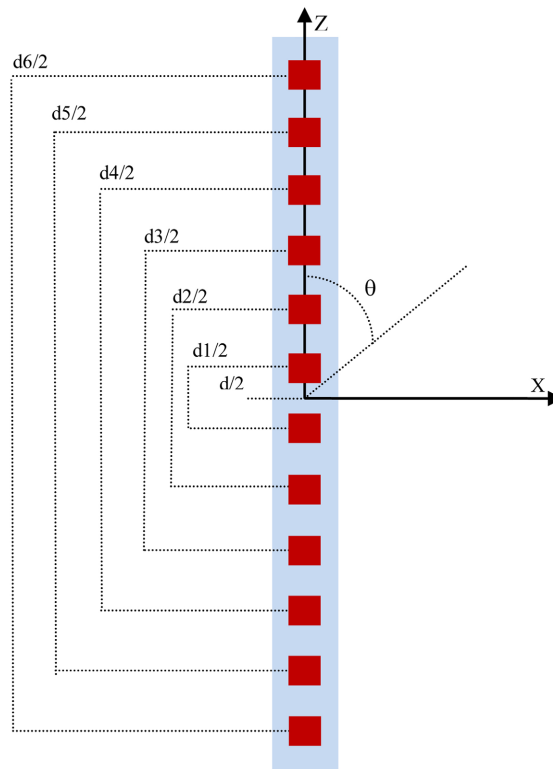


Figure 1. Linear array antenna.

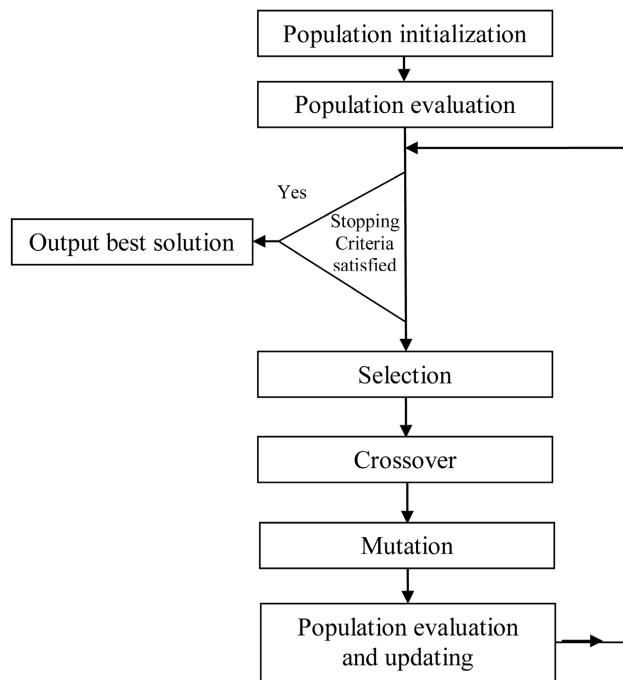


Figure 2. Genetic algorithm description.

The basic steps are the following:

- 1) 1st step, we initialize the particle swarm in the search space.
- 2) 2nd step, the speed is also initialized at random.
- 3) 3rd step Assess fitness for each particle.
- 4) 4th step Update velocity based on its best performance in the best performance of its neighbors.
- 5) 5th step Finish if the stop condition is satisfied.
- 6) 6th step Back to Step 2, this formula is used for updating velocity and the particle.

$$v_{id} = w_i v_{id} + c_2 (p_{id} - x_{id}) + c_3 (p_{gd} - x_{id})$$

$$x_{id} = x_{id} + v_{id}$$

With:

- d : dimension.
- c_2, c_3 : random value.
- w_i : Constant.
- p_{id} : the best performance i .
- p_{gd} : the best performance of neighbors.
- x_i : the particle No. i .
- v_i : the velocity of the particle No. i .

4.3. Chebyshev

In mathematics, the Chebyshev polynomials, named after Pafnuty Chebyshev, are a sequence of orthogonal polynomials [33]. The Chebyshev polynomials of the first kind are defined by the recurrence relation:

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$T_{(n+1)}(x) = 2xT_{(n)}(x) - T_{(n-1)}(x)$$

- $T_0(x) = 1$
- $T_1(x) = x$
- $T_2(x) = 2x^2 - 1$
- $T_3(x) = 4x^3 - 3x$
- $T_4(x) = 8x^4 - 8x^2 + 1$
- $T_5(x) = 16x^5 - 20x^3 + 5x$
- ...

Our bibliographic research shows that the previous three algorithms have largely proven their performance. Most newly developed algorithms are improvements and derivations of these algorithms **Table 1** illustrates all variables used in this article.

5. Result and Discussion

A 4, 8, 12, 16, 20, and 24-element linear antenna array of isotropic radiating

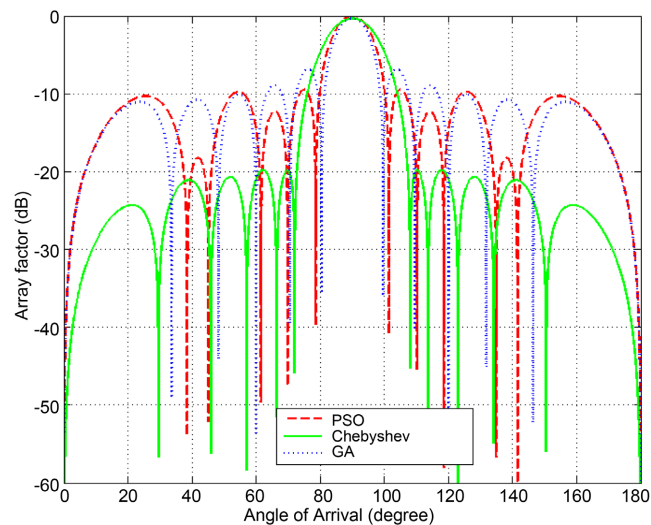
Table 1. Variable nomenclatures.

Variable	Description
<i>SAR</i>	Specific Absorption Rate
<i>BFN</i>	Beamforming Network
<i>SSL</i>	Side Lobe Level
<i>PSO</i>	Particle Swarm Optimization
<i>ABC</i>	Number of servers
<i>JS</i>	File index
<i>MA</i>	server index
<i>TLBO</i>	Teaching-Learning-Based Optimization
<i>HPBw</i>	Half Power BeamWidth
<i>FNBw</i>	First Null BeamWidth

elements, with inter-element spacing is considered for reference. Genetic Algorithm, PSO, and Chebyshev are applied to get deeper nulls and to reduce Side Lobe Levels (SSLs). Every time, these algorithms ran with 400 iterations. The population size was fixed to 100. GA algorithm is initialized using random values of excitation ($0 < I_n < 1$) and spacing. The programming has been written in Matlab language.

5.1. Radiation Pattern

Figure 3 shows the comparison between the three algorithms, The Chebyshev algorithm showed the best values because it depicts the substantial reductions in the maximum peak of the SSL with non-uniform current excitation, as compared to the non-uniform current excitation of genetic algorithm and non-uniform current excitation of PSO. Indeed the Chebyshev algorithm showed an SSL equal to -19.52 dB while PSO showed an SSL level equal to -9.49 dB. With a genetic algorithm, the SSL level is equal to -6.82 dB.

**Figure 3.** Radiation pattern using GA, PSO and Chebyshev.

In the next step, the variation of the SSL with the number of elements will be presented in **Figure 4**.

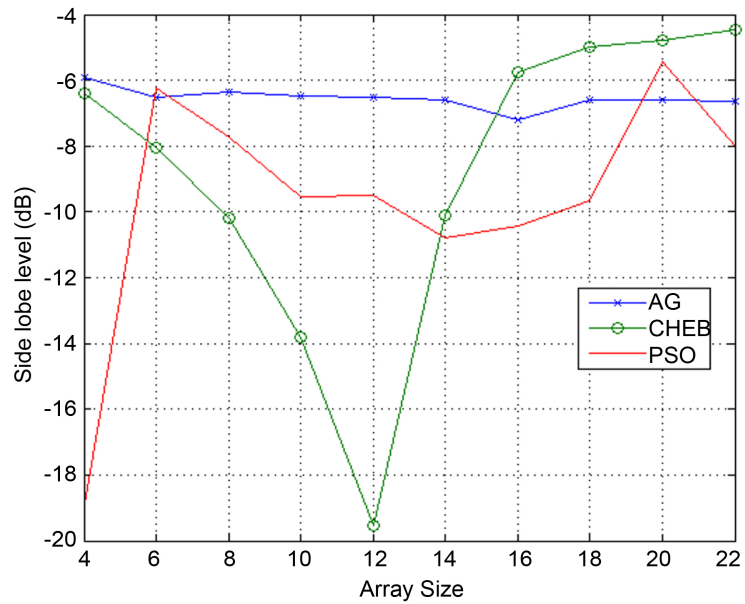


Figure 4. SSL variation with the array dimension.

For this scenario, the best values are obtained with the number of elements equal to 12. **Figure 5** shows the running time with the variation of the array size. For N elements equal to 4, 6, 8, 12, 16, and 18-element arrays, the Chebyshev algorithm showed less running time than others.

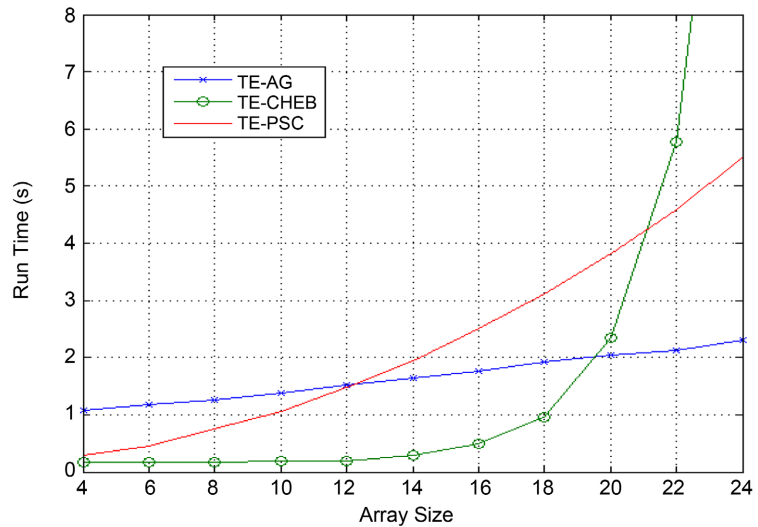


Figure 5. Running time variation.

5.2. Comparison of the Effects of Excitation and Number of Elements N

Table 2 shows the numerical results and a comparative study between the three

algorithms.

Table 2. Numerical results.

Number of element	Parameters	GA	PSO	Chebyshev
4	Excitation	0.5555	0.67792	0.9611
		0.70712	0.74205	0.8472
	SSL (dB)	-5.91	-19	-6.4
	FNBw (degree)	59.58	47.01	62.84
	HPBw (degree)	35.87	36.79	37.03
8	Excitation	0.30137	0.90579	0.9611 0.8472
		0.70993	0.63236	0.6600 0.4364
		0.45978	0.54688	
		0.24117	0.15761	
	SSL (dB)	-6.37	-7.73	-10.17
	FNBw (degree)	29	30.02	36.42
	HPBw (degree)	17.49	15.2	20.41
12	Excitation	0.20064	0.93401	0.9611 0.8472
		0.19728	0.46939	0.6600 0.4364
		0.35707	0.16218	
		0.27514	0.462	
		0.31669	0.26297	
		0.69138	0.74815	
	SSL (dB)	-6.82	-9.49	-19.52
	FNBw (degree)	19.21	22.81	36.01
	HPBw (degree)	11.04	13.04	16.06

The comparative parameters include the excitation, the SSL, the FNBW, and the HPBW as well as the number of elements (4 elements, 8 elements, and 12 elements non-uniformly excited). Because of the symmetry of the linear antenna array design, the excitation value numbers are divided by 2, for example, $N = 4$ corresponds to two excitation values, etc. For $N = 4$, PSO shows the best SSL (-19 dB) but for $N = 8$ and $N = 12$, it's the Chebyshev algorithm that dominates. The variation of FNPw and HPBw parameters can be discussed but our interest is focalised on minimizing SSL.

This work is a comparative study between three optimization algorithms, the results may depend on several parameters including the capacity of the resources allocated in the simulation tests.

6. Conclusion

In this paper, Three optimization algorithms are applied to reduce the second

side lobe SSL generated from a typical linear antenna. SSL is a result of antenna radiation. The reduction of these electromagnetic fields especially in local zones can contribute to minimizing interference and keep the human body safe. A comparison study of the three algorithms is also carried out, the goal is to identify the best algorithms in the optimization process. Results showed a dominance for the Chebyshev algorithm in reducing SSL values. This work can be enhanced by considering newly developed heuristics and algorithms, and proposed practice solutions to convict mobile operators' services to use such algorithms and reduce SSL values.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] World Health Organization (2024) Specific Absorption Rate. [https://www.who.int/data/gho/data/indicators/indicator-details/GHO/specific-absorption-rate-\(sar\)-\(w-kg\)](https://www.who.int/data/gho/data/indicators/indicator-details/GHO/specific-absorption-rate-(sar)-(w-kg))
- [2] European Regulations (1999) 1999/519/EC: Council Recommendation of 12 July 1999 on the Limitation of Exposure of the General Public to Electromagnetic Fields (0 Hz to 300 GHz).
- [3] Denden, M., Jemmali, M., Boulila, W., Soni, M., Khan, F. and Ahmad, J. (2024) Clustering-Based Resource Management for Consumer Cost Optimization in IoT Edge Computing Environments. *IEEE Transactions on Consumer Electronics*. <https://doi.org/10.1109/tce.2024.3414929>
- [4] Laufs, J., Borrion, H. and Bradford, B. (2020) Security and the Smart City: A Systematic Review. *Sustainable Cities and Society*, **55**, Article 102023. <https://doi.org/10.1016/j.scs.2020.102023>
- [5] Eljack, S., Jemmali, M., Denden, M., Turki, S., Khedr, W.M., Algashami, A.M., *et al.* (2023) A Secure Solution Based on Load-Balancing Algorithms between Regions in the Cloud Environment. *PeerJ Computer Science*, **9**, e1513. <https://doi.org/10.7717/peerj-cs.1513>
- [6] Jemmali, M., Denden, M., Boulila, W., Srivastava, G., Jhaveri, R.H. and Gadekallu, T.R. (2022) A Novel Model Based on Window-Pass Preferences for Data Emergency Aware Scheduling in Computer Networks. *IEEE Transactions on Industrial Informatics*, **18**, 7880-7888. <https://doi.org/10.1109/tii.2022.3149896>
- [7] Kurdi, M.H., Denden, M. and Paul, D. (2024) A Study on the Challenges of Human-Centric Cyber-Security and the Guarantee of Information Quality. *Journal of Information Security*, **15**, 218-231. <https://doi.org/10.4236/jis.2024.152013>
- [8] Jayarama, K. and Chen, C.H. (2024) Enhanced Wideband Frequency Estimation via FFT: Leveraging Polynomial Interpolation and Array Indexing. *Journal of Computer and Communications*, **12**, 35-48. <https://doi.org/10.4236/jcc.2024.121003>
- [9] Mohsen, D., Ghannay, N. and Samet, A. (2009) A Half Hollow Cylindrical Antenna (HHCA) Analysis Using the CFDTD Algorithm. *Progress in Electromagnetics Research C*, **11**, 51-60. <https://doi.org/10.2528/ pierc09090804>
- [10] Romdhani, F., Denden, M. and Samet, A. (2009) A Printed Reconfigurable Antenna for Communication System. 2009 *Mediterranean Microwave Symposium (MMS)*,

- Tangiers, 15-17 November 2009, 1-3. <https://doi.org/10.1109/mms.2009.5409789>
- [11] Mailloux, R.J. (2017) Phased Array Antenna Handbook. Artech House.
- [12] Al-Zoubi, A.S., Amaireh, A.A. and Dib, N.I. (2022) Comparative and Comprehensive Study of Linear Antenna Arrays' Synthesis. *International Journal of Electrical and Computer Engineering (IJECE)*, **12**, 2645-2654. <https://doi.org/10.11591/ijece.v12i3.pp2645-2654>
- [13] Liang, Q., Chen, B., Wu, H., Ma, C. and Li, S. (2021) A Novel Modified Sparrow Search Algorithm with Application in Side Lobe Level Reduction of Linear Antenna Array. *Wireless Communications and Mobile Computing*, **2021**, Article 9915420. <https://doi.org/10.1155/2021/9915420>
- [14] Song, Q., Lei, S., Chen, M., Tian, J., Yang, W. and Sun, K. (2024) Peak Sidelobe Level Suppression via Reconfigurable Element Pattern Selection for Beam Scanning Linear Array Antenna. *Digital Signal Processing*, **145**, Article 104333. <https://doi.org/10.1016/j.dsp.2023.104333>
- [15] Khalili, H., Mohammadpour-Aghdam, K. and Alamdar, S. (2024) Sidelobe Level Reduction in Pattern Synthesis of a Non-Uniform Series-Fed Microstrip Antenna Array Using the TLBO Algorithm. *AEU-International Journal of Electronics and Communications*, **176**, Article 155143. <https://doi.org/10.1016/j.aeue.2024.155143>
- [16] Amer, S.M., Khalaf, A.A.M., Hussein, A.H., Alqahtani, S.A., Dahshan, M.H. and Kassem, H.M. (2024) New Antenna Array Beamforming Techniques Based on Hybrid Convolution/Genetic Algorithm for 5G and Beyond Communications. *Computer Modeling in Engineering & Sciences*, **138**, 2749-2767. <https://doi.org/10.32604/cmescs.2023.029138>
- [17] Li, W., Zhang, Y. and Shi, X. (2019) Advanced Fruit Fly Optimization Algorithm and Its Application to Irregular Subarray Phased Array Antenna Synthesis. *IEEE Access*, **7**, 165583-165596. <https://doi.org/10.1109/access.2019.2953544>
- [18] Federico, G., Caratelli, D., Theis, G. and Smolders, A.B. (2021) A Review of Antenna Array Technologies for Point-to-Point and Point-to-Multipoint Wireless Communications at Millimeter-Wave Frequencies. *International Journal of Antennas and Propagation*, **2021**, Article 5559765. <https://doi.org/10.1155/2021/5559765>
- [19] Zardi, F., Nayeri, P., Rocca, P. and Haupt, R. (2021) Artificial Intelligence for Adaptive and Reconfigurable Antenna Arrays: A Review. *IEEE Antennas and Propagation Magazine*, **63**, 28-38. <https://doi.org/10.1109/map.2020.3036097>
- [20] Zhang, J., Ge, X., Li, Q., Guizani, M. and Zhang, Y. (2017) 5G Millimeter-Wave Antenna Array: Design and Challenges. *IEEE Wireless Communications*, **24**, 106-112. <https://doi.org/10.1109/mwc.2016.1400374rp>
- [21] Guo, Y.J., Ansari, M. and Fonseca, N.J.G. (2021) Circuit Type Multiple Beamforming Networks for Antenna Arrays in 5G and 6G Terrestrial and Non-Terrestrial Networks. *IEEE Journal of Microwaves*, **1**, 704-722. <https://doi.org/10.1109/jmw.2021.3072873>
- [22] Xu, Q., Zeng, S., Zhao, F., Jiao, R. and Li, C. (2021) On Formulating and Designing Antenna Arrays by Evolutionary Algorithms. *IEEE Transactions on Antennas and Propagation*, **69**, 1118-1129. <https://doi.org/10.1109/tap.2020.3016181>
- [23] Durmus, A., Kurban, R. and Karakose, E. (2021) A Comparison of Swarm-Based Optimization Algorithms in Linear Antenna Array Synthesis. *Journal of Computational Electronics*, **20**, 1520-1531. <https://doi.org/10.1007/s10825-021-01711-w>
- [24] Guerra, A., Guidi, F., Dardari, D. and Djuric, P. (2021) Near-Field Tracking with Large Antenna Arrays: Fundamental Limits and Practical Algorithms. *IEEE Transactions on Signal Processing*, **69**, 5723-5738.

- <https://doi.org/10.1109/tsp.2021.3101696>
- [25] Nabil, G., Mohsen, D., Faouzi, R. and Abdelaziz, S. (2008) A Novel Technique for Calculating Moment Method Impedance Matrix. *IEEE Mediterranean Microwave Symposium 2008*, Damascus, 14-16 October 2008, 77-80.
- [26] Mohamed, H., Fethi, C., Mohsen, D., Tan Hoa, V. and Jacques, D. (2011) A Novel Compact Ultra-Wideband Rectangular Shaped Antenna. *Progress in Electromagnetics Research Symposium Proceedings*, Marrakesh, 20-23 March 2011, 381-385.
- [27] Alnasser, N., Alabssi, R., Faran, B., Alessa, L. and Nagy, N. (2024) The Implementation of Ray Tracing Algorithm with Openmp Parallelization. *Journal of Computer and Communications*, **12**, 120-130. <https://doi.org/10.4236/jcc.2024.121008>
- [28] Alkhalifah, A. and Denden, M. (2023) Investigating the Impact of COVID-19 on the Morale of Deaf and Hearing-Impaired Students in Saudi Arabia Technical Colleges: Lessons Learned and Future Implications. *Journal for Educators, Teachers and Trainers*, **14**, 420-428.
- [29] Denden, M. and Alkhalifah, A. (2023) Assessing the Impact of COVID-19 on the Psychology of Saudi Technical College Students: Lessons and Tips. *Creative Education*, **14**, 518-529. <https://doi.org/10.4236/ce.2023.143036>
- [30] Balanis, C.A. (2016) *Antenna Theory: Analysis and Design*. John Wiley & Sons.
- [31] Mitchell, J. (1998) Introduction to Melanie Klein. In: Phillips, J. and Stonebridge, L., Eds., *Reading Melanie Klein*, Routledge, 11-31. <https://doi.org/10.4324/9780203360538-1>
- [32] Freitas, D., Lopes, L.G. and Morgado-Dias, F. (2020) Particle Swarm Optimisation: A Historical Review up to the Current Developments. *Entropy*, **22**, Article 362. <https://doi.org/10.3390/e22030362>
- [33] Rivlin, T.J. (2020) *Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory*. Dover Publications.