

# Modeling $VO_{2max}$ with Relative Fitness and BMI

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## Abstract

Cardiorespiratory fitness (CRF) is considered a foundation for good health and longevity. The variable,  $VO_{2max}$ , is considered a reliable measure of cardiorespiratory health. The measurement of  $VO_{2max}$  is stressful and is not a common practice, thus, it is desirable to relate  $VO_{2max}$  to other variables including age, gender, Body Mass Index (BMI), body fat percentage (BFP), and physical activity. Mathematical models are presented to relate  $VO_{2max}$  to age, gender, BMI, body fat percentage, and aerobic exercise. Predictions match well with measured  $VO_{2max}$  data published in the literature. The robust models are stable for all boundary conditions including young adults to old age and lower sedentary to Olympic-trained levels of relative fitness for both male and female applications. The relationship may be especially helpful for older people who are dealing with elevated values of BMI. The models build on two levels of relative fitness (sedentary and endurance-trained) and boundary conditions of Olympic-trained athletes and lower sedentary conditions. Thus, the model responds to a full range of fitness conditions. Four equations are provided: one with relative fitness related to distance walked or ran each week, one using BMI to estimate relative fitness, and two using both BMI and BFP to estimate relative fitness (one interacting with age). An equation is also provided to evaluate the dynamic change in relative fitness using input of distance walked or ran each day, which can be estimated by a smart watch. The equation that only uses age, gender, and BMI as inputs overestimates  $VO_{2max}$  for people with low BMI and who are physically inactive. This weakness is partially overcome by adding BFP as a variable when available. The most accurate equation is the one that uses distance walked or ran each day to estimate relative fitness based on the individual's physical activity. Unfortunately, this method requires the individual to keep track of their physical activity daily. Relative fitness may be a more appropriate variable than  $VO_{2max}$  to indicate fitness because it is independent of age.

## Keywords

$VO_{2max}$ , BMI, Body Mass Index, Exercise, Health, Body Fat Percentage

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## 1. Introduction

There is overwhelming evidence that health and longevity are associated positively with  $VO_{2max}$  [1] [2]. Cardiorespiratory fitness (CRF) refers to the health of the heart and lungs to perform physical activity. Low CRF is associated with high blood pressure, coronary artery disease, type 2 diabetes, and some cancers [2]-[4]. Physical inactivity or low CRF “ranks as the number three actual cause of death in the United States according to the Centers for Disease Control and Prevention.” [4]. The  $VO_{2max}$  measurement or estimate evaluates the maximum uptake of oxygen at peak physical performance. Rao *et al.* [2] report that  $VO_{2max}$  is the most accurate parameter evaluating cardiorespiratory fitness in reference to the American College of Sports Medicine’s Guideline for Exercise Testing and Prescription [5].

The  $VO_{2max}$  evaluation is an extreme and stressful test of a person’s heart and lung capacity. Athletes use this measurement to evaluate their training progress. Non-athletes as well as athletes and healthcare providers can benefit from knowing a reasonable estimate of  $VO_{2max}$ .

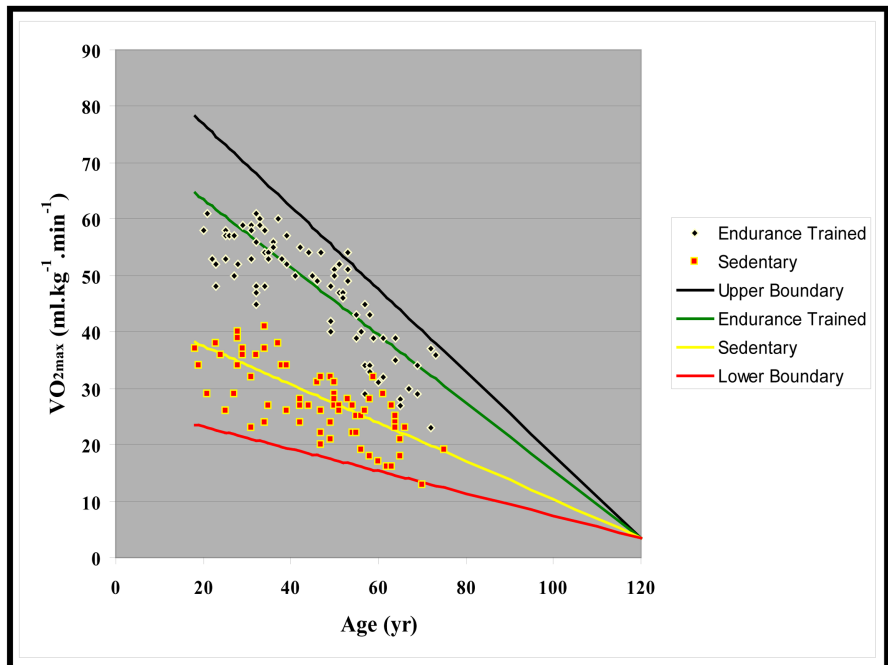
Gregory [1] used measured data from Tanaka *et al.* [6] for women (**Figure 1**) and data from Pimentel *et al.* [7] for men (**Figure 2**) to develop a general mathematical model for  $VO_{2max}$  using age, relative fitness, and gender as variables. The measured data trend and the mathematical model predictions converge to one value at age 120 years for each gender. The model includes a gender difference of  $7 \text{ ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$  at this age [1]. While the measured data from these two references was limited to adults, the model was verified to work for children using data reported by Astrand *et al.* ([8], p. 261) [9]. While these data sets are relatively old, they represent one of the more complete measurements in terms of age, lifestyle, and gender.

Based on the scatter in **Figure 1** and **Figure 2**, there is considerable uncertainty with both the sedentary and endurance-trained lifestyles. A 60-year-old male for example on average is expected to have a  $VO_{2max}$  of 30 for a sedentary lifestyle but can range from 20 to 40-a 30 percent uncertainty. An endurance trained male at age 60 can vary from 40 to 54. Females (**Figure 1**) have a similar uncertainty. Thus, while there is strong evidence of a linear decline of  $VO_{2max}$  that occurs with aging, there is much uncertainty about  $VO_{2max}$  without additional information.

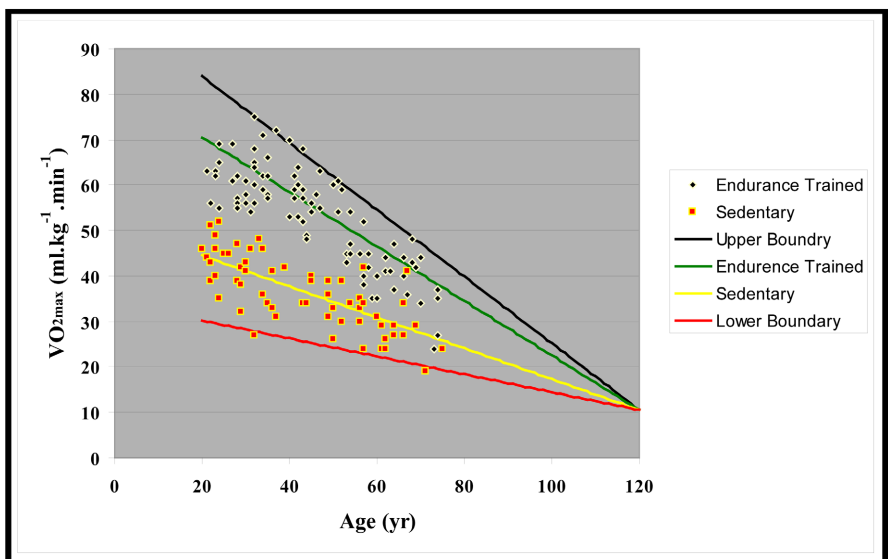
An older classic data set [10] provides information on the dynamics of changes in  $VO_{2max}$ . This study contrasted changes associated with extended bedrest to changes due to exercise. The decline of  $VO_{2max}$  during bedrest may emulate the decline in  $VO_{2max}$  and related health risks as adults adopt a low-physical-activity-sedentary lifestyle. This research provides strong evidence that lack of physical activity over time will result in a reduced  $VO_{2max}$ .

Two common and simple measurements often made when we visit our healthcare providers are weight and height from which BMI is computed. The BMI variable has been used [11] in an equation to estimate  $VO_{2max}$  for both men and women. While this is a useful equation, it fails at boundary conditions, especially for high BMI values and old ages. Their model includes a gender difference of

10.99 ml·kg<sup>-1</sup>·min<sup>-1</sup> compared to the 7.0 difference determine by Gregory [1]. This model includes a linear slope of -0.38 with age. It is obvious from **Figure 1** and **Figure 2** that one linear slope should not predict well when considering the age variable.



**Figure 1.** Reduction of VO<sub>2max</sub> for women as a function of aging and fitness lifestyle. Data from Tanaka *et al.* [6].



**Figure 2.** Reduction of VO<sub>2max</sub> for men as a function of aging and fitness lifestyle. Data from Pimentel *et al.* [7].

In a general sense, athletics at any age should be in the endurance-trained region of **Figure 1** and **Figure 2**, and obese individuals should be in the low

sedentary region of **Figure 1** and **Figure 2**. Athletics tend to have low BMI and obese individuals tend to have high BMI with limited athletic ability.

Afshari *et al.* [12] reported a linear model to predict  $VO_{2max}$  as a function of BMI. Their model was calibrated based on young men with an average age of 21.36 years. They had an excellent  $R^2$  value of 0.919 but did not include age as a variable. They also measured body fat percentage and reported a model for  $VO_{2max}$  as a linear function of BFP ( $R^2 = 0.756$ ). The same young men were involved in the BFP evaluation. Thus, BFP is an additional variable to consider when modeling  $VO_{2max}$ . Their  $VO_{2max}$  data ranged between a high of about 50 and a low of about 30—essentially the full range of sedentary males in **Figure 2** for age of 20 years. Thus, their data suggests that either BMI or BFP information might improve the prediction of  $VO_{2max}$ .

$VO_{2max}$  data for men and women with an average age of 20.09 years were reported by Mondal and Mishra [13] as a function of BMI. Their data has much scatter because of the average difference of  $11.6 \text{ ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$  between men and women. The lower data points (probably for women) reveal a non-linear trend of  $VO_{2max}$  as a function of BMI. There appears to be some evidence that a linear function may not be a correct model for  $VO_{2max}$  when considering a wide range of BMI values. There also seems to be a consistent average difference between men and women (10.99, 11.6) that is greater than the gender difference of 7.0 reported by Gregory [1].

## Objective

There appears to be a need for a more complete mathematical model to estimate  $VO_{2max}$  over a wide range of ages, BMI values, and BFP measurements. This model should use variables that are easily measured to facilitate usage by lay people and primary healthcare professionals.

The objective of this paper is to present a set of mathematical models to estimate  $VO_{2max}$  using age, relative fitness, gender, BMI, BFP, and training or conditioning through physical activity. All equations are to be stable and valid for all boundary conditions of adult ages and variations in BMI, BFP, and physical activity.

## 2. Methods

### 2.1. Development

The Gregory [1] equation for  $VO_{2max}$  will be used as the starting point in this development:

$$VO_{2max} = 107.4 \text{ RF} \left( 1 - \frac{A}{120} \right) + G \quad (1)$$

where  $VO_{2max}$  = aerobic fitness ( $\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$ );

RF = relative fitness (fraction of upper limit).

- (1.00) upper limit, Olympic class skier or runner
- (0.82) approximate upper limit for endurance-trained

- (0.67) endurance-trained
- (0.50) active (upper boundary for sedentary)
- (0.45) estimate for control or average population (1/4 endurance trained & 3/4 sedentary)
- (0.38) sedentary
- (0.22) approximate lower limit for sedentary
- (0.00) non active bed rest

$A$  = age (years);

$G$  = gender coefficient (males: 10.5; females: 3.5)  $\text{ml} \cdot \text{kg}^{-1} \cdot \text{min}^{-1}$ .

This equation is based on two major data sets, one for females with a variety of conditions primarily of endurance-trained and sedentary women [6] and a similar study for men [7]. Gregory added an upper and lower boundary as boundary conditions and a middle division (RF = 0.5) dividing sedentary data from endurance-trained data. The mathematical model for each of the two data sets was reduced to one equation using a gender factor of 10.5  $\text{ml} \cdot \text{kg}^{-1} \cdot \text{min}^{-1}$  for men and 3.5  $\text{ml} \cdot \text{kg}^{-1} \cdot \text{min}^{-1}$  for women. The data sets are shown in **Figure 1** and **Figure 2** with linear lines illustrating relative fitness (RF) values.

The RF of 0.0 does not indicate zero oxygen uptake. Instead, it indicates a flat line in **Figure 1** and **Figure 2** associated with the intersection point at age 120 years. The  $\text{VO}_{2\text{max}}$  value at this RF value is the minimum or close to the minimum uptake of oxygen needed to sustain life as a human. It is interesting that the RF of 0.5 that is halfway between the top athletic people male or female and the RF of 0.0 is also the dividing boundary between athletic and sedentary lifestyles.

The labels used to describe relative fitness refer to activity or lack of physical activity. Athletics generally keep track of distance ran, swam, bicycled, etc. as part of their training process. The average person, however, usually is not interested in this detail. They may know that they are sedentary but not much more. They may be on a church or community softball or soccer team but fail to add much conditioning.

Therefore, it is desirable to look to other variables that are easily defined and generally available to add to the  $\text{VO}_{2\text{max}}$  model. The model expressed in Equation (1) includes the effect of age and gender. It also provides physical limits of boundary conditions: upper limit for the athlete and lower limit for the sedentary lifestyle. The RF limits provide a descriptive and robust set of limits to the human condition of  $\text{VO}_{2\text{max}}$ .

The Gregory equation was applied to heart risk data with four quartiles of fitness reported by Sandvik *et al.* [14]. The calibration of the heart risk data resulted in the highest fitness group in the low range of endurance trained. The other three quartiles were in the sedentary range [1]. The control or average value of 0.45 for relative fitness is based on 1/4 endurance trained and 3/4 sedentary. The value of 0.45 was determined from data for longevity [1] [15] for a control data set in comparison to running club members with an endurance-trained relative fitness (0.67). Predictions of  $\text{VO}_{2\text{max}}$  from Equation (1) for men are shown in **Table 1**.

**Table 1.** Predicted  $\text{VO}_{2\text{max}}$  ( $\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$ ) for men as a function of age and relative fitness using Equation (1).

Age	$\text{VO}_{2\text{max}}$							
	Relative Fitness							
	1.00	0.82	0.67	0.50	0.45	0.40	0.38	0.22
0	117.9	98.6	82.5	64.2	58.8	53.5	51.3	34.1
10	109.0	91.2	76.5	59.7	54.8	49.9	47.9	32.2
20	100.0	83.9	70.5	55.3	50.8	46.3	44.5	30.2
25	95.5	80.2	67.5	53.0	48.8	44.5	42.8	29.2
30	91.1	76.6	64.5	50.8	46.7	42.7	41.1	28.2
35	86.6	72.9	61.5	48.5	44.7	40.9	39.4	27.2
40	82.1	69.2	58.5	46.3	42.7	39.1	37.7	26.3
45	77.6	65.5	55.5	44.1	40.7	37.4	36.0	25.3
50	73.2	61.9	52.5	41.8	38.7	35.6	34.3	24.3
55	68.7	58.2	49.5	39.6	36.7	33.8	32.6	23.3
60	64.2	54.5	46.5	37.4	34.7	32.0	30.9	22.3
65	59.7	50.9	43.5	35.1	32.7	30.2	29.2	21.3
70	55.3	47.2	40.5	32.9	30.6	28.4	27.5	20.3
75	50.8	43.5	37.5	30.6	28.6	26.6	25.8	19.4
80	46.3	39.9	34.5	28.4	26.6	24.8	24.1	18.4
85	41.8	36.2	31.5	26.2	24.6	23.0	22.4	17.4
90	37.4	32.5	28.5	23.9	22.6	21.2	20.7	16.4
95	32.9	28.8	25.5	21.7	20.6	19.5	19.0	15.4
100	28.4	25.2	22.5	19.5	18.6	17.7	17.3	14.4
105	23.9	21.5	19.5	17.2	16.5	15.9	15.6	13.5
110	19.5	17.8	16.5	15.0	14.5	14.1	13.9	12.5
115	15.0	14.2	13.5	12.7	12.5	12.3	12.2	11.5
120	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5

With the exception of relative fitness of 0.40, relative fitness values shown in this table are from the calibration values with Equation (1); The relative fitness of 0.40 was determined by matching predictions with Equation (2) from Jackson *et al.* [11]; Predictions shown in bright yellow are a reasonable match with predictions from Equation (2) for the descriptions provided by Jackson *et al.* [11].

The next step in development was to use the regression equation from Jackson *et al.* [11] to obtain a relationship between  $\text{VO}_{2\text{max}}$  and BMI. They report the

following equation for men that includes BMI:

$$\text{VO}_{2\text{max}} = 67.350 + 1.92(\text{PA-R}) - 0.381A - 0.754\text{BMI} \quad (2)$$

where PA-R = the NASA/JSC physical activity scale (0 - 7);

BMI = body mass index ( $\text{kg}/\text{m}^2$ ).

The 67.350 in Equation (2) can be replaced by 56.363 for females: a difference of 10.99.

Note that Equation (2) has only one slope ( $-0.381$ ) associated with age in contrast to the data presented in **Figure 1** and **Figure 2** and Equation (1). Data in **Figure 1** and **Figure 2** clearly show multiple slopes with age for different lifestyles. Eight slopes expressed as RF in Equation (1) are defined to describe the measured data in **Figure 1** and **Figure 2**. Thus, the equation provided by Jackson *et al.* [11] does not deal with the observed boundary limits or lifestyle choices for human performance expressed in terms of  $\text{VO}_{2\text{max}}$ . It only matches the sedentary relative fitness.

The PA-R and BMI components in Equation (2) are independent variables with respect to each other and age. Equation (2) adjusts  $\text{VO}_{2\text{max}}$  as a linear function of BMI. Walsh *et al.* [16] reported that master athletics have BMI values that ranged from 20.8 to 27.3 depending on the sport. They reported that 27.3 was for soccer players. The average value for BMI was 23.8 for master athletics. Thus, it is reasonable that the lower limit for BMI for athletic adults is around  $20 \text{ kg}/\text{m}^2$ . A value of 18.5 is used as the lower BMI value in the normal range. Because muscle mass is denser than fat mass, the lower limit for athletic adults may be higher than that for the average person.

The upper limit for BMI is not as easily defined. There is evidence that a small percentage of people can exceed a BMI of 40 [17]. As BMI values increase, the associated RF values should not dip below the observed lower limit. The function relating  $\text{VO}_{2\text{max}}$  to BMI must asymptotically approach the lower limit. Thus, an exponential decay function was assumed to predict RF as a function of BMI that would provide a lower limit for RF of about 0.2, slightly below the observed low sedentary RF of 0.22.

Newport *et al.* [18] from a Gallop survey reported an average BMI value of 27 for the US population (men and women). Women had slightly lower BMI values than men. If we assume that 1/4 of the people in the study by Jackson *et al.* [11] were somewhat athletic (BMI = 23.8) and 3/4 were sedentary (BMI = 27), an estimated BMI of 26 results.

The equation from Jackson *et al.* [11] was used next to determine the RF value associated with a BMI of 26. A midpoint of 3.5 (0 - 7) was used for the PA-R value. They reported that the main age group was between 35 and 49 years. The midpoint age of 42 years was used as a guide in selecting  $\text{VO}_{2\text{max}}$  data for the next analysis. The results are shown in **Table 2**. They also reported that their results were best between  $\text{VO}_{2\text{max}}$  values of 36 and 55. This range is highlighted (horizontal bright yellow box) in **Table 1** and **Table 2**. The vertical column for BMI of 26 is highlighted from ages 20 to 80. The RF value was varied in **Table 1** to provide an

approximate match between Equations (1) and (2). The best match occurred for a RF value of 0.40 shown in **Table 1**. Both equations seem to work in agreement for the highlighted yellow age, RF, and BMI values.

**Table 2.**  $VO_{2max}$  ( $ml \cdot kg^{-1} \cdot min^{-1}$ ) predictions from Equation (2) as a function of age and BMI.

Age	$VO_{2max}$									
	BMI									
	20	22	25	26	27	28	30	33	36	40
0	59.0	57.5	55.2	54.5	53.7	53.0	51.5	49.2	46.9	43.9
10	55.2	53.7	51.4	50.7	49.9	49.2	47.6	45.4	43.1	40.1
20	51.4	49.9	47.6	46.8	46.1	45.3	43.8	41.6	39.3	36.3
25	49.5	48.0	45.7	44.9	44.2	43.4	41.9	39.7	37.4	34.4
30	47.6	46.1	43.8	43.0	42.3	41.5	40.0	37.8	35.5	32.5
35	45.7	44.2	41.9	41.1	40.4	39.6	38.1	35.9	33.6	30.6
40	43.8	42.2	40.0	39.2	38.5	37.7	36.2	34.0	31.7	28.7
45	41.8	40.3	38.1	37.3	36.6	35.8	34.3	32.0	29.8	26.8
50	39.9	38.4	36.2	35.4	34.7	33.9	32.4	30.1	27.9	24.9
55	38.0	36.5	34.3	33.5	32.8	32.0	30.5	28.2	26.0	23.0
60	36.1	34.6	32.4	31.6	30.9	30.1	28.6	26.3	24.1	21.1
65	34.2	32.7	30.5	29.7	29.0	28.2	26.7	24.4	22.2	19.1
70	32.3	30.8	28.6	27.8	27.0	26.3	24.8	22.5	20.3	17.2
75	30.4	28.9	26.6	25.9	25.1	24.4	22.9	20.6	18.4	15.3
80	28.5	27.0	24.7	24.0	23.2	22.5	21.0	18.7	16.4	13.4
85	26.6	25.1	22.8	22.1	21.3	20.6	19.1	16.8	14.5	11.5
90	24.7	23.2	20.9	20.2	19.4	18.7	17.2	14.9	12.6	9.6
95	22.8	21.3	19.0	18.3	17.5	16.8	15.3	13.0	10.7	7.7
100	20.9	19.4	17.1	16.4	15.6	14.9	13.4	11.1	8.8	5.8
105	19.0	17.5	15.2	14.5	13.7	13.0	11.4	9.2	6.9	3.9
110	17.1	15.6	13.3	12.6	11.8	11.1	9.5	7.3	5.0	2.0
115	15.2	13.7	11.4	10.7	9.9	9.1	7.6	5.4	3.1	0.1
120	13.3	11.8	9.5	8.7	8.0	7.2	5.7	3.5	1.2	-1.8

Based on the graphs and data in **Figure 1** and **Figure 2**, there appears to be a lower limit for relative fitness that is near or maybe slightly below 0.22. Notice the two points on **Figure 2** that are slightly below the lower boundary line. Logically,

as BMI increases, the associated relative fitness asymptotically approaches a lower limit of about 0.2. Certainly, the function cannot be linear over a wide range of BMI values because of boundary conditions.

Equations (1) and (2) both have a linear component that approximately matches at a RF value of 0.40 and a BMI of 26. Also, predicted results from Equation (2) dip below the observed lower boundary from Equation (1) for age 60 and older for BMI values of 40 kg/m<sup>2</sup> illustrating the limitations of Equation (2).

The World Health Organization gives the following BMI classifications [19]:

Underweight	BMI < 18.5
Normal	18.5 ≤ BMI < 25
Pre-obesity	25 ≤ BMI < 30
Stage 1 obesity	30 ≤ BMI < 35
Stage 2 obesity	35 ≤ BMI < 40
Stage 3 obesity	BMI ≥ 40

The observed lower BMI value for athletes is 20.8 compared to the lower normal BMI range of 18.5. Values below 20 for BMI may indicate low muscle weight and low VO<sub>2max</sub>.

While a lower BMI value of 18.5 is reasonable for most adults, there is no real upper value for BMI. For example, a 5' 10" man with a weight of 200 pounds has a BMI of 28.7. The same man with a weight of 300 pounds has a BMI of 43.1. A 350-pound man with the same height has a BMI of 50.3. Fortunately, these high BMI values are rare; nevertheless, they are possible. As a reference for extreme BMI, Saad *et al.* [17] in a study of hypogonadal men observed 14 percent with a BMI more than 40 kg/m<sup>2</sup>. It is reasonable to assume that these high BMI values would be associated with a low sedentary relative fitness lifestyle.

There is also a normal weight obesity term defined in the literature [20] [21]. This term is defined as a BMI below 25 and body fat larger than 23 percent for men and larger than 30 percent for women [21]. There is clear evidence that BMI is not a precise predictor of VO<sub>2max</sub> or health risks. Nevertheless, even with these limitations, BMI improves the prediction of VO<sub>2max</sub> as reported by Jackson *et al.* [11].

The following equation was formulated to adjust the RF from the upper boundary condition down to that associated with the lower relative fitness of about 0.2:

$$RF = 0.2 + 0.8e^{-c(BMI - BMI_0)} \quad (3)$$

where  $c$  = calibration coefficient (0.183) m<sup>2</sup>/kg;

BMI<sub>0</sub> = reference value for calibration (BMI 18.5 kg/m<sup>2</sup>).

## 2.2. Initial Results

Data were obtained from **Figure 2** for measured points on or near the upper and lower boundaries for relative fitness. Data associated with the upper boundary for endurance trained were associated with a BMI of 20. A BMI value of 40 was used for the lower boundary.

Your author has a 50-year-old neighbor who trains for and competes in Iron-man events. He has a BMI of 21 kg/m<sup>2</sup> and a measured VO<sub>2max</sub> of 54 ml·kg<sup>-1</sup>·min<sup>-1</sup>.

The new model predicted a  $VO_{2max}$  of  $54.75 \text{ ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$  (1.39 percent error). In comparison, the model from Jackson *et al.* [11] predicted a value of  $46 \text{ ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$  (14.9 percent error).

Also, a midpoint for a BMI of 26 and an estimated  $VO_{2max}$  of 38.25 (average of 39.2 and 37.3) was used from matching Equations (1) and (2) using the results from Jackson *et al.* [11]. Values for  $c$  and  $BMI_0$  were varied to obtain a maximum  $R^2$  of 0.99 and a 1 to 1 slope between measured and predicted  $VO_{2max}$  data. The results are highly significant ( $p < 0.001$ ). Results are shown in **Table 3**.

The reference  $BMI_0$  that resulted from calibration with this data set is 18.5, which is also the same value as the lower limit that the World Health Organization uses for the normal range in BMI. A BMI value of 20 is where RF value is 0.808 based on the calibration of Equation (3). This value is close to 0.82 in the original calibration by Gregory [1] for the upper boundary condition of endurance trained.

Similar to Jackson *et al.* [11] the function for BMI is independent of age. Notice that two men were 32 years of age: one was on the top boundary with a  $VO_{2max}$  of 75; one was on the bottom boundary with a  $VO_{2max}$  of 27. Both predicted points had less than 2 percent error. Notice also that the RF value of 0.808 occurred for ages ranging from 32 to 64 years.

**Table 3.** Comparison of predicted and measured  $VO_{2max}$  ( $\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}$ ) values for various ages, BMI, and boundary conditions.

BMI	RF	$VO_{2max}$	Age	Measured $VO_{2max}$	Percent Error	Source
20	0.808	82.81	20			
20	0.808	74.14	32	75	-1.15	Upper boundary points from <b>Figure 2</b>
20	0.808	72.69	34	71	2.38	Upper boundary points from <b>Figure 2</b>
20	0.808	70.52	37	72	-2.06	Upper boundary points from <b>Figure 2</b>
20	0.808	68.35	40	70	-2.36	Upper boundary points from <b>Figure 2</b>
20	0.808	60.40	51	61	-0.99	Upper boundary points from <b>Figure 2</b>
20	0.808	51.00	64	47	8.50	Upper boundary points from <b>Figure 2</b>
21	0.706	54.75	50	54	1.39	Ironman neighbor
26	0.403	38.62	42	38.25	0.96	Jackson <i>et al.</i> (1990) equation
40	0.216	27.48	32	27	1.79	Lower boundary points from <b>Figure 2</b>
40	0.216	22.66	57	24	-5.59	Lower boundary points from <b>Figure 2</b>
40	0.216	19.96	71	19	5.04	Lower boundary points from <b>Figure 2</b>

The equation for the predicted results is given as follows:

$$VO_{2max} = 107.4 \left( 0.2 + 0.8e^{-0.183(BMI-18.5)} \right) \left( 1 - \frac{A}{120} \right) + G \quad (4)$$

These results provide strong evidence that RF limits should be used to limit the effects of BMI in predicting  $VO_{2max}$ . The regression equation from Jackson *et al.* [11] provides evidence that a general population seems to follow this relationship between BMI and  $VO_{2max}$ . An individual, however, can be thin and trim without much aerobic exercise investment. They, thus, should be much lower than the predicted  $VO_{2max}$  for a low BMI. This condition will be illustrated in the next section.

### 3. Results

Next, additional data sets were analyzed to test the model independent of data used to develop Equation (4). Mondal and Mishra [13] presented BMI data as a function of  $VO_{2max}$  for both young men and women with average age of 20.09 years. This information was digitized and  $VO_{2max}$  data expressed as a function of BMI (solid green circles in Figure 3). Mondal and Mishra [13] lumped their data together for males and females. There is a general linear decline of  $VO_{2max}$  as a function of BMI ( $R^2 = 0.20$ ). Both Gregory [1] and Jackson *et al.* [11] observed a difference in  $VO_{2max}$  associated with gender. The inclusion of data for men and women together is one reason for the major scatter in  $VO_{2max}$  data in Figure 3. Using the observed difference of 7 to 10.99  $ml \cdot kg^{-1} \cdot min^{-1}$  between men and women for  $VO_{2max}$  [1] [11], the lower data points appear to be for women. Mondal and Mishra [13] also reported that their average  $VO_{2max}$  for men was 43.25 and 31.65 for women, further suggesting that the upper green circles are for men and the lower green circles are for women.

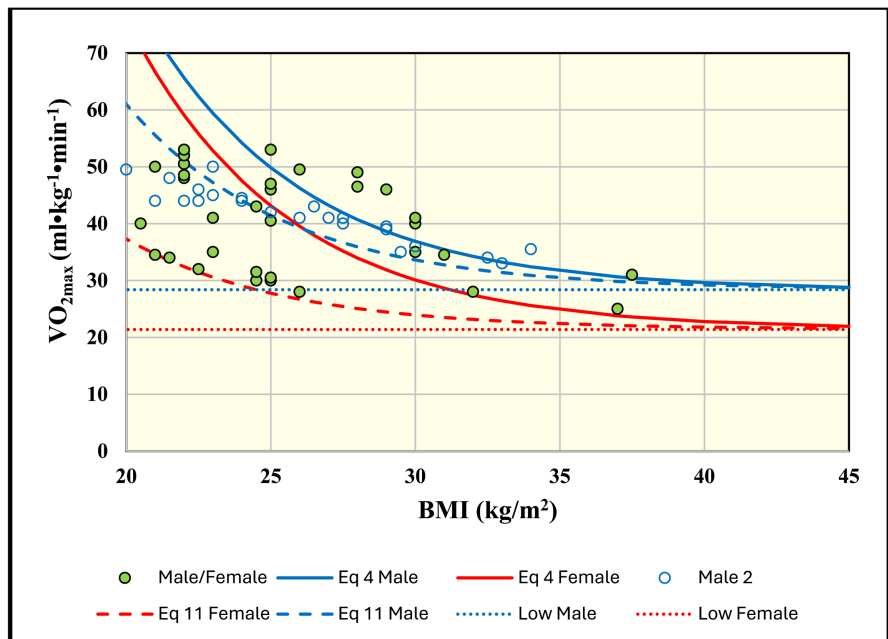


Figure 3. Illustration of the variability in predicting  $VO_{2max}$  as a function of BMI.

First, note that at age 20 and a BMI of 20, both males and females have the potential to achieve a  $VO_{2max}$  above their respective low limit all the way to the top

of this graph. None of the male or female data points approach the upper boundary as discussed in earlier sections of this paper. Furthermore, all the measured data points are well on their way to the respective lower limits for BMI values above 30 kg/m<sup>2</sup>. Measured data points can be above or below the predictions.

Predictions from Equation (4) are shown as the solid blue line for men and solid red line for women. The lower boundaries for VO<sub>2max</sub> (relative fitness = 0.20) are shown as a dotted blue line for men and red for women. All simulated lines are for age 20.09 years.

A second data set [12] is also shown in **Figure 3** as open blue circles for men. The average age for this data set is 21.36 years, essentially the same age as the previous data set. This data set has a strong linear trend. Afshari *et al.* [12] report an R<sup>2</sup> of 0.919 for their linear equation. The lower end of this data set is a close match with the predicted blue line for men from Equation (4). Most of the blue points are below the solid blue line. The solid blue line (Equation (4)) does a reasonable job of predicting both the upper green circles and the blue circles for BMI values of 25 and larger. Equation (4) overestimates VO<sub>2max</sub> for both the green and blue data sets for BMI values less than 25.

The solid blue line is a relatively close match for the points for men at a BMI of 26. This is a close match to the equation prediction from Jackson *et al.* [11]. See **Table 3**. Also, Equation (4) seems to be adequate in predicting VO<sub>2max</sub> for BMI values up to 38 implying that the model is both stable (not crossing lower boundary limit) and adequate for high BMI values. There seems to be no need for further improvement for BMI values above 25. Adding BFP as a variable improved predictions for BMI values less than 25 as shown in the dashed blue line for men and red for women. The development of this modification will be discussed next.

It was observed that in a few papers body fat percentage and VO<sub>2max</sub> were presented [22]-[24]. In some of the papers, BMI data were not presented. Thus, there was a need to predict BMI from body fat percentage (BFP). Afshari *et al.* [12] present VO<sub>2max</sub> as a function of BMI and in a second graph as a function of body fat percentage. The following equation describes their linear data to predict VO<sub>2max</sub> as a function of BMI:

$$VO_{2max} = 70 - \frac{32}{30}BMI \quad (5)$$

Next, the following function was developed from their data to describe the effects of body fat percentage on VO<sub>2max</sub>:

$$VO_{2max} = 55 - \frac{22}{40}BFP \quad (6)$$

Equating the right sides of these two equations to each other leads to

$$BMI = 14.06 + 0.516BFP \quad (7)$$

Rounding reduces the equation to

$$BMI = 14 + 0.5BFP \quad (8)$$

This equation should be used with some caution. Newport *et al.* [18] in their Gallup survey found an average trend with age. They observed a 24.9 BMI for ages

18 - 24, 26.8 for ages 25 - 29, and 28.2 for ages 30 - 69. As reported earlier, their average BMI was about 27. Thus, the 14 probably should be increased to 15 for ages 25 - 29, and 16 for ages 30 - 69. After age 70 BMI starts to decrease as high BMI, obese people start to die, and people start to have loss of muscle and bone mass with age.

Bellissimo *et al.* [21] did a study of males and females aged 43 to 50 years showing that BMI has limitations in defining health risk. They defined three categories: lean, normal weight obesity, and overweight obesity. Lean was defined as BMI less than 25 and body fat percentage less than 23 for men and less than 30 for women. Normal weight obesity was defined as BMI less than 25 but body fat percentage larger than 23 for men and larger than 30 for women, a 7 percent difference. Overweight-Obesity was defined as BMI larger than 25 and body fat percentage larger than 23 for men and larger than 30 for women. From these results [12] [21] it became clear that body fat percentage might be a missing variable.

While different studies use different cutoff values as their study design, in general women are reported to have about 7 percent more BFP than men. The study by Bellissimo *et al.* [21] gives some justification for adding BFP as an additional variable, especially for BMI values less 25 where Equation (4) seems to over predict the measured results.

In Equation (4), the change in BMI (BMI—18.5) was used to adjust the prediction of  $VO_{2max}$ . The change in BMI as a function of body fat percentage is the term, 0.5 BFP. By adding both changes and subtracting a calibration variable to adjust for the additional change, Equation (4) was adjusted to include body fat percentage as follows:

$$VO_{2max} = 107.4 \left( 0.2 + 0.8e^{-0.183(BMI-18.5+0.5BFP-K)} \right) \left( 1 - \frac{A}{120} \right) + G \quad (9)$$

where  $K$  = a calibration adjustment (5.0) for adding the BFP variable.

Lindberg and Luo [25] report a range for BFP of 6 to 13 for male athletes. The value of 10 is near the middle of this range. A 10 was used for BFP for top male athletes in Equation (9). The  $K$  calibration value was determined to be 5 when Equation (9) matched predictions from Equation (4).

When a 10 is used for BFP for men, the dashed blue line in **Figure 3** is superimposed over the solid blue line. A BFP of 10 for females causes the dashed red line for females to be superimposed over the solid red line. The only difference between Equation (4) based on a calibration for men and the predicted solid red line was the change in gender factor.

The dashed lines in **Figure 3** are predicted with Equation (11) presented later as a modification to Equation (9). Lindberg and Luo [25] report a range for BFP of 14 to 20 for female athletes. The middle value is 17. This value is 7 percent higher than the male athletics. The predicted dashed red curve for this BFP value is above the lower green circles as expected for female athletics. The dashed red line as shown is for a BFP of 22. This value for BFP is in the range of 21 to 24 fitness category given by Lindberg and Luo [25]. The dashed blue line shown for

men is for a BFP value of 15, which is in the fitness category (14 to 17) given by Lindberg and Luo [25] for men and 7 percent lower than the female value. Finally, when a 6 (lowest BFP value given for men by Lindberg and Luo [25]) is used for BFP, the predicted curve goes through the upper green circles. In conclusion, Equation (11), that includes both BMI and BFP, seems to cover the full range of measured data from Mondal and Mishra [13]. Including the BFP variable is especially important for predicting  $VO_{2max}$  for young women.

It was noted earlier that both Jackson *et al* [11] and Mondal and Mishra [13] reported a difference of about 11 for  $VO_{2max}$  compared to the difference of 7 for Gregory (2022). Because of the effect of BFP, the difference in both data and prediction curves at a BMI of 25 is about 11.

The testing of Equation (9) up to this point has been for male and female data at or near the age 20.09 years. Does it work for older adults? The Ironman 50-year-old neighbor has a  $VO_{2max}$  of 54 and a BMI of 21. He also has a measured BFP of 9.5. Equation (9) predicted a  $VO_{2max}$  value of 56.2, a 4.07 percent error, worse than Equation (4). If a BFP value of 10 is used, the prediction is 54.7 (1.3 percent error). The difference between 9.5 and 10 for BFP is well within measurement error even for whole body scans. It obviously seems to work for this highly trained older man. Nevertheless, these results may indicate a problem with Equation (9) for older people. This problem led to the following investigation.

It is unknown if Equation (8) works for older men or females. A publication by Woolcott and Bergman [26] is useful in providing an indirect method to assess the validity of Equation (8) for older men and women. They present the following equation:

$$RFM = 64 - 20 \frac{H}{W} + 12S \quad (10)$$

where RFM = relative fat mass (estimate of BFP);

$H$  = height of individual;

$W$  = waist circumference;

$S$  = sex variable (0 for males; 1 for females).

This equation seems to suggest that women have 12 percent more BFP than men compared to the difference of 7 percent discussed earlier. This 12 percent value, however, adjusts for the 7 percent as discussed earlier and the different in the ratio of  $H/W$  between men and women. Even with this adjustment, the 12 percent seems high.

Your author (77 years of age) used Equation (10) to estimate his BFP. This resulted in a value of 24.6. Next, Equation (8) ( $16 + 0.5$  BFP) was used to estimate BMI. A value of 28.3 was obtained. Your author weighs most mornings before his shower. His average BMI over the last few months is 27.4. The resulting error is 3.3 percent. While this is only one point, it is a result for an old man and supports the probability that Equation (8) adjusted for age is valid for older adult men. The predicted BMI with Equation (8) for the Ironman neighbor using his measured BFP of 9.5 is 20.75. His measured BMI is 21. The prediction error is 1.2 percent,

another indication that Equation (8) works for older men.

In a study of untrained 26-year-old male students [22], an average BFP of 18.0 was reported. The height and weight were reported resulting in a measured BMI of 25.9. Equation (8) was used resulting in a predicted BMI of 24. This result underestimated BMI by 7.3 percent. Equation (8) seems to work over a wide range of ages for men.

Next, Equations (8) and (10) were used with your author's wife. The results from Equation (10) were  $22 + 12 = 34$ . Using the BFP of 34 in Equation (8) resulted in a BMI of 33. Her measured BMI is 26.6, producing a 24.1 percent error. In a Gallup survey [18], it was reported that men and women had about the same average BMI and that value was about 27. Women on average are both shorter and lighter than men; thus, it makes sense that the average value for both sexes might be approximately the same. Yet, women have on average about 7 percent more BFP. This extra fat certainly does not affect height and appears not to have much effect on weight in terms of BMI compared to men. Thus, Equation (8) was used with a 22 value for RFM, neglecting the extra 12 percent associated with being female. This calculation resulted in a predicted BMI of 27.0. This prediction is within 1.5 percent of the measured value. This is only one point, but it seems logical.

Next, Equation (9) was used to predict  $VO_{2max}$  for your author. A relative fitness of 0.36 and a  $VO_{2max}$  of 24.2 resulted from Equation (4). A relative fitness of 0.24 and a  $VO_{2max}$  of 19.9 resulted from Equation (9) including the BFP estimate from Equation (10). While your author does not know his  $VO_{2max}$  value, the Equation (9) prediction seems low, especially since your author walks 10 to 20 miles per week when he is not traveling. There is over 18 percent difference between Equation (4) and Equation (9) predictions for your author.

There is strong evidence as shown in **Figure 3** that including BFP for young people helps to determine their  $VO_{2max}$ . The variable BFP may not, however, be independent of BMI for older adults. In fact, as BFP goes up for older people, it is likely that BMI will also increase more or less in sync with lifestyle choices. Remember, it was observed that Equation (4) without BFP as a variable seemed to perform well for BMI values greater than 26. Equation (9) was modified to decrease the effect of BFP on the prediction of  $VO_{2max}$  as a function of age:

$$VO_{2max} = 107.4 \left( 0.2 + 0.8e^{-0.183(BMI-18.5+e^{-0.123(A-21)}(0.5BFP-K))} \right) \left( 1 - \frac{A}{120} \right) + G \quad (11)$$

This modification and calibration are based on data from Newport *et al.* [18] for a decrease in changes in BMI as a function of age. Using Equation (11) instead of Equation (9) reduced the prediction error of 4.07 percent to 1.48 percent for the Ironman neighbor. For your author's age, Equation (11) predicts the same as Equation (4). It appears that Equation (11) should be used instead of Equation (9) when considering a wide range of ages.

Data from Shete *et al.* [24] for both athletic and non-athletic women were

analyzed next. They did not report BMI. The reported age range was 17 - 22 years. They measured BFP using skin fold calipers (not always done with accuracy). A BFP of 24.11 and a  $VO_{2max}$  of 39.62 were reported for the athletic women. Using the measured BFP of 24.11 minus 12 in Equation 7 provided 20.1 as an estimate of BMI. This value is slightly below the range (20.8 - 24.7) of BMI values reported by Walsh *et al.* [16] for young athletic women. The predicted  $VO_{2max}$  for a BMI of 20.1, BFP of 24.11, and age of 19.5 years is 36.3 compared to the measured 39.62 (8.3 percent under prediction). A reduction of approximately 1 percent (24.11  $\rightarrow$  23) in BFP changed the BMI to 19.5 and the predicted  $VO_{2max}$  to 39.7, a near perfect match. This 1.11 percent change in BFP measurement is certainly reasonable.

A similar analysis was made for the non-athletic women. The non-athletic measurements were 23.54 for  $VO_{2max}$  and 29.31 for BFP. The predicted BMI is 22.7 using Equation (8). The predicted  $VO_{2max}$  is 27.2 (15.5 percent over prediction). If the 12 percent in BFP is not subtracted, then a BMI of 28.7 results and the predicted  $VO_{2max}$  is 23.4 producing only a 0.60 percent error. The reported 23.54  $VO_{2max}$  is very near the lower sedentary boundary in **Figure 1** for age 19.5 years.

If the 12 for female is not subtracted in Equation (10), then Equation (8) predicts a high BMI that increases the underprediction of  $VO_{2max}$  for the athletic women. If the 12 for sex is added in Equation (10), then the  $VO_{2max}$  prediction is close for the nonathletic women, but the BMI is way off for the athletic women and your author's wife.

Either Equation (8) does not work for these women; the BFP measurement is too low; or something else is governing the result. The reported 23.54 is low compared to the measured green points of women in **Figure 3** for a BMI of 22.7. One simple explanation is that these women did in fact have both a low BMI and a low  $VO_{2max}$  and that the low  $VO_{2max}$  was due to lack of exercise, and they maintained a slim body through diet. Another possibility is that BMI varies with body type (stocky versus skinny). Wash *et al.* [16] reported that athletes varied from 20.8 to 27.3. If athletics vary by 6.5, maybe nonathletes also vary by 6.5 units. A BMI of 27.3 produces a  $VO_{2max}$  of 23.9, which is close to the measured 23.54 resulting in only 1.63 percent error. This prediction point is close to the dashed red line in **Figure 3** that matches the lower green points. Even with the improvements with Equation (11), there is still some uncertainty in predicting  $VO_{2max}$ .

In a more recent study, Ross and Myers [27] listed 16 studies with equations to predict  $VO_{2max}$ . Fourteen out of the 16 included age as a variable. Twelve studies used age as a linear function and two studies used age as a quadratic function. It is interesting that Ross and Myers [27] included an equation from Jackson *et al.* [11] that included BFP ( $R^2 = 0.812$ ) but did not include the equation from Jackson *et al.* [11] that included BMI with a slope with age of  $-0.38$  ( $R^2 = 0.783$ ). Ross and Myers [27] reported an  $R^2$  of 0.66 for the equation with BFP. The range in slopes with age for studies with a wide range of ages included in the study was  $-0.175$  to  $-0.381$ . Even with the variability of the slopes reported and the confusion about  $R^2$  values, it seems obvious that most of the studies made measurements on

sedentary individuals. Athletic men and women were not measured and grouped individually. There also appears to be evidence that the slope on age with the linear regression development was also dependent on what other variables were included in the model. None of the developments addressed upper or lower boundary conditions. Ross and Myers [27] state “The available equations have tended to underestimate CRF among higher fit individuals and overestimate CRF among lower fit individuals.” The nonlinear relationship between relative fitness and BMI in Equation 3 and included with modifications in Equation 4, 9, and 11 facilitate predictions over a wide range of fitness conditions. This relationship seems to address the problems described above.

#### 4. Discussion

Equation (1) [1] was used as a starting model to predict  $VO_{2max}$  as a function of age, gender, and relative fitness. Using only two lifestyles for relative fitness (endurance-trained and sedentary) resulted in an  $R^2$  value of 0.858 for women for the data in **Figure 1** and 0.775 for men in **Figure 2** [28]. While these are reasonable  $R^2$  values, there is still much uncertainty in predictions.

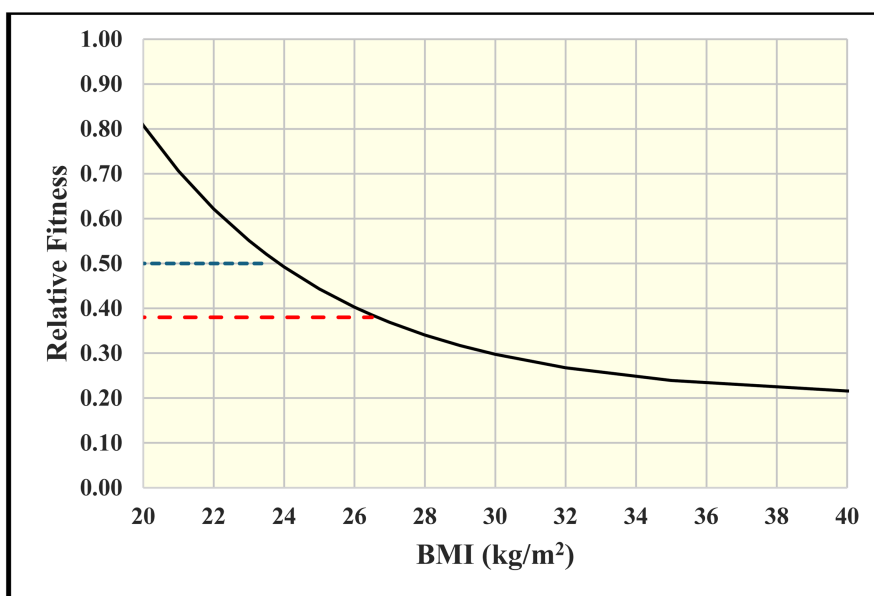
For comparison, Matthews *et al.* [29] presented a model using multiple regression development with a similar age range of 19 to 79 years. An  $R^2$  of 0.74 and 0.73 for two slightly different but similar models including variables of age, gender, weight, height, and BMI were included. It is interesting that including BMI in their model was no better as measured with  $R^2$  values than knowledge of being in a running club (endurance-trained) or defaulting to an average sedentary relative fitness using Equation (1).

The mathematical model expressed in Equation (11) provides considerable improvement over Equation (1) by Gregory [1] or Jackson *et al.* [11]. Two new variables were added to Equation (1): BMI and BFP. The boundary conditions with upper and lower limits on relative fitness keeps the model robust preventing unreasonable predictions in  $VO_{2max}$  over the full range of adult ages for both men and women. At mid-range in sedentary lifestyle (RF = 0.40) associated with a BMI of 26, the new model provides  $VO_{2max}$  predictions in line with the regression equation of Jackson *et al.* [11]. Unlike the equation of Jackson *et al.* [11], the new model predicts well the  $VO_{2max}$  values over a wide range of ages for sedentary and endurance-trained men and women. For high BMI values of 40 and higher, the model predicts well for people at lower relative fitness. For low BMI values, the model predicts the full range of endurance-trained relative fitness.

Adding BFP as a variable helped to adjust the relative fitness to predict  $VO_{2max}$  for young men and women as illustrated in **Figure 3**. It appears as reflected in Equation (11) that BFP has little value in predicting  $VO_{2max}$  for older adults. Age seems to buffer the effects of BFP after considering BMI. It also is obvious that BMI and BFP are not sufficient to fully predict  $VO_{2max}$  for people with low BMI values.

Thus, exercise needs to be considered in evaluating  $VO_{2max}$  for BMI values less

than 27 kg/m<sup>2</sup>. The uncertainty is illustrated in **Figure 4**. The dashed red line is a relative fitness of 0.38 associated with an average sedentary lifestyle. For BMI values larger than 27, the relative fitness (age independent) is most likely to be sedentary and at or below the solid black line. As BMI increases above 27, the relative fitness moves to values less than the average sedentary level. For BMI values less than 27, the relative fitness can still be sedentary but can also be much higher for those who exercise on a regular basis.



**Figure 4.** Relationship between BMI and relative fitness. Solid line is an estimate of the maximum relative fitness for a given BMI and BFP in balance with BMI. The difference between the black line and red dashed line depends on the amount of exercise achieved on a regular basis. Relative fitness values above the dashed green line are in the range of endurance-trained men and women.

For exercise to be effective, it needs to occur on a regular basis over a long period of time. It needs to become a habit. The value of regular exercise of running or walking in terms of relative fitness is shown in **Table 4**. A good runner can complete a 26.2-mile marathon in less than three hours. In other words, they can run a long distance at an average speed of 8.7 mi/hr. At this speed, they can complete 87 miles of training in 10 hours per week. An aged or a less trained individual will run at a slower speed and achieve fewer miles per week. **Table 4** was prepared by decreasing the time per week available from 10 to 0 hours. As training decreases, speed was reduced to 4 mi/hr that can be achieved by an older person 75+ years of age. Unless you are a speed walker, the maximum walking speed for most people is about 4 mi/hr. This speed was reduced for the older or leisure walker down to 2 mi/hr. Note that at 5 hours per week, both the runner and walker achieve about the same distance of 14 miles for a relative fitness of 0.3. Note that it takes time, usually about two months, to adjust to a new relative fitness. Either Equation (1) or **Figure 1** and **Figure 2** can be used to estimate  $VO_{2max}$  from age,

relative fitness, and gender.

**Table 4.** Estimate of the distance and time per week needed to achieve a desired relative fitness.

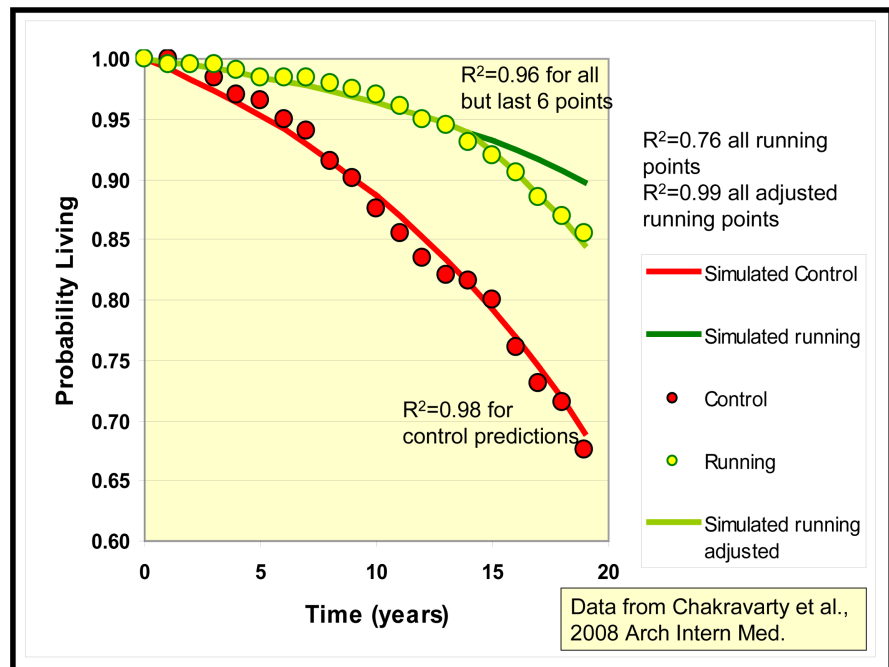
Hours per week	Distance (miles)	Running RF	Distance (miles)	Walking RF
10	87	0.82	40	0.48
9	68	0.68	36	0.46
8	52	0.57	32	0.43
7	39	0.47	28	0.40
6	27	0.39	21	0.35
5	18	0.33	18	0.32
4	14	0.30	14	0.30
3	11	0.28	9	0.26
2	7	0.25	5	0.24
1	4	0.23	2	0.21
0	0	0.20	0	0.20

This table is a rough estimate that should be used for planning exercise or to realize how little most of us exercise. Based on a smartwatch that counts steps, your author averages about 20 miles per week doing senior exercise at a gym, feeding pets, and doing yardwork, house cleaning etc. On a two-week trip, which included flying and driving, this average dropped to only 14 miles. It is very easy to not move much while flying, driving, and attending meetings.

These somewhat hidden low-intensity miles count as exercise. How much they should count is unknown. It is assumed that these miles count the same as a walk or run.

Since that trip, your author has averaged 15.4 miles of walking per week plus the 20 miles of doing normal activities for a total of about 35 miles per week. In theory, he has increased his relative fitness from about 0.36 to about 0.45. In one week, he walked 20.6 miles for an estimated relative fitness of 0.48. He has a goal of a relative fitness level of 0.50, which would require about 43 miles walking (20 + 23). Unfortunately, his BMI has not changed much. This lack of change can be explained in part by the fact that while exercise reduces fat, it also increases muscle. It takes a long time to lose weight by exercising.

These relative fitness values seem reasonable compared to data from Chakravarty *et al.* [15] and an analysis by Gregory [1]. The results are shown in **Figure 5**. The last six years of data for people (70+ years of age) in a running club seemed to have a reduced relative fitness (0.45 compared to 0.67) for younger years of age. The 0.45 relative fitness level is equal to the 0.45 discussed above for an exercise week of walking 15.4 miles. Running club members often run or walk about 30 miles per week in preparation and participation in 5 and 10 K competitions. Hence their relative fitness is generally in the endurance-trained range with a relative fitness above 0.5.



**Figure 5.** Probability of living as a function of age and maximum oxygen uptake. Data from Chakravarty [15]. The relative fitness for the control population is 0.45. The relative fitness for the running group is 0.67 for endurance-trained conditions. It is apparent from the above graph that the endurance-trained group became more like the control group as people aged (70+ years) in the endurance-trained group.

The value of walking a few miles per week is real. Booth and Neuffer [4] report that female nurses aged 40 to 65 who walked three or more hours per week, the equivalent of 9 or 10 miles per week based on **Table 4**, “had 30 percent less coronary artery disease, ischemic stroke and type-2 diabetes compared to sedentary nurses.” If they, like your author, walked 20 miles per week during normal activity or maybe 30 miles per week doing chores at home and checking on patients at work, their deliberate exercise walk represents an increase of about 1/3. The benefit seems to be about a one-to-one result for the increase in distance walked. This observed result was from a study of nurse’s health with 70,000 nurses involved. Exercise has value.

In addition to our deliberate exercise, we need to be active in our choices that affect our sort-of-hidden exercise. For example, going to the grocery store or mall to shop adds several steps even miles to our routine. People who use shopping services to do these activities are cheating themselves from important movement activities. Our society has many options for convenience that indirectly hurt our health by reducing our hidden exercise.

BMI values greater than 27 are associated with limited exercise, excessive eating, metabolism issues, such as high cholesterol and elevated risk for type 2 diabetes and heart disease. It is important to have lab work completed to check for health issues.

It is uncertain how a person with BMI higher than 27 should go about a

reduction in BMI. Diet seems obvious but is difficult to do and sustain. Reducing sitting and inactivity is important to change metabolism problems.

Research by Haider *et al.* [30] may give some insight into the nature of the problem. They studied men with low testosterone. In their study, men were divided into two groups. One group received testosterone treatment to increase testosterone to normal levels. The other group did not. Both groups were educated and encouraged to exercise and work to improve health issues. The study lasted 11 years.

The group receiving testosterone reduced their BMI from 36.5 to 29.2 kg/m<sup>2</sup>. In contrast, the non-treated group increased their BMI from 33.4 to 35.6 kg/m<sup>2</sup> over the same length of time. Other medical factors also changed. Improvements occurred in fasting glucose, triglycerides, total cholesterol, HbA1c, and fasting insulin for the testosterone group and worsened for the untreated group. Thirty-four percent of the treated group had remission from type 2 diabetes. Finally, the treated group developed fewer cases of prostate cancer. Many of the variables that changed were associated with metabolic health. Getting the testosterone back to a normal range seems to have been the solution to men's health and reduced BMI for the low testosterone men.

Gregory [1] presented evidence with data published in the literature showing that VO<sub>2max</sub> is important for heart health, sleep quality, and longevity. There is little doubt about the value of maintaining a high VO<sub>2max</sub> relative to our age. It takes physical work!

We should be careful in computing VO<sub>2max</sub> using BMI. A stocky body build will have a relatively high BMI compared to a slim build with possibly the same VO<sub>2max</sub>. Soccer is an active aerobic sport with lots of running but also quick stops and turns. Soccer players also must have strong muscles to kick to pass or score. It is reasonable to expect their VO<sub>2max</sub> values to be comparable with the average for all sports (BMI 23.8). A three-unit difference in BMI [16] for the same relative fitness results in about a 12 percent error of probably underestimating VO<sub>2max</sub> using BMI for physically active and stocky athletes. This problem should only exist for people involved with exercise or work with lots of physical activity.

It has been reported that BMI overestimates the health risks of body fat by roughly 10 percent for a large, tall frame person and underestimates by 10 percent for the short stature person [31]. These observations suggest that the uncertainty of BMI may be about 10 percent for estimating VO<sub>2max</sub>. Therefore, the uncertainty of predicting VO<sub>2max</sub> using Equations (4) or (11) should be about 10 percent. The BMI value also starts to have limited effect on VO<sub>2max</sub> for large values of BMI because the relative fitness factor in Equations (4) and (11) approaches the lower sedentary relative fitness. This lower relative fitness boundary, however, has high risk for health issues [1]-[3] [28].

Four Equations (1) (4) (9) (11) are presented in this paper. Which one is best? There is no one best equation. It depends on the data available. Equation (1) using **Table 4** has the advantage of including exercise data as input. As an example, your

author with the 20 weekly low intensity miles plus the 15 walking miles results in a relative fitness of 0.45 and a calculated  $VO_{2max}$  of 27.8. Using the BMI information of 27.4 and Equation (4), the calculated relative fitness and  $VO_{2max}$  are 0.36 and 24.2 respectively. Your author does not have a measurement of his body fat percentage. Using an estimate from Equation (10) of 24.6 and the BMI of 27.4 with Equation (9), the predicted relative fitness and  $VO_{2max}$  are 0.24 and 19.8. The results from Equation (9) are 20.5 percent lower than from Equation (1). Equation (9) probably should not be used for older adults. Equation (11) appears to be the better choice.

Equations (1) (4) (9) (11) are designed to calculate  $VO_{2max}$  for static or steady-state conditions. In mid-January, your author had three severely herniated disks removed from his neck. Before this surgery, your author lost the ability to tie shoes, button shirts, and walk up or down stairs without the aid of a railing. This condition persisted two months before the surgery. While the nerve damage healed well, he did not go for walks for several months. He certainly was at the bottom of **Table 4**. Equation (1) with the limited exercise condition or Equation (9) would have predicted the low  $VO_{2max}$  better than Equation (4) or (11). During this period his BMI had little change. Obviously, Equations (4) and (11) do not have enough information to respond to the dynamic that occurred.

The study by Haider *et al.* (2020) discussed above lasted 11 years. It took years for the changes to slowly occur. In contrast, changes in  $VO_{2max}$  from the study by Saltin *et al.* [10] occurred within two months or less.

Booth and Neuffer (2005) reported that it takes about a week for 50 percent of the genes to be replaced that control skeletal muscles. The following equation predicts the change in relative fitness as people change their daily exercise routine:

$$RF = W_{ex} RF_{old} + (1 - W_{ex}) RF_{new} \quad (12)$$

where  $W_{ex}$  = weight based on exercise;

= 0.905 for increasing fitness from the previous day;

= 0.980 for decreasing fitness from the previous day;

$RF_{old}$  = old relative fitness from previous day;

$RF_{new}$  = new relative fitness for the current day.

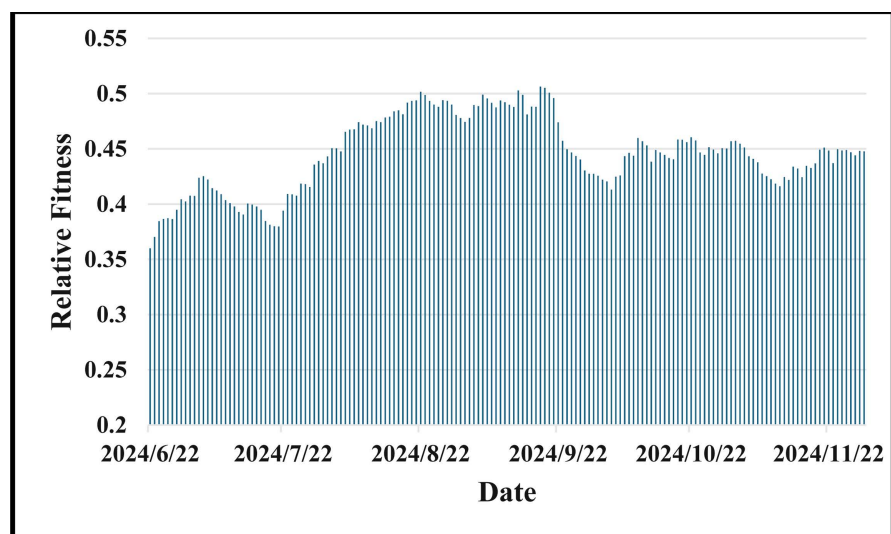
This function provides the same dynamic as a saturating exponential for increasing relative fitness or exponential decay for decreasing relative fitness. The advantage of using a weighting average like this equation is that it can increase or decrease relative fitness using daily input. The  $W_{ex}$  value of 0.905 is based on a half-life of 7 days in genes that affect skeletal muscles after exercise as reported by Booth and Neuffer [4]. This calibration also matches measured data from Saltin *et al.* [10] for men in exercise training after a period of active bedrest. The  $W_{ex}$  value of 0.980 is a calibration based on three weeks of active bedrest from the study of Saltin *et al.* [10]. This calibration is in line with the report by Booth and Neuffer [4] that it is not uncommon for an immobile leg to lose a third of muscle mass after only a few weeks.

As an example, your author uses the total number of steps per day from his watch to estimate the total distance associated with movement. He uses the following equation to predict the  $RF_{new}$  variable to use in Equation (12):

$$RF_{new} = 0.0000214Steps + 0.2 \quad (13)$$

where Steps = Steps counted with a wristwatch per day.

An example of the dynamic effects of exercise is shown in **Figure 6** for your author starting on June 22, 2024. The two major downward periods are associated with driving, flying, and nonexercised activities away from home. The recovery and upward trend and leveling off near a relative fitness level of about 0.5 is associated with walking 15 to 20 miles per week. The relative fitness predicted agrees with the reference or control population reported in **Figure 5**.



**Figure 6.** Effects of walking on the estimate of relative fitness for your author.

In conclusion, each of the three Equations (1) (4) (11) to predict  $VO_{2max}$  has value. Health care providers and their patients have access to BMI and most likely will not have exercise data or body fat percentage data. Individuals, who are willing to track the distance they walk or run, can plan what it takes to improve or maintain their  $VO_{2max}$ . The relative fitness presented in this paper is independent of age and may be a more encouraging variable to evaluate fitness for older adults as their  $VO_{2max}$  declines with age.

As one ages, it is important to stay physically active for body and mind health. Kiskac *et al.* [19] found that the optimum BMI for older people is higher than 25  $kg/m^2$ . A BMI of 27 may be an optimum value for older people but not for young people. Klatsky *et al.* [32] in their **Figure 1** provide context for understanding BMI effects on mortality for people of all ages. They show that odds of death are relatively flat in terms of BMI for people 60 + years of age. They, however, show a very rapid increase in risk for death as BMI increases above the 18.5 - 25 range for people less than 40 years of age. Furthermore, they show that the risk of death from cardiovascular causes for all ages increases as BMI increases above 18.5

kg/m<sup>2</sup>. The risk of non-cardiovascular death, however, starts to increase beyond the 25 - 30 BMI range. Generally, it is desirable to maintain BMI in the 18.5 to 25 range. Aune *et al.* [33] completed a major literature search including 207 publications and determined that the minimum risk for death in general occurs for BMI values between 20 and 24 kg/m<sup>2</sup>. While BMI has limitations, it nevertheless has value in understanding health risk. Equation (4) relates relative fitness to BMI measurements. Relative fitness facilitates the prediction of VO<sub>2max</sub> and CRF health risk assessment, which is well known.

## 5. Summary and Conclusions

The objective of this development was to provide an expanded mathematical model to include BMI as a variable but to retain a robust nature of accurate boundary conditions. From the data of Tanaka *et al.* [6] and Pimentel *et al.* [7], there appears to be both an upper and lower boundary that Gregory [1] defined with the term relative fitness. The upper boundary associates with only very high aerobic condition of Olympic training in sports that have very high demand on heart and lungs. The lower limit has a numeric value of about 0.20 relative fitness. Even a very high BMI does not seem to cause relative fitness to go much below this value (See **Figure 3**). These upper and lower relative fitness boundaries seem to apply for all ages. Equations (1) (4) (9) and (11) all predict within these boundary conditions and have more stability than Equation (2) from Jackson *et al.* [11].

The relative fitness variation between these two boundaries seems to be an exponential decay function of BMI that asymptotically approaches the lower boundary as BMI increases. By relating the difference in relative fitness between the upper and lower limits to an exponential decay function of BMI, the equation from Gregory [1] was expanded to include BMI as a variable. The current calibration is based on boundary conditions from measure data [6] [7] and predictions from an equation from Jackson [11] for sedentary conditions. The new model satisfies all boundary conditions for both BMI and VO<sub>2max</sub>. It also predicted the VO<sub>2max</sub> with only a 1.39 percent error for a 50-year-old neighbor in extreme training for Ironman competition with a BMI of 21 and a VO<sub>2max</sub> of 54 ml·kg<sup>-1</sup>·min<sup>-1</sup>. The equation from Jackson *et al.* [11] had a 14.9 percent error for this man.

While this new model is not perfect in predictions, it does facilitate people to estimate their VO<sub>2max</sub> over a wide range of VO<sub>2max</sub> values, age, and BMI. The three independent variables are age, gender, and BMI. Body fat percentage was also added to the model. It is not completely independent of BMI but seems to supplement the BMI variable for young adults with BMI less than 26. The model fails to accurately predict VO<sub>2max</sub> for people with low BMI who are naturally slim without engaging in intense physical activity. To overcome this limitation an exercise chart of distance walked or ran, or time spent walking or running is provided as an additional check on aerobic fitness.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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