Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(1), 499-512
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Study of the Stability of a Compressible Gas Layer in a Gravity Field

Igor Palymskiy[1,2]

[1]Institute of Optics and Information Security Technologies, Siberian State University of Geosystems and Technologies, Plakhotnogo Str. 10, Novosibirsk, Russia
[2]Department of Physics, Siberian State University of Telecommunications and Information Sciences, Kirova str. 86, Novosibirsk, Russia
Email: palymsky@yandex.ru

How to cite this paper: Palymskiy, I. (2026) Study of the Stability of a Compressible Gas Layer in a Gravity Field. *Journal of Applied Mathematics and Physics*, **14**, 499-512.
https://doi.org/10.4236/jamp.2026.141026

Received: November 18, 2025
Accepted: January 27, 2026
Published: January 30, 2026

Abstract

The stability of the equilibrium of a compressible gas layer in a gravity field is investigated. The same temperature is specified on all boundaries of the computational domain. The stability of the static equilibrium state of a compressible gas layer in a gravity field is analyzed in the linear approximation. It is shown in the linear approximation that at a sufficiently large height of the layer, the equilibrium static solution becomes unstable. The obtained data are supplemented by the results of solving a system of complete nonlinear equations describing the flows of compressible gas. The features of the obtained non-stationary solution are discussed.

Keywords

Gas, Turbulence, Compressibility, Gravitational Force, Numerical Simulation

1. Introduction

Let us consider the equilibrium of a gaseous or liquid continuous medium in a gravity field in a rectangular region, the boundaries of which are all rigid with the no-slip condition for velocity and are isothermal.

It is known [1] that in the incompressible medium considered within the framework of the Boussinesq approximation, in the absence of a vertical temperature gradient, only monotonically decaying motions are observed. With this approach, the density depends only on the temperature, and the dependence on pressure is ignored [1]. Thus, in the absence of a vertical temperature gradient, there is also no vertical density heterogeneity, which is the reason for the decay of the perturbations of the static solution.

However, in a compressible medium, the dependence of density on pressure can become significant [2]. In this case, pressure plays an active role, and its change, in principle, can generate non-uniformity of density along the vertical with the development of instability of the static solution. However, due to the extremely poor study of slow flows of a compressible medium (at low Mach numbers) due to technical difficulties, the question of the possibility of implementing such a scenario is not discussed in the literature [3].

However, the calculations carried out in this work convincingly showed that during the development of instability of the static equilibrium state of a layer of compressed gas in a gravity field, some heating of the medium is observed, which makes the flow in it physically similar to Rayleigh-Benard convection, which occurs when a gas layer is heated from below [4].

However, in the overwhelming majority of works, Rayleigh-Benard convection is considered as an incompressible fluid flow in the Boussinesq approximation [1] [4]. And convection in a compressible gas medium, even in such a relatively simple case, has not been studied sufficiently. It has been shown that on a laboratory scale (with a layer height of the order of several centimeters), the compressibility of the medium is weakly manifested and convection of a compressible gas medium can be considered in the Boussinesq approximation as an incompressible fluid flow. However, when the height of the region exceeds the critical value (17.3 cm for air under normal conditions), a relatively large change in hydrostatic pressure allows adiabatic processes to develop in the gas, which significantly changes the characteristics of convective processes [3] [5]. For example, the possibility of developing adiabatic processes makes it possible to develop convective instability even with neutral and stable density stratification of the medium [2].

The features of convection in a compressible medium have been discussed in a number of monographs and discussions [6]-[10]. It is traditionally believed that gas compressibility during convection on a laboratory scale is insignificant and manifests itself only on large (planetary) scales. In this case, both scales (laboratory and planetary) are considered as asymptotic, and their intersection is not taken into account.

In [8], the planetary atmosphere is treated as a compressible medium in which the flow is assumed to be adiabatic. It is shown that the convective flow in the atmosphere is unstable with sufficiently strong heating from below.

In [11], gas convection in a horizontal layer with horizontal boundaries free of shear stresses is considered analytically in a linear approximation and numerically in a nonlinear one. It is stated that the static equilibrium state is stable in the absence of a vertical temperature gradient and unstable with sufficiently strong heating from below.

The results of this work convincingly show that at a sufficiently large height of the compressible gas layer in the gravity field, instability of the static equilibrium mode develops. However, the amplitudes of the disturbances of the static solution are very small, which explains why such flows have not been studied previously.

To illustrate the importance of studying such flows, we point out their significance for the issue of explosion safety when storing hydrocarbons in large tanks, for example, at automobile filling stations.

An explosive situation occurs when the tank is almost empty, but a small amount of hydrocarbon remains at the bottom. In the presence of any flow, the fuel (vaporized hydrocarbon vapor) mixes with the oxidizer (air), forming a potentially explosive vapor-gas environment. The key point here is the question of the presence or absence of movement (mixing) of the medium, and the intensity of the flow does not play a special role in this context. The formation of an explosive mixture has been studied in many studies [12]-[15].

In this paper, the stability of the equilibrium of a compressible gas layer in a gravity field is investigated. To simplify the study, the problem is considered in a two-dimensional formulation, and the flow region is considered rectangular. All horizontal and vertical boundaries of the region are assumed to be rigid with the no-slip condition for velocity and isothermal, so that the same temperature is specified at all boundaries of the region. First, the stability of the static equilibrium of a compressible gas layer is analyzed in the linear approximation. The obtained data are supplemented by the results of solving a system of complete nonlinear equations describing the flows of compressible gas. The features of the emerging non-stationary system are discussed.

2. Numerical Model and Problem Statement

The convective flow of a compressible, viscous and heat-conducting gas in a gravity field can be described by the following system of equations [1] [16]:

$$
\begin{aligned}
&\rho_t + \rho\, div\vec{u} + u\cdot\rho_x + v\cdot\rho_y = M\,\nabla^2(\rho-\rho_h),\\
&u_t + u\cdot u_x + v\cdot u_y = -\frac{1}{\gamma\rho}(\rho T)_x + M\,(\frac{4}{3}u_{xx} + u_{yy} + \frac{1}{3}v_{xy}),\\
&v_t + u\cdot v_x + v\cdot v_y = -\frac{1}{\gamma\rho}(\rho T)_y + M\,(v_{xx} + \frac{4}{3}v_{yy} + \frac{1}{3}u_{xy}) - C_F,\\
&T_t + u\cdot T_x + v\cdot T_y = \frac{M}{\mathrm{Pr}}\nabla^2 T - \frac{\gamma-1}{\gamma}T\,div\vec{u},\ \ P=\rho T.
\end{aligned}
\tag{1}
$$

Here u, v, P, ρ and T are dimensionless components of velocity, pressure, density and temperature, $M = \nu/((\gamma T_0 R)^{0.5}H) = 4.608\cdot 10^{-8}\cdot H^{-1}$ is the Mach number, where the velocity calculated from kinematic viscosity is related to the adiabatic speed of sound, $T_0 = 300^\circ$K is taken as the characteristic value for temperature, the selected values of specific gas constant $R = 287$ J/(kg·K), adiabatic index $\gamma = 1.4$, kinematic viscosity $\nu = 16\cdot 10^{-6}$ m^2/s and Prandtl number $\mathrm{Pr} = \nu/\chi = 0.71$ correspond to air, where χ denotes the gas diffusivity and $C_F = gH/(\gamma RT_0) = 8.130\cdot 10^{-5}\cdot H$ is the hydrostatic compressibility and g is standard acceleration of free fall. As the length scale, we chose the height of the domain H, for temperature and density—their values T_0 and ρ_0 at the lower horizontal boundary, for the velocity—adiabatic speed of sound $(\gamma RT_0)^{0.5}$, for the pressure—$R\rho_0 T_0$ and time—$H/(\gamma RT_0)^{0.5}$. The dependence of viscosity and thermal conductivity coefficients on temperature is neglected in the calculations. The height of the flow region in the calculations varies from 0.003 m to 0.5 m.

The equation for temperature (the fourth equation of system (1)) in the case of a region of low altitude asymptotically transforms into the equation for temperature in an incompressible medium in the Boussinesq approximation [1].

Figure 1 shows the formulation of the problem in dimensionless form. The horizontal size of the region, referred to the vertical, is equal to π in all simulations. All vertical and horizontal boundaries of the region are considered rigid with the no-slip condition for velocity and isothermal.

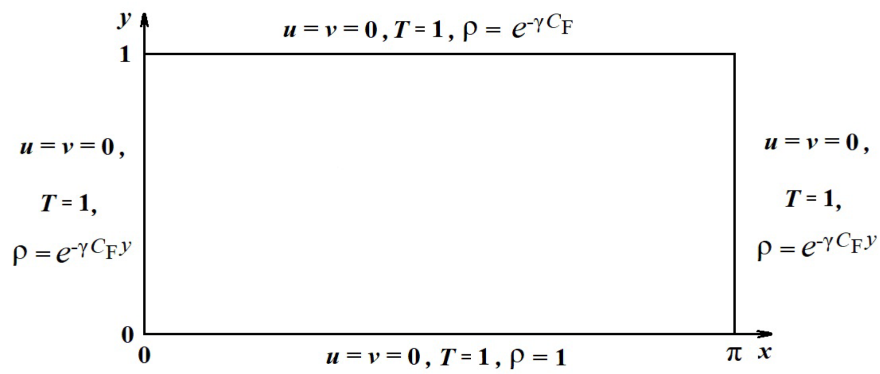

Figure 1. Formulation of the problem.

This problem has a static solution:

$$u = 0,\ v = 0,\ T = 1,\ \rho_h(y) = e^{-\gamma C_F y} \approx 1 - \gamma C_F y.$$

The relationships for static density are derived from the system of Equation (1) taking into account the absence of motion, the smallness of the value of C_F, and the equality of the dimensionless value of density to 1 at the lower horizontal boundary.

The calculations were performed using the explicit scheme in time, and since the appearance of shock waves in the solution was not expected, a non-divergent formulation of the system of equations was used. The convective nonlinear and diffusion terms were approximated by the monotonic scheme of A.A. Samarskii [4], and thus the numerical method used was of the first order of approximation in time and the second order in space.

All calculations were performed on a grid of (241·81) nodes with a dimensionless time step of 0.01. Test calculations on more detailed space and time grids showed sufficient accuracy and stability of the algorithm used.

All calculations were carried out near the stability threshold with the Reynolds number value as follows:

$$\mathrm{Re} = \sqrt{2Ek / \pi} \cdot M^{-1},$$

which had values of the order of 10^{-3}. Here, Ek denotes the total kinetic energy of the entire moving mass of gas [2].

Note, that the height of the region for clarity is always a dimensional value.

3. Linear Stability Analysis of a Static Solution

For infinitely small perturbations of the static solution from system (1), we can obtain (for simplicity of presentation, we use the same notations) system (2), which is given below.

$$\begin{aligned} &\rho_t + div\,\vec{u} - \gamma\, C_F v = M\,\nabla^2 \rho, \\ &u_t + (\rho_x + T_x)/\gamma = M\,(\frac{4}{3}u_{xx} + u_{yy} + \frac{1}{3}v_{xy}), \\ &v_t - C_F(T-\rho) + (T_y + \rho_y)/\gamma = M\,(v_{xx} + \frac{4}{3}v_{yy} + \frac{1}{3}u_{xy}), \\ &T_t + \frac{\gamma-1}{\gamma} div\,\vec{u} = \frac{M}{\Pr}\nabla^2 T. \end{aligned} \tag{2}$$

When deriving system (2), the hydrostatic compressibility C_F were considered small compared to 1.

We will consider the solutions of system (2) in the form that is usually used in studying the stability of convective flows [1]:

$$(\rho,u,v,T) = (\rho_0,u_0,v_0,T_0)\,e^{-\lambda t} e^{i(\alpha x+\beta y)}.$$

Here λ, u_0, v_0, ρ_0 and T_0 are complex constants, α and β are real constants and the amplitudes of the disturbances increase for real part $\lambda_r < 0$ and decay for $\lambda_r > 0$. In this section, the solution is considered to be periodic in the horizontal and vertical directions with wave numbers α and β. This formulation of the problem is physically close to the formulation of the problem of flow in a region with free horizontal boundaries [11].

From system (2), we can obtain a system of equations for the amplitudes:

$$\begin{aligned} &-\lambda\rho_0 + i(\alpha u_0 + \beta v_0) - \gamma C_F v_0 + M\,(\alpha^2+\beta^2)\rho_0 = 0, \\ &-\lambda u_0 + \frac{i\alpha}{\gamma}(\rho_0 + T_0) + M(\frac{4}{3}\alpha^2 u_0 + \beta^2 u_0 + \frac{1}{3}\alpha\beta\, v_0) = 0, \\ &-\lambda v_0 - C_F(T_0-\rho_0) + i\,\beta\,(T_0+\rho_0)/\gamma + M(\alpha^2 v_0 + \frac{4}{3}\beta^2 v_0 + \frac{1}{3}\alpha\beta u_0) = 0, \\ &-\lambda T_0 + (\gamma-1)\,i\,(\alpha u_0 + \beta v_0)/\gamma + M\,(\alpha^2+\beta^2)T_0/\Pr = 0. \end{aligned} \tag{3}$$

3.1. Development of Disturbances in the Absence of Gravity

Let us consider the solutions of the systems of Equations (2) and (3) in the special case, in the absence of gravity $C_F = 0$, and we obtain system (4):

$$\begin{aligned} &-\lambda\rho_0 + i(\alpha u_0 + \beta v_0) + M\,(\alpha^2+\beta^2)\rho_0 = 0, \\ &-\lambda u_0 + \frac{i\alpha}{\gamma}(\rho_0 + T_0) + M(\frac{4}{3}\alpha^2 u_0 + \beta^2 u_0 + \frac{1}{3}\alpha\beta\, v_0) = 0, \\ &-\lambda v_0 + i\,\beta\,(T_0+\rho_0)/\gamma + M(\alpha^2 v_0 + \frac{4}{3}\beta^2 v_0 + \frac{1}{3}\alpha\beta u_0) = 0, \\ &-\lambda T_0 + (\gamma-1)\,i\,(\alpha u_0 + \beta v_0)/\gamma + M\,(\alpha^2+\beta^2)T_0/\Pr = 0. \end{aligned} \tag{4}$$

System (4) was written in matrix form and from the condition that the determinant of the system is equal to zero, an algebraic equation of the fourth order was derived to determine λ:

$$\lambda^4 - \frac{13}{3} M\,S \cdot \lambda^3 + \frac{S(7M^2\gamma^2 S + 2\gamma - 1)}{\gamma^2} \cdot \lambda^2 -$$
$$-\frac{MS^2(5M^2\gamma^2 S + 4\gamma - 2)}{\gamma^2} \cdot \lambda + \frac{1}{3} M^2 S^3 \frac{4M^2\gamma^2 S + 6\gamma - 3}{\gamma^2} = 0,\ S = \alpha^2 + \beta^2.$$

Despite some cumbersomeness, the resulting equation has four solutions, which are written out analytically and divided into two groups:

$$\lambda_{1,2} = M\,S, \quad \lambda_{3,4} = \frac{7M}{6} S \pm \frac{i}{\gamma}\sqrt{S(2\gamma - 1)}.$$

It can be found that the first two roots correspond to a two-parameter family of solutions that are monotonically damped under the action of viscosity, where C_1 and C_2 are two arbitrary constants:

$$\rho_0 = C_1,\ T_0 = -C_1,\ \ u_0 = 0,\ v_0 = 0;$$
$$\rho_0 = 0,\ T_0 = 0,\ u_0 = C_2,\ v = -\alpha\, C_2 / \beta.$$

This solution describes the flow of a viscous incompressible fluid, since the continuity equation $\alpha u + \beta v = 0$ ($u_x + v_y = 0$) is satisfied exactly here.

In this part, the situation is similar to that observed in the convection of an incompressible fluid in the Boussinesq approximation. The obtained solutions determine the convective mode, since they are the ones that, in the presence of heating from below and gravity, lead to the development of convective motion in an incompressible fluid in the Boussinesq approximation. Note that in a compressible medium, the calculated neutral curve also corresponds to the convective regime [2] [3] [5].

However, the second group of solutions (roots $\lambda_{3,4}$) corresponds to motions of a more complex structure.

To avoid cumbersome calculations, we will limit ourselves to a numerical example, calculating the solution with the choice of specific values of the parameters $\alpha = 3$, $\beta = \pi$, $H = 0.5$ and $\gamma = 1.4$.

Carrying out obvious simplifications associated with discarding small terms, we obtain:

$$\lambda = i \cdot 4.16284,\ T_0 = 0.4317,\ \ \rho_0 = 1.511,\ \ u_0 = 1,\ \ v_0 = 1.0472.$$

The solution written out defines a rapidly oscillating motion of a compressed gas, since the continuity equation $\alpha u + \beta v = 0$ ($u_x + v_y = 0$) is not satisfied here. This solution defines a thermoacoustic mode, since it corresponds to thermoacoustic waves, which are analogs of pressure waves.

The oscillation frequency of the thermoacoustic mode is determined by the imaginary part of the roots $\lambda_{3,4}$ and is determined only by the Poisson adiabatic index γ and the wave number $k = S^{0.5} = (\alpha^2 + \beta^2)^{0.5}$. Neglecting the relatively weak attenuation, we have:

$$\lambda_{3,4} = \frac{i\,k}{\gamma}\sqrt{2\gamma - 1}, \quad k = \sqrt{\alpha^2 + \beta^2}.$$

Methodological considerations have shown that the dependence on other parameters, such as the altitude of the region, the presence or absence of gravity or heating, is very week here.

It can be shown that the propagation speed of the thermoacoustic wave is equal to the adiabatic speed of sound. Or, in other words, the characteristic time scale of the thermoacoustic mode is equal to 1.

3.2. Linear Analysis with Gravity

Now, let us consider the development of linear disturbances taking into account the gravity force $C_F > 0$. Similar to the procedure described above, a system of equations for the amplitudes is derived from system (3), the resulting system is rewritten in matrix form, and from the equality of the system determinant to zero, we obtain an equation for the increment λ. The resulting fourth-order algebraic equation for λ is solved numerically.

Test calculations have shown that the solutions corresponding to the convective mode in the absence of heating are always damped, however, what is very interesting and unusual, the solutions corresponding to the thermoacoustic mode become increasing at a sufficiently large height of the region.

Figure 2 shows the real part of the increment of the solution corresponding to the thermoacoustic mode λ_{3r} at $H = 0.5$ m and $\beta = \pi$ as a function of α. It is evident that in the range of wave numbers $0 \le \alpha < 4.6$ the solution corresponding to the thermoacoustic mode increases in time. The fastest growth of the solution is observed at $\alpha = 0$.

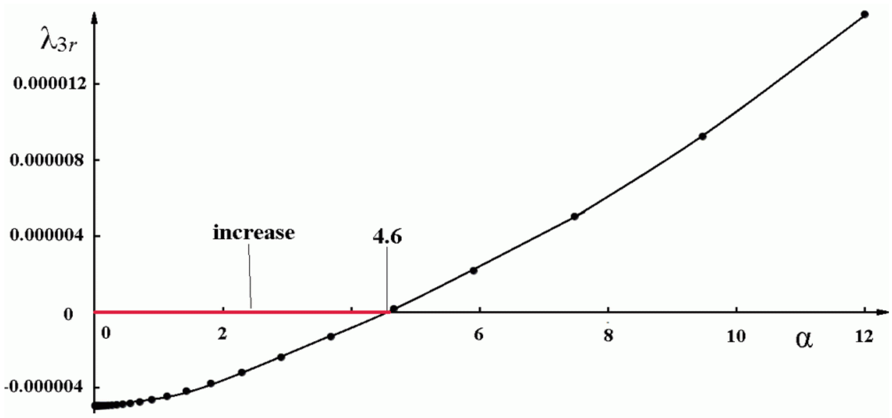

Figure 2. Real part of the thermoacoustic mode growth rate as a function of α.

Figure 3 shows the real part of the solution growth rate corresponding to the thermoacoustic mode λ_{3r} at $\alpha = 0$ and $\beta = \pi$ as a function of *H*. It can be seen that the thermoacoustic mode becomes increasing at a region height greater than 0.2134 m.

4. Results of Numerical Simulation

Now, let us consider the results of numerical modeling using the complete nonlinear system of Equation (1).

Figure 4 shows the velocity field at $H = 0.5$ m, the two-vortex flow structure is clearly visible. Gas particles descend along the vertical boundaries, where the density is highest, and rise in the center of the region, where the density of the medium is lowest.

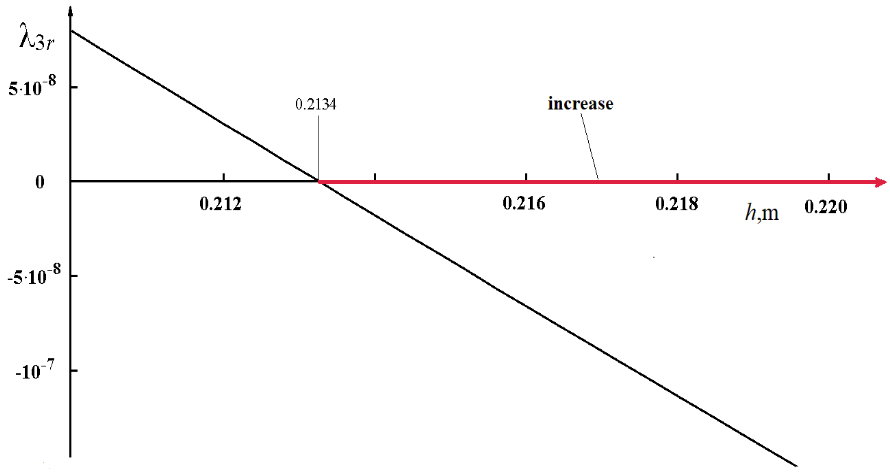

Figure 3. Real part of the thermoacoustic mode growth rate as a function of H at $\alpha = 0$.

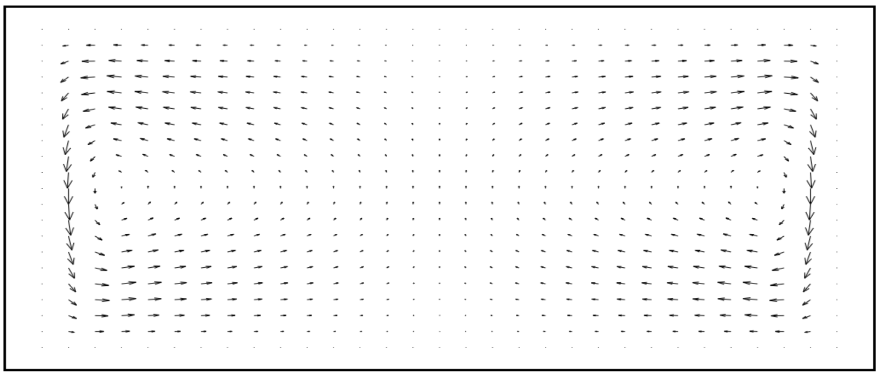

Figure 4. Velocity field.

A small velocity of motion (about 0.1 μm/s) corresponds to the Reynolds number shown in **Figure 5** as a function of the height of the region H. The signs in **Figure 5** show the results of numerical calculations. The break clearly visible in **Figure 5** at $H = 0.02$ m corresponds to a change in the flow regime from two-vortex to single-vortex. According to the linear analysis data (**Figure 3**), instability of the compressible gas layer should be observed at a layer height greater than 0.2134 m. However, the simulation results shown in **Figure 5**, performed using a complete nonlinear system of equations, clarify the conclusion of the linear theory and show that motion will be observed at any height of the compressible gas layer.

The asymptotic formula for the Reynolds number constructed from the data in **Figure 5**.

$$\mathrm{Re} = 0.001 \cdot e^{1.9 \cdot H}$$

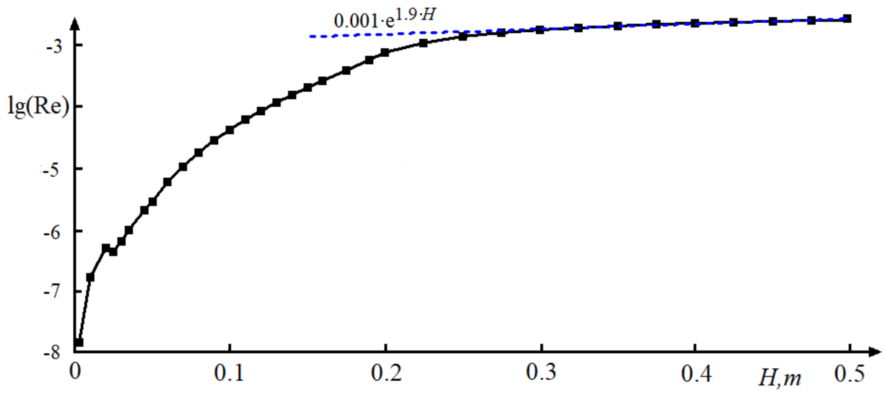

Figure 5. Reynolds number.

Of course, at best, can only give an ordinal estimate of its value. Nevertheless, this asymptotic formula shows that a layer height of 6 m corresponds to a Reynolds number equal to 100, which in the physically close problem of Rayleigh-Benard convection corresponds to a developed and complex flow regime [4].

Figure 6 shows the dimensional standard deviation of temperature as function of the height of the region *H*. It is also evident that movement is observed at any height of the region. At $H = 0.25$, a maximum is observed, its position approximately corresponds to the height of the gas layer when adiabatic processes begin to appear in convection [2].

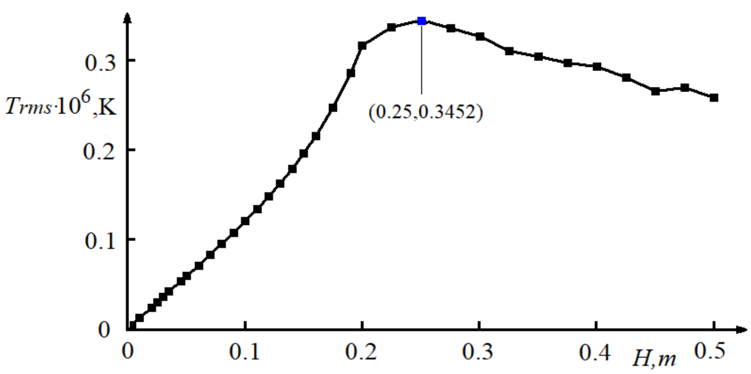

Figure 6. Dimensional root mean square temperature.

Figure 7 shows the dimensionless temperature profile at $H = 0.5$ m, obtained by averaging the temperature field along the horizontal coordinate *x*. The shape of the temperature profile reflects the fact that heating is observed inside the region, with a maximum temperature of about 0.26 μK.

Let us emphasize that inside the region the gas density decreases according to the ratio between the deviations of density and temperature from the static solution $\Delta\rho = -\Delta T$, which follows from the equation of state [2]. The density profile, taken with the minus sign, coincides with graphical accuracy with the temperature profile in **Figure 7**.

Figure 8 shows the dependence of the total kinetic energy *Ek*, calculated over the entire region, on time. This dependence is large-scale, and periodicity is not observed. Note that the source of motion is instability of the thermoacoustic mode. However, the characteristic time scale of the resulting large-scale motion is approximately five orders of magnitude larger than the characteristic scale of the thermoacoustic mode.

In **Figure 9**, the energy spectrum of the dependence of *Ek* on time is shown. It is evident that the large-scale simplicity of the dependence of the kinetic energy

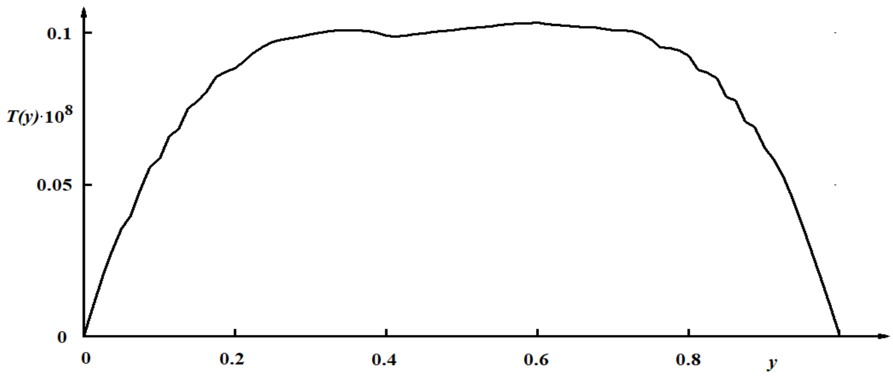

Figure 7. Temperature profile.

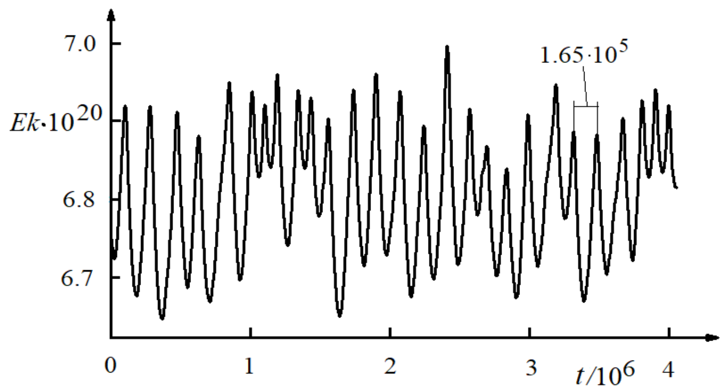

Figure 8. Kinetic energy as a function of time.

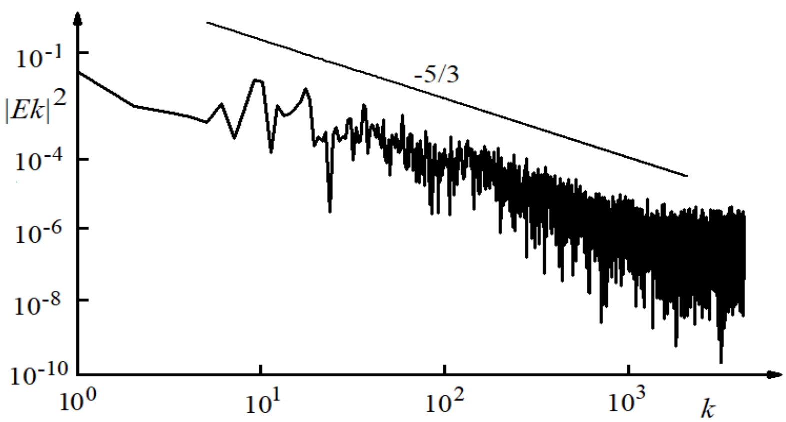

Figure 9. Kinetic energy spectrum.

on time is deceptive, since the given spectrum is complex and similar to turbulent. The power law -5/3 shown in **Figure 9** was observed in all calculations at $H > 0.2$ m.

Figure 10 shows the projection of the solution at $H = 0.5$ m onto the plane of the Reynolds number Re and the standard deviation of the temperature *Trms*. The blurriness of the trajectory, characteristic of stochastic processes, is clearly visible.

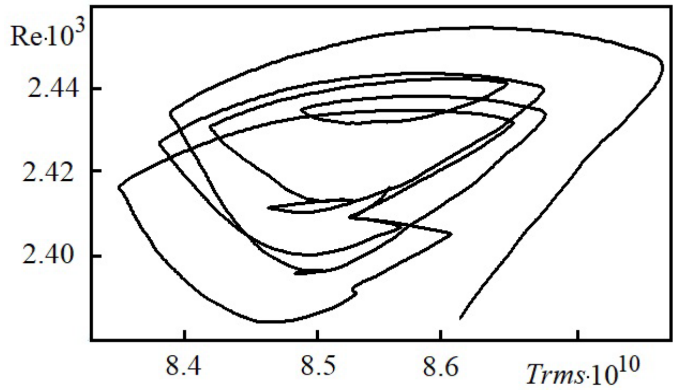

Figure 10. Projection of the solution onto a plane Re - *Trms*.

5. Discussion

As linear analysis shows, the equilibrium state of a compressible gas layer in a gravity field (thermoacoustic mode) is unstable at a layer height greater than 0.2134 m. However, calculations performed using a complete nonlinear system of equations show that non-stationary regimes are observed at any non-zero layer height.

Apparently, this difference is due to the limitations of linear analysis. As has been shown earlier, the instability of the static equilibrium regime in a compressible gas layer (convective mode) develops in the linear approximation at a region height greater than 0.268 m with the emergence of convective motion [12]. However, a final answer to the question about the stability boundary of the static solution requires additional research.

The analysis of the linear stability of the static solution shows that the birth of a large-scale flow is associated with the instability of a rapidly oscillating thermoacoustic mode. However, the characteristic scale of the forming large-scale flow is at least five orders of magnitude greater than the characteristic time scale of the thermoacoustic mode. The dependence of the kinetic energy on time is also large-scale, but this simplicity is deceptive. The time spectrum of the kinetic energy is complex and resembles a turbulent one; the trajectory of the solution exhibits a blurring characteristic of stochastic processes. At a sufficiently large altitude of the region, the energy spectrum of the time dependence of the kinetic energy corresponds to a power law of -5/3 with greater or lesser accuracy, the cause of which, as well as the energy (cascade) processes of the formation of a large-scale flow from a small-scale (in time) one, require additional research.

Large-scale flow cannot exist without the energy supply of thermoacoustic waves, where instability originates, therefore large-scale flow cannot be realized without the existence of a cascade (red) process of energy transfer [4]. Moreover, it is clear that due to small values of the Reynolds number, such a cascade process can only be realized in time. It is clear that this interesting situation requires further consideration.

The intensity of large-scale motion increases as expected with increasing layer height. However, the mean square temperature reaches a maximum and then decreases with increasing layer height. This decrease may be due to the development of adiabatic processes [2]. Of course, the final answer to this question requires additional research.

In order to partially reduce the influence of small solution amplitudes on the accuracy of the obtained solution, it seems important in the long term to perform a similar series of calculations for the full nonlinear system of equations, but written in deviations from the static solution. It is possible that such an approach will be more effective in calculations with a small height of the region, where extremely small amplitudes of deviations from the static solution are observed.

It is also important to note the wide variety of forms of the obtained numerical solutions and their strong dependence on the details of the calculation organization. The structure of the obtained solutions can depend on the notation of the system of initial equations and even on the accuracy of the defining parameters. In all our calculations, the values of the defining parameters M, C_F, γ, and Pr were specified with an accuracy of four to five significant digits. However, as already noted, the results of test calculations demonstrate the dependence of the obtained solution on the accuracy of the defining parameters. On the other hand, even the accuracy of the defining parameters adopted in this study is completely unrealistic for practical problems. Therefore, the observed diversity of the obtained solutions requires further study; therefore, at this stage, the numerical modeling results are primarily illustrative and can be further refined.

6. Conclusions

In this paper, we study the stability of the equilibrium of a compressible gas layer in a gravity field. To simplify the numerical modeling, the gas flow is considered in a limited region. All boundaries of the region are assumed to be rigid and isothermal.

The constancy of the temperature at all boundaries and the absence of unstable density stratification determine the absence of solutions corresponding to the convective regime. However, the results of the linear analysis show that for a region height of more than 0.2134 m, the solutions corresponding to the rapidly oscillating thermoacoustic mode become growing.

The development of instability of the thermoacoustic mode leads to the formation of a large-scale flow with a characteristic time scale that is five orders of magnitude greater than the original thermoacoustic one.

A study of the dependence of the kinetic energy of the flow on time shows that the large-scale simplicity of the forming flow is deceptive. In fact, the kinetic energy spectrum has a complex character and resembles a turbulent one. The trajectory of the solution is blurred, which is typical for stochastic processes. With increasing height of the gas layer, the intensity of motion increases, while the temperature of the layer reaches a maximum at a height of 0.25 m and then falls.

The results obtained show that, contrary to the generally accepted idea that the compressibility of a gas medium manifests itself only when moving in it at speeds of the order of sound or at a large height of the gas layer in a gravity field [7], the compressibility of a gas medium can play a significant role even when considering disturbances of an equilibrium static solution of infinitely small amplitude in layers of compressible gas of any height.

Let us present considerations explaining the development of instability of a compressible gas layer in a gravity field.

The highest pressure (hydrostatic) in a gas layer is always observed at the lower horizontal boundary. Gas particles are compressed there and, accordingly, have the greatest potential energy.

However, the state of the system with the greatest potential energy is unstable, so the particle rises upward with a partial transition of potential energy into the kinetic energy of internal waves and into the internal energy of the gas with an increase in temperature.

It should be noted that there are technical devices that use physical principles close to those described in this work [17]. We are talking about the perpetual clock of J. Cox, which showed the time from 1774 to 1961 and was never wound manually. The role of the engine in it was performed by a mercury barometer, in which mercury, under the action of atmospheric pressure, rose from a glass vessel at the bottom of the clock along a glass tube. At the same time, a spring located inside the clock was compressed, storing energy for the operation of the clock. The driving force here is the daily pressure difference. In this paper, the driving force is the dependence of hydrostatic pressure on altitude, which, in the presence of compressibility of the medium, leads to the development of instability.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Gershuni, G.Z. and Zhukhovitskii, E.M. (1976) Convective Stability of Incompressible Fluid. Israel Program for Scientific Translations, Jerusalem.

[2] Palymskiy, I.B. (2024) On the Features of Thermal Convection in a Compressible Gas. *Fluid Dynamics & Materials Processing*, **20**, 957-974. https://doi.org/10.32604/fdmp.2024.048829

[3] Palymskiy, I.B. and Palymskiy, V.I. (2021) Convection of Compressible Gas. *Proceedings of Analytical and Numerical Methods in Differential Equations Conference*, Nakhon Ratchasima, 23-26 August 2021, 1-8.

[4] Palymskiy, I.B. (2011) Turbulent Rayleigh-Benard Convection. Numerical Method and Calculation Results. LAP. (In Russian)

[5] Palymskiy, I.B. (2022) On the Features of Convection in a Compressible Gas. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, **11**, 29-35.

[6] Lapin, Y.V. and Strelets, M.H. (1989) Internal Gas Mixtures Flows. Science, Moskow. (in Russian)

[7] Landau, L.D. and Lifshitz, E.M. (2013) Course of Theoretical Physics: Vol. 6 Fluid Mechanics. Elsevier Science & Technology Books.

[8] Tritton, D.J. (2012) Physical Fluid Dynamics. Springer.

[9] Smits, A.J. and Dussauge, J.P. (2006) Turbulent Shear Layers in Supersonic Flow.2nd Edition, Springer.

[10] Verma, M.K. (2018). Physics of Buoyant Flows: From Instabilities to Turbulence. World Scientific. https://doi.org/10.1142/10928

[11] Gorbunov, A.A. and Polezhaev, V.I. (2008) Perturbation Method and Numerical Modeling of Convection for the Rayleigh Problem in Liquids with an Arbitrary Equation of State, preprint № 897. Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences (IPMech RAS). (In Russian)

[12] Palymskiy, I.B. and Fomin, P.A. (2024) Modeling of Atmospheric Geophysical Processes of Explosive Nature. SGUGIT. (In Russian)

[13] Trotsyuk, A.V. and Fomin, P.A. (2019) Reduced Model of Chemical Kinetic and Two-Dimensional Structure of Detonation Wave in Rich Mixtures of Methane with Oxidizer. *Journal of Physics*: *Conference Series*, **1261**, Article ID: 012037. https://doi.org/10.1088/1742-6596/1261/1/012037

[14] Yu, J., Hu, Y., Wu, C., Li, Y. and Palymskiy, I.B. (2020) Direct Numerical Simulations of Rayleigh-Bénard Convection of a Gas-Liquid Medium near Its Density Maximum. *Applied Thermal Engineering*, **175**, Article ID: 115387. https://doi.org/10.1016/j.applthermaleng.2020.115387

[15] Fomin, P.A., Palymskiy, I.B., Gharehdash, S. and Ayrapetyan, V.S. (2021) Explosive and Detonation Processes in Chemically Active Bubble Media. SGUGIT. (In Russian)

[16] Polezhaev, V.I. (1967) Numerical Solution of the Navier-Stokes Equations for Flow and Heat Transfer in a Closed Two-dimensional Domain. Ph.D. Thesis, Research Institute of Thermal Processes. (In Russian)

[17] Ord-Hume, A.W.J.G. (1977) Perpetual Motion: The History of an Obsession. George Allen & Unwin LTD.