

Spatial, Temporal and Coherent Superposition of Quantum States, a Reinterpretation of the Quantum Superposition Principle

Ivan Georgiev Koprinkov

Department of Applied Physics, Technical University of Sofia, Sofia, Bulgaria
Email: igk@tu-sofia.bg

How to cite this paper: Koprinkov, I.G. (2024) Spatial, Temporal and Coherent Superposition of Quantum States, a Reinterpretation of the Quantum Superposition Principle. *Journal of Applied Mathematics and Physics*, 12, 4010-4019.
<https://doi.org/10.4236/jamp.2024.1211244>

Received: October 16, 2024

Accepted: November 26, 2024

Published: November 29, 2024

Abstract

Spatial, temporal and coherent superposition of quantum states is considered. A consistent interpretation of the simultaneous superposition of stationary quantum states within material wave packets is proposed.

Keywords

Quantum Superposition, Dressed Quantum States, Quantum Coherence, Quantum Wave Packets

1. Introduction

The principle of superposition is one of the fundamental principles in physics. It takes place in various fields of physics for physical system having linear response to external perturbations. This includes the important class of wave phenomena due to the linearity of the basic wave equation. The principle of superposition, in particular, the quantum superposition principle, holds for the quantum phenomena. According to the quantum superposition principle, if a number of quantum states $|\psi(\vec{r}, t)\rangle_i$ are states of a given quantum system, any linear combination of these states is also a state $|\psi(\vec{r}, t)\rangle$ of the same quantum system:

$$|\psi(\vec{r}, t)\rangle = \sum_i c_i(t) |\psi(\vec{r}, t)\rangle_i \quad (1)$$

Equation (1) represents the conventional superposition principle of quantum states, which is correct from a mathematical point of view due to the linearity of the Schrödinger equation.

It is widely accepted that the superposition of quantum states in Equation (1) is simultaneous, coherent and holds for any kind of states of a given quantum

system. It means that the quantum states are superposed simultaneously, but not consequently. The characteristic time of the electron motion inside atoms falls in the attosecond time range, $1\text{as} = 10^{-18}\text{s}$. The current ultrafast laser technology can generate attosecond pulses, but no experimental test on the simultaneity of superposition of quantum states has been done so far. The coherence is related to the ability of the quantum system to interfere. In order to interfere, the quantum system must be simultaneously in the superposed quantum states. Hence, the simultaneity is a key point in the superposition of quantum states. More precisely speaking, the coherence means that the superposed states must have a definite and stable in time phase relations, which will result in a stable interference pattern. It is also considered that any kind of states can be superposed simultaneously, including the most widely used stationary eigenstates of the quantum systems. Finally, for completeness, one more aspect to the quantum superposition must be added, requiring, as in the superposition of the macroscopic waves, not only temporal but also spatial overlapping of the quantum states. The importance of the spatial, temporal and coherence superposition of the quantum states is confirmed by the experiments on the interference of intraatomic [1] and intramolecular wave packets [2] [3]. The superposition of quantum states is an inherent feature of the quantum phenomena and its correct understanding has crucial importance for the quantum theory and its application in the fields of quantum computing, quantum communications, quantum information science, etc.

In this work we show that the above mentioned features of the quantum superposition, *i.e.*, the simultaneity and the coherence of the real physical superposition of quantum states is not automatically guaranteed according to Equation (1). More particularly, it does not take place if the superposed states are eigenstates of adiabatic or stationary Hamiltonians. We have found quantum states which can be superposed simultaneously and coherently. The present results are based on the adiabatic theorem of quantum mechanics [4], phase-sensitive nonadiabatic dressed states [5], and experimental studies on the population dynamics of real and virtual quantum states [6]-[8]. A reinterpretation of the quantum superposition principle is proposed in order to understand the superposition of quantum states in intraatomic and intramolecular wave packets.

2. Spatial Superposition of Quantum States

The spatial overlapping is required for the interference of any kind of waves, including the quantum states. Here we will focus our attention on the internal states of atomic type quantum systems (atoms, ions, molecules). While the subatomic constituents of such quantum systems (electrons, nuclei) are highly localized objects, their internal stationary eigenstates, *e.g.*, the electronic bound states, are nonlocal and become distributed in a definite way within the space occupied by the quantum systems. This, in principle, strongly simplifies the problem of spatial superposition because there is a good natural space overlapping of the stationary states within such quantum systems. More attention must be paid to the interference

of intraatomic [1] and intramolecular wave packets [2] [3], which are much stronger confined in space than the stationary eigenstates.

3. Temporal Superposition of Quantum States

The temporal overlapping of the quantum states is the main problem of the quantum superposition because the real time tracing of, e.g., the electron motion, is still a hard and challenging experimental task. This is why, our approach to this problem will be based on general physical principles and theorems, suitable quantum states and experimental studies, which allow making decisive conclusions about the temporal dynamic of the different quantum states.

3.1. Adiabatic Theorem of Quantum Mechanics

The adiabatic theorem of quantum mechanics [4] states that a quantum system remains in an instantaneous eigenstate $\psi(t)$ of its time dependent Hamiltonian $\hat{H}(t)$, i.e., $\hat{H}(t)\psi(t) = E(t)\psi(t)$, if the Hamiltonian $\hat{H}(t)$ changes slow enough, i.e., adiabatically, due to a given perturbation, and if there is an energy gap between the energy of this state and the rest part of the Hamiltonian spectrum. At the end of the perturbation, the quantum system will be in the same quantum state, from which the adiabatic evolution begins, and no transition to other state will occur. This, in fact, means that the quantum system cannot be simultaneously in more than one eigenstate of the adiabatic Hamiltonian. The adiabatic theorem, with even higher power, can also be applied to the particular case of stationary eigenstates of a closed quantum system, e.g., the bare states (BS) ψ_0 of the unperturbed Hamiltonian \hat{H}_0 , i.e., $\hat{H}_0\psi_0 = E_0\psi_0$, because the Hamiltonian $\hat{H}_0(t)$ is not simply adiabatic but it is even stationary, $\hat{H}_0(t) = const$. The adiabatic theorem in this perfect adiabatic case claims that if a quantum system is in given BS, it will remain in that state and no transition to other BS will occur. Consequently, *a quantum system cannot be simultaneously in more than one eigenstate of adiabatic or stationary Hamiltonian.*

Extrapolating further the adiabatic theorem, the physical reasons for quantum transitions are nonadiabatic factors acting on the quantum system. *The nonadiabatic factors* are rapid (nonadiabatic) variations of electromagnetic field, collisions with other particles (atoms, molecules, etc.), zero point vacuum fluctuations, etc., which, in principle, have a stochastic character and *does not, in general, preserve the coherent properties of the quantum states.*

3.2. Phase-Sensitive Nonadiabatic Dressed States

The two-level counterpart of the quantum superposition, Equation (1), for the case of stationary BS, e.g., ground $|g\rangle$ and excited $|e\rangle$ BS, of a quantum system, **Figure 1**, is:

$$|\psi(\vec{r}, t)\rangle = c_g(t)|g\rangle + c_e(t)|e\rangle \quad (1a)$$

In agreement with the properties specified above, the superposition of the most

widely used quantum states, *i.e.*, the stationary BS, Equation (1) or Equation (1a), is neither simultaneous nor coherent. In order to reconcile the quantum superposition principle and the adiabatic theorem, we have to find quantum states that can be superposed simultaneously and coherently. We will show that such features possess (but, probably, not only these states) the phase-sensitive nonadiabatic dressed states (PSNADS) [5] [9]¹. The simultaneity has been already proved [10] for the case of nonadiabatic dressed states (NADS) [11] but as they do not take into account all phase contributions, the PSNADS will be used in the present considerations. In addition, a mutual relation between the quantum superposition (creation of superposition of quantum states) and the collapse of wave function (destruction of superposition of quantum states) as two opposite directions of same physical process has been formulated for the first time in [10].

The PSNADS are derived from an analytic solution of the Schrödinger equation for an open two-level quantum system subject to electric-dipole interaction with a near-resonant electromagnetic field and the environment. The relevant physical factors are taken into account by the Hamiltonian of the quantum system [5]. The nonadiabatic factors from the field (field time derivatives) and the environment (damping) as well as all phase contributions from the field and the matter, including the initial phases φ_g and φ_e of the ground $|g\rangle$ and excited $|e\rangle$ BS, respectively, participate explicitly in the PSNADS. Each PSNADS, ground $|\tilde{G}\rangle$ and excited $|\tilde{E}\rangle$, Equations (2), represents a linear superposition of a real (index “r”) and a virtual (index “v”) components (states), **Figure 1**:

The real and virtual components of PSNADS (at ground states initial conditions) are:

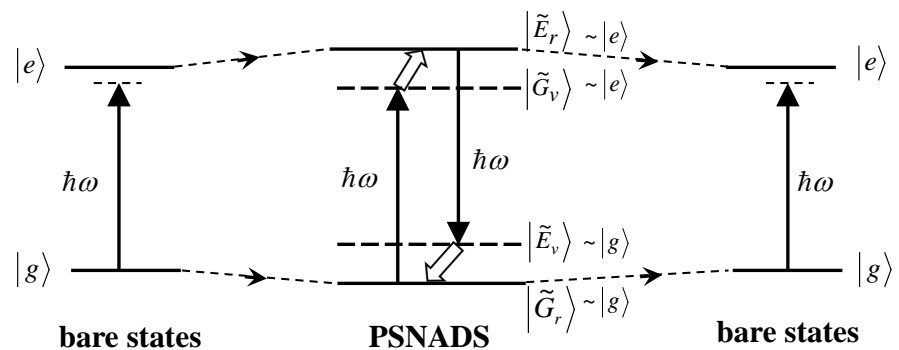


Figure 1. Evolution of the initial BS toward PSNADS and back to the BS with switching on/off the electromagnetic field. The full lines show the real components, the broken lines show the virtual components. The full arrows show radiative transitions, the hollow arrows show nonadiabatic transitions.

$$|\tilde{G}\rangle = \cos(\theta/2)|\tilde{G}_r\rangle + \sin(\theta/2)|\tilde{G}_v\rangle \quad (2a)$$

¹The PSNADS were initially called “*adiabatic*” because a generalized adiabatic condition is used in the derivation of these states but as the nonadiabatic factors from the field and the environment present in the PSNADS, they have been latter called “*nonadiabatic*”, see, e.g., [9].

$$|\tilde{E}\rangle = \text{COS}(\theta/2)|\tilde{E}_r\rangle - \text{SIN}(\theta/2)|\tilde{E}_v\rangle \quad (2b)$$

The real and the virtual components of the PSNADS, e.g., at ground state initial conditions, are given by Equations (3):

$$|\tilde{G}_r\rangle = |g\rangle \exp\left\{-i\left[\varphi_g + \int_0^t \tilde{\omega}'_G dt'\right]\right\} \quad (3a)$$

$$|\tilde{G}_v\rangle = |e\rangle \exp\left\{-i\left[\varphi_g + \varphi(t) + \int_0^t (\tilde{\omega}'_G + \omega) dt'\right]\right\} \quad (3b)$$

$$|\tilde{E}_r\rangle = |e\rangle \exp\left\{-i\left[\varphi_g + \varphi(t) + \int_0^t \tilde{\omega}'_E dt'\right]\right\} \quad (3c)$$

$$|\tilde{E}_v\rangle = |g\rangle \exp\left\{-i\left[\varphi_g + \int_0^t (\tilde{\omega}'_E - \omega) dt'\right]\right\} \quad (3d)$$

The superposition coefficients $\text{COS}(\theta/2)$ and $\text{SIN}(\theta/2)$ determine the “intensiveness” of the real and virtual components, respectively, in a given PSNADS and play the key role for the simultaneity of the superposition. At zero field, $E_0(t) = 0$, the partial representation of the virtual components in the superposition given by Equations (2), is zero, $\text{SIN}(\theta/2) = 0$, and the PSNADS consist of a real component only—the initial BS from which it originates. Increasing the field magnitude, the partial representation of the virtual component increases while this of the real components decreases and at extremely strong fields, $E_0(t) \rightarrow \infty$, they become equal. Thus, the virtual components appear with switching the field on and disappear with switching the field off. Consequently, *the superposition of a real and a virtual component in a given PSNADS, Equations (2), is simultaneous within the overlap time between the lifetime of the real component and the time of action of the field that creates the virtual components.*

3.3. Experimental Studies on the Population of Quantum States

There are some key experiments [6]-[8], which, while having only nanosecond time resolution, allow distinguishing some general features of the population dynamic of real and virtual states based on spectral resolution and difference in the time behavior of the states. To get a closer relation to such a work [6], we will apply our notations and terms to the energy levels involved in this study, **Figure 2**. Atoms are excited adiabatically by a pump laser pulse at frequency ω from the ground state real component $|\tilde{G}_r\rangle$ to the ground state virtual component $|\tilde{G}_v\rangle$ of the ground PSNADS $|\tilde{G}\rangle$. Two other laser fields at frequency ω_{p1} and ω_{p2} probe the population of the virtual $|\tilde{G}_v\rangle$ and the real $|\tilde{E}_r\rangle$ components, respectively, by means of absorption to a high-lying state $|n\rangle$.

The results from these experiments can be summarized in the following. Real population on the virtual state $|\tilde{G}_v\rangle$ (an order of magnitude larger than that of the real state $|\tilde{E}_r\rangle$, in this case) is detected experimentally, which means that the virtual components of the PSNADS are real physical states but not simply a mathematical construct. The population of the virtual state $|\tilde{G}_v\rangle$ is proportional to the intensity of the laser pulse following “coherently” (for the strict phase

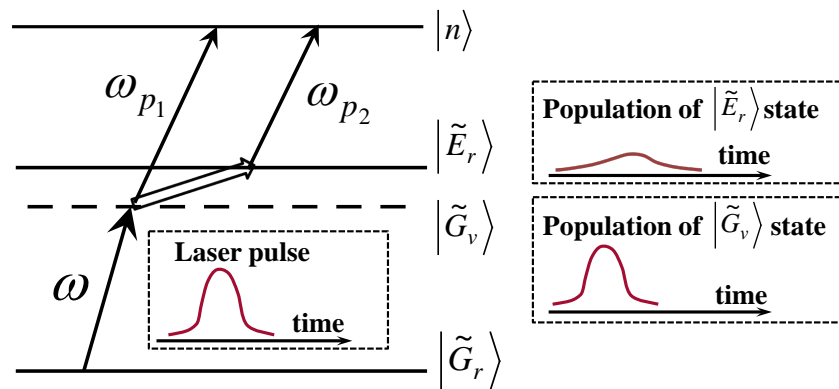


Figure 2. Energy level diagram for experimental measurement of population of real and virtual components of dressed quantum states [6]. The insets (broken line rectangles) show the time shape of the laser pulse, which creates the virtual component $|\tilde{G}_v\rangle$, and the corresponding population of the real $|\tilde{E}_r\rangle$ and virtual $|\tilde{G}_v\rangle$ components. The decreasing of population of the real component, once the pulse terminates, results from the decay of the states.

coherence, see below) its time shape. Although the laser pulse in this experiment is adiabatic, population of the real state $|\tilde{E}_r\rangle$ is observed due to nonadiabatic factors from the environment (collisions with other atoms), which transfer incoherently population from $|\tilde{G}_v\rangle$ to $|\tilde{E}_r\rangle$. The population on the real state $|\tilde{E}_r\rangle$ is proportional to the time integral from the intensity of the laser pulse because the population in $|\tilde{E}_r\rangle$ is captured and accumulated there within the lifetime of the state. Capturing the population in the real state $|\tilde{E}_r\rangle$ brakes the evolution of the quantum system within the ground PSNADS $|\tilde{G}\rangle$ (the central left arrow in **Figure 1**) and initiates similar evolution within the excited PSNADS $|\tilde{E}\rangle$ (the central right arrow in **Figure 1**). Thus, the evolution of the quantum system within ground PSNADS $|\tilde{G}\rangle$ and excited PSNADS $|\tilde{E}\rangle$ is decoupled in time and cannot develop simultaneously in both states. The same takes place for the ground and excited BS, $|g\rangle$ and $|e\rangle$, because each of these BS form the real component of ground and excited PSNADS, $|\tilde{G}\rangle$ and $|\tilde{E}\rangle$, respectively, **Figure 1**. Any stationary BS, being a (quasi)stable state, may capture the quantum system and does not allow a simultaneous and coherent superposition with other stationary BS because the population of the latter takes place by means of well separated in time incoherent nonadiabatic process.

Above theoretical and experimental results are in a mutual agreement and show that the quantum system cannot be simultaneously in two (or more) different BS or PSNADS. Simultaneous evolution takes place between the real and virtual components within given PSNADS. On the other hand, the quantum superposition expressed by Equation (1) or Equation (1a) is a perfectly legitimate mathematical operation. Thus, it turns out that *the superposition of quantum states is a well-defined physical process having a well-definite physical mechanism rather than a pure mathematical formalism.*

4. Coherent Superposition of Quantum States

The coherence means a well-defined phase relation between the states of particular physical objects (different kind of waves, fields, etc.), which thus are capable to interfere. The same takes place for the quantum states. The direct application of such an approach to the quantum states, however, contradicts to the most widely accepted interpretation of quantum mechanics, the Copenhagen interpretation, according to which only the amplitude, but not the phase, of wave function has physical meaning. The present approach, however, can be well understood within dynamics-statistical interpretation of quantum mechanics, which recognizes the physical meaning not only of the amplitude, but also of the phase of wave function, based on a number of theoretical and experimental evidences for the causal relation of the phase of wave function with the physical reality [12]. In agreement with such a concept, quantum states which account for all phase contributions as, *e.g.*, the PSNADS, must be used.

Closed form solution for the PSNADS is known for a two-level quantum system. To better understand the quantum coherence, the PSNADS will be extrapolated toward the multilevel case [13]. The multilevel PSNADS consists of a single real component (the initial BS from which its formation begins) and a number of virtual components arising from the contribution of each electric-dipole allowed state (from the initial real state) of the energy spectrum. The structure of, *e.g.*, ground multilevel PSNADS and the phases of the respective components are shown in **Figure 3**. All ground state virtual components $|\tilde{G}_{vi}\rangle$, $i = 1, 2, 3, \dots$, have same energy (Bohr frequency) $\tilde{\omega}'_G + \omega$ because the energy of each virtual component $|\tilde{G}_{vi}\rangle$ is equal to the energy (Bohr frequency) $\tilde{\omega}'_G$ of the ground state real component $|\tilde{G}_r\rangle$ plus one photon energy (frequency) ω . The same Bohr frequency of all virtual components means same time rate of phase acquisition of these components. As the phase is a time integral of Bohr frequency, this means that the total phases $\Phi_{Gv,i}$ of the different virtual components $|\tilde{G}_{vi}\rangle$ may differ only by a constant phase shift $\varphi_{Gv,i}$, **Figure 3**. Taking into account the general understanding for coherence of oscillating fields, *i.e.*, same frequency and

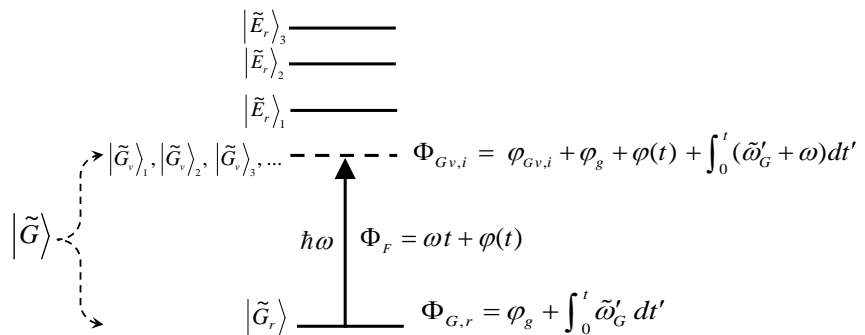


Figure 3. The energy structure and phases of multilevel PSNADS due to the contribution of a number of excited states. The real/virtual states are shown with full/broken lines, respectively.

constant phase difference, one may conclude that *all virtual components* $|\tilde{G}_{vi}\rangle$ *are mutually coherent*.

5. Reinterpretation of the Quantum Superposition Principle

To formalize the requirements for physical simultaneity and coherence of the superposition of quantum states, the quantum superposition principle has been reformulated accordingly [13]. For a reason of general validity, the quantum superposition principle must encompass all possible scenarios. In this relation, a careful consideration of experiments with material wave packets [1]-[3] is required because the wave packets exist even after termination of action of the field when the virtual states do not exist and the wave packets are considered as a simultaneous superposition of stationary eigenstates. This is illustrated in **Figure 4** by a wave packet within a molecule. According to the ordinary understanding, the laser pulse excites simultaneously a number of stationary vibrational eigenstates $|\psi(\vec{r}, t)\rangle_i$ of energies $E_1, E_2, E_3 \dots$, respectively, which fall within the spectral bandwidth $\Delta\omega$ of the laser pulse and it forms in this way a vibrational wave packet $|\psi(\vec{r}, t)\rangle$. Below, we propose an interpretation that is not based on a simultaneous superposition of stationary states. The laser pulse creates a multi-level PSNADS, whose virtual states fall within the laser bandwidth $\Delta\omega$ and form a wave packet of virtual states excited around the stationary vibrational states. The virtual wave packet appears and disappears following the time dependence of the laser pulse. If the quantum system does not absorb irreversibly a photon from the field, it will be in the ground electronic state after the laser pulse terminates. If the quantum system absorbs irreversibly a photon from the field, it will be in the excited electronic state after the laser pulse terminates. As the wave packet of virtual state is localized due to the time localization of the laser pulse, it creates similarly localized quantum state, a wave packet $|\psi(\vec{r}, t)\rangle$, on the excited electronic state. This wave packet is not an eigenstate of the unperturbed Hamiltonian of the quantum system and it will evolve and change during the propagation over the excited electronic state, **Figure 4**, until after amplitude and phase nonadiabatic perturbations (collision with other atoms, zero-point vacuum fluctuations, etc.), it will be finally captured in some of the stationary vibrational states of the excited electronic state. Although, mathematically, the wave packet

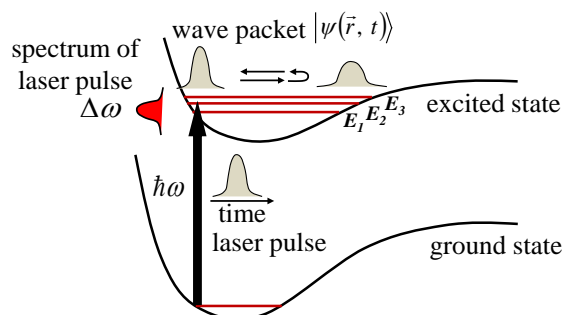


Figure 4. Creation and evolution of wave packet within a molecule.

$|\psi(\vec{r}, t)\rangle$ can always be expressed as a superposition of stationary vibrational eigenstates $|\psi(\vec{r}, t)\rangle_i$, Equation (1), because the eigenstates of a Hermitian Hamiltonian form a basis, *physically, the quantum system is always in a single quantum state $|\psi(\vec{r}, t)\rangle$ but not in all stationary vibrational eigenstates $|\psi(\vec{r}, t)\rangle_i$ simultaneously.* This interpretation of the quantum superposition principle is in agreement with the adiabatic theorem of quantum mechanics and it can be extended beyond the case under consideration.

6. Conclusion

Spatial, temporal and coherent superposition of real and virtual components within given multilevel PSNADS created under a forcing coherent physical factor, e.g., electromagnetic field, is established. A consistent interpretation of the quantum superposition principle with the adiabatic theorem of quantum mechanics is proposed.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Noel, M.W. and Stroud Jr., C.R. (1995) Young's Double-Slit Interferometry within an Atom. *Physical Review Letters*, **75**, 1252-1255. <https://doi.org/10.1103/physrevlett.75.1252>
- [2] Scherer, N.F., Ruggiero, A.J., Du, M. and Fleming, G.R. (1990) Time Resolved Dynamics of Isolated Molecular Systems Studied with Phase-Locked Femtosecond Pulse Pairs. *The Journal of Chemical Physics*, **93**, 856-857. <https://doi.org/10.1063/1.459456>
- [3] Gerdy, J.J., Dantus, M., Bowman, R.M. and Zewail, A.H. (1990) Femtosecond Selective Control of Wave Packet Population. *Chemical Physics Letters*, **171**, 1-4. [https://doi.org/10.1016/0009-2614\(90\)80039-g](https://doi.org/10.1016/0009-2614(90)80039-g)
- [4] Born, M. and Fock, V. (1928) Proof of the Adiabatic Theorem. *Zeitschrift für Physik*, **51**, 165-180. <https://doi.org/10.1007/bf01343193>
- [5] Koprinkov, I.G. (2000) Phase Sensitive Adiabatic States, or Does the Material Phase Recognize the Physical Reality? *Physics Letters A*, **272**, 10-19. [https://doi.org/10.1016/s0375-9601\(00\)00387-x](https://doi.org/10.1016/s0375-9601(00)00387-x)
- [6] Grischkowsky, D. (1976) Coherent Excitation, Incoherent Excitation, and Adiabatic States. *Physical Review A*, **14**, 802-812. <https://doi.org/10.1103/physreva.14.802>
- [7] Carlsten, J.L., Szöke, A. and Raymer, M.G. (1977) Collisional Redistribution and Saturation of Near-Resonance Scattered Light. *Physical Review A*, **15**, 1029-1045. <https://doi.org/10.1103/physreva.15.1029>
- [8] Courtens, E. and Szöke, A. (1977) Time and Spectral Resolution in Resonance Scattering and Resonance Fluorescence. *Physical Review A*, **15**, 1588-1603. <https://doi.org/10.1103/physreva.15.1588>
- [9] Koprinkov, I.G. (2010) Material Phase Causality or a Dynamics-Statistical Interpretation of Quantum Mechanics. *AIP Conference Proceedings*, **1301**, 573-586. <https://doi.org/10.1063/1.3526659>
- [10] Koprinkov, I.G. (2023). Quantum Superposition, Collapse of Wave Function,

-
- Quantum Measurement and Nonadiabatic Dressed States. *AIP Conference Proceedings*, **2939**, Article ID: 090001. <https://doi.org/10.1063/5.0179154>
- [11] Koprnikov, I.G. (2001) Non-Adiabatic Semiclassical Dressed States. *Journal of Physics B: Atomic, Molecular and Optical Physics*, **34**, 3679-3688. <https://doi.org/10.1088/0953-4075/34/18/306>
- [12] Koprnikov, I.G. (2016) Causality of Phase of Wave Function or Can Copenhagen Interpretation of Quantum Mechanics Be Considered Complete? *Journal of Modern Physics*, **7**, 390-394. <https://doi.org/10.4236/jmp.2016.74039>
- [13] Koprnikov, I.G. (2023) The Quantum Superposition Principle: A Reconsideration. arXiv: 2311.02391.