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On the Features of the Development of Convective Flows in Compressible Gas

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Abstract

The stability of the static mode of compressible gas convection is analyzed in the linear approximation with heating from below. The obtained data are compared with the results of solving the system of complete nonlinear equations describing convective flows of compressible gas. The features of the constructed neutral curve are discussed.

Keywords

Rayleigh-Benard Convection, Gas, Linear Analysis, Temperature Gradient, Adiabatic Temperature Gradient

1. Introduction

Rayleigh-Bénard convection is a classical problem on fluid and gas mechanics, which has mostly been considered in terms of the Boussinesq approach for an incompressible fluid [1] [2]. However, in the case of gas convection the situation is much more complicated due to gas compressibility. As a result, when studying gas convection, we can use Boussinesq approach only in laboratory studies when the layer height does not exceed ten centimeters (for air under normal conditions). In this case the small height of the layer causes a relatively little change in the hydrostatic pressure and coupled with it change of density; while calculations of convection in dozens and more centimeters high domains due to relatively big change of hydrostatic pressure and corresponding variation of density require taking into account gas compressibility described by full nonlinear equations of the gas dynamic theory [3]-[5].

It is necessary to consider convective flows in gas mixtures taking place in large domains when we study explosion safety issues while keeping hydrocarbons in storage reservoirs at gas stations. An explosive situation occurs when the tank is almost empty, but some small amount of hydrocarbon remains at the bottom of the tank. In the presence of convective flow, the fuel (vapors of vaporized hydrocarbon) mixes with the oxidizer (air), forming a potentially explosive gas-vapor medium. Such gas-vapor mixture is explosive if the volumetric concentration of fuel vapor is between the lower and upper concentration limits of ignition. In this case, any accidental initiation (for example, a spark when turning on a switch) can lead to an emergency, the scale of which is determined by the amount of explosive gas-steam mixture formed. Since the degree of explosiveness of the mixture is determined by the concentrations of the fuel and the oxidizer, and convective processes lead to mutual movement and mixing of the fuel and oxidizer masses with changes in their local concentrations in certain areas, an convecting explosion-proof mixture can become locally explosive and vice versa, an explosive gas mixture can become explosion-proof. The latter processes require a detailed study, and the mode, structure and intensity of convective motion are of decisive importance here, since they determine the mutual arrangement, mixing of fuel, oxidizer and their concentrations [6]-[8]. Thus, it is the convective flow in this case that forms a potentially explosive gas-vapor medium from vaporized residues of liquid fuel and gaseous oxidizer. Features of the development of detonation processes in such medium have been studied in numerous works [9]-[15].

The features of convection in a compressible medium have been discussed in a number of monographs and discussions [16]-[20]. It is traditionally considered that gas compressibility under convection at laboratory scales is insignificant, and it is essential only at large (planetary) scales. In this case, both scales (laboratory and planetary) are considered as asymptotic and their intersection is not taken into account.

In [18], the planetary atmosphere is treated as a compressible medium in which the flow is assumed to be adiabatic. It is shown that convective flow in the atmosphere is unstable if the Schwarzschild number Sc is less than 1 and stable otherwise. The Schwarzschild number is defined in the usual way, *i.e.*, as the ratio of the adiabatic temperature gradient to a given temperature gradient.

In this connection, we note that gas convection based on the full nonlinear equations of gas dynamics has been not adequately investigated [16], which is due to technical difficulties associated, firstly, with the large rigidity of the system of equations and with an extremely low relative pressure change [3] [16].

Such rigidity of the system of equations, which leads to the need to perform calculations with an unreasonably small time step, is due to the coexistence of two types of motion—thermoacoustic waves propagating at sonic speed (analogs of pressure waves) [21] and relatively slow convective motion developing against their background. The performed methodological calculations have shown that the limitations on the time step, which ensure the stability of calculations when using explicit time schemes, are precisely determined by the need to correctly reflect the development of "fast" thermoacoustic waves, while the accuracy of the calculation is determined by both "slow" and "fast" motions. In this connection, we note that the use of implicit schemes makes it possible to multiply the time step of the calculation [2], but due to the inevitable "smearing" of "fast" thermoacoustic waves, the use of such methods is in a certain sense equivalent to the use of numerical methods with a filtering algorithm.

Some compromise is achieved using of simplified models of gas convection. Derived from the equations of gas dynamics under the assumption of small Mach number and hydrostatic compressibility parameter, the corresponding systems of equations describe the medium in which sound perturbations propagate at an infinitely large velocity. Mathematically, the structure of the obtained systems is in a certain sense similar to the structure of the viscous incompressible fluid equations with additional terms to account for compressibility [22], and, as a consequence, such a system can be successfully used in calculations of convection in domains of low heights with large variations in temperature and density [22] [23]; however, calculations of convective flows in domains of bigger heights, where the compressibility of the medium begins to appear fully, require the use of full nonlinear gas dynamics equations.

Jeffrey H. expanded the scope of application of the Navier-Stokes equations to the case of compressible flows by taking into account the work of compression forces in the heat transfer equations [4] [5]. In this case, the critical Rayleigh numbers for an incompressible medium in the Boussinesq approximation Ra_b and for a compressible gas Ra are related by the relation:

$$Ra = Ra_b / (1 - Sc),$$

where Schwarzschild number Sc is the ratio of adiabatic gradient to the given. The proposed relation describes the calculated dependence of the critical Rayleigh number on the height of the layer in the case of convection of a compressible gas with qualitatively correct (with an error up to 30%) and predicts adiabatic suppression of convective motion at its large value. By adiabatic suppression of convection we mean the growth of the critical value of the Rayleigh number as the height of the layer increases.

In [18], when considering planetary-scale convective flows, it is proposed to replace a given temperature gradient by an effective temperature gradient equal to the given one but reduced by the adiabatic temperature gradient. This substitution leads to the Jeffrey's relation. The authors observed the stabilizing effect of the adiabatic gradient in atmospheric currents.

It is emphasized in [18] [24] that the Boussinesq approximation is applicable when several conditions are satisfied, of which the most restrictive is the smallness of the Schwarzschild number. This restriction is much more stringent than the restriction on the value of the product of the thermal expansion coefficient and the temperature difference commonly used in experiments [25]. It can be shown that the condition of smallness of the Schwarzschild number is equivalent to the requirement of constancy of the critical value of the Rayleigh number.

The compressibility is very significant in near-critical gas convection and becomes decisive even at a layer thickness of about 1 mm [24]. The near-zero value of the thermodiffusion coefficient (large Prandtl numbers) here determines almost complete adiabaticity of the gas flow, and the proximity to the critical point determines a large variation of density with temperature change. Due to this, a new form of convective heat transfer in near-critical medium, known as the piston effect, becomes possible. In [26], the author mentions the existence of a domain with density-stable stratification at a temperature gradient larger than adiabatic. However, its study in near-critical gas is difficult due to the vanishing relative width of this domain as $1/\gamma$, where the adiabatic exponent γ approaching infinity in the vicinity of the critical point [26].

In numerical simulations, the convection of near-critical gas was considered on the basis of the gas dynamics equations with the van der Waals equation of state [26]. However, the value of these works is somewhat reduced by the use of the thermoacoustic wave filtering algorithm for save computational time, due to which the model used is similar to the small Mach number approximation described above with its inherent limitations.

In [27], gas convection in a horizontal layer with horizontal boundaries free of tangential stresses is considered analytically in the linear approximation and numerically in the nonlinear approximation. Due to misinterpretation derived analytical results, it is argued that convection can develop when the Schwarzschild number is strictly less than 1. In [18] it is argued also that convection is impossible in the adiabatic mode and convective motion can develop only when the temperature gradient exceeds the adiabatic gradient.

In this regard, we note that the numerical simulations carried out in the works convincingly show that convection in a compressible gas can occur at a Schwarschild number value significantly greater than 1, or, in other words, convection in a compressible gas can develop at a temperature difference less than the adiabatic one.

Numerous works [27] [28] on numerical study of convection in compressible gas have shown qualitatively correctly the presence of adiabatic suppression of convection; however, due to technical difficulties, the given values of the determining dimensionless parameters in this series of works are far from the real ones. For instance, the value of the hydrostatic compressibility criterion characterizing the relative pressure change and the resulting relative density change is overestimated by three orders of magnitude.

The necessity to take into account the adiabatic temperature gradient (gas compressibility) with corresponding corrections of the measurement results is indicated by numerous experimental works of a French group of scientists, where the temperature convection in gaseous helium at a cryogenic temperature of about 5 K was studied. It was shown that even at a relatively small height of 0.2 m, the correction for the adiabatic temperature gradient significantly changes the measured value of heat transfer [25].

Consideration of gas compressibility is particularly relevant in the experimental study of asymptotic modes of convection at extremely large values of the Rayleigh number because of the use in such experiments of domains up to 3.3 m high [29].

We should note that new technical possibilities have now appeared. The unconditional validity of the remark about the rigidity of the system of equations and the necessity to perform calculations with a small time step is partially leveled by the possibility to perform calculations according to an explicit scheme with massively parallel data processing on multicore CPUs or/and GPUs using OpenMP or CUDA technology [4] [5]. The advantages of massively parallel data processing appear the stronger the larger the amount of information processed, so such computations are especially promising for detailed two-dimensional and three-dimensional calculations.

Note also that in terms of special geometry of the domain and not very high supercriticality the convection develops as a roll quasi-two-dimensional [5], which gives grounds for consideration of compressible and incompressible convective flows in two-dimensional formulation.

In the author's previous works [3]-[5] it was shown that during convection in a compressible gas there is a critical height of the convection domain, above which adiabatic processes become decisive and the isobaric convection regime is replaced by an adiabatic or, at a sufficiently large temperature difference, a super adiabatic one. An estimate was derived for the height of the domain above which the Boussinesq approximation becomes inapplicable.

In this paper, the problem statement is specified and the stability of the static mode of compressible gas convection is analyzed in the linear approximation. The results obtained are compared with the results of solving the system of complete nonlinear equations describing convective flows of compressible gas.

2. Numeric Models and Methodology

Preliminarily, we consider the stability characteristics of the static Rayleigh-Benard convection mode of an incompressible fluid in the Boussinesq approximation. It is obtained that the critical Rayleigh number is equal to 1971.4 at the relative horizontal extent of the flow domain equal to π. For calculations of convective flows, the numerical method on the basis of finite-difference representation of the solution was used. All boundaries of the domain were considered rigid, horizontal boundaries were isothermal, linear temperature distribution was set at the lateral boundaries, and calculations were performed on a grid of (257·65) nodes. The stream function and vorticity values at the rigid boundaries were determined using the fast Fourier transform algorithm [2].

Now consider the convective flow in a compressible, viscous, and thermally conductive gas in the field of gravity, which can be described by the following system of equations [1] [28]:

$$\rho_t + \rho\, div\vec{u} + u\cdot\rho_x + v\cdot\rho_y = M\,\nabla^2(\rho-\rho_h),$$
$$u_t + u\cdot u_x + v\cdot u_y = -\frac{1}{\gamma\rho}(\rho T)_x + M\,(\frac{4}{3}u_{xx} + u_{yy} + \frac{1}{3}v_{xy}),$$
$$v_t + u\cdot v_x + v\cdot v_y = -\frac{1}{\gamma\rho}(\rho T)_y + M\,(v_{xx} + \frac{4}{3}v_{yy} + \frac{1}{3}u_{xy}) - C_F, \tag{1}$$
$$T_t + u\cdot T_x + v\cdot T_y = \frac{M}{\mathrm{Pr}}\nabla^2 T - \frac{\gamma-1}{\gamma}T\, div\vec{u},\;\; P = \rho T.$$

Here u, v, P, ρ and T are dimensionless components of velocity, pressure, density and temperature, $\mathrm{M} = \nu/((\gamma T_0 R)^{0.5} H) = 4.608\times10^{-8}\cdot H^{-1}$ is the Mach number, where the velocity calculated from kinematic viscosity is related to the adiabatic speed of sound, $T_0 = 300\,^\circ\mathrm{K}$ is taken as the characteristic value for temperature, the selected values of specific gas constant R = 287 J/(kg·K), adiabatic index $\gamma = 1.4$, kinematic viscosity $\nu = 16\times10^{-6}$ m^2/s and Prandtl number $\mathrm{Pr} = \nu/\chi = 0.71$ correspond to air, where χ denotes the gas diffusivity and $C_F = gH/(\gamma R T_0) = 8.130\times10^{-5}\cdot H$ is the hydrostatic compressibility and g is standard acceleration of free fall. As the length scale we chose the height of the domain H, for temperature and density—their values T_0 and ρ_0 at the lower horizontal boundary, for the velocity—adiabatic speed of sound $(\gamma R T_0)^{0.5}$, for the pressure—$R\rho_0 T_0$ and time—$H/(\gamma R T_0)^{0.5}$. The dependence of viscosity and thermal conductivity coefficients on temperature is neglected in the calculations. The height of the convection domain in the calculations is equal to 0.5 m.

In perspective it is supposed to consider convection of not homogeneous gas mixture in which molecular masses of gas mixture components (fuel and oxidizer) differ significantly, for example, vapor densities of gasoline and air differ in 3.5 times. Therefore, in the equation for density (the first equation of the system (1)) a term describing mass diffusion is introduced taking into account that in a perfect gas the coefficients of kinematic viscosity and mass diffusion coefficients coincide in value. Note, that to the analogous equation for density one can come also in the Boussinesq approximation [1], if to take into account the linear dependence of density on temperature and to substitute the corresponding relation in the equation for temperature. Such regularization of the density equation is not essential for relatively slow convective flows, where viscosity and heat conduction are already fully manifested due to the smallness of the Reynolds number.

As in the case above, all boundaries of the domain were considered rigid with no-slip conditions for velocity, and the lower and upper horizontal boundaries were considered isothermal with temperatures T_0 and T_0-ΔT, respectively. Here ΔT is dimensionless difference of temperatures between lower and upper horizontal boundaries. And the temperature at the vertical boundaries, its initial and equilibrium (static) distributions were considered linear along the vertical coordinate without dependence on the horizontal coordinate. At all boundaries of the domain, the density was assumed to be equal to its values in the static state according to the equation that is derived from the system of Equation (1) taking into account the absence of motion:

$$(\rho_h T_h)_y = -\gamma\rho_h C_F, \quad T_h(y) = 1 - \Delta T \cdot y, \tag{1}$$

where ρ_h and T_h are the density and temperature in a static state.

Figure 1 shows the formulation of the problem in dimensionless form. The horizontal size of the domain referred to the vertical is equal to π.

The calculations were performed using the explicit scheme in time, and since the appearance of shock waves in the solution was not expected, a non-divergent formulation of the system of equations was used. The convective nonlinear and diffusion terms were approximated by the monotonic scheme of A.A. Samarskii [2], and thus the numerical method used was of the first order of approximation in time and the second order in space.

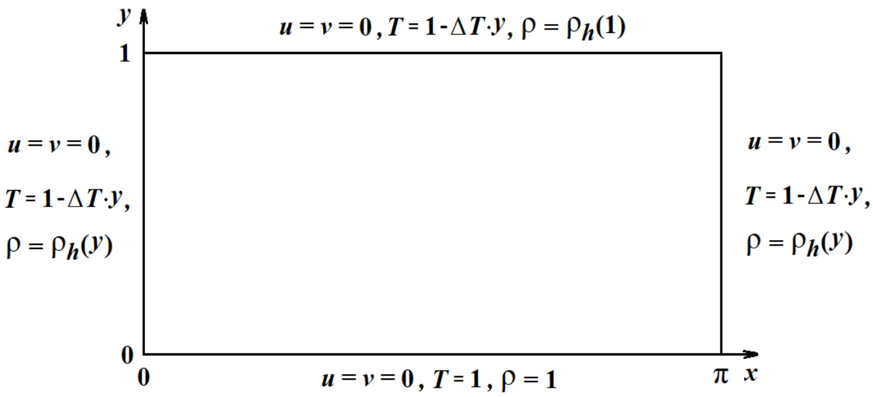

Figure 1. Formulation of the problem.

All calculations were performed on a grid of (241.81) nodes with a dimensionless time step of 0.01. Test calculations on more detailed space and time grids showed sufficient accuracy and stability of the algorithm used.

All calculations were carried out near the stability threshold with the Reynolds number value as follows:

$$\mathrm{Re} = \sqrt{2Ek/\pi} \cdot M^{-1},$$

which did not exceed values of the order of 10. Here *Ek* denotes the total kinetic energy of the entire convective motion of the gas mass [2].

Due to the low velocity of convective flows, the main contribution to the pressure variation is made by its hydrostatic component [5].

The numerical study to determine the neutral curve was organized according to the following scheme.

By calculations with different ΔT with using extrapolation, the critical value of the temperature difference was determined, at which the solution has zero increment, *i.e.*, it does not increase or decay in time. Then, the critical Rayleigh number and Schwarzschild number were calculated from the obtained temperature difference as follows:

$$Ra = gH^3\Delta T / (\chi\nu) = \Pr C_F M^{-2}\Delta T,$$
$$Sc = \Delta T_{ad} / \Delta T = (\gamma - 1)gH / (\gamma\,\Delta T\,R T_0).$$

Here $\Delta T_{ad} = (\gamma - 1) C_F$ is adiabatic gradient [3].

We also present the relationship between the Rayleigh and Schwarzschild numbers:

$$Ra = (\gamma - 1)\Pr C_F^2 / (M^2 Sc).$$

Note, that we use always the dimensionless temperature difference, *i.e.*, the temperature difference related to the temperature value of the lower horizontal boundary of the domain. And the height of the domain for clarity is always a dimensional value.

3. Linear Analysis of Stability of Convective Flows

For infinitely small perturbations of the static solution, from system (1) we can obtain (for simplicity of presentation we use the same notations):

$$\begin{aligned}
&\rho_t + div\vec{u} + v(-\gamma C_F + \Delta T) = M\nabla^2\rho, \\
&u_t + (\rho_x + T_x)/\gamma = M(\frac{4}{3}u_{xx} + u_{yy} + \frac{1}{3}v_{xy}), \\
&v_t + ((-\gamma C_F + \Delta T)T - \Delta T\rho + T_y + \rho_y)/\gamma = M(v_{xx} + \frac{4}{3}v_{yy} + \frac{1}{3}u_{xy}), \\
&T_t - v\Delta T + \frac{\gamma - 1}{\gamma}div\vec{u} = \frac{M}{\Pr}\nabla^2 T.
\end{aligned} \tag{2}$$

When deriving system (2), the dimensionless temperature difference ΔT and hydrostatic compressibility C_F were considered small compared to 1.

Solutions of system (2) are considered in the form:

$$(\rho, u, v, T) = (\rho_0, u_0, v_0, T_0)e^{-\lambda t}e^{i(\alpha x + \beta y)}.$$

Here λ, u_0, v_0, ρ_0 and T_0 are real constants, and the amplitudes of the disturbances increase for $\lambda > 0$ and decay for $\lambda < 0$. In this section, the solution is considered to be periodic in the horizontal and vertical directions with wave numbers α and β. This formulation of the problem is physically close to the formulation of the problem of convection with free horizontal boundaries [27].

From system (2) we can obtain a system of equations for the amplitudes:

$$\begin{aligned}
&-\lambda\rho_0 + i(\alpha u_0 + \beta v_0) + v_0(-\gamma C_F + \Delta T) + M(\alpha^2 + \beta^2)\rho_0 = 0, \\
&-\lambda u_0 + \frac{i\alpha}{\gamma}(\rho_0 + T_0) + M(\frac{4}{3}\alpha^2 u_0 + \beta^2 u_0 + \frac{1}{3}\alpha\beta v_0) = 0, \\
&-\lambda v_0 + (-\gamma C_F + \Delta T)T_0/\gamma - \Delta T\rho_0/\gamma + i\beta(T_0 + \rho_0)/\gamma \\
&+M(\alpha^2 v_0 + \frac{4}{3}\beta^2 v_0 + \frac{1}{3}\alpha\beta u_0) = 0, \\
&-\lambda T_0 - v_0\Delta T + (\gamma - 1)i(\alpha u_0 + \beta v_0)/\gamma + M(\alpha^2 + \beta^2)T_0/\Pr = 0.
\end{aligned} \tag{3}$$

A neutral curve corresponding to monotonic stability with $\lambda = 0$ is considered.

System (3) was written in matrix form and from the condition of equality of the system determinant to zero, an equation for determining ΔT was derived. After obvious simplifications associated with neglecting relatively small quantities, the resulting second-order algebraic equation takes the form:

$$1.3120\alpha^2\Delta T^2 - 1.0785\cdot10^{-4} H\alpha^2\Delta T + 2.5695\cdot10^{-15}\alpha^6/H^2 + 7.6081\cdot10^{-14}\alpha^4/H^2 + 7.5088\cdot10^{-13}\alpha^2/H^2 + 2.4703\cdot10^{-12}/H^2 + 1.8885\cdot10^{-9}\alpha^2 H^2. \quad (4)$$

To determine ΔT from Equation (4), the following analytical formulas follow:

$$\Delta T_{1,2} = (1.0785\cdot10^{-4} H\alpha^2 \pm \sqrt{Discr})/2.624\alpha^2,$$
$$Discr = 10^{-12}\cdot(1720.6\,\alpha^4 H^2 - 0.013484\,\alpha^8/H^2 - 0.39926\,\alpha^6/H^2 - 3.9405\,\alpha^4/H^2 - 12.964\,\alpha^2/H^2). \quad (5)$$

To determine the boundaries of the instability domain, the discriminant (5) is set to zero (**Figure 2**, curve 1). As can be seen from **Figure 2**, at $H > 0.268$ m, the height of the domain determines the minimum and maximum values of the wave numbers that determine the range of unstable wave numbers α in the linear approximation.

The minimum value of wave numbers, which determines the lower boundary of the instability domain in wave numbers, with increasing H reaches a power-law asymptotic (curve 2 in **Figure 2**):

$$\alpha_{\min} = 0.0868\,H^{-2},$$

and the maximum, which determines the upper boundary of the instability domain, reaches a linear asymptotic:

$$\alpha_{\max} = -1.33 + 19.87\,H.$$

The smallness of the dimensionless temperature difference ΔT and hydrostatic compressibility C_F compared to 1 determines the fact that system (3) correctly reflects the development of convective instability of the static solution only at a sufficiently large height of the domain, since at a small height of the domain ΔT is proportional to H^{-3} and can take large values. Instability is not observed at a domain height less than $H = 0.268$ m (see **Figure 2**).

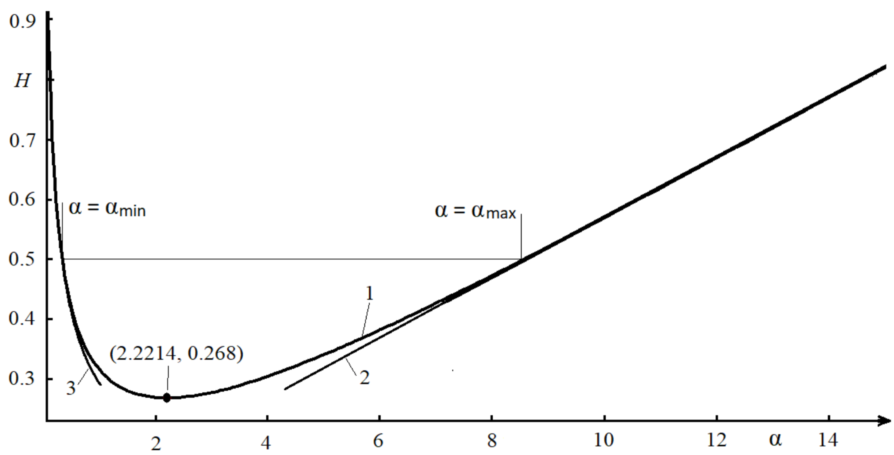

Figure 2. Determination of minimum and maximum wave numbers as functions of the height of the domain.

It can be assumed that the domain height should be greater than its critical value. In this case, adiabatic processes become decisive, this causes a change in the convection regime, namely, the replacement of the isobaric convection regime by adiabatic or superadiabatic. The critical height for air under normal conditions is 17.3 cm [3].

Figure 3 shows the neutral curve in the Sc – α plane at H = 0.5 m and $\beta = \pi$. The neutral curve consists of two branches—upper and lower. The points correspond to numerical calculations with α = 3 and 4.

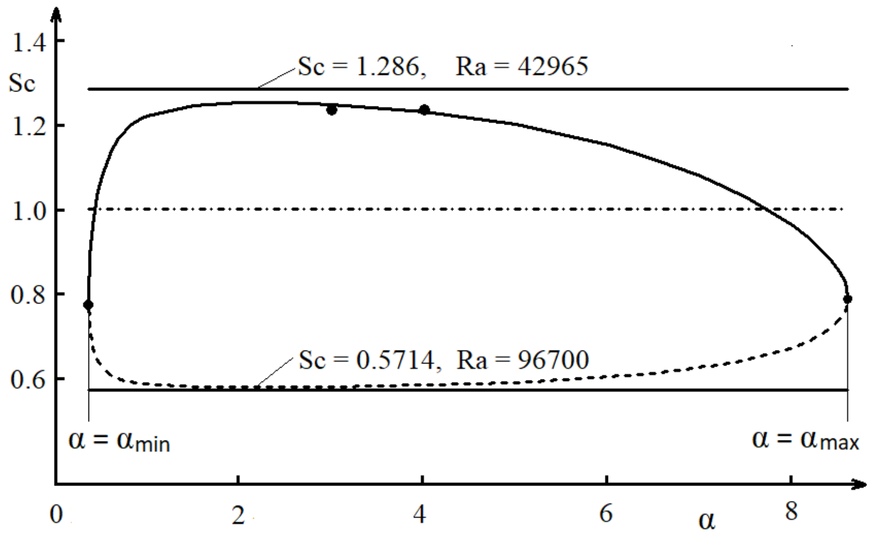

Figure 3. Neutral curve on the plane Sc-α.

The solid lines in **Figure 3** show the asymptotic values of the Sc number (for an infinitely high altitude of the domain, equivalently, for an infinitely small value of the Mach number) for the lower:

$$Sc_l = 2(\gamma - 1)/\gamma$$

and upper branches

$$Sc_{up} = (2\gamma - 1)/\gamma$$

of the neutral curve. It can be seen that the asymptotic expression for the upper branch always gives a value greater than 1. And the asymptotic expression for the lower one gives a value less than 1 for $\gamma < 2$, which is always true for gases under real conditions [30]. The Rayleigh number values shown in **Figure 3** correspond to a domain altitude of H = 0.5 m. All analytical calculations in this paper were performed using the Maple 15 symbolic computation program.

We will briefly describe the method of extrapolating the value of the Sc_n number, which determines the position of the neutral curve. The neutral curve can be determined by the Nusselt number or the Reynolds number; below, both of these options are implemented simultaneously to control the accuracy. The position of the neutral curve is characterized by the Sc_n value, which is calculated by the last three (closest to the neutral curve) points from the relations:

$$Nu = 1 + A(Sc_n - Sc)^B, \quad \mathrm{Re} = A_1(Sc_n - Sc)^{B_1}.$$

Despite the difference in the power laws for the Nu and Re numbers, the obtained Sc_n values coincide with an accuracy of up to thousandths of a percent, which shows good accuracy in determining the Sc_n value.

Table 1 shows the exact Sc values obtained according to the linear theory for the specified wave number values and the data obtained by extrapolation from the calculated data. It can be seen that the calculated data correspond with good accuracy to the analytically obtained exact values. Some discrepancy between the numerical and analytical data is due to the difference in the boundary conditions (**Figure 1**).

Table 1. Comparison of the results of linear theory and numerical simulation.

α	Linear theory	Numerical (deviation in %)
3	1.2483	1.2347 (1.10%)
4	1.2316	1.2346 (0.24%)

The analytical formula of Jeffreys [3]-[5] gives a value of 0.9655, which is less than 1 and differs by about 30% from the values obtained by the linear theory and numerically in this work.

The difference in the boundary conditions also explains the fact that calculating the position of the neutral curve at a smaller or larger value of the wave number causes difficulties. The fact is that any convective vortex structure, due to the selected boundary conditions, is represented by a set of harmonics (wave packet), in which, when approaching the neutral curve, a kind of filtration occurs with the selection of the most stable mode and a jump of the solution to it.

The development of convective instability is determined only by the upper branch of the neutral curve (the temperature difference is less than the adiabatic one, as follows from the definition of the Sc number, the solid line in **Figure 3**), and the presence of the second lower branch (the temperature difference is greater than the adiabatic one) is not manifested in any way in the obtained numerical data, which is presumably associated with the ambiguity of the solution to the convection problem. In this regard, we note that in [28], the development of convective instability is erroneously associated with the lower branch of the neutral curve, shown in **Figure 3** by the dotted line.

As can be seen from **Figure 2**, the neutral curve in the form of a distorted ellipse is observed at a domain height of $H > 0.268$ m, and at a lower domain height, convective instability is not observed in this model. This fact, which contradicts observations, is associated with the above-mentioned inappropriateness of using this model at a low convection domain height. The neutral curve originates at a domain height $H = 0.268$ m and a wave number $\alpha = 2.2214$ (see **Figure 2**). As the domain height increases, the extent of the instability domain in terms of wave number increases, and the values on the upper and lower branches of the neutral curve approach their asymptotic values.

Figure 4 shows the neutral curve on the Ra–α plane. With this representation, the lower branch of the neutral curve (the temperature difference is less than the adiabatic one) is responsible for the development of convective instability. The dotted line in **Figure 4** for comparison shows the neutral curve in the case of convection of an incompressible fluid in the Boussinesq approximation with free horizontal boundaries [1]; when constructing it, the corresponding values of Ra were multiplied by 67. The minima of the neutral curves are observed at the same value of the wave number α = 2.2214.

The neutral curve shown for a compressible gas in **Figure 4** has the shape of a well with vertical walls and a practically flat bottom. It follows that in the linear approximation convective flows with any horizontal wave number from the interval between its minimum and maximum values can be realized in a compressible gas.

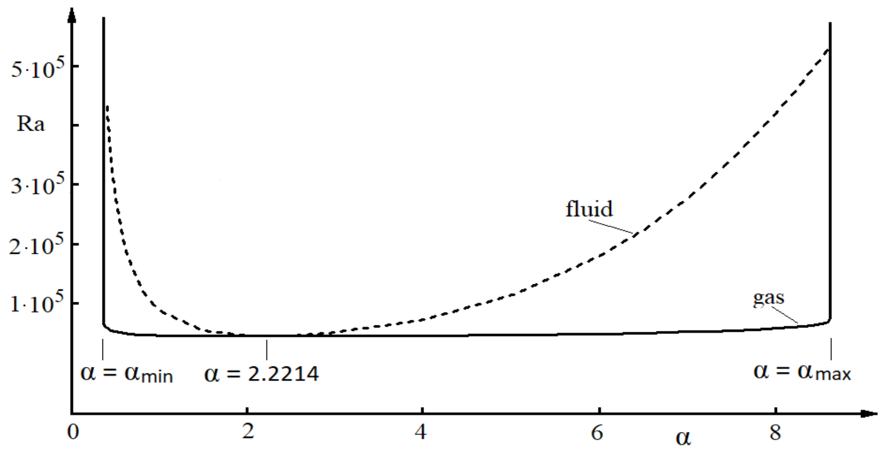

Figure 4. Neutral curve on the plane Ra-*α*.

The same conclusion can be made for solutions of the complete nonlinear system of equations (1) at low supercriticality, when nonlinear effects are still weakly expressed, which is confirmed by the analysis of data from numerous methodological calculations.

4. Discussion

As shown above, the written systems of linear equations (2) and (3) describing the linear development of disturbances do not provide a correct description of the development of convective instability at low altitude. Formally, mathematically, this is due to the fact that when deriving system (2), the dimensionless temperature difference ΔT and hydrostatic compressibility C_F were considered small compared to 1, and ΔT increases with decreasing H. Physically, the appearance of convective instability can be associated with the excess of the domain's critical value by altitude, when, with the determining role of adiabatic processes, a change in the convection regime occurs, namely, the isobaric convection regime is replaced by an adiabatic or superadiabatic one. The critical altitude for air under normal conditions is 17.3 cm [3]. Of course, the physical mechanisms of the appearance of convective instability require more in-depth study and understanding.

The neutral curve shown for a compressible gas in **Figure 4** has the shape of a well with vertical walls and a practically flat bottom. It follows that in the linear and weakly nonlinear approximations, convective flows with any horizontal wave number from the interval between its minimum and maximum values can be realized in a compressible gas. However, this conclusion is based on studying the behavior of linear two-dimensional disturbances. It is not yet clear how this conclusion will change when taking into account the three-dimensionality of the flow, developed nonlinearity, and the dependence of the thermal conductivity and viscosity coefficients on temperature.

It is shown that the development of convective instability is associated with the upper branch of the neutral curve on the Sc-α plane. The presence of the second lower branch of the neutral curve is not traced in any way in the results of numerical modeling. In this regard, we note that in [28] the development of convective instability is erroneously associated with the second branch of the neutral curve, shown in **Figure 3** by the dotted line. The appearance of the second branch of the neutral curve, not realized in numerical calculations, is presumably associated with the non-uniqueness of the solution to the convection problem, but this conclusion requires additional research.

5. Conclusions

Let us formulate the main conclusions:

1) For the case of a solution periodic in all directions, a linear analysis of the monotonic stability of the static convection regime is performed, and an analytical formula is obtained for the corresponding neutral curve of the temperature difference on the horizontal boundaries. For the lower and upper branches of the neutral curve, asymptotics are derived that correspond to an infinitely large height of the domain.

2) It is shown that neglecting the dimensionless temperature difference and the coefficient of hydrostatic compressibility compared to 1 leads to a non-physical conclusion about the absence of instability at a height of the convection domain less than $H \le 0.268$ m. Namely, in this model instability arises at $H = 0.268$ m, $\alpha = 2.2214$ and is observed at all values of $H \ge 0.268$ m.

3) It is shown that the development of convective instability is associated with the upper branch of the neutral curve (on the Sc-α plane, **Figure 3**). The presence of the second branch of the neutral curve is not traced in any way in the results of numerical modeling. In this regard, we note that in [28] the development of convective instability is erroneously associated with the second branch of the neutral curve, shown in **Figure 3** by the dotted line. The above-mentioned error in interpreting the obtained results also led to the erroneous statement about the existence of a minimum positive value of the Reynolds number during convection of a compressible gas.

4) It follows from the shape of the neutral curve that in the linear and weakly nonlinear approximations in a compressible gas, convective flows with any horizontal wave number from the interval between its minimum and maximum values can be realized. This conclusion is confirmed by the data of numerical methodological calculations of convective flows based on a complete nonlinear system of equations at low supercriticality.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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