

Binary Phase Shaping and SHG Contrast Ratios

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How to cite this paper: Xiong, T.Y. (2024) Binary Phase Shaping and SHG Contrast Ratios. *Journal of Applied Mathematics and Physics*, 12, 3026-3035.
<https://doi.org/10.4236/jamp.2024.128181>

Received: August 12, 2024

Accepted: August 27, 2024

Published: August 30, 2024

Abstract

Finding binary sequences with Large SHG ratios is very important in the field of ultrafast science, biomedical optics, high-resolution microscopy and label-free imaging. In this paper, we have demonstrated the relation between the SHG contrast ratio and the traditional Merit Factor values. And in the light from known results in Merit Factor Problems, we have shown that Legendre Sequences or Jacobi Sequences, are still the best candidates to obtain binary sequences with large SHG contrast ratios. The authors also discussed the SHG behaviors on some sequences obtained from cyclotomic classes over the finite field $GF(2^l)$.

Keywords

Binary Phase Shaping, Binary Sequences, SHG Contrast Ratio, Merit Factor

1. Introduction

In the use of laser pulses to control chemical reactions, it is worthwhile to manage interference between pairs of available frequencies. A successful management technique called binary phase shaping (BPS) has been introduced and discussed in ([1]-[3]). A given frequency can be masked out completely or present with one of two phases, 0 and π , and these choices are then be represented by a sequence of 0's, -1's, and +1's. The BPS choice criteria turn out to be related to aperiodic correlation properties of the associated sequences and related ± 1 -sequences. Preferred sequences have good correlation and (what we will refer to as) focusing properties. While there is a large literature (for instance ([4]-[10]) on aperiodic correlation, the special needs of focusing do not seem to have been addressed.

2. Binary Phase Shaping

Given a laser pulse, described by having intensity $E(\omega)$ at frequency ω , we are interested in the intensities that result from two types of nonlinear, second order

interference. For two photon excitation or second harmonic generation (SHG), the interference is positive with the second order intensity at frequency ω given by

$$E^{SHG}(\omega) = \int_{\alpha+\beta=\omega} E(\alpha) + E(\beta) d\alpha$$

In binary phase shaping the situation is simplified by considering a pulse that only contains a finite number of equally spaced frequencies (indexed by $0 \leq k \leq n-1$ with each frequency admitting only a limited set of intensities. Specifically, a given frequency can be masked to 0 but otherwise it is at a fixed level, the only choice allowed being whether the phase is left unchanged or is switched by π . The shape of a given laser pulse can thus be encoded by the shaping sequence $e = (e_i | 0 \leq i \leq n-1)$ where $e_i = 0$ if frequency i has been masked out, and otherwise e_i is +1 or -1 depending on whether the phase change is 0 or π . (For all other i , set $e_i = 0$). In this case the above intensity measures at the k_{th} frequency become

$$e^{SRS}(k) = \sum_{a-b=k} e_a e_b, \quad 0 \leq k \leq n-1$$

and

$$e^{SHG}(k) = \sum_{a+b=k} e_a e_b, \quad 0 \leq k \leq 2n-2$$

The authors have discussed the SRS problem in [8]. Thus in this paper, we will only focus on SHG problem as defined as follows. The nonzero values of $e^{SHG}(k)$ occur for $0 \leq k \leq 2n-2$ and all are relevant. Set

$$t^{SHG} = \sum_{k=0}^{2n-2} |e^{SHG}(k)|^2, \quad 0 \leq k \leq 2n-2,$$

the SHG total energy. Then for a chosen focus frequency h we wish to choose a shaping sequence e to accomplish the following desired properties:

- 1) maximize the h-foreground energy $|e^{SHG}(h)|^2$;
- 2) minimize the h-background energy $b^{SHG}(k) = t^{SHG} - |e^{SHG}(k)|^2$;
- 3) maximize the h-contrast ratio

$$CR^{SHG}(h) = \frac{|e^{SHG}(h)|^2}{b^{SHG}(k)}$$

In this paper, we will focus on maximizing the contrast ratio $CR^{SHG}(h)$.

Example 2.1. Suppose a sequence $e = (e_0, e_1, e_2, e_3, e_4, e_5)$ is of length 6, then by definition

$$e^{SHG}(7) = \sum_{a+b=7} e_a e_b = e_2 e_5 + e_3 e_4 + e_4 e_3 + e_5 e_2$$

$$e^{SHG}(8) = \sum_{a+b=8} e_a e_b = e_4 e_4 + e_3 e_5 + e_5 e_3$$

The following property, which will be referred frequently later, gives an equivalent definition of $e^{SHG}(k)$ at the k -th foreground frequency:

Property 2.2. For any sequence $e = (e_i | 0 \leq i \leq n-1)$ of length n , $e^{SHG}(k)$

has the following equivalent form

$$e^{SHG}(k) = \sum_{a=c_k}^{\min(k,n)} e_a e_{k-a},$$

where $c_k = k - n + 1$ if $n \leq k \leq 2n - 2$, and 0 otherwise.

Definition 2.3. We call a sequence $e = (e_0, e_1, \dots, e_{n-1})$ a symmetric sequence if $e_i = e_{n-1-i}$, and an antisymmetric sequence if $e_i = -e_{n-1-i}$, where $i = 0, 1, \dots, n - 1$.

The following definition gives a method of constructing a symmetric or antisymmetric sequence given any base sequence c .

Definition 2.4. Let sequence $c = (c_0, c_1, \dots, c_{m-1})$ of length m , define

$$\tilde{c} = (c_{m-1}, c_{m-2}, \dots, c_1, c_0)$$

Define

$$e = \{c \mid \tilde{c}\} = (c_0, c_1, \dots, c_{m-1}, c_{m-1}, c_{m-2}, \dots, c_1, c_0); \tag{1}$$

and

$$e = \{c \mid -\tilde{c}\} = (c_0, c_1, \dots, c_{m-1}, -c_{m-1}, -c_{m-2}, \dots, -c_1, -c_0); \tag{2}$$

Note that in the constructed sequences (1) and (2), the $2m$ -foreground energy takes the maximum value

$$|e^{SHG}(2m)| = 2m$$

The following property describes a general “symmetric” property of $e^{SHG}(k)$ values of sequences defined as in Definition 2.4.

Property 2.5 Given a symmetric or antisymmetric sequence $e = (e_0, e_1, \dots, e_{n-1})$ of length n , that is, $e_i = e_{n-1-i}$, or $e_i = -e_{n-1-i}$, where $i = 0, 1, \dots, n - 1$. Then

$$e^{SHG}(k) = e^{SHG}(2n - 2 - k), \quad k = 0, 1, \dots, 2n - 2$$

Proof.

$$\begin{aligned} e^{SHG}(k) &= \sum_{a+b=k} e_a e_b = \sum_{a+b=k} e_{n-1-a} e_{n-1-b} = \sum_{a+b=2n-2-k} e_a e_b \\ &= e^{SHG}(2n - 2 - k) \quad \blacksquare \end{aligned}$$

Property 2.5 shows that if a sequence e is symmetric or antisymmetric, then we have

$$\sum_{k=0}^{2n-2} e^{SHG}(k)^2 = 2 \times \sum_{k=0}^{n-1} e^{SHG}(k)^2, \tag{3}$$

With the equation (3), we are ready to address the SHG problem to be studied in details in this paper.

Problem SHG: Find a binary sequence c of length m , construct sequence $e = \{c \mid \pm\tilde{c}\}$ of even length $2m$ as defined in Definition 2.4, with the result in (3), we want the contrast ratio

$$CR^{SHG}(2m-1) = \frac{|e^{SHG}(2m-1)|^2}{2 \times \sum_{k=1}^{2m-2} e^{SHG}(k)^2} = \frac{m^2}{2 \times \sum_{k=1}^{m-1} e^{SHG}(k)^2}$$

to be large.

3. SHG Problem

In certain cases, Chemists or Physicists are also interested in binary sequences with small $|e^{SHG}(k)|$ values for k even or odd. For instance, Chemists or Physicists are also looking for certain binary sequences $e = (e_0, e_1, \dots, e_{n-1})$, such that all the following $|e^{SHG}(2k)|$ values are small except one particular position k :

$$e^{SHG}(2k) = \sum_a e_{k-a} e_{k+a}, \quad 0 \leq k-a, k+a \leq n-1$$

From the discussion before, we have known that a sequence taking for $e = \{c | \tilde{c}\}$ would maximize the $|e^{SHG}(2m)| = 2m$. To minimize the other $e^{SHG}(2k)$ values at even shifts, we can construct the sequence c as follows.

Property 3.1. Given a positive integer m , let the alternating sequences

$$c = (+1, +1, -1, -1, +1, +1, \dots) \quad \text{or} \quad c = (+1, -1, -1, +1, +1, -1, \dots)$$

be sequences of length m as defined in ([9] [10]). Let $b = \{c | \pm \tilde{c}\}$ be the sequence of length $2m$ as defined above, then $|e^{SHG}(2m-1)| = 2m$, and $|e^{SHG}(k)| \leq 1$ with k even and $1 \leq k \leq 2m-1$.

Proof. By Property 2.2, $e^{SHG}(2k) = \sum_{a=0}^{2m-1} e_a e_{2m-1-a} = 2 \times \sum_{a=0}^{m-1} e_a^2 = 2m$.

Now we will only prove that the rest $|e^{SHG}(k)| \leq 1$ for even k values.

If $b = \{c | \pm \tilde{c}\}$ with $c = (+1, +1, -1, -1, +1, +1, \dots)$ or $c = (+1, -1, -1, +1, +1, -1, \dots)$, then $b_i = b_{2m-1-i} = (-1)^{k_i}$, where $k_i = i(i-1)/2$, if $0 \leq i \leq m-1$; or $k_i = (2m-1-i)(2m-2-i)/2$, if $m \leq i \leq 2m-1$.

By Property 2.2, if $0 \leq k \leq 2m-1$,

$$|e^{SHG}(k)| = \left| \sum_{a=0}^k b_a b_{k-a} \right| = \left| \sum_{a=0}^k (-1)^{a^2-ka} \right| = \left| \sum_{a=0}^k (-1)^{a^2} \right| = \left| \sum_{a=0}^k (-1)^a \right| \leq 1,$$

The last equality in the calculation above follows from the fact that k is even and a and a^2 have the same even or odd parity. ■

The following property shows that the contrast ratio CR^{SHG} value is closely related to the merit factor value as defined in ([9] [10]).

Property 3.2. Given a symmetric (or antisymmetric) binary sequence $e = \{c | \pm \tilde{c}\}$ of even length n , then

$$A_e(k) = e^{SHG}(n-1-k), \quad 1 \leq k \leq n-1,$$

where $A_e(k)$ is aperiodic autocorrelation function of sequence e at shift i as defined in ([9] [10]).

Proof. $A_e(k) = \sum_{j=0}^{n-1-k} e_j e_{j+k} = \sum_{j=0}^{n-1-k} e_j e_{n-1-j-k} = e^{SHG}(n-1-k)$

The last equality in the calculation above follows from the fact that e is symmetric (or antisymmetric) and Property 2.2. ■

Property 3.2 shows that for symmetric or antisymmetric sequences of even length, the contrast ratio CR^{SHG} can be classified as the Merit Factor problem firstly introduced in [4]. Thus naturally we will start with Legendre Sequences defined in [6] which is well-known for its high asymptotic merit factor values.

Based on Property 3.2, we are ready to prove the first important Theorem of this paper.

Theorem 3.3. Given an odd prime p , let $\alpha = (\alpha_0, \alpha_1, \dots, \alpha_{p-1})$ be the Legendre sequence as defined in {[6] [7]}. Construct a sequence $e = (\alpha_1, \dots, \alpha_{p-1})$ be of even length $p-1$. Then the family of such sequences has the asymptotic central contrast ratio $CR_e^{SHG}(p-2)$ is 1.5.

Proof. It is well known that e is symmetric if $p \equiv 1 \pmod{4}$, and antisymmetric if $p \equiv 3 \pmod{4}$. Or in other words,

$$e_i = (-1)^{(p-1)/2} e_{p-2-i}$$

Therefore, $|e^{SHG}(p-2)| = p-1$. Meanwhile, by Properties 2.5 and 3.2,

$$\begin{aligned} \lim_{p \rightarrow \infty} CR_e^{SHG}(p-2) &= \lim_{p \rightarrow \infty} \frac{e^{SHG}(p-1)^2}{2 \sum_{k=1}^{p-2} e^{SHG}(k)^2} = \lim_{p \rightarrow \infty} \frac{(p-1)^2}{2 \sum_{k=0}^{p-3} e^{SHG}(k)^2} \\ &= \lim_{p \rightarrow \infty} \frac{(p-1)^2}{2 \sum_{k=1}^{p-2} A_e(k)^2} = \lim_{p \rightarrow \infty} F_\alpha = 1.5 \quad \blacksquare \end{aligned}$$

The authors have also considered rotating the Legendre sequence α to obtain the base sequence c . For instance, let $c = \alpha^r$ where r is the rotation ratio, or $c_i = \alpha_{i+r}$, where $i=0,1,\dots,p-1$.

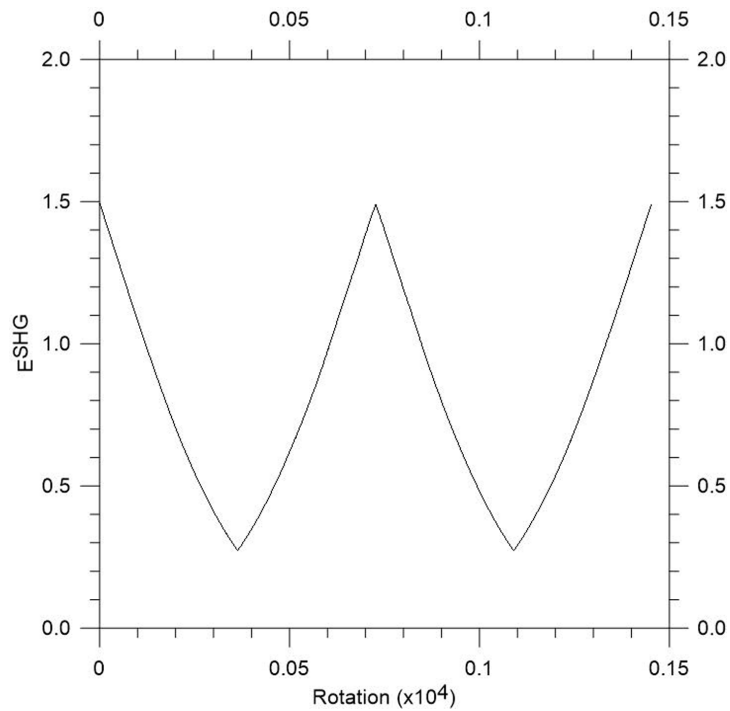


Figure 1. An example that $p = 3931$, the highest $CR^{SHG} = 1.5$ at $r=0$.

Property 3.4. For a positive integer m , a binary sequence $c = (c_0, c_1, \dots, c_{m-1})$ of length m , if a sequence $e = \{c \mid \delta \tilde{c}\}$ where $\delta = \pm 1$. Then

$$e^{SHG}(k) = \begin{cases} c^{SHG}(k), & \text{if } 0 \leq k \leq m-1; \\ c^{SHG}(k) + 2\delta + A_c(2m-1-k), & \text{if } m \leq k < 2m-1; \\ \delta \times 2m, & \text{if } k = 2m-1; \end{cases}$$

Proof. When $0 \leq k \leq m-1$, by Property 2.2, we have

$$e^{SHG}(k) = \sum_{a=0}^k e_a e_{k-a} = \sum_{a=0}^k c_a c_{k-a} = c^{SHG}(k)$$

When $m \leq k < 2m-1$,

$$\begin{aligned} e^{SHG}(k) &= \sum_{a=0}^k e_a e_{k-a} = \sum_{a=0}^{k-m} e_a e_{k-a} + \sum_{a=k-m+1}^{m-1} e_a e_{k-a} + \sum_{a=m}^k e_a e_{k-a} \\ &= \delta \times \sum_{a=0}^k c_a c_{a+2m-1-k} + \sum_{a=k-m+1}^{m-1} c_a c_{k-a} + \delta \times \sum_{a=m}^k c_{2m-1-a} c_{k-a} \\ &= 2\delta \times A_e(2m-1-k) + e^{SHG}(k) \end{aligned}$$

Finally when $k = 2m-1$,

$$e^{SHG}(2m-1) = \sum_{a=0}^{2m-1} e_a e_{2m-1-a} = 2\delta \sum_{a=0}^{m-1} c_a^2 = \delta \times 2m \quad \blacksquare$$

Both Properties 3.2 and 3.4 discussed the distribution of foreground energy values $e^{SHG}(k)$ s for even-length-sequences e . For the sequences $e = \{c | e_m | \delta \tilde{c}\}$, of odd length $2m+1$, we have the following result which is also based on Legendre sequences.

Theorem 3.5. Let $\alpha = (\alpha_0, \alpha_1, \dots, \alpha_{p-1})$ be the Legendre sequence of odd prime length p . Construct a sequence $e = (\alpha_1, \dots, \alpha_{p-2}, \alpha_{p-1} | \delta | \delta \alpha_{p-1}, \delta \alpha_{p-2}, \dots, \delta \alpha_1)$ of odd length $2p-1$, where $\delta = (-1)^{(p-1)/2}$. Then the family of such sequences has the asymptotic central contrast ratio CR_e^{SHG} is 1.2.

We need the following Lemma to give the foundation of Theorem 3.6.

Lemma 3.6. Given p odd prime, let $e = (\alpha_1, \dots, \alpha_{p-1} | \delta | \delta \alpha_{p-1}, \delta \alpha_{p-2}, \dots, \delta \alpha_1)$ be of odd length $2p-1$ be as defined in Theorem 3.6, then

$$|e^{SHG}(k)| = \begin{cases} |A_\alpha(p-2-k)|, & 0 \leq k \leq p-3 \\ |p-1|, & k = p-2 \\ \left| 2p-1 \pm \frac{\delta-1}{2} \right|, & k = 2p-2 \\ |P_\alpha(2p-2-k) + A_\alpha(2p-2-k) + c_k|, & O.W. \end{cases}$$

where $\delta = (-1)^{(p-1)/2}$, and c_k 's are small integers satisfying $|c_k| \leq 3$.

Proof. When $0 \leq k \leq p-3$. $e^{SHG}(k) = \alpha^{SHG}(k) = A_\alpha(p-2-k)$ by Prop. 3.2; If $k = p-2$, by Property 2.2,

$$e^{SHG}(p-2) = \sum_{a=0}^{p-2} e_a e_{p-2-a} = \sum_{j=1}^{p-1} \alpha_j \alpha_{p-1-j} = \delta \times (p-1),$$

Since the truncated Legendre sequence $(\alpha_1, \dots, \alpha_{p-1})$ is symmetric or antisymmetric depending on the value $p \pmod{4}$.

If $k = 2p-2$, by Property 2.2,

$$e^{SHG}(2p-2) = \sum_{a=0}^{2p-2} e_a e_{2p-2-a} = a_0^2 + 2\delta \sum_{j=1}^{p-1} a_j^2 = 1 - 2\delta \times (p-1)$$

Now the most complicated case is $p-1 \leq k < 2p-2$. By Property 2.2,

$$\begin{aligned} e^{SHG}(k) &= \sum_{a=0}^k e_a e_{k-a} = \sum_{a=0}^{k-p} e_a e_{k-a} + e_{k-p+1} e_{p-1} + \sum_{a=k-p+2}^{p-2} e_a e_{k-a} \\ &\quad + e_{p-1} e_{k-p+1} + \sum_{j=p}^k e_{p-1} e_{k-p+1} \end{aligned}$$

$$\begin{aligned}
 &= \sum_{j=0}^{k-p} \alpha_{j+1} \alpha_{j+2p-1-k} + \alpha_0 \alpha_{k-p+2} + \sum_{j=k-p+2}^{p-2} \alpha_{j+1} \alpha_{k-j+1} \\
 &\quad + \sum_{j=p}^k \alpha_{2p-1-j} \alpha_{k-j+1} + \alpha_0 \alpha_{k-p+1} \\
 &= I + II + III + IV + V
 \end{aligned}$$

$$I = \sum_{j=0}^{k-p} \alpha_{j+1} \alpha_{j+2p-1-k} = A_\alpha(2p-2-k) \pm 1$$

$$II + III = A_\alpha(k+p+2)$$

Note that by the well-known property of the periodic and aperiodic autocorrelations of the binary sequences ([4] [7]), we can derive

$$I + II + III = A_\alpha(2p-2-k) \pm 1 + A_\alpha(k+p+2) = P_\alpha(2p-2-k) \pm 1$$

And in term *IV*, re-order the subscript $k-j+1 = a$, then

$$IV + V = \sum_{a=1}^{k-p+1} \alpha_a \alpha_{a+2p-k-2} + \alpha_0 \alpha_{k-p+1}$$

$$= A_\alpha(2p-2-k) + \alpha_0 \alpha_{k-p+1} - \alpha_0 \alpha_{2p-k-2} \cdot \blacksquare$$

Now we are ready to prove Theorem 3.5.

Proof of Theorem 3.5 Without loss, we only assume $p \equiv 1 \pmod{4}$, since the proof to case $p \equiv 3 \pmod{4}$ is essentially identical.

When $p \equiv 1 \pmod{4}$, $\delta = 1$. Then by Property 2.5,

$$\frac{1}{CR^{SHG}} = \frac{2 \sum_{k=1}^{2p-1} [A_\alpha(k)^2 + (P_\alpha(k) + A_\alpha(k) + c_k)^2 + (p-1)^2]}{(2p-1)^2},$$

where c_k are small integers with $|c_k| \leq 3$. Therefore,

$$\lim_{p \rightarrow \infty} \frac{1}{CR^{SHG}} = \frac{1}{2F_\alpha} + \frac{1}{2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

Thus

$$\lim_{p \rightarrow \infty} CR^{SHG} = \frac{6}{5} = 1.2 \quad \blacksquare$$

With the inspiration of Property 3.4, the authors also conducted the numerical analysis of CR^{SHG} values on m -sequences. In other words, we are considering the sequences in form of $e = \{c | 1 | \delta \tilde{c}\}$, where c is an m -sequence of length $2n-1$. Unlike Legendre sequences, the structure of m -sequence is more complicated. For instance, the different choice of primitive polynomials, different initial state for a fixed primitive polynomial, etc., may all influence the behavior of SHG contrast ratios. Our numerical analysis has the following discoveries:

1) Fix a positive integer n , different primitive polynomials of degree n generates non-cyclically-equivalent m -sequences length $2n-1$. But we did not find significant differences among the highest CR^{SHG} values of these sequences. For instance, we have chosen $n=10$, and tested the highest CR^{SHG} from cyclically shifting m -sequences generated by all distinct 15 primitive polynomials:

$$f_1 = x^{10} + x^3 + 1, f_2 = x^{10} + x^8 + x^3 + x^2 + 1, f_3 = x^{10} + x^4 + x^3 + x + 1,$$

$$\begin{aligned}
 f_4 &= x^{10} + x^6 + x^5 + x^3 + x^2 + x + 1, f_5 = x^{10} + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x + 1, \\
 f_6 &= x^{10} + x^9 + x^4 + x + 1, f_7 = x^{10} + x^4 + x^3 + x + 1, f_8 = x^{10} + x^8 + x^4 + x^3 + 1, \\
 f_9 &= x^{10} + x^8 + x^5 + x^4 + 1, f_{10} = x^{10} + x^8 + x^5 + x + 1, \\
 f_{11} &= x^{10} + x^8 + x^7 + x^6 + x^5 + x^2 + 1, f_{12} = x^{10} + x^9 + x^8 + x^6 + x^3 + x^2 + 1, \\
 f_{13} &= x^{10} + x^9 + x^8 + x^6 + x^5 + x + 1, f_{14} = x^{10} + x^9 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1, \\
 f_{15} &= x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + 1,
 \end{aligned}$$

Figure 2 shows that both maximum and minimum CR^{SHG} values are not significantly different for m -sequences c generated by f_1, f_2, \dots , etc., and $e = \{c | 1 | \delta \tilde{c}\}$. In particular, the maximum CR^{SHG} values are ranging between 1.1199 - 1.2; and the minimum CR^{SHG} is from 0.8575 - 0.9352. And we have obtained the similar conclusion to n values other than 10.

2) For a fixed primitive polynomial, different initial state will give different sequences. Although these sequences are cyclically equivalent, there is no reason to believe that they all give the same CR^{SHG} values. **Figure 2** also shows that different initial stages obviously cause different sequence behavior on CR^{SHG} values. Since there are $2^{10} - 1$ different choice of non-zero initial input, the authors have not found a clear answer to determining the “right” initial states for a given primitive polynomial to generate the ideal m -sequences which yield the best SHG contrast ratio.

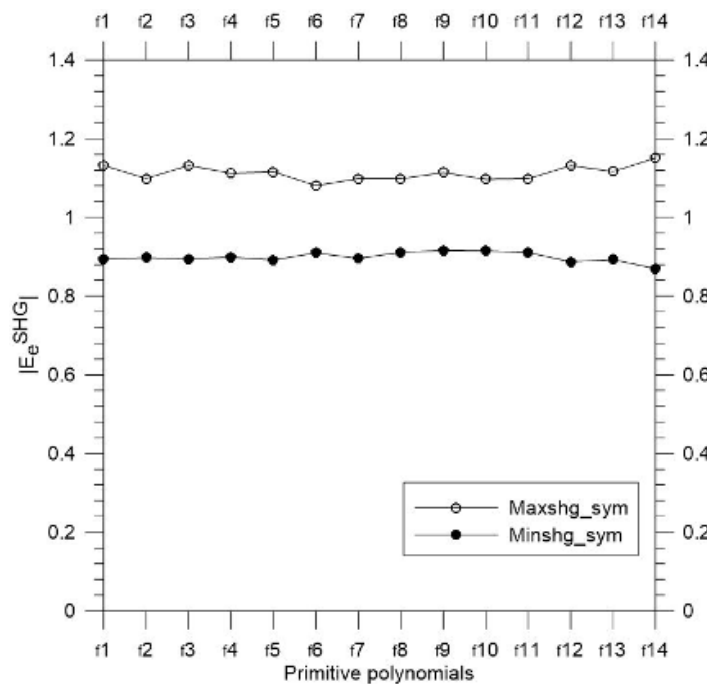


Figure 2. The maximum and minimum CR^{SHG} ratios of $e = \{c | 1 | \delta \tilde{c}\}$ with c m -sequences generated by $f_1 - f_{15}$.

3) Given an m -sequence c , $e = \{c | 1 | \tilde{c}\}$, and $e = \{c | 1 | -\tilde{c}\}$, yield different

CR^{SHG} ratios like Legendre sequences? Our numerical experiments have shown that opposite to the case in Legendre sequences, the CR^{SHG} values for $e = \{c | 1 | \pm \tilde{c}\}$ are not significantly different. And we have obtained the similar conclusion to n values other than 10.

4. Conclusions

In this paper we have studied the SHG contrast ratios on several well-known families of binary sequences. Here is a summary of all the results presented in previous sections:

- For $e = \{c | \delta \tilde{c}\}$, $\delta = \pm 1$ of even length $2N$, the central contrast ratio $CR_e^{SHG} = O(N)$ when N is large.
- For $e = \{\alpha | \delta | \delta \tilde{\alpha}\}$, where α is the Legendre sequence of odd prime length p , and $\delta = (-1)^{(p-1)/2}$. Then the family of such sequences has the asymptotic central contrast ratio $CR_e^{SHG} \approx 1.2$.
- For $e = \{m | 1 | \pm \tilde{m}\}$, of length $2^{n+1} - 1$, where m is an m -sequence generated by a primitive polynomial of degree n , then at an ideal initial state, the asymptotic $CR_e^{SHG} \approx 1.2$, when $n \rightarrow \infty$.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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