

Natural Convection of MHD over a Vertical Wavy Surface in Presence of Porous Media

Rasha Adel

Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt

Email: roshaadel23@gmail.com

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Abstract

This paper shows the natural convective heat transfer in porous media over the vertical wavy surface and it assumes that the fluid is viscous and in-compressible. This model shows the effects of the inverse of Darcy number. The dimensional partial differential equations are converted into a dimensionless form. The non-linear system of equations is obtained and these equations are solved numerically by the finite difference method. The results are obtained for inverse Darcy number, magnetic parameter, Prandtl number, amplitude of surface, parameter of heat generation and parameter of thermal conductivity, and their effects on the velocity, temperature of the fluid and Nusselt number.

Keywords

Natural Convection, Magnetohydrodynamics (MHD), Porous Media, Wavy Surface

1. Introduction

This paper studies the effects of porous medium of a vertical wavy surface in presence of the magnetic field and heat transfer, and uses a different method of numerical solution to solve the problem (finite difference-fully implicit method). Heat generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity were investigated by Md. Abdul Alim *et al.* [1].

The study of this paper has a wide application in the fields of geology and biology. Many authors presented solutions for this type of flow just like that, mass and Heat transfer in porous media was studied by Vadasz, P. (ed) [2]. Wang, C.Y. [3] investigated the free convection over a vertical stretching surface. Yao [4] studied the natural convection over a vertical complex wavy surface. The natural convection flow of Non-Darcy over a vertical wavy plate embedded in a

non-Newtonian fluid-saturated porous medium was investigated by Hady, F.M., *et al.* [5]. Cheng, P. and Minkowycz, W. J. [6] studied the free convection along a vertical flat plate with application to transfer from a dike embedded in a porous medium. The free convection flow of boundary layer along a vertical surface in a porous medium with Newtonian heating is also investigated by Lesnic, D., Ingham, D. B., *et al.* [7]. The effectiveness of convection heat transfer on a flat plate in porous media for power-law fluids has been investigated by Hady, F. M. and Ibrahim, F. S. [8]. Tashtoush and Al-odat [9] investigated the effect of Magnetic field on fluid flow with a variable heat flux over a wavy surface. The natural convection flow with uniform surface temperature over a vertical curvy surface in presence of heat generation/absorption was investigated by Molla *et al.* [10].

The system of equations will be converted from partial differential equations into dimensionless form. The nonlinear system which is obtained is solved numerically by finite difference (fully implicit method) [11]. The effectiveness of inverse Darcy number, wavy surface amplitude, parameter of magnetic field, Prandtl number, variation of thermal conductivity and heat generation on velocity, temperature and Nusselt number will be obtained.

Formulation of Mathematic

It is considered the free convection boundary layer flow in two-dimensions is steady over vertical wavy surfaces and porous media. It is a viscous incompressible fluid and electrically conducting. There is magnetic field of strength B_0 . The uniform temperature at the surface T_w^* and T_∞^* is the temperature at the fluid, where $T_w^* > T_\infty^*$.

Assume that the surface geometry function $\sigma^*(\bar{x}_1)$ is arbitrary. Consider the equation of the wavy surface could be determined as:

$$\bar{x}_2 = \sigma^*(\bar{x}_1) = a \sin\left(\frac{n\pi\bar{x}_1}{l}\right) \quad (1)$$

whereas \bar{x}_1 is the dimensional axis along the curvy surface, \bar{x}_2 is dimensional axis perpendicular to the curvy surface, l is the distance related to the curvy surface and a is the amplitude of curvy surface. It is shown in a **Figure 1** that the wavy

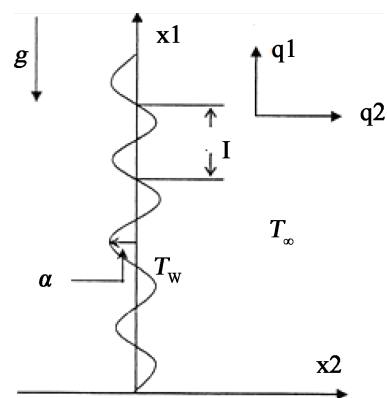


Figure 1. Physical model and coordinate system

surface and the system of Cartesian coordinates. Consider that there is no Joule heat effect and neglect the heat generation, viscous dissipation and body forces of the system.

With introducing Boussinesq approximation, the governing equations are steady, laminar and in two-dimensions. Then the following equations are the continuity equation, momentum equations and energy equation are written as:

$$\frac{\partial \bar{q}_1}{\partial \bar{x}_1} + \frac{\partial \bar{q}_2}{\partial \bar{x}_2} = 0 \tag{2}$$

$$\bar{q}_1 \frac{\partial \bar{q}_1}{\partial \bar{x}_1} + \bar{q}_2 \frac{\partial \bar{q}_1}{\partial \bar{x}_2} = -\frac{1}{\rho^*} \frac{\partial \bar{p}^*}{\partial \bar{x}_1} + \nu^{\otimes} \nabla^2 \bar{q}_1 + g^* \beta^{\otimes} (T^{\otimes} - T_{\infty}^{\otimes}) - \frac{\sigma_o \beta_o^2}{\rho^*} \bar{q}_1 - \frac{\nu^{\otimes}}{k^*} \bar{q}_1 \tag{3}$$

$$\bar{q}_1 \frac{\partial \bar{q}_2}{\partial \bar{x}_1} + \bar{q}_2 \frac{\partial \bar{q}_2}{\partial \bar{x}_2} = -\frac{1}{\rho^*} \frac{\partial \bar{p}^*}{\partial \bar{x}_2} + \nu^{\otimes} \nabla^2 \bar{q}_2 - \frac{\nu^{\otimes}}{k^*} \bar{q}_2 \tag{4}$$

$$\bar{q}_1 \frac{\partial T^{\otimes}}{\partial \bar{x}_1} + \bar{q}_2 \frac{\partial T^{\otimes}}{\partial \bar{x}_2} = -\frac{K^{\oplus}}{\rho^* c_p^*} \nabla^2 T^{\otimes} + \frac{Q_o^* (T^{\otimes} - T_{\infty}^{\otimes})}{\rho^* c_p^*} \tag{5}$$

The boundary conditions

$$\begin{aligned} \text{at } \bar{x}_2 = \bar{x}_{2w} = \bar{\sigma}^* (\bar{x}_1): \bar{q}_1 = 0, \bar{q}_2 = 0, T^{\otimes} = T_w^{\otimes} \\ \text{at } \bar{x}_2 \rightarrow \infty: \bar{q}_1 = 0, T^{\otimes} = T_{\infty}^{\otimes}, \bar{p}^* = p_{\infty}^* \end{aligned} \tag{6}$$

where \bar{q}_1, \bar{q}_2 are the velocity components in direction of \bar{x}_1, \bar{x}_2 respectively, the density ρ^* , \bar{p}^* is the dimensional pressure of fluid, ∇^2 the Laplacian operator, β_o^2 the magnetic induction, σ_o the electrical conduction, β^{\otimes} the thermal expansion coefficient, ν^{\otimes} the kinematic viscosity, g^* the acceleration of gravity, K^{\oplus} the thermal conductivity of fluid, c_p^* the specific heat at the constant pressure, k^* the saturated porous Medium permeability, p_{∞}^* the outside pressure of the boundary layer of the fluid and Q_o^* is the constant of heat generation.

Transformed the curvy surface into a flat surface and boundary layer approximation by Using Prandl's transposition theorem. The dimensionless variables are:

$$\begin{aligned} x_1 = \frac{\bar{x}_1}{l}, \quad x_2 = \frac{\bar{x}_2 - \sigma^*}{l} Gr^{\otimes \frac{1}{4}}, \quad p^* = \frac{l^2}{\rho^* \nu^{\otimes 2}} Gr^{\otimes -1} \bar{p}^* \\ q_1 = \frac{\rho^* l}{\mu^{\otimes}} Gr^{\otimes \frac{1}{2}} \bar{q}_1, \quad q_2 = \frac{\rho^* l}{\mu^{\otimes}} Gr^{\otimes \frac{1}{4}} (\bar{q}_2 - \sigma_{x_1}^* \bar{q}_1), \quad \sigma_{x_1}^* = \frac{d\sigma^*}{dx_1} \\ Gr^{\otimes} = \frac{g^* \beta^{\otimes} (T_w^{\otimes} - T_{\infty}^{\otimes})}{\nu^{\otimes 2}} l^3, \quad \theta^{\otimes} = \frac{T^{\otimes} - T_{\infty}^{\otimes}}{T_w^{\otimes} - T_{\infty}^{\otimes}} \end{aligned} \tag{7}$$

whereas x_1, x_2 are the dimensionless coordinates and q_1, q_2 are the dimensionless velocity components in x_1, x_2 directions, θ^{\otimes} the function of dimensionless temperature, μ^{\otimes} the dynamic viscosity, Gr^{\otimes} the Grashof number and p^* the dimensionless pressure by substituting the previous dimensionless variables into the equations (2)-(5). After ignoring terms of smaller orders of magnitude of Gr^{\otimes} (the Grashof number), the system of equations are ob-

tained, then the governing equations become in form:

$$\frac{\partial q_1}{\partial x_1} + \frac{\partial q_2}{\partial x_2} = 0 \quad (8)$$

$$q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} = -\frac{\partial p^*}{\partial x_1} + Gr^{\otimes/4} \sigma_{x_1}^* \frac{\partial p^*}{\partial x_2} + (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} + \theta^{\otimes} - M^* q_1 - Da^{\oplus-1} q_1 \quad (9)$$

$$\sigma_{x_1}^* \left(q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} \right) = -Gr^{\otimes/4} \frac{\partial p^*}{\partial x_2} + \sigma_{x_1}^* (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} - \sigma_{x_1 x_1}^* q_1^2 - Da^{\oplus-1} \sigma_{x_1}^* q_1 \quad (10)$$

$$q_1 \frac{\partial \theta^{\otimes}}{\partial x_1} + q_2 \frac{\partial \theta^{\otimes}}{\partial x_2} = \frac{1}{Pr^{\oplus}} (\sigma_{x_1}^{*2} + 1) (\gamma^* \theta^{\otimes} + 1) \frac{\partial^2 \theta^{\otimes}}{\partial x_2^2} + q^* \theta^{\otimes} + \frac{1}{Pr^{\oplus}} (\sigma_{x_1}^{*2} + 1) \gamma^* \left(\frac{\partial \theta^{\otimes}}{\partial x_2} \right)^2 \quad (11)$$

whereas γ^* the parameter of thermal conductivity, q^* the parameter of heat generation, M^{\otimes} the magnetic field, Prandtl number is Pr^{\oplus} the and Da^{\oplus} is Darcy number.

By multiplying Equation (10) by $\sigma_{x_1}^*$. The equation becomes:

$$\sigma_{x_1}^{*2} \left(q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} \right) = -\sigma_{x_1}^* Gr^{\otimes/4} \frac{\partial p^*}{\partial x_2} + \sigma_{x_1}^{*2} (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} - \sigma_{x_1}^* \sigma_{x_1 x_1}^* q_1^2 - Da^{\oplus-1} \sigma_{x_1}^{*2} q_1 \quad (12)$$

In this problem, the inviscid flow field is at rest, therefore $\frac{\partial p^*}{\partial x_1} = 0$. By adding the Equations (9), (10), thus can be eliminate $\frac{\partial p^*}{\partial x_2}$ from these equations.

The equation is:

$$q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} = (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} - \frac{\sigma_{x_1}^* \sigma_{x_1 x_1}^*}{\sigma_{x_1}^{*2} + 1} q_1^2 - Da^{\oplus-1} q_1 - \frac{M^*}{\sigma_{x_1}^{*2} + 1} q_1 + \frac{1}{\sigma_{x_1}^{*2} + 1} \theta^{\otimes} \quad (13)$$

The governing equations could be in form:

$$\frac{\partial q_1}{\partial x_1} + \frac{\partial q_2}{\partial x_2} = 0 \quad (14)$$

$$q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} = (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} - \frac{\sigma_{x_1}^* \sigma_{x_1 x_1}^*}{\sigma_{x_1}^{*2} + 1} q_1^2 - Da^{\oplus-1} q_1 - \frac{M^{\otimes}}{\sigma_{x_1}^{*2} + 1} q_1 + \frac{1}{\sigma_{x_1}^{*2} + 1} \theta^{\otimes} \quad (15)$$

$$\begin{aligned}
 & q_1 \frac{\partial \theta^\otimes}{\partial x_1} + q_2 \frac{\partial \theta^\otimes}{\partial x_2} \\
 &= \frac{1}{Pr^\oplus} (\sigma_{x_1}^{*2} + 1) (\gamma^* \theta^\otimes + 1) \frac{\partial^2 \theta^\otimes}{\partial x_2^2} + q^* \theta^\otimes + \frac{1}{Pr^\oplus} (\sigma_{x_1}^{*2} + 1) \gamma^* \left(\frac{\partial \theta^\otimes}{\partial x_2} \right)^2
 \end{aligned} \tag{16}$$

by using the dimensionless formulation (7) in The boundary conditions (6), it becomes in form:

$$\begin{aligned}
 & \text{at } x_2 = 0 : q_1 = 0, q_2 = 0, \theta^\otimes = 1 \\
 & \text{at } x_2 \rightarrow \infty : q_1 = 0, \theta^\otimes = 0
 \end{aligned} \tag{17}$$

the physical quantities such as τ_w is the shearing stress concerning the coefficient of skin friction C_f^* and the rate of heat transfer with reference to Nusselt number Nu^* , their equations:

$$C_f^* = \frac{2\tau_w}{\rho^* U^2} \quad \text{and} \quad Nu^* = \frac{q_w x_1}{k_\infty (T_w^\otimes - T_\infty^\otimes)} \tag{18}$$

By using the dimensionless formulation (7), the coefficient of local skin friction C_f^* and the heat transfer rate with reference to Nusselt number Nu will be like that:

$$C_f^* \left(\frac{Gr^\otimes}{x_1} \right)^{1/4} / 2 = \sqrt{1 + \sigma_{x_1}^{*2}} \frac{\partial^2 q_1}{\partial x_2^2} \tag{19}$$

$$Nu^* (Gr^\otimes / x_1)^{-1/4} = -(1 + \gamma^*) \sqrt{1 + \sigma_{x_1}^{*2}} \frac{\partial \theta^\otimes}{\partial x_2} \tag{20}$$

whereas $\tau_w = (\mu^* \bar{n} \cdot \nabla \bar{q}_1)_{x_2=0}$ and $q_w = -k(\bar{n} \cdot \nabla T^\otimes)_{x_2=0}$.

2. Method of Solution

The Equations (14)-(16) are nonlinear partial differential and boundary conditions (17) can be solved numerically by finite difference fully (implicit method). By using the implicit method The derivatives w.r.t x_1 and x_2 are approximated by using central difference.

For example the momentum equation of q_1 :

$$\begin{aligned}
 & q_1 \frac{\partial q_1}{\partial x_1} + q_2 \frac{\partial q_1}{\partial x_2} \\
 &= (\sigma_{x_1}^{*2} + 1) \frac{\partial^2 q_1}{\partial x_2^2} - \frac{\sigma_{x_1}^* \sigma_{x_1 x_1}^*}{\sigma_{x_1}^{*2} + 1} q_1^2 - Da^{\oplus-1} q_1 - \frac{M^\otimes}{\sigma_{x_1}^{*2} + 1} q_1 + \frac{1}{\sigma_{x_1}^{*2} + 1} \theta^\otimes \\
 & \frac{\theta q_1^{n+1} (q_1^{n+1}{}_{i+1,j} - q_1^{n+1}{}_{i-1,j}) + (1-\theta) q_1^n (q_1^n{}_{i+1,j} - q_1^n{}_{i-1,j})}{2\Delta x_1} \\
 & + \frac{\theta q_2^n (q_1^{n+1}{}_{i,j+1} - q_1^{n+1}{}_{i,j-1}) + (1-\theta) q_2^n (q_1^n{}_{i,j+1} - q_1^n{}_{i,j-1})}{2\Delta x_2} \\
 &= \frac{1}{\Delta x_2^2} \left[\theta \left[(\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n (q_1^{n+1}{}_{i,j+1} - q_1^{n+1}{}_{i,j}) - (\sigma_{x_1}^{*2} + 1)_{i,j-\frac{1}{2}}^n (q_1^{n+1}{}_{i,j} - q_1^{n+1}{}_{i,j-1}) \right] \right]
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 & + (1 - \theta) \left[(\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n (q_{1,i,j+1}^n - q_{1,i,j}^n) - (\sigma_{x_1}^{*2} + 1)_{i,j-\frac{1}{2}}^n (q_{1,i,j}^n - q_{1,i,j-1}^n) \right] \\
 & - \frac{(\sigma_{x_1}^*)_{i,j} (\sigma_{x_1 x_1}^*)_{i,j}}{1 + (\sigma_{x_1}^{*2})_{i,j}} (q_{1,i,j}^n)^2 - Da^{\oplus-1} (q_{1,i,j}^n) - \frac{M^{\otimes}}{1 + (\sigma_{x_1}^{*2})_{i,j}} (q_{1,i,j}^n) + \frac{1}{(\sigma_{x_1}^{*2})_{i,j} + 1} \theta_{i,j}^{\otimes n}
 \end{aligned}$$

In the above, θ^{\otimes} is a weighting factor. Take the value of $\theta^{\otimes} = 0.9$.

This equation will be written in form:

$$\begin{aligned}
 & A_{i,j} q_{1,i-1,j}^{n+1} + B_{i,j} q_{1,i,j-1}^{n+1} + C_{i,j} q_{1,i,j}^{n+1} + D_{i,j} q_{1,i+1,j}^{n+1} + E_{i,j} q_{1,i,j+1}^{n+1} \\
 & = F_{i,j} q_{1,i-1,j}^n + G_{i,j} q_{1,i,j-1}^n + K_{i,j} q_{1,i,j}^n + Q_{i,j} q_{1,i+1,j}^n + R_{i,j} q_{1,i,j+1}^n + O_{i,j}
 \end{aligned}$$

where

$$\begin{aligned}
 A_{i,j} &= \frac{-\theta q_{1,i,j}^{n+1}}{2\Delta x_1} \\
 B_{i,j} &= \frac{-\theta q_{2,i,j}^n}{2\Delta x_2} - \frac{\theta (\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n}{\Delta x_2^2} \\
 C_{i,j} &= \frac{\theta (\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n}{\Delta x_2^2} + \frac{\theta (\sigma_{x_1}^{*2} + 1)_{i,j-\frac{1}{2}}^n}{\Delta x_2^2} \\
 D_{i,j} &= \frac{\theta q_{1,i,j}^{n+1}}{2\Delta x_1} \\
 E_{i,j} &= \frac{\theta q_{2,i,j}^n}{2\Delta x_2} - \frac{\theta (\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n}{\Delta x_2^2} \\
 F_{i,j} &= \frac{(1 - \theta) q_{1,i,j}^n}{2\Delta x_1} \\
 G_{i,j} &= \frac{(1 - \theta) q_{2,i,j}^n}{2\Delta x_2} + \frac{(1 - \theta) (\sigma_{x_1}^{*2} + 1)_{i,j-\frac{1}{2}}^n}{\Delta x_2^2} \\
 K_{i,j} &= \frac{(\theta - 1) (\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n}{\Delta x_2^2} + \frac{(\theta - 1) (\sigma_{x_1}^{*2} + 1)_{i,j-\frac{1}{2}}^n}{\Delta x_2^2} \\
 & - \frac{(\sigma_{x_1}^*)_{i,j} (\sigma_{x_1 x_1}^*)_{i,j}}{1 + (\sigma_{x_1}^{*2})_{i,j}} q_{1,i,j}^n - Da^{\oplus-1} - \frac{M^{\otimes}}{1 + (\sigma_{x_1}^{*2})_{i,j}} \\
 Q_{i,j} &= \frac{(\theta - 1) q_{1,i,j}^n}{2\Delta x_1} \\
 R_{i,j} &= \frac{(\theta - 1) q_{2,i,j}^n}{2\Delta x_2} + \frac{(1 - \theta) (\sigma_{x_1}^{*2} + 1)_{i,j+\frac{1}{2}}^n}{\Delta x_2^2} \\
 O_{i,j} &= \frac{1}{(1 + \sigma_{x_1}^{*2})} \theta_{i,j}^{\otimes n}
 \end{aligned}$$

3. Results

The numerical results are represented by graphs for the velocity q_1 and temperature θ and Nusselt number Nu' . Use the different values of parameters to illustrate their effective on the velocity, temperature and Nusselt number. **Figure 2** reveals the influence of magnetic parameter M^\oplus to velocity profiles q_1 , this figure shows that the increasing of magnetic parameter tends to increase the velocity q_1 at the other parameters are constant as $pr^\oplus = 0.7$, $q^* = 0.1$, $a = 0.2$, $Da^{\oplus-1} = 0.1$ and $\gamma^* = 2$. **Figure 3** indicates to the effective of increase the inverse Darcy number tends to increase velocity q_1 , at $pr^\oplus = 0.7$, $q^* = 0.1$, $a = 0.2$, $M^\oplus = 0.2$ and $\gamma^* = 2$. **Figure 4** shows that the heat generation q^* increase tends to the velocity q_1 increase at $pr^\oplus = 0.7$, $Da^{\oplus-1} = 0.1$, $a = 0.2$, $M^\oplus = 0.2$ and $\gamma^* = 2$. **Figure 5** illustrated that the thermal conductivity variation increase γ tends to the velocity q_1 increase at $pr^\oplus = 0.7$, $Da^{\oplus-1} = 0.1$, $a = 0.2$, $M^\oplus = 0.2$ and $q^* = 0.1$. **Figure 6** indicates

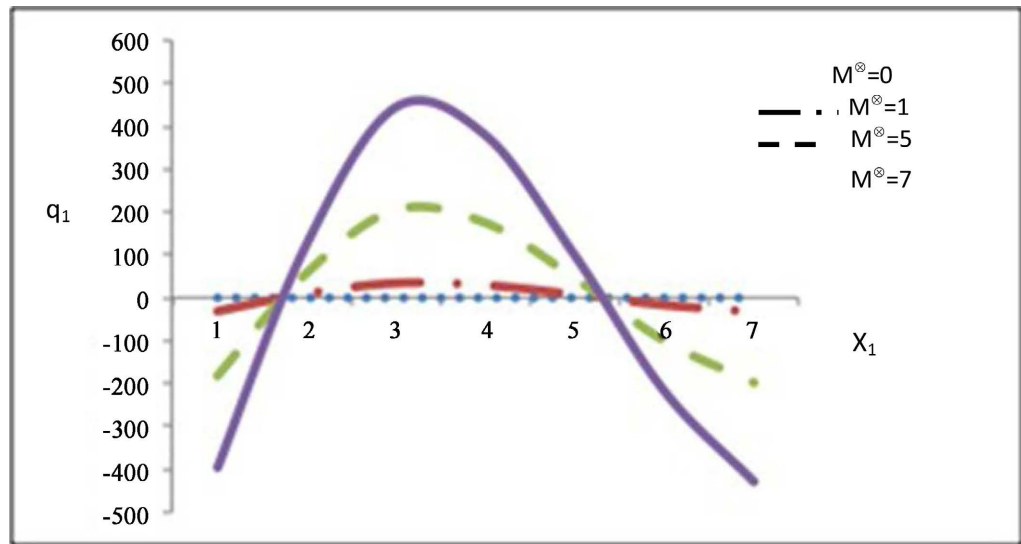


Figure 2. Velocity distribution q_1 for different magnetic parameter M^\oplus .

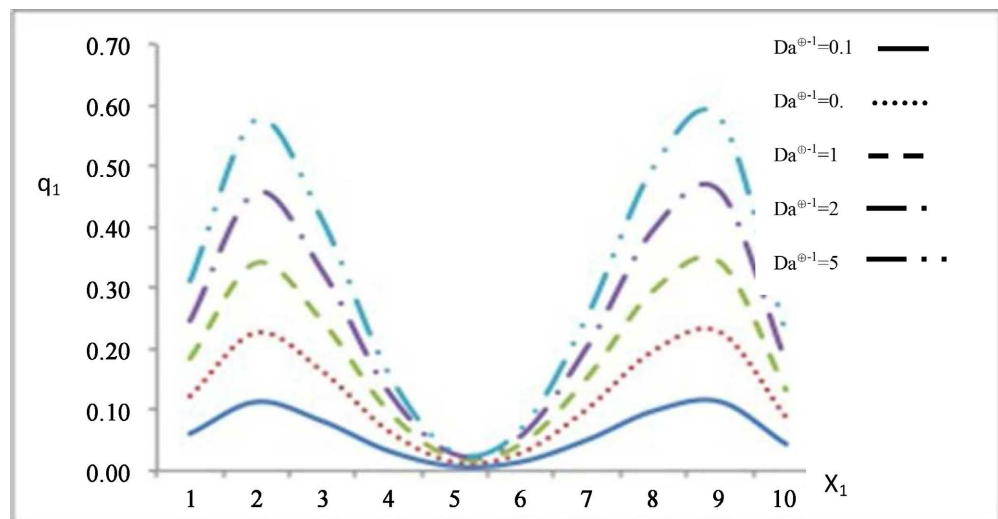


Figure 3. Velocity distribution q_1 with various inverse Darcy number.

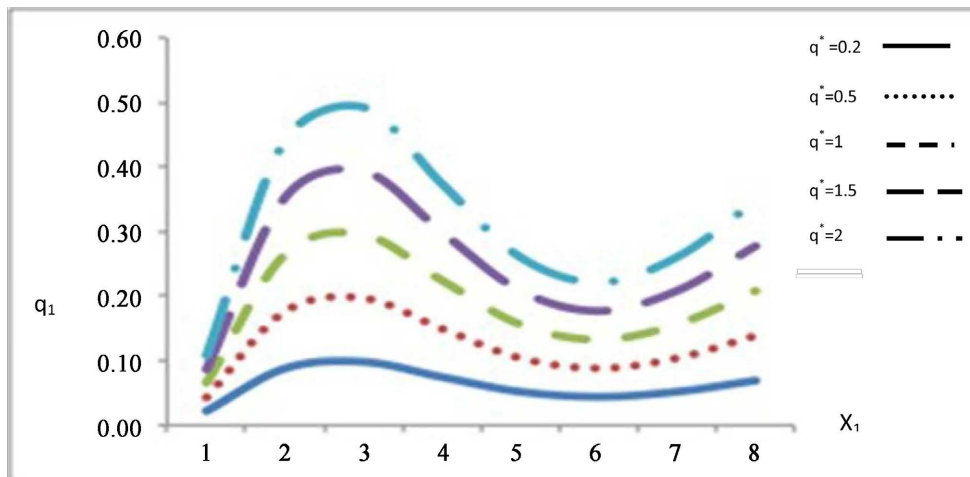


Figure 4. Velocity distribution q_1 for different heat generation parameter q^* .

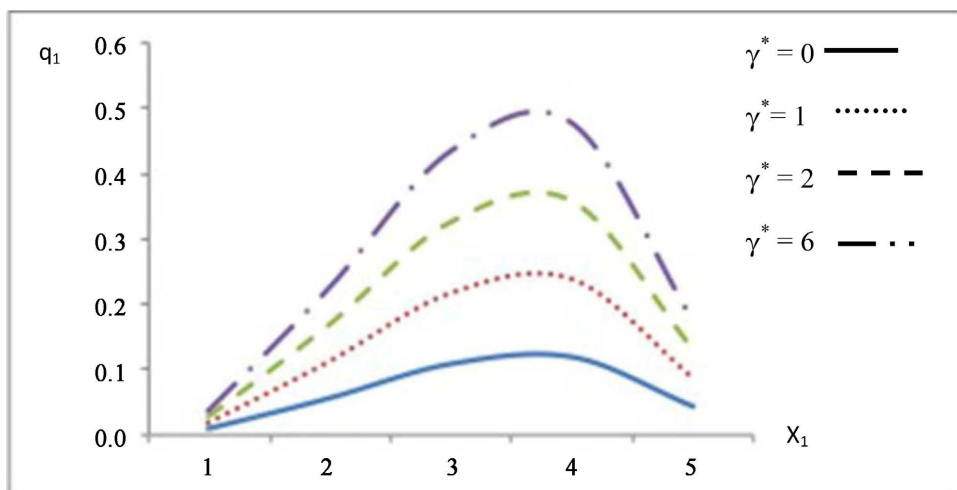


Figure 5. Velocity distribution q_1 for different thermal conductivity variation γ^* .

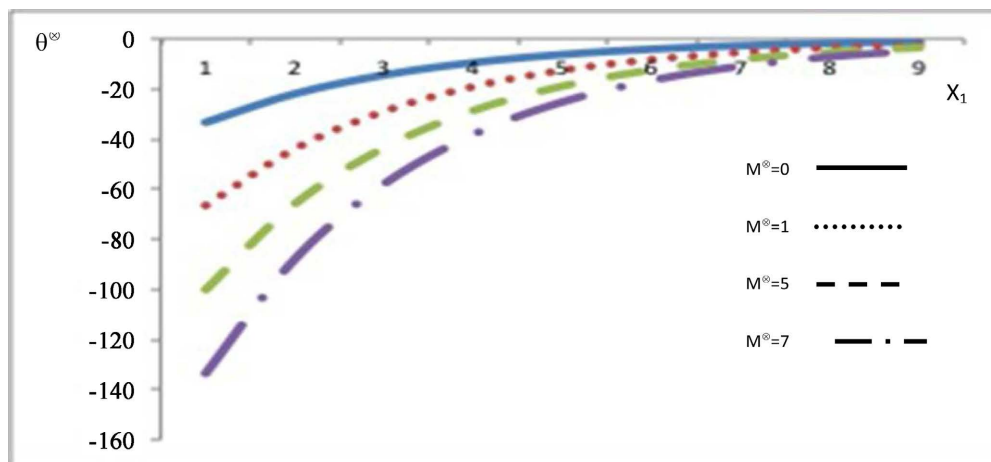


Figure 6. The temperature θ^∞ with various magnetic.

to the influence of increase the magnetic field tends to decrease the temperature θ^∞ , at $pr^\infty = 0.7$, $q^* = 0.1$, $a = 0.2$, $Da^{\infty-1} = 0.1$ and $\gamma^* = 2$. **Figure 7** and **Figure 8**

present the temperature θ decrease with increase both the inverse of Darcy number and prandtle number Pr^\oplus at $q^* = 0.1$, $a = 0.2$, $M^\oplus = 0.2$ and $\gamma^* = 2$. **Figure 9** and **Figure 10** indicate to influence of prandtle number and magnetic field on Nusselt number, the increase of prandtle number and magnetic field parameter tends to increase the Nusselt number Nu^* at $q^* = 0.1$, $a = 0.2$, $Da^{\oplus-1} = 0.1$ and $\gamma^* = 2$. **Figure 11** and **Figure 12** present increase the Nusselt number cause increase both of inverse Darcy number and heat generation parameter q^* at $Pr^\oplus = 0.7$, $a = 0.2$, $M^\oplus = 0.2$ and $\gamma^* = 2$. **Figure 13** shows that the increase in thermal conductivity variation parameter γ tends to increase the Nusselt number Nu^* at $M^\oplus = 0.2$, $Da^{\oplus-1} = 0.1$, $a = 0.2$, $q^* = 0.1$, and $Pr^\oplus = 0.7$.

4. Conclusion

This paper presents the influence of porous medium wavy vertical surface of

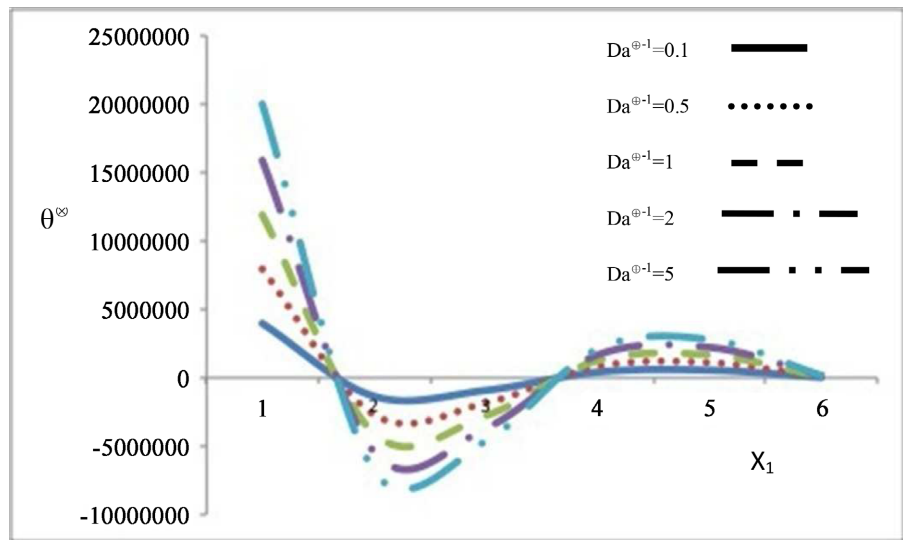


Figure 7. The temperature θ^∞ with various inverse Darcy number $Da^{\oplus-1}$.

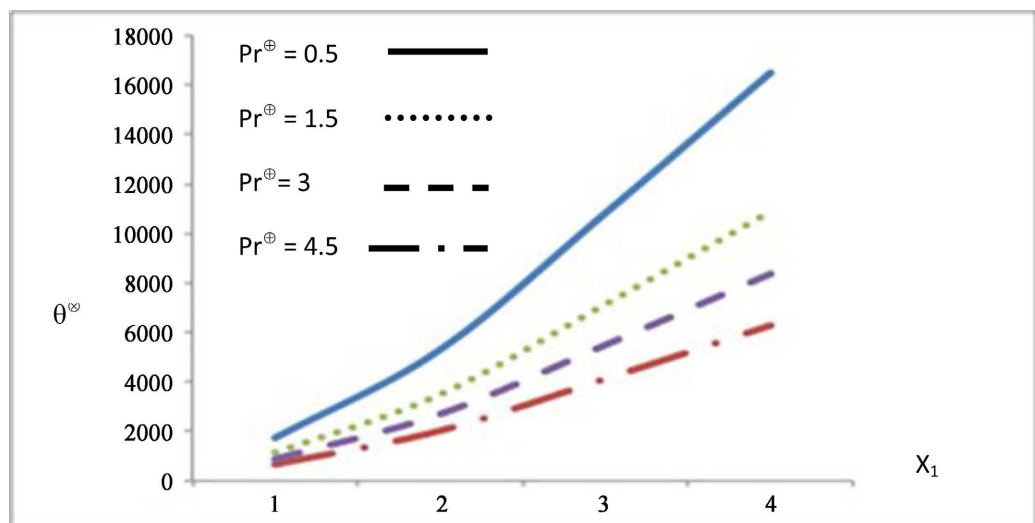


Figure 8. The temperature θ^∞ for different prandtle number Pr^\oplus .

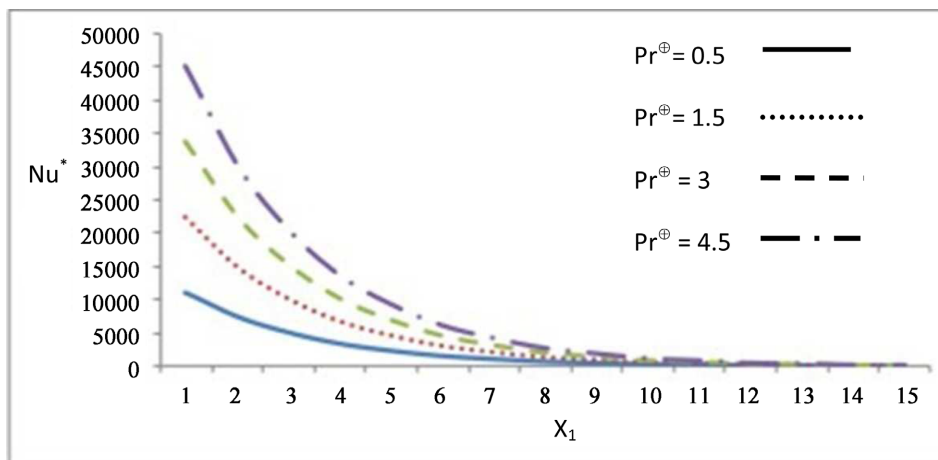


Figure 9. Nusselt number with various prandtle number.

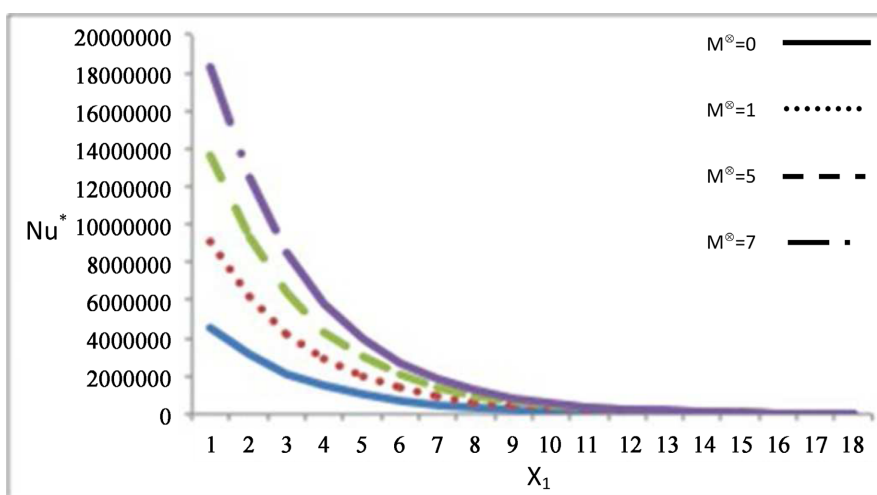


Figure 10. Nusselt number with various magnetic parameter.

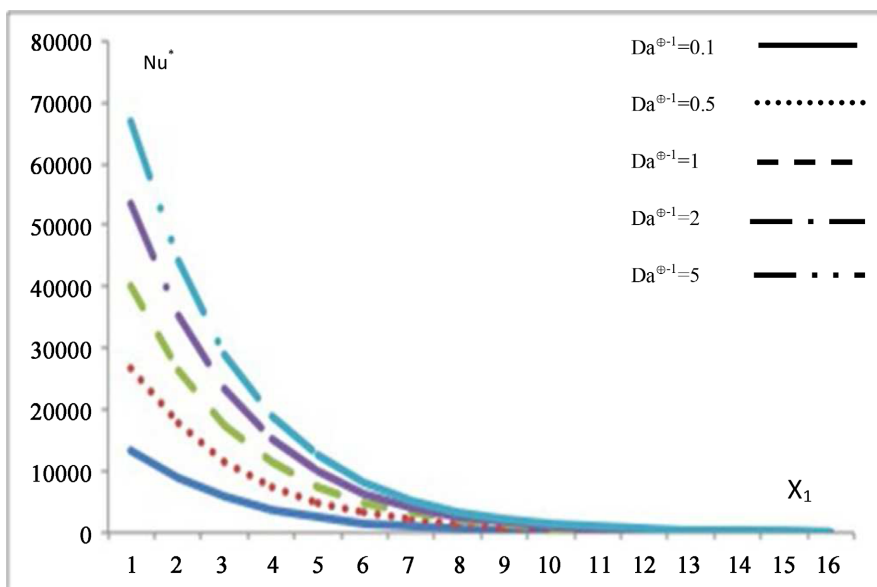


Figure 11. Nusselt number with various inverse Darcy number.

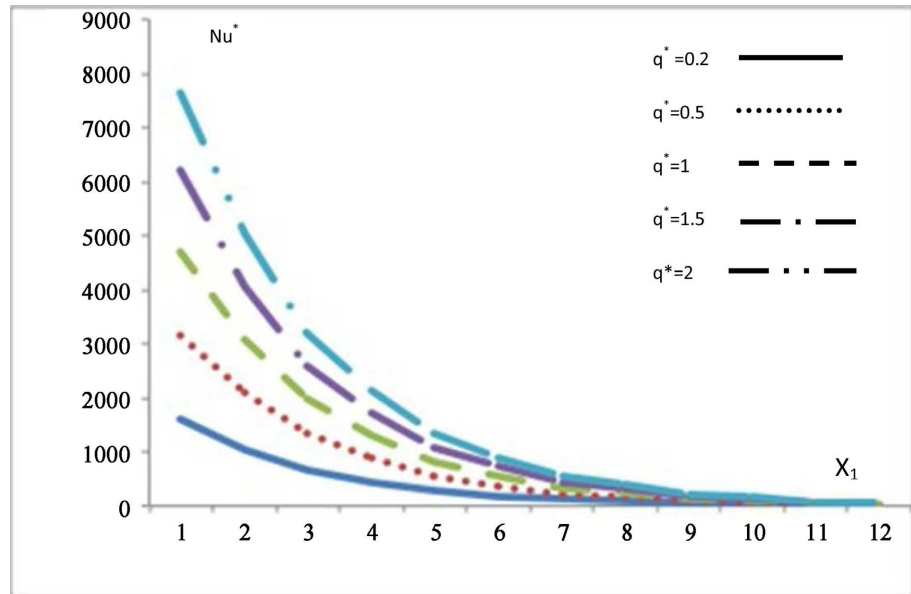


Figure 12. Nusselt number with various heat generation.

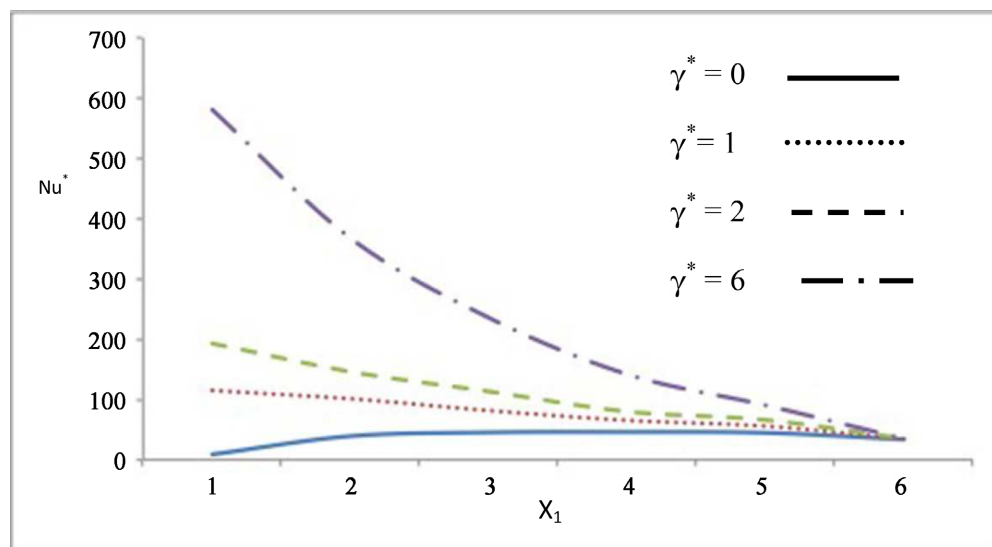


Figure 13. Nusselt number with various thermal conductivity.

natural convection flow with thermal conductivity variable. The system of dimensional partial differential equations is turned into equations of partial differential whose non-linear, they are solved numerically by using finite difference (fully implicit method). This paper has a new result of influence of inverse Darcy number with the velocity, temperature and Nusselt number. The results have been obtained to velocity, temperature and Nusselt number with various parameters, such as magnetic field, Prandtl number, amplitude of wavelength, heat generation parameter and thermal conductivity variation parameter.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Nomenclature

a	the dimensionless amplitude of the wavy surface
B_o	magnetic induction
M^*	magnetic field parameter
l	characteristic reference wave-length
Nu^*	local Nusselt number
\bar{p}^*	the dimensional pressure of fluid
Pr^{\oplus}	Prandtl number
Gr^{\otimes}	Grashof number
σ_o	electrical conductivity
g^*	gravitational acceleration
T^{\otimes}	temperature
q_1, q_2	axial and normal dimensionless velocity Components, respectively
\bar{q}_1, \bar{q}_2	the velocity components along
(\bar{x}_1, \bar{x}_2)	
x_1, x_2	dimensionless coordinates
\bar{x}_1, \bar{x}_2	dimensional coordinates
Da^{\oplus}	Darcy number
p^*	the dimensionless pressure.
q^*	heat generation parameter
Q_o^*	heat generation constant
c_p^*	specific heat at constant pressure
k^*	permeability of the saturated permeable Medium
K^{\oplus}	thermal conductivity

Greek Symbol

ν^{\otimes}	kinematics viscosity
ρ^*	density
$\sigma^*(\bar{x}_1)$	surface geometry function
θ^{\otimes}	dimensionless temperature
μ^{\otimes}	the dynamic viscosity
β^{\otimes}	coefficient of thermal expansion
γ^*	thermal conductivity variation parameter
τ_w	the shearing stress

Superscripts

-	dimensional quantity
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Subscripts

w	wall surface
∞	free stream
x_1	derivative with respect to x