Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2022, 10, 2649-2689
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

The Maximum Ordinality Principle (MOP) and Its Formal Language, *Namely* the "*Incipient*" Differential Calculus (IDC), *Open*, and at the Same Time *Offer*, A Radically New Perspective to Modern Science

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How to cite this paper: Giannantoni, C. (2022) The Maximum Ordinality Principle (MOP) and Its Formal Language, *Namely* the "*Incipient*" Differential Calculus (IDC), *Open*, and at the Same Time *Offer*, A Radically New Perspective to Modern Science. *Journal of Applied Mathematics and Physics*, **10**, 2649-2689.
https://doi.org/10.4236/jamp.2022.109178

Received: August 3, 2022
Accepted: September 11, 2022
Published: September 14, 2022

Abstract

The aim of the paper, as explicitly indicated by its title, is to show that The Maximum Ordinality Principle (MOP) and its Formal Language, namely The "Incipient" Differential Calculus (IDC), open, and at the same time offer, a Radically New Perspective to Modern Science. The methodology adopted, in this respect, is articulated in two parts: 1) firstly, by recalling the general characteristics of the Self-Organizing Systems, whose behavior is described by the Maximum Ordinality Principle and, at the same time, the reasons for the introduction of a different Formal Language termed as IDC ("Incipient" Differential Calculus); 2) afterwards, by comparing the specific properties of the MOP and IDC, and the Fundamental Differences they introduce in describing the surrounding world, with respect to any Scientific Discipline based on TDC. Self-Organizing Systems, in fact, always show an unexpected "excess" with respect to their phenomenological premises. An "excess" can be termed as Quality (with a capital Q) because it is not a simple "property" of a given phenomenon. In fact, it is never reducible to the usual categories of the Traditional Science: efficient causality, logical necessity, functional relationships. Consequently, the description of such an "Emerging Quality" requires new mental categories (Generative Causality, Adherent Logic, Ordinal Relationships) and, correspondingly, a new Formal Language, termed IDC. The result of such a comparison is that the New Scientific Perspective, in spite of the different ways of describing Physical, Biological and Human System, always remains, by itself, a "com-possible" option with respect to the Traditional one. The two Perspectives, in fact, are not in contrast between them. They

simply "co-exist", because of the absence of Perfect Induction in their corresponding Logics. However, in spite of the fact that the two Perspectives are "com-possible" in principle, their differences become significantly "marked" at the operative level, as shown by the several study cases, analyzed in the second part of the Paper, in the light of a General Methodology for Strategic Decisions based on both MOP and IDC. The conclusions thus would (apparently) be that TDC and IDC can be operatively adopted only separately (or, at the most, both "in parallel") in order to research for the optimal solutions, according to their respective operative validity. Without forgetting, however, that in all cases: 1) TDC "reflects" the presuppositions of the Traditional Scientific Approach, so that, by itself, it is "self-referential"; 2) IDC, vice versa, "reflects" the presuppositions of the New Scientific Approach, so that, by itself, it is "hetero-referential". In reality, the paper also shows that there exists an ulterior possibility, which represents an Over-Conclusion. In fact, in adherence to the General Methodology for Strategic Decisions presented in the paper, TDC and IDC can be "seen" as generating a Unique Feed-Back Logical Process. Like "Two Brothers", that cooperate between them. Consequently, and surprisingly, TDC and IDC "reveal" that, in the context of the abovementioned Logical Structure, they are "nothing but" a unique "casket" of "precious pearls".

Keywords

IDC, TDC, Theoretical Com-Possibility, Operative In-Equivalence, Harmonious Over-Ordinal Com-Possibility

1. Introduction

In order to show the various aspects anticipated in the Abstract, we will first present the Fundamental Characteristics of the Traditional Scientific Approach and, in parallel, those pertaining to the New Scientific Approach, based on both the *Maximum Ordinality Principle* (MOP) and the correlative *"Incipient" Differential Calculus* (IDC), which, as already anticipated, originate from the analysis of Self-Organizing Systems.

2. Fundamental Characteristics of the Two Scientific Approaches

These characteristics, which will be illustrated and analyzed in more detail in the contest of the paper, are here synthetically shown, *in parallel*, in Table 1. A synoptic picture that, *at the same time*, represents *both* the *structure* and the *synthesis* of the paper.

2.1. Fundamental Characteristics of the Traditional Scientific Approach

Modern Science is characterized by a persistent and progressively ascendancy

Table 1. Synoptic comparison between the fundamental characteristics of the two Scientific Approaches.

Basic Presuppositions	***"Emerging Quality"* of Self-Organizing Systems**
1) causality principle (efficient causality) 2) classical logic (necessary logic) 3) functional relationships	1') Generative Causality 2') Adherent Logic 3') Ordinal Relationships

d/dt is the corresponding formal translation $f(t)$ represents a *functional relationship*	**Development of an appropriate Language** - L. Boltzmann, A. Lotka - H. T. Odum: Emergy Algebra and MEmPP - Mathematical Formulation of the MEmPP - Introduction of the "Incipient" derivative $\tilde{d}/\tilde{d}t$ - Re-proposition of the MEmPP as the MOP

- Thermodynamic Principles (1st , 2nd, 3rd) **- Physical Laws (specific for each Discipline)** **Every System is a "Mechanism"** **Hypotheses** ↓ **Mathematical Formalization** ↓ **Conclusions** ↓ **Confirmation by experimental results**	**The Maximum Ordinality Principle** **- is applicable to *any* Field of analysis: *non-living* Systems, *living* Systems, "*thinking*" Systems (e.g. Human Systems)** **- at *any space-time scale* and in *variable conditions*** **- it also offers a *more appropriate* description of any given System and its surrounding habitat** **Every System is a "Self-Organizing System" as the Exit of its *Specific Generative* Process**

toward ever more general Physical Laws and Principles.

However, before any formulation of a single hypothesis or a physical theory, Modern Science (let us say, from Newton on) adopts three fundamental *pre-suppositions* (see Table 1): the *causality principle* (also termed as "efficient causality"), *classical logic* (also termed as "necessary logic"), and *functional relationships* (between the various parts of any System analyzed).

On the basis of such fundamental presuppositions, and only after having developed *a strictly conform* consequential *Formal Language*, that is the *Traditional Differential Calculus* (TDC), Modern Science progressively ascends toward ever more general Physical Laws and Principles:

1) from Phenomenological Laws (e.g. Kepler's Laws); 2) to Physical Laws specific of each Discipline (e.g. Newton's Laws, Maxwell's Equations, etc.); 3) up to the three well-known Thermodynamic Principles.

Such a progressive development has given origin to a hierarchy of a *multiplicity* of *quantitative* Physical Laws and Principles, in particular as a consequence of the first basic presupposition: the *causality principle*. This Principle, in fact, has led Modern Science to introduce "different causes" in different Disciplines. The Principle of causality, in fact, tends to "sub-divide" the entire phenomenology (at present known) in different "branches", precisely because, on the basis of such a presupposition, it leads Scientists to research for the most "appropriate causes" pertaining each specific set of phenomena each time considered.

In this way, Modern Science persistently propends to show that: "*Every System is a mechanism*". *Especially because of its Formal Language adopted*.

Such a conclusion, however, although *confirmed by experimental results*, can be considered as being valid *only* from an *operative* point of view, but not from an *absolute point of view*. This is because "necessary logic" (adopted as a second basic presupposition) does not admit any form of "*Perfect Induction*" (see Popper's *Falsification Principle*).

In fact, as synthetically illustrated in Table 1, in the strict contest of "necessary logic":

1) after having formulated a single or more hypotheses (such as in the case of a Theory);

2) after having formalized them in *the Formal Language adopted*, faithfully *conform* to the three above-mentioned basic presuppositions;

3) after having drawn the consequential *formal* conclusions in terms of "necessary" logic;

4) and after having also obtained experimental confirmations of the previous formal conclusions;

5) it is impossible, *in any case whatsoever*, to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* and *unique* hypotheses capable to explain those experimental results.

This is precisely because of the *absence*, in "*necessary*" logic, of any form of *perfect induction*.

In fact, only in the presence of a *perfect induction* it would be possible to assure the *uniqueness* of the *inverse* process and, thus, to transform the adopted hypotheses into an *absolute* perspective.

This means that Modern Science, precisely because based on *necessary logic*, should always be "open" to recognize that *there always exist* many other *possible* Approaches (in principle *infinite*) capable to interpret the same experimental results.

At this stage, after having synthetically recalled the basic characteristics of Modern Science, we can now analyze in more detail the fundamental properties of the New Perspective, synthetically indicated in parallel (for a better comparison) in the right hand side of Table 1.

2.2. Fundamental Characteristics of the New Scientific Approach

The best way of illustrating such a New Scientific Approach is that of recalling, step by step, its *Process of Genesis*, synthetically summarized in the second "box" in the right column of Table 1.

Such a Scientific Approach, as already anticipated, originates from the analysis of Self-Organizing Systems.

These Systems and their "emerging properties" began to be studied by L. Boltzmann toward the end of XIX century. Several other Authors (e.g. A. Lotka) successively dealt with such a theme.

L. Boltzmann was in fact the first who attempted at describing Self-Organizing Systems in more appropriate *formal* terms, by proposing the adoption of a new

Thermodynamic Principle: "The Principle of Maximum Exergy *Inflow* to the System" [1].Some years later, A. Lotka reformulated such a Principle in the form of: "The Principle of Maximum Exergy *Flow through* the System" [2] [3] [4].

Both such attempts, however, were not perfectly successful, because still based on the concept of Exergy, which is a quantity that is strictly pertaining to Classical Thermodynamics. Consequently, it re-proposes the concepts of *efficient causality*, *logical necessity*, *functional relationships*, which are not properly adequate to represent the "*Emerging Quality of Self-Organizing Systems*".

Such an expression, in fact, wants to indicate that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So that they usually say: "*The Whole is much more than its parts*".

This is precisely the reason why such an "excess" can be termed as *Quality*, with a capital *Q*, to point out that it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*.

This evidently suggests a *radically new gnosiological* perspective, which corresponds to recognize that: *There are processes*, *in Nature*, *which cannot be considered as being pure* "*mechanisms*".

This also leads, *in adherence*, to the adoption of "*new mental categories*"[1] and, correspondently, to the development of a completely *new formal language*, so that the description of Self-Organizing Systems might result as being faithfully conform to their "Emerging Quality".

A really *new formal language* only appears with H. T. Odum, with the genial introduction of Emergy (*Em*), defined as Exergy (*Ex*) by Transformity (*Tr*) [5] [6] [7]

$$Em = Ex \cdot Tr \tag{1}$$

Such a definition clearly shows that Emergy is *still* based on "Exergy". However:

1) *Quality Factor Tr* "*Transforms*" *Ex* into a *new physical quantity*: Emergy.

2) The latter in fact is not defined in "functional terms", but only by "*assignation Rules*" [8].

3) This is precisely because *Tr* is expressed by means of a *non-conservative Algebra* (ib.).

4) Thus the output "excess" of the three Fundamental Processes (Co-Production, Inter-Action, Feed-Back) is always understood as being "irreducible" to its cor-

[1]These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are chosen only "a posteriori", on the basis of the "Emerging Quality" previously recognized. "*Generative Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Adherent Logic*", correspondently, refers to the capacity of our mind to draw "*emerging conclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, which will be illustrated in more details later on, in the case of a Co-Production Process.

relative inputs in *mere functional terms*.

This means that Emergy is able to represent the "Emerging Quality" of Self-Organizing *Processes*.

This is why Prof. Odum, on the basis of such a concept of Emergy, enunciated the Maximum Em-Power Principle [5] [6] [7], which asserts that: "*All self-organizing systems tend to maximize their rates of Emergy use or Empower, and those systems that maximize Empower will prevail*" (Odum, 1988, Odum, 1996) [9] [10].

As a direct corresponding consequence, Prof. Odum proposed such a Principle as the *Fourth Thermodynamic Principle*, to be understood, however, in terms of Thermodynamics of *Quality*.

This is precisely because the general enunciation of the *Maximum Em-Power Principle*, given at a phenomenological level, can directly be referred, as a *corresponding tendency*, to the "*Emerging Quality*" of *Self-Organizing Systems*.

However, in spite of such valid bases, the Maximum Em-Power Principle had not received a Mathematical Formulation *corresponding to its general enunciation*, and this, for years, has prevented from considering such a Principle as a possible Fourth Thermodynamic Principle "*of Quality*", as Prof. Odum usually asserted.

On the other hand, *such a Mathematical Formulation could not be given in terms of the Traditional Differential Calculus*, because *traditional derivatives*, as a consequence of their conceptual basic presuppositions (see **Table 1**), are not properly apt at representing the "*generative*" behavior on behalf of "Self-Organizing Systems".

At this stage, it is then worth synthetically recalling, in the next paragraphs, the *logical and historical* steps of the process that led us to the Mathematical Formulation of the Maximum Em-Power Principle (MEmPP) and, some years later, to its *more general* Mathematical Formulation, as the "Maximum Ordinality Principle" (MOP), substantially based on the introduction of a *new concept of Derivative*: the "*Incipient*" *Derivative*.

3. Historical Presuppositions of the Mathematical Formulation of the Maximum Em-Power Principle

As a simple introduction I would like to recall that I "became aware" of the "existence" of Self-Organizing Systems *only* in 1993, by reading a scientific paper prepared for an official publication[2], sent to me by Prof. Sergio Ulgiati (University of Siena).

In fact, during my personal scientific training as a Nuclear Engineer, I had never heard of "Self-Organizing Systems" and, contextually, of the possibility of their description by means of a new physical quantity termed as "Emergy".

Nonetheless, I was *favorably surprised* of that and, during the following months

[2]Ulgiati S., Odum H. T., Bastianoni S. *Emergy use, environmental loading and sustainability. An emergy analysis of Italy*. Ecological Modeling 73 (1994) 215-268 (received 9 December 1992, accepted 10 August 1993).

I had several email contacts with Prof. Sergio Ulgiati (first author of the paper), whom I continuously asked for additional documents to better understand the *logical and physical formal origin* of such a new physical concept, its rigorous definition, together with the concept of Self-Organizing Systems.

In this respect, I want to thank Prof. Ulgiati for his willingness to deal with such a fastidious interlocutor.

In fact, what first impressed me was *the special Algebra* adopted (termed as "*Emergy Algebra*"), which was *very different* from the traditional one, but at the same time so "similar" (in a certain sense) to the *Differential Fractional Calculus* I had begun to study six years before. In fact, the first time I met Prof. Ulgiati, I did not miss the opportunity of asking him: "Why don't you adopt Fractional Calculus in Emergy Analysis?"

Some weeks later Prof. Ulgiati invited me to attend a special course on Emergy Analysis given by Prof. Mark T. Brown in Siena (September 1993), during which I had the opportunity of meeting another extremely important person in my life. My collaboration with Prof. Brown, in fact, started precisely there, in Siena: together we prepared a general scheme which reproduced, by means of the special *System Diagrams* adopted in Emergy Analysis, the Italian Energy Supply System in terms of Thermodynamics, Economics and Rights (Norms and Laws), which was successively presented at the International Workshop of Porto Venere in 1998 [11].

I must also thank Prof. Brown, in a particular way, because he encouraged my attempts at introducing the "*mathematically Equivalent Source Terms*" in Emergy Algebra, which represented one of the fundamental steps for the successive mathematical formulation of the Maximum Em-Power Principle.

In May 24, 1995 I personally met Prof. Howard T. Odum, who had been invited by our ENEA's Energy Savings Division to give two Lectures (in reality *two splendid* Lectures) at ENEA's Headquarters. On that occasion Prof. Odum was so kind as to spend the whole following day with me (in the presence of his Lady Prof. Elisabeth Odum and Prof. Ulgiati), answering my questions, dissolving my doubts, and suggesting possible new lines of research.

I cannot fail to mention that, among other things, he also gave me five of his most famous books as a present.

At the end of that day, after having experienced such a profound willingness shown by Prof. Odum, I frankly expressed my *perplexity* about the Maximum Em-Power Principle when termed as "Thermodynamic" Principle. In fact, I observed, without a general Mathematical Formulation of such a Principle it is rather difficult to decide whether the Maximum Em-Power Principle is a "Thermodynamic" Principle or not.

Prof. Odum, with a sweet and delicate smile, promptly answered: "You should do it". "In fact I agree with you, and I believe you can succeed in this task".

That hope, so openly manifested by Prof. Odum, indelibly marked the "*official birth*" of my research for a Mathematical Formulation of the Maximum

Em-Power Principle.

For years, in fact, his paternal look encouraged me in this difficult task. His faithful invitation sustained me especially when the major difficulties arose, during the successive stages of the formulation.

Thank you Prof. Odum. Such a Mathematical Formulation is more your merit than mine. Without your sweet smile and your paternal trusting invitation, I would have never found the courage to face such a hard task.

4. Mathematical Formulation of the Maximum Em-Power Principle

The Mathematical Formulation of the Maximum Em-Power Principle was given in 2001, and it was contextually presented at the 2nd Biennial Emergy Conference (September 20-22, 2001), [12].

The Mathematical Formulation is here synthetically reproduced

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3 V = \frac{\mathrm{d}}{\mathrm{d}t} \int_{D^*(t)} em_v^* d_3 V \to Max\,, \quad \forall D^*(t) \subseteq S_U(t) \tag{2}$$

which faithfully translates in formal terms its corresponding verbal enunciation: "*Every System tends to organize its internal structure to generate progressively increasing spring-Emergy levels in order to maximize the flow of processed* (*or "useful"*) *Emergy*".

In Equation (2) in fact:

φ_v^* = the "*equivalent*" Source Term per unit volume.

Γ = the *local* structural amplification and re-normalization factor (corresponding to the product of the coefficients γ_k^* and γ_k in the case of discrete form), which also accounts for the structural variations with time.

$$em_v^* = em_{v,m} + em_{v,q}^+ + em_{v,w}^+ \tag{3}$$

where

$em_{v,m} = C \cdot \rho \cdot ex$ is the Emergy per unit volume associated to the *mass* (thus transportable by mass flows).

$em_{v,q}^+$ = the Emergy per unit volume associated to *heat* source terms.

$em_{v,w}^+$ = the Emergy per unit volume associated to *work* source terms.

In this way, the formulation is valid for *any Domain* (D^*) belonging to *Universal Space* ($S_U(t)$).

Additional details about such a formulation can be found in the paper presented at the 2nd Biennial Emergy Conference [ib.], which is also directly available on the Author's website http://www.ordinality.org/ and, even more, in the book titled "*The Maximum Em-Power Principle as the basis for Thermodynamics of Quality*" [13].

In fact, at the end of the 2nd Emergy Conference, Prof. Odum, who had appreciated such a formulation, invited me to write a book, that was effectively published the year after (2002).

In the preface of this book Prof. Mark Brown, after having pointed out the ba-

sic characteristics of the book and the correlative fundamental results achieved, explicitly mention two aspects:

a) "The Center for Environmental Policy, through a generous gift from H.T. Odum has made funds available to help with the book's publication"; b) "Unfortunately, Odum died on September 11, 2002, before the publication of this book. During his last months, we discussed this work, and his feeling was that it represents a major contribution to the field, and that it was a very important next step in the formulation of his life work".

5. The Fundamental Results Presented in the Book on the Maximum Em-Power Principle

The Fundamental results presented in the book can be synthesized as follows:

- The *First Principle of Thermodynamics*, *The Second Principle of Thermodynamics*, *The Third Principle of Thermodynamics*, together with *The Minimum Action Principle* have been reconsidered in the light of The Maximum Em-Power Principle.
- Such a Process has clearly shown that all these Principles can be "obtained" from The Maximum Em-Power Principle. However, not by "*deduction*", but only by "*reduction*".

These results thus *confirm*, *sustain* and *reinforce* Odum's assertion that The Maximum Em-Power Principle has to be considered as The Fourth Thermodynamic Principle. However, not in terms of *quantity*, but in terms of *Quality*.

6. Subsequent Researches on the Maximum Em-Power Principle in the Years 2003-2009

All the researches pertaining to this period were essentially devoted to *two different lines of research*, analyzed in parallel, however strictly related:

A. The *Relevance* of The Maximum Em-Power Principle in *several scientific fields*.

B. The research for *a more general formulation* of the Rules of Emergy Algebra as *Generative Rules*.

As far as the results pertaining to point A), apart from that published by Applied Mathematics [14], and those presented at the various Biennial Emergy Conferences [15] [16] [17], they were synthesized in two books (in Italian), titled "*Lightness of Quality*" (2007) [18] and "*Ascendency of Quality*" (2008) [19], respectively.

The first book presented the novelties introduced by The Maximum Em-Power Principle in the various Scientific Felds listed at column 1 of the following **Table 2**, while the second book presented the correlative novelties in the fields listed at column 2.

At the same time, *in parallel* (sometimes *also contextually*) to the abovementioned line of research, a particular attention was persistently devoted to a possible

Table 2. Scientific Fields in which the MEmPP shows its Relevance.

"Lightness" of Quality	"Ascendency" of Quality
Classical Mechanics	Electromagnetism
Relativistic Mechanics	Inorganic Chemistry
Quantum Mechanics	Organic Chemistry
	Biological Chemistry
	Molecular Biology
	Ontogenetic Biology
	Phylogenetic Biology
	Cosmology

improvement of the *Formal Language* adopted in the MEmPP, as it will be illustrated in the next paragraph.

7. The Research for a More Appropriate "*Formal Language*" in the Context of the MEmPP

Such a research was mainly focused on the cases in which The Maximum Em-Power Principle is adopted in order to describe a given System *under transient conditions* (see previous References [14] [15] [16] [17]).

This is because Emergy is an "*Emerging Property*" of Self-Organizing Systems. Consequently, when the System is analyzed under steady state conditions, the Rules of Emergy Algebra are faithfully accounted for by the Mathematical Formulation of the Maximum Em-Power Principle.

Under transient conditions, however, the adoption of the Traditional Differential Calculus (TDC) tends to "filter", in *a more or less* marked measure, the "Emerging Properties" of the System described by the concept of Emergy.

This is because the correlative *Non-Conservative* Algebra manifests that "Emergy" is an "*Emerging Property*" of Self-Organizing Systems, which cannot be expressed in terms of the traditional mental categories, typical of the Traditional Differential Calculus (TDC): *efficient causality*, *logical necessity*, *functional relationships*. Consequently, the "*Emerging Properties*" of the System cannot be described in terms of the *traditional derivatives* that, at the level of *formal language*, represent the *perfect reflex* of such "a priori" mental categories.

This evidently suggests the adoption of a *Different Formal Language*, so that the description of Self-Organizing Systems, by means of the Mathematical Formulation of the Maximum Em-Power Principle, could possibly result as being faithfully conform to their "Emerging Quality".

This is why the adoption of *a new concept of derivative*, the "*Incipient Derivative*", already developed during a couple of years before, as a *possible contribution* to a *further development* of the *Fractional Calculus* [20] [21] [22] and, for completeness, contextually presented in Appendix 9 of the book devoted to Mathematical Formulation of the Maximum Ordinality Principle [13].

The introduction of a *new concept* of derivative, in fact, is suggested by *the same formulation* (2). This is because the presence (in it) of the sign (=) and that of the symbol of the *traditional first order derivative*, ends up by "describing" a "*necessary*" *relationship* between "efficient causes" (first member) and "consequential effects" (second member), while, in a context of Quality, we should only speak of "*generative causes*" and consequential "*adherent effects*".

This suggests that the same formulation (2) should *be thought of* as better written as follows

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3 V \overset{\rightarrow}{=} \frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t} \int_{D^*(t)} em_v^* d_3 V \to Max\,, \quad \forall D^*(t) \subseteq S_U(t) \tag{4}$$

where the symbol $\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}$ *represents the already mentioned "incipient" derivative and the symbol "*$\overset{\rightarrow}{=}$*" represents either a* logical *consequence* (*second side*) which is "adherent" *to its premises* (*first side*) *or* physical effects (*second side*) *which are* "adherent" *to their* "spring" causes (*first side*), *or both*.

Such a *new formulation* does not change, *by itself*, the *quantitative* aspects pertaining to the M. Em-P. Principle as formulated in Equation (2), because such a Mathematical Formulation only makes use of the *first order* derivative (see next par. 7.1).

In reality, such a *formal-logical passage* from an *a posteriori* mathematical description to a mathematical description conceived of as being "*incipient*" (that is capable of "following" the Emerging Quality in its continuous *Process of Genesis*) represents *one of the most important aspects* concerning the Mathematical Formulation of the M. Em-P. Principle.

7.1. Definition of the "Incipient" Derivative of Ordinality $\tilde{q}$

Such a *new concept* of "*Incipient Derivative*" is defined as follows

$$\left(\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{\tilde{q}} f(t) = \widetilde{\lim_{\tilde{\Delta}t:0\to 0^+}} \left(\frac{\tilde{\delta}^{\sim} - 1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \quad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \tag{5}$$

in which the sequence of the symbols is now interpreted according to the *direct priority* of the order of the three elements that constitute its definition (that is *from left to right*).

It is precisely such an *inversion of "reading*" that which *transforms* a "*functional" definition* into a "*generative" definition*.

Definition (5) in fact, as more widely illustrated both in the previously mentioned References [20] [21] [22] and, especially in [14] [15], clearly shows that the "*Incipient Derivative*" is not an "operator", like the derivative ($\mathrm{d}/\mathrm{d}t$), that describes "necessary" processes according to the Traditional Differential Calculus (TDC).

The "*Incipient Derivative*", vice versa, could properly be termed as a "*generator*", because it describes a *Generative Process*, *in its same act of being born*, and

this will represent *the fundamental basis* of the Mathematical Formulation of the Maximum Ordinality Principle [23], dealt with at par. 9.

Consequently:

1) The three symbols adopted *acquire a completely different meaning* with respect to the traditional ones.

2) Mainly because they do not represent "three" distinct operations, but *a unique and sole Generative Process*.

3) The symbol $\widetilde{Lim}$, in fact, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "*threshold*" of that "*ideal window*" from which we observe and describe the considered *Generative Process*, *in its same act of being born*.

4) The symbol $\tilde{\Delta}t:0\to 0^{+}$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) in *its proper act of being born*.

5) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). Thus the ratio which appears in Equation (5) indicates not only a quantitative variation in time, but both the variation in *Quality* and *quantity*.

6) Consequently, when we take the incipient (or "prior") derivative of *Ordinality* $\tilde{q}$ of any $f(t)$, the *generative Exit* of such a process *will keep "memory"* of its *genetic origin* because it will result as being structured according the indication of such an exponent. The latter in fact is properly termed as *Ordinality*, because it precisely expresses "*how each part of the output is* genetically Ordered *to* the Whole *and*, *at the same time*, *how each part is related to all the others in terms of* Ordinal *Relationships*".

7) In this way the "Incipient" Derivative represents the *Generativity* of the considered Process, that is the output "*Excess*" (per unit time) characterized by both its *Ordinality* and its related *cardinality*, while the sequence of the symbols, in definition (Equation (5)), can be interpreted as representing a *unique inter-action process* between *the three correlative concepts*.

8) The above-mentioned aspects clearly show why the "Incipient" Derivative is able to *unify* (and, at the same time, to *specify*) the description of the various Self-Organizing Processes, understood as the *Exit* of their *Generative Nature*.

9) This also means that the "Incipient" Derivative presents an *Exit* that is generally different from the "*result*" of the corresponding derivative in TDC, even when its Ordinality is reduced to *a mere cardinality*. For example, the traditional derivative of order *n* of the function $e^{\alpha(t)}$, evaluated according to Faà di Bruno's Formula [24], and the corresponding "Incipient" Derivative, respectively give

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^{n} e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1!k_2!\cdots k_n!} \cdot \left(\frac{\dot{\alpha}}{1!}\right)^{k_1} \left(\frac{\ddot{\alpha}}{2!}\right)^{k_2} \cdots \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \tag{5.1}$$

and

$$\left(\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{n} \mathrm{e}^{\alpha(t)} \overset{*}{=} \mathrm{e}^{\alpha(t)} \cdot \left[\overset{\circ}{\alpha}(t)\right]^{n} \tag{5.2}$$

where $\overset{\circ}{\alpha}(t)$ represents the *first order* "Incipient" Derivative. And even if they in some cases coincide (for instance when $\alpha(t)$ is linear), such a "*coincidence*" has to be seen in the light of the symbol $\overset{*}{=}$ in Equation (5.2), which reminds us that any "Incipient" Derivative is always the *Exit* of a *Generative* Logical Process and not of a *necessary* logical process.

7.2. The Rules of Emergy Algebra Formulated in Terms of the "Incipient" Derivative

The first *fundamental* attempt at adopting such a new form of Derivative was obviously focused on the possible reformulation of the Rules of Emergy Algebra *in "incipient" differential terms*. That is, in terms of the "Incipient" Derivative, when the latter is characterized by an appropriate Ordinality.

In this case, the Rules of Emergy Algebra can be reformulated as "Generative Rules" in *Transient Conditions* as follows (see [14] [15] [16]), now re-written with reference to the exponential function $\mathrm{e}^{\alpha(t)}$, because they can be referred to *any function* $f(t)$, and thus they can always be written in such an exponential form:

An *Inter-action Process* can be represented by a "*duet*" Ordinal Relationship:

$$\left(\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{\tilde{2}} \mathrm{e}^{\alpha(t)} = \left\{\overset{\circ}{\alpha}_1(t), \overset{\circ}{\alpha}_2(t)\right\} \cdot \mathrm{e}^{\alpha(t)} \tag{6}$$

where the brackets { } stand for *a Whole*. In fact, it is like a "*Singing Duet*", whose "*Exit*" is much more than the "sum" of the contributions of the two singers.

A *Co-production Process* can be represented by a "*binary*" Ordinal Relationship:

$$\left(\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{\frac{1}{\tilde{2}}} \mathrm{e}^{\alpha(t)} = \left\{\begin{matrix}\overset{\circ}{\alpha}_1(t) \\ \overset{\circ}{\alpha}_2(t)\end{matrix}\right\} \cdot \mathrm{e}^{\alpha(t)} \tag{7}$$

It is like the *Generation of "Two Brothers"*, related between them *by Ordinal Relationships* of *Generative Nature*.

In fact, "*brothers*" are termed as such not because of their "direct reciprocal relationships" (that is, because they love each other, they respect each other, etc.), but because of their *direct reference* to *the same genetic principle*: their father (or their mother or both).

A *Feed-back Process* can be represented by a "*duet-binary*" Ordinal Relationship:

$$\left(\frac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{\frac{\tilde{2}}{\tilde{2}}} \mathrm{e}^{\alpha(t)} = \left\{\left\{\begin{matrix}\overset{\circ}{\alpha}_1(t) \\ \overset{\circ}{\alpha}_2(t)\end{matrix}\right\}, \left\{\begin{matrix}\overset{\circ}{\alpha}_2(t) \\ \overset{\circ}{\alpha}_1(t)\end{matrix}\right\}\right\} \cdot \mathrm{e}^{\alpha(t)} \tag{8}$$

that is, as "*Two Brothers*" that *always cooperate* between them, and *always* in *the form* of a "*singing Duet*", even in both (possible) cases of their inverted "*role of priority*".

Contextually, *in all the previous equations* $\overset{\circ}{\alpha}(t)$ always represents the *first order* incipient derivative of $\alpha(t)$.

It is then easy to recognize that, while the left hand sides of the previous equations have an *identical formal structure*, always in the form $\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)^{\tilde{q}}$, where $\tilde{q}$ indicates the Ordinality of the specific *Generative Process* (*i.e.* $\tilde{2}$, $1/\tilde{2}$, $\tilde{2}/\tilde{2}$ *respectively*), the right hand sides of Equations (6)-(8) represent, in "*formal terms*", the specific *Ordinal Structure* of an *Inter-action*, a *Co-production* and a *Feed-back* Process, respectively, according to what previously said at paragraph 7.1 (in particular items vi) and vii)), with reference to *the components* of each system, which, in these cases, are always 2 in number.

In such a way, the Rules of *non-conservative* Emergy Algebra have their direct correspondence with the three *Generative Processes* characterized by their specific *Ordinalities* ($\tilde{2}$, $1/\tilde{2}$, $\tilde{2}/\tilde{2}$, respectively), where the latter, apart from their formal expression and the "tilde" notation, always indicate that the number of components of each *Generative Process* equals 2.

This suggested that the Incipient Derivative $\left(\dfrac{\tilde{\mathrm{d}}}{\tilde{\mathrm{d}}t}\right)^{\tilde{q}}$ (Equation (4)) could represent an appropriate *Mathematical Concept* in order to express the *Generative Activity* of *any* Self-Organizing System.

In fact, the Rules of Emergy Algebra, reformulated in terms of the "Incipient" Derivative, could represent a *valid basis* for a possible generalization of the description of *any System*, made up of an *arbitrary number of components*.

8. Toward a Mathematical Formulation of the Maximum Ordinality Principle

As is well known, the Rules of Emegy Algebra refer to different possible Interactions between *two sole sub-Systems* and, as previously shown, they can all be represented in *the general form*

$$\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)_{s}^{\tilde{k}/\tilde{l}}\left\{\tilde{r}\right\} \tag{9}$$

where $\tilde{k}/\tilde{l}$ stands for $\tilde{2}$, $1/\tilde{2}$, $\tilde{2}/\tilde{2}$, while $\left\{\tilde{r}\right\}$ is the *Relational Space* of the considered System, which in Equations (6)-(8) is represented by $\mathrm{e}^{\alpha(t)}$. This then suggests that is also possible to generalize such a description to a more articulated System, made up of *an arbitrary number* of *sub-Systems*. For example, it is possible to write

$$\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)_{s}^{\left(\tilde{1}/\tilde{1}\right)}\left\{\mathrm{e}^{\tilde{\alpha}(t)}\right\}\overset{[\rightarrow}{=}\left\{\tilde{0}\right\} \tag{10}$$

which, as we will immediately see, it can represent *the Evolution* of the Solar System, *even in explicit terms* (as it will be shown later on), precisely because its

description is formulated in terms of an "Incipient" derivative of a *specific Ordinality*.

In fact, the number 11 can refer to the number of the bodies in the Solar Systems, that is: *Sun* + 9 *Planets* + *asteroid belt*, while the symbol $\overset{[\rightarrow}{=}\{\tilde{0}\}$ indicates that the Solar System, during its time evolution, is *always adherent* to its initial and habitat conditions.

Equation (10) can then represent the Mathematical Formulation of the Maximum Ordinality Principle [23], *with specific reference to the Solar System*. In fact, when formulated in this way, it allowed us to solve several problems that, up to now, are still considered as being *unsolvable* in Classical Mechanics. For example:

a) The distribution of the Planets in the Solar System

Such distribution, in fact, is still only approximately described by the semi-empirical Bode's Law (1778), which, however, has never found a physical foundation, neither in Classical Mechanics nor in General Relativity. Even less, it is the result of a Mathematical "Explicit Solution", because the dynamic description of the Solar System is, by itself, *intrinsically unsolvable*. In fact, the problem, as demonstrated by H. Poincaré in 1889 [25], is *intrinsically unsolvable* even if the System is made up of *three sole* bodies.

Vice versa, if the Solar System is modeled as a Self-Organizing System according to Equation (10), the distribution of the Planets can easily be obtained as the *Explicit Solution* to the same Equation (10), which can also easily evaluated on the basis on an "Emerging Quality Simulator" (EQS), formally shown in [26]. A Simulator that translates, in *operative terms*, the Maximum Ordinality Principle and, in particular, the *Harmony Relationships* of the same Maximum Ordinality Principle, which are illustrated in detail in Appendix 1.

The corresponding results are represented in **Table 3**, where it is also possible to recognize the *improvement* of the description when the Solar System is more

Table 3. Distribution of the planets in the solar system.

Planets	Bode's Law	Astronomic Data	"Isolated" System	System + Habitat
Mercury	0.4 AU	0.39 AU	0.39	0.39
Venus	0.7 AU	0.72 AU	0.6	0.70
Earth	1.0 AU	1.00 AU	0.9	0.98
Mars	1.6 AU	1.52 AU	1.4	1.50
Ceres	2.8 AU	2.77 AU	2.5	2.74
Jupiter	5.2 AU	5.20 AU	4.7	5.05
Saturn	10.0 AU	9.54 AU	8.0	9.50
Uranus	19.6 AU	19.2 AU	16.0	19.0
Neptune	38.8 AU	30.1 AU	24.0	28.9
Pluto		39.5 AU	34.0	38.0

appropriately considered *in relation with its proper Habitat*. That is, when it is understood as a part of our Galaxy [27] (cap. 3).

b) The azimuthal angular distribution of the planetary orbital planes with respect to the Ecliptic

This phenomenon also has never found a satisfactory explanation, neither in Classical Mechanics nor in General Relativity. The main reason fundamentally depends on the fact that, in the absence of any explicit solution to the "*Three-body Problem*", it is impossible to evaluate the exact influence between the reciprocal orbits of the Planets.

The various angles of the orbital planes, in fact, are distributed in a cone of a rather large width (20˚), which reduces to 10˚ only if the extreme Planets (Neptune and Pluto) are "excluded" (because the latter are usually considered as being rather "anomalous") (ib.).

c) Precessions of the Planets

The Maximum Ordinality Principle is also able to describe another "Irreducible Excess" concerning the Solar System: *The Precessions of the Planets* (ib.).

General Relativity, in fact, which has given *a preliminary* answer to this phenomenology, asserts that their values are always "constant" in time and they can be evaluated in terms of a *direct interaction* between *two sole* celestial bodies, such as in the case of Sun and Mercury.

The MOP, on the contrary, shows that the Precession of *any* Planet is not "constant" in time and it has to be considered in the context of the Solar System understood *as a Whole*, that is, when modeled as *one sole* "Self-Organizing System" (ib.).

On the basis of the Example of the Solar System, and the previous concept of "Incipient" Derivative, we gave *the General Mathematical Formulation* of the Maximum Ordinality Principle, which will be presented at the next paragraph 8, which was also adopted in several other fields of Analysis shown here below.

The MOP, in fact, do not "restrict" its valid applicability to the sole Solar System. It is also able to give an *elegant solution* to several other problems (*unsolvable*, *intractable* or *both*, as those shown in **Table 4**) concerning both *non-Living* Systems, *Living* Systems, and "*Thinking*" Systems (that is *Human* Systems), the majority of them dealt with as specific sections in [28] and [29].

At this stage it is then worth presenting the General Mathematical Formulation of the Maximum Ordinality Principle, enunciated *for the first time* in 2010 [23].

9. Mathematical Formulation of the Maximum Ordinality Principle

Such a Formulation, as already anticipated, always leads to *Explicit Solutions*, as clearly shown in [27].

9.1. First Fundamental Equation of the Maximum Ordinality Principle

The First Fundamental Equation is formulated as follows

Table 4. Adoption of the MOP in other fields of analysis.

Non-Living Systems	Living Systems	Conscious Systems (Human Systems)
i) The "*Three-Body Problem*". Specific section in [28] ii) *The azimuthal angular distribution of the planetary orbital planes with respect to the Ecliptic*. Specific section in [28] iii) *Precessions of the Planets*. Specific section in [28] iv) *The angular velocities of the Stars in a Galaxy, without "Dark Energy" and "Dark Matter"* [30] v) "*The Accelerated Expansion of the Universe, in the light of the Maximum Ordinality Principle, in the absence of "Dark Energy" and "Dark Matter"* [31]	i) *Protein Folding*. One of the most intractable problems in Biology [32] ii) *Protein-Protein Interaction*. An analogous intractable problem as the previous case [33] iii) *The Duchenne Muscular Dystrophyn* (DMD) *[34]* [35] iv) *New oncological therapies. The immune-targeted therapies*. As a pecific section in [29]	i) *The Three-good two-factor Problem in Economics* [36], also dealt with as a specific section in [29] ii) *The research for equilibrium conditions in a free-market economy*. Specific section in [29] iii) *The Stability of Smart Grids* [37], also dealt with as a specific section in [29] iv) *Inter-Actions between Man and the Environment* (for example, *The "unexplained" sea level rise over the period* 1900-2000, dealt with as a specific section in [29])

$$\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)_{s}^{(\tilde{m}/\tilde{n})}\left\{\tilde{r}\right\}\overset{[\rightarrow}{=}\left\{\tilde{0}\right\} \tag{11}$$

$$\left(\tilde{m}/\tilde{n}\right)\rightarrow Max\rightarrow\left\{\tilde{2}/\tilde{2}\right\}\uparrow\left\{\tilde{N}/\tilde{N}\right\} \tag{11.1}$$

where $\left\{\tilde{r}\right\}$ is the *Relational Space* of the System under consideration, while $\left(\tilde{m}/\tilde{n}\right)$ represents its corresponding Ordinality, characterized by $\tilde{m}$ Ordinal Interactions and $\tilde{n}$ Ordinal Co-productions, which reaches its *maximum* when it equals $\left\{\tilde{2}/\tilde{2}\right\}\uparrow\left\{\tilde{N}/\tilde{N}\right\}$ (as indicated in Equation (11.1)).

In this respect, it is worth noting that:

i) The *underlined* symbol $\left(\underline{\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t}\right)_{s}$ explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*. This is because it is precisely that which gives origin to its Self-Organization as a Whole;

ii) The symbol "$\overset{[\rightarrow}{=}\left\{\tilde{0}\right\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (11);

- while the symbol "$\overset{[\rightarrow}{=}$", as already anticipated, indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

9.2. The Second Fundamental Equation of the Maximum Ordinality Principle

This Equation is formulated as follows

$$\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)^{(\tilde{2}/\tilde{2})}\left\{\left\{\tilde{r}\right\}\circledR\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)^{(\tilde{2}/\tilde{2})}\left\{\tilde{r}\right\}\right\}\overset{[\rightarrow}{=}\left\{\tilde{0}\right\} \tag{12}$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System. Equation (12), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® (defined in [26]) with *its proper Generative Capacity* $\left(\tilde{\mathrm{d}}/\tilde{\mathrm{d}}t\right)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive* Generative Capacity, which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being its proper habitat.

The Maximum Ordinality Principle, formulated in its *two fundamental equations*, then represents *an Over-deduction* with respect to MEmPP. In particular, when the MOP is formulated in such a form, it always presents an *Explicit Solution*, which can also be structured in a more *operative form* by means of an "*Emerging Quality Simulator*" (EQS), formally shown in [26], and already adopted *in all* the various Examples pertaining to *non-Living* Systems, *Living* Systems, and "*Thinking*" Systems (*Human* Systems) previously recalled.

10. The Radically New Perspective *Opened* by the Maximum Ordinality Principle

On the basis of what already anticipated in the previous paragraphs, we can surely assert that the Maximum Ordinality Principle "*opens*" (and at the same time *offers*) a *Radically New Perspective* to Modern Science. Not only because of the *new Mental Categories* adopted to describe the surrounding world but, especially, for *its new and specific Formal Language*: in particular, the "*Incipient*" *Derivative* and its *correlative Generative Consequences*.

In fact, w*hile* all the Traditional Scientific Disciplines affirm that "Every System is a *mechanism*" (at a phenomenological level), the New Approach affirms that "*Every System is a Self-Organizing System*" (always at a phenomenological level). Such two different conclusions are exactly *the clear exit* of the adoption of *two corresponding different Formal Languages* (as previously shown in **Table 1**), which also lead to some other associated *extremely important consequences*.

In fact, as we have already seen, the adoption of TDC on behalf of the Traditional Scientific Approaches leads to:

i) *Unsolvable Problems* (in explicit formal terms). Such as, for instance, the famous "Three body Problem" [25].

ii) *Intractable Problems* (even by using the most advanced computers). For example: *Protein Folding* [32] and *Protein-Protein Interaction* [33].

iii) *Problems characterized by experimental "drifts"*, which always represent an indication of possible "side effects".

iv) In addition, it is worth pointing out that the description in terms of TDC can present some "side effects" *even in the case of accurate experimental confirmations*. Such "side effects", in fact, can result as being "*masked*" by the same fact that all the experimental confirmations are always based on the adoption of *methods*, *instrumentation and measurements* that are conceived (and designed)

in a perfect conformity with the fundamental presuppositions of TDC [28].

Vice versa, the adoption of IDC in the context of the New Ordinal Approach *does not present such problems*, whereas, in turn, *it presents several advantages*.

In fact, the adoption of the *Formal Language* IDC, finalized to describe the "Emerging Quality" of "Self-Organizing Systems", leads to the formulation of the MOP, which is precisely that Principle which is able to offer *a radically New Perspective* to Modern Science, that is: "*Every System is a Self-Organizing System*" (see **Table 1**).

This is because IDC results as being *the most appropriate language* able to describe the fundamental characteristics of "*Self-Organizing Systems*". In fact:

i) It is able to represent, in appropriate formal terms, the "Emerging Quality" of Self-Organizing Systems as an "*Irreducible Excess*".

ii) In this way IDC "*guides*" to the formulation of a very general *Principle*, the Maximum Ordinality Principle (MOP), which can be understood as "*One Sole Reference*" *Principle* of *General Validity* [38].

iii) The Maximum Ordinality Principle, in fact, results as being valid *in any Field* of Analysis (from *non-living* Systems, to *living* Systems and *human social* Systems too).

iv) In addition, it *always* leads to *explicit formal solutions* in the *Proper Space* and *Proper Time* of the System [39].

v) At *any topological scale* (e.g. from atoms to galaxies).

vi) Both *under steady state and variable conditions*.

vii) What's more, the corresponding Solution to *any* mathematical model based on the MOP always results as being an "*Emerging Solution*" **[37]**. That is, a Solution whose *Ordinal Information content* is always *much higher* than the Ordinal content corresponding to the initial formulation of the problem.

viii) And, as a direct consequence, this leads to the fact that any "Emerging Solution" can never be reduced to mere "functional relationships".

xi) This is also means that the adoption of the MOP *does not require any specific reference* to the traditional Physical Laws or to the well-known Thermodynamic Principles (precisely because the latter are always "*conceived and formulated*" in terms of "efficient causality", "necessary logic" and "functional relationships").

xv) Finally, the adoption of the MOP (and its associated IDC) never leads to "side effects". This is because, even when an "Emerging Solution" might manifest some related "Emerging Exits" (see [38]), the latter can always be interpreted as being corresponding "*Extra Benefits*", initially not recognized as such [26] [27] [37].

xvi) "Emerging Solutions" that, in addition, manifest that *each System* evolves in its *Proper Time* and *Proper Space*. Or *better*, its *Proper Space-Time* [39].

It should be then easy to recognize the validity of what initially anticipated, that is: "the Maximum Ordinality Principle and its correlative *Formal Language* (IDC) open, and at the same time *offer*, a Radically New Perspective to Modern Science".

And this *not only* for the *New Mental Categories* adopted, but *especially*, but in *particular way*, for *the new faithfully corresponding Formal Language* (*i.e.* the *Incipient Derivative* and its *correlative IDC*) which *effectively* enables us to speak, and even more to "*think*", *according to* those Mental Categories.

11. Two "Com-Possible" Formal Languages, Albeit "Not Equivalent" between Them

The two Formal Languages, TDC and IDC, respectively, when considered with reference to their corresponding "presuppositions", result as being two different formal *descriptive modalities* which are always "*com-possible*" between them. In the sense that they *do not exclude each other*. They simply *co-exist*.

This is because, as already anticipated, the Traditional Scientific Approach, which leads to TDC, *cannot exclude* (in principle) the adoption of a different Formal Language (e.g. IDC), because of the *absence* in its presuppositions (especially its "necessary" logic) of any form of *Perfect Induction*.

On the other hand, the same happens in the case of the adoption of IDC, precisely because of the *same reason*, although the latter is based on presuppositions characterized by a different form of Logic (the "*Adherent*" Logic).

Consequently, the two formal languages, TDC and IDC, can *always* be adopted independently from one another. Although this "com-possibility" *in principle* does not mean that they are "*equi-valent*" (in particular, from an *operative* point of view).

Their *operative* "in-equivalence", in fact, can easily be shown by comparing the different *consequences* of their respective adoption, when such consequences are obviously considered in the light of their corresponding "pre-suppositions", as previously shown at par. 10.

In this respect, in fact, it is worth mentioning some advantages, from *an Operative point of view*, on behalf of the MOP and its IDC (with respect to the Traditional Scientific Approach). For example, the *particular* and *specific Strategic Decision Process* "they suggest" in *any* Field of Analysis.

In fact, apart from the advantages due to the *availability* of *always Explicit Solutions* to such a Decisional Process, there are also, in addition, the *specific advantages* of the same *General Methodology for Strategic Decisions* in itself considered, which "*reflects*" its *Generative Nature* of "*Quality Benefits*" on *all* the *specific* Strategic Decisions in *any Filed* considered.

As an Ostensive Example of such *a very particular aspect*, we will consider some different *case studies*, pertaining to *three* substantially different Fields of Analysis.

12. General Methodology for *Strategic Decisions* in *any* Field of Analysis, based on both the Maximum Ordinality Principle (MOP) and its Formal Language (IDC)

In this respect, we will consider some specific case studies, as Ostensive Exam-

ples of *progressive ascending* Generality and corresponding *increasing levels* of Ordinality.

12.1. General Methodology for Strategic Decisions Able to "Extend" the Validity of EMA to the Case of Transient Conditions

We will here synthetically *re-propose* such a General Methodology, officially presented for the first time at the Emergy Conference held at University of Florida [40] (on last June 29, 2022), already adopted in the past on the occasion of all the other case studies (of different nature) that will be presented later on.

Such a General Methodology, in fact, was explicitly and *formally* presented at such a Conference, precisely because understood as a specific contribution finalized to the main object of the same Conference: *Advances* in Emergy Analysis (EMA).

On that occasion we then showed how it is possible, on the basis of such General Methodology, to "extend" the validity of Emergy Analysis (EMA) to *transient conditions*. In fact, at present, it is substantially "limited" (we could also say "blocked") to the sole case of *steady state conditions*. This is because, the correlative Analysis *in transient conditions*, *if* dealt with in term of TDC, the latter Formal Language will practically "filter" the "Emerging Quality" described by the *Non-Conservative* Rules of Emergy Algebra, typical and specific of EMA in steady state conditions, as a consequence of *the basic presuppositions* of TDC.

So that, in order to overcome such a situation of "*impasse*", the General Methodology, in this specific case, is based on a *joint adoption* of EMA and MOP. In this respect, it is worth starting from a short premise.

We have already shown that the MEmPP (which under *steady state conditions* condition substantially "coincides" with EMA) and the MOP are *intimately related* to each other, when they are considered in their Historical-Logical Origin (as shown in the first part of the paper). The MOP, in fact, is nothing but the Re-proposition (in terms of Generative "Incipient" Derivatives) of the MEmPP, formulated in terms of *Non-Conservative* Algebra.

Consequently, it is also possible, in principle, *to adopt them jointly*. However, not in "functional" terms, but in Ordinal Terms. In fact, in adherence to their subjacent Generative Adherent Logic, they *can give origin* to a *Unique Ordinal Feed-Back Process* of *Generative Nature*. That is, as a *Self-Organizing Logical System* of Ordinality $\tilde{2}/\tilde{2}$.

In this respect we will now present the *Logical* General Methodology for Strategic Decisions when adopted in the *Energy Field* (understood in its widest meaning), and, as *an Ostensive example*, when, in particular, it *is based on both EMA and MOP*, *in the form of a Feed-Back Logical and Generative Process of Ordinality* 2/2. *That is*, *as "Two Brothers" that cooperate between them*, *also even in the case of their successive phases of reciprocal Inverse Priority*.

EMA, *in fact*, *which (as we know) is nothing but the MEmPP in steady state conditions*, *in this case represents the first step and*, at the same time, *the last step of the considered* Unique Ordinal Feed-Back Process of Generative Nature:

$$\textit{In Formal Terms} \quad \left\{\begin{pmatrix}\text{EMA}\\ \text{MOP}\end{pmatrix},\begin{pmatrix}\text{MOP}\\ \text{EMA}\end{pmatrix}\right\} \tag{13}$$

In order to show *how* the "*Logical*" General Methodology "reflects" (itself) *in operative terms*, we will start from considering a given System whose corresponding EMA has led to the Emergy Diagram represented in **Figure 1**.

In this respect, we will now *preliminary* show how the results of such an Analysis can be *made apt* for a *successive analysis* in terms of the Maximum Ordinality Principle.

To this *specific finality*, the considered Emergy Diagram can preliminarily be transformed into the corresponding *Relational Space of the System*, which is *a specific concept* pertaining to the MOP, usually represented by the Ordinal Matrix in **Figure 2**, as clearly shown in [26]. A Matrix which obviously is not a *traditional matrix*, because it does not represent "functional relationships" between its elements, but only the specific *Ordinal Relationships* between them.

Such elements, in fact are, by themselves, already of *Ordinal Nature* (as explicitly indicated by the symbols of "tilde" and the adoption of "curly brackets", which, as usual, indicate that *the System is understood as a Whole*).

More precisely, the various sub-Systems of the Emergy Diagram, after having been specifically numbered according to a given sequence, which, however is essentially discretionary, they can be considered (and thus represented) in terms of *couples*. This is because, in an Ordinal Context, the *minimum form* of Relationship is represented by a "*Couple*".

On these bases, the MOP furnishes, always in the form of *an explicit formal solution* [26], all the *Ordinal Relationships* between the various couples of the

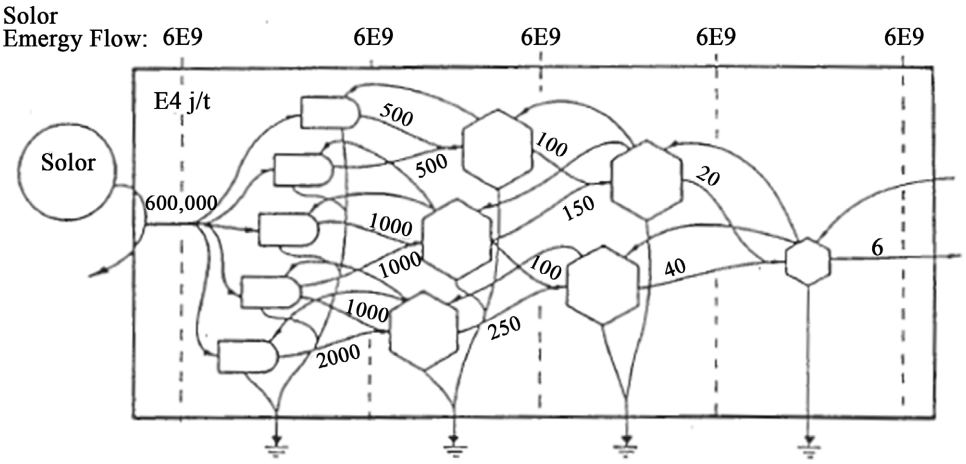

Figure 1. Example of odum's emergy diagram [5].

$$\begin{Bmatrix}\tilde{\alpha}_{12}(t) & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1N}(t)\\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{23}(t) & \cdots & \tilde{\alpha}_{2N}(t)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\alpha}_{N1}(t) & \tilde{\alpha}_{N2}(t) & \cdots & \tilde{\alpha}_{NN}(t)\end{Bmatrix}$$

Figure 2. *Ordinal* matrix representing the relational space of the system.

System, when the latter has actually reached its corresponding *Maximum Ordinality*.

Under these conditions, in fact, all the various couples that characterize the Ordinal Space of the System become of Ordinality $\left\{\tilde{2}/\tilde{2}\right\}$, independently from their corresponding *initial* Ordinalities that they had in the basic Emergy Diagram. In addition (and this, as we will see, represents *extremely important aspect*), they satisfy the following Ordinal Relationships, which are the *Faithfull* "*Reflex*" of a *Diffusive Generativity that manifests* inside the *Self-Orgnizing* System:

$$\left\{\tilde{\alpha}_{i,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{i,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left(\sqrt[N-1]{\left\{\tilde{1}\right\}}\right)_j \circledR \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad (14)$$

for $j = 1, 2, 3, \cdots, N-1$.

Such Relationships (widely illustrated in Appendix 1) are properly termed as "*Harmony Relationships*" because, when the System is *at its Maximum Ordinality*, they show that all the various couples are related between them *in Generative Terms* (and *not* in "functional terms"), on the basis of the *reference* to a *unique* and *sole arbitrary* couple, usually indicated as $\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$.

The various couples, in fact, are related on the basis of some additional *correlation factors* $\left\{\tilde{\lambda}_{i,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, which are not of "functional nature" (see the symbol $\overset{*}{=}$). At the same, the various couples are also related in terms of the *Ordinal Roots of Unity* $\left(\sqrt[N-1]{\left\{\tilde{1}\right\}}\right)_j$ (illustrated in Appendix 2), where the Unity $\left\{\tilde{1}\right\}$ does not indicate a simple "figure", but it specifically indicates the "*Unity*" of the System understood *as a Whole*.

As an *immediately clear Ostensive Example* of such an *Ordinal Description*, it is sufficient to recall that it is exactly what happens in the case of the Solar System (previously illustrated), when as *reference couple* is assumed, for example, Sun and Mercury. Nonetheless, the *Ordinal Description* of the Solar System is *always the same* even if we assume *whatever else* couple as reference (for instance Earth and Mars).

The Methodology here proposed clearly offers some Significant Advantages, both in *steady state and in transient conditions*, especially for the properties of the above-mentioned Harmony Relationships.

In fact such General Methodology clearly offers an *increased rapidity* of the Strategic Decision Process, because the researched *Optimal values* for the *operative* Ordinal Re-Configuration of the System are *co-instantaneously* given by the Simulator EQS, even in the case of hundreds of sub-Systems.

At the same time, the same General Methodology offers a correlative *higher reliability* of the Re-Qualification Process of System analyzed, because the Decision Maker correlatively knows *all* the "optimal values" that are properly apt a requalifying the Whole System *in adherence* to its *Maximum Ordinality*, this is because they are precisely those furnished by the Harmony Relationships of the System.

The same General Methodology, In addition, offers very decisive *Ordinal and operative Advantages* with reference to those *particular transient conditions* that, for the sake of brevity, can synthetically be termed as *Perspective A* and *Perspective B*.

The latter, in fact, refers to the case in which the Habitat represents a "Disturbance" to the System.

For example, sudden variations of external market conditions, scarcity of Energy Resources, sometimes due to external political changes (for example the present war Russia-Ukraine), which "modify" the *external conditions* of the System, as initially assumed, described and represented in the *specific and characteristic* terms of EMA.

In such a case, the Methodology shows how it is possible to increase (according to a predetermined desired measure) the *Resistance* and, at the same time, the *Resilience* of the System against the considered *Adversities*.

Vice versa, when the Methodology is adopted in *Perspective A*, it suggests how to research for the Maximum Ordinality of the Relationship between the System and its pertaining Habitat, because Perspective A specifically refers to the case of a *collaborating Inter-Action Process* between them.

Such two latter *particular transient conditions* will be analyzed, in more detail, in the two successive paragraphs.

12.2. *Requalification* Process of an Urban System, From a Seismic Point of View, Against Calamities

This second Ostensive example clearly refers to a completely different filed of analysis, which however, precisely because of its specific difference, can contribute to show the *General Applicability* of the Methodology, which is, by itself, deeply rooted on the *Generality of MOP* and *its correlative Formal Language IDC*.

Such an Example, which is carefully illustrated in all its details in [41], refers to a *Requalification Process* of an Urban System, from a Seismic point of view, against Calamities.

An Example that *equally* shows (in its specific context) the corresponding increase of *Resistance* and *Resilience* of the Urban System analyzed.

The description of the village is based on the so called "Minimum Urban Structure" which, as analyzed by V. Fabietti and Others [42], is composed by 39 elements (buildings, roads, utilities) that, according to the available data [ib.], can be characterized by means of *three Fundamental Indicators*: *Vulnerability*, *Index of Urban Complex*, *Strategic Valence*. Consequently, it is describable by means of 29 × 3 Indicators. More precisely:

- The "Vulnerability" is the probability that a threat event can induce a more or less marked loss of integrity.
- The "Index of Urban Complex" (or, more simply, "*Index of Complex*") represents the level of physical and functional interconnections with neighboring elements.

- The "Strategic Valence", according to a conventional scale from 1 to 12, indicates the relevance of the role assigned to each specific element, for example: administrative center, hospital, commercial center and so on. It is worth adding that, for practical reasons, in our analysis we have also adopted the inverse concept of "Vulnerability", defined as "*Persistence to Integrity*" (or, more synthetically, "*Persistence*").

The results of the analysis, synthetically shown in Table 5, refer to the *Final Configuration* of the System as a consequence of the hypothesized earthquake. It is then possible to recognize that such an Ordinal Exit represents a clear manifestation of the recovery of the *Internal Stability* on behalf of the System, as explicitly foreseen by Equation (12), that is the *Second Fundamental Equation* of the MOP.

In fact, on the basis of the results of the analysis, although here represented in an *extremely synthetic form* with respect to that widely shown in [41], it is possible to immediately recognize the *reduction* and *mitigation* of the effects of the earthquake, with reference to the values of the three fundamental Indicators. Both with respect to their *maximum* values and their *minimum* values respectively.

For the sake of brevity, but also for clarity, the corresponding "mitigations" of the effects are reproduced in Table 5 in the form of percentage changes.

The values in Table 5 clearly shows how, on the basis of *the same* General Methodology, it is possible to requalify the structures of an Urban System, in order to increase its *resistance* and its correlative *resilience* against natural calamities (earthquakes, hurricanes, etc.), by always adopting as reference *fundamental criterion* the *Maximum Ordinality Principle (MOP) and its correlative IDC*.

In fact, in the specific study case adopted as an Ostensive Example, we have a reduction of the *earthquake foreseen effects* of a little more than 50%.

In this sense we can surely assert that the MOP, especially for its Formal Language, *opens* a radically New Perspective also in this field.

In fact the village assumed as a case study was modelled *as a Self-Organizing System*. This is because, although the village is usually considered as being solely

Table 5. Reduction and mitigation of the effects of the earthquake.

Indicator of Reference	Initial Value without Requalification	Final value with Ordinal Requalification
High Values of Persistence	−4.82%	−2.30%
High Values of Complex	−11.54%	−5.82%
High Values of Valence	−5.49%	−2.68%
Low Values of Persistence	−9.92%	−4.95%
Low Values of Complex	−6.53%	−3.19%
Low Values of Valence	−5.50%	−2.68%

made of buildings, streets, places and so on, in reality it has been *conceived*, *planned* and *realized* by human beings during several centuries. In addition, the people who actually leave in such an urban center, systematically deal with its *maintenance*, in order to possibly increase its *functionality*. This justifies the assumption of the village as being a *Self-Organizing System* and, consequently, it has been analyzed in the light of the MOP, which, as we already know, represents a valid and General Reference Principle for analyzing both "non-living", "living" and "conscious" Self-Organizing systems.

All such results were obtained, as usual, by adopting the same *Emerging Quality Simulator* (EQS), described in [26], which, as already anticipated, represents the *operative version* of the MOP in terms of IDC.

As far as the specific Methodology pertaining to the *Perspective A* is concerned, that is the case of *a positive cooperation* between the System and its surrounding Habitat, it is also illustrated, from a general point of view, in the same Reference previously mentioned [ib.]. In which it is also shown how the General Methodology can even be adopted at successive increasing Levels of Generality and associated *Higher Levels* of Ordinality.

However, in order to illustrate such increasing Level of Ordinality, always based on the adoption of the MOP (and especially of its Formal Language IDC), it is surely much *more meaningful* to show such an increasing Role of the General Methodology for Strategic Decisions, with reference to the Concept of "*Empowering Communities*", which will be presented at the next paragraph**.**

12.3. *Empowering Communities* in the Light of the Maximum Ordinality Principle, Well Beyond Energy Scarcity

This section, in fact, is a synthesis of the paper presented at the European Round Table on Sustainable Consumption and Production Graz (September 8-10, 2021) [43]. It was specifically finalized to show how Empowering Communities may progressively *increase their role* by adopting the Maximum Ordinality Principle (MOP) and its correlative *Formal Language* (IDC) as *basic reference criterion* to this specifically finality.

Such an adoption on behalf of a given Community, in fact, generates an increasing sense of being *a real Community*, at successive and progressively *higher levels*, together with the corresponding responsibilities of Role.

This aspect will be here synthetically re-proposed by considering the following *different* Levels of Analysis:

a) As an introductory and preliminarily stage of the Ascending Process previously mentioned, we will consider the case of *external adverse conditions*, such as, for instance, the case of "energy scarcity".

However, the Process previously mentioned becomes much *more intensive* when the MOP and its IDC are adopted to deal with aspects that go *well beyond* "energy scarcity". For example, the following cases:

b) Energy Saving and Renewable Energy Sources. The role of Smart Grids.

c) Economics Stability of a System of Nations and their *increasing conscious-*

ness as being a real Community.

d) Increase of Ordinality and Consciousness of a "Community" with reference to both its *Internal* and *External* Relationships.

e) Relationships between Man and Environment. Climate change forecasts. For example, the Sea Level Rise over the Period 1900-2000.

However, taking into account the "practical" impossibility of presenting *all* the various Ascending cases, we will limit to present that could be termed as the *basic level*. This in order to show how the General Methodology can be adopted in those selected *case studies* well beyond energy scarcity and, in the end, it will also enable us to recall the General Conclusions of the their corresponding Analyses.

In all the considered study cases, the System assumed as "*Reference System*" is always the same, made up of five European Countries (more precisely, Italy, France, Spain, Germany, Austria), which were modelled as *a unique and sole Self-Organizing System*, that is, understood as being a *Real Community*.

On the basis of such an assumption, as a preliminary stage the above mentioned "*ascending*" General Methodology, we will consider the case study characterized by *a given amount* of *energy scarcity*, in order to show the increase of *Resistance* and *Resilience* of such an Ordinal Community adverse such a *hypothesized* even.

12.3.1. General Methodology in the Case Study Pertaining to a Given Adverse Energy Scarcity

In this perspective, each Nation is characterized by three indicators, shown in Table 6, whose values were taken from World Data Bank (2020).

As far as the *energy crisis* is concerned, this is thought of as an *energy scarcity*, which may be due either to a reduction of fossil fuels production or to an increase of their prices (or both) and, as a work hypothesis, it was supposed characterized by an "incidence" of the order of 20% on the values of Indicator 1.

The incidence on the values of the other two Indicators is strictly correlative to the Ordinal Reconfiguration of the System (shown in Table 6).

Consequently, the corresponding effects on the System, simulated by means of

Table 6. Ordinal Requalification of the System in *operative terms* (elaboration of data from World Bank).

Progressive Number	Nation	1. Imported equiv. oil per person (ton)	2. Imported equiv. oil per person/ Occupational level (%)	3. GDP per person/ 1000/Occupational level (%)
1	France	1.8	2.53	0.570
2	Italy	2.0 * (−)	3.21 * (−)	0.533 * (+)
3	Germany	2.4 * (−)	3.03 * (−)	0.586 * (+)
4	Spain	2.5 * (−)	3.81 * (−)	0.452 * (++)
5	Austria	2.5	3.35	0.734

EQS, in this case show an incidence of the order of 10% on the values of Indicator 2 and an incidence of the order of 5% on the values of Indicator 3.

More precisely, Table 7 sows that while the resulting "incidence" on the values of Indicator 1 is equal to 20%, as precisely supposed by hypothesis, the correlative "incidence" on the values of Indicator 2 is equal to 9.75%, while the "incidence" on the values of Indicator 3 evenly ranges from 5.10% and to 5.16%.

It is then easy to recognize the corresponding *reduction* and *mitigation* of the effects due to the reduction of imported energy, with reference to *all the values* of *the three Indicators*, both with respect to their *maximum* and *minimum* values.

For the sake of synthesis, but also for clarity, the corresponding "mitigations" of the effects are given in Table 7 in the form of percentage changes.

Such results unable us to surely affirm that the System manifests a higher *Resistance* with respect to the corresponding conditions of a *total absence* of an Ordinal Requalification.

At the same time, it is also possible to recognize a correlative increase of its *Resilience*.

In fact, in the presence of a *prior* Ordinal Requalification, the System, *after having mitigated the effects* of the energy scarcity, still *keeps an Ordinality* level sufficiently high to adequately, and progressively, reacquire its specific *Role*, both in terms of Ordinal Relationships *within itself* and, even more, with respect to its Ordinal Relationships with its *surrounding Habitat*.

The general conclusions of this (for the moment) preliminarily step of analysis can be synthesized as follows:

- In view of a possible *energy scarcity* (or, more in general, *energy crises*), any System of Nations should provide, in advance, to improve its *Ordinal Requalification*, appropriately commensurate to the "foreseeable" energy crises pertaining to its specific case.
- This is because, from such an Ordinal Requalification, it will result a "*Rebound*" of its "Resistance" and at the same time, of its correlative level of "*Resilience*".

This evidently becomes even truer, for example, in the case of the European Community (made up of 27 Nations) and, even more, in the case of USA (made

Table 7. "Mitigations" of the effects in the form of *percentage changes*.

Indicator of Reference	Initial Value without Requalification	Final value with Ordinal Requalification
High Values of Indicator 1	−20.0%	−10.0%
High Values of Indicator 2	−9.75%	−4.67%
High Values of Indicator 3	−5.16%	−2.18%
Low Values of Indicator 1	−20.0%	−10.10%
Low Values of Indicator 2	−9.74%	−4.67%
Low Values of Indicator 3	−5.10%	−2.67%

up of 50 States), with particular reference to their relevant specific Role in the World.

In these cases, in fact, there is a progressive increase of the corresponding *Ordinality* of the Systems, because it is directly associated to *the increasing number of their States*, as clearly shown by Eq. (1.1), which is formulated, of course, in Ordinal Terms.

As far as a synthetic exposition of the results obtained in correspondence of the *successive higher levels of analysis*, we can briefly recall that.

12.3.2. Energy Saving and Renewable Energy Sources. The Role of Smart Grids

The *diffusion* of the Smart Grids should be *uniform between the various Nations* and, contextually, *among their pertinent Regions*, always for the respect of the above mentioned *Harmony Relationships* and the correlative reduction of the "vulnerability" to energy scarcity.

12.3.3. Economics Stability of a System of Nations and Their Increasing Consciousness as Being a Real Community

The results pertaining to this aspect clearly shows that *a free market economy* cannot be considered as being *a simple "mechanism"*, as it is usually supposed as being on behalf all the Traditional Theories of Economics.

A "*free market*", in fact, is characterized by "Initiative", "Inventiveness" (understood as a "continuous development of new products"), without considering that *any transaction* always generates "*Extra*" *Benefits of Ordinal Nature* [44], which are *irreducible* to a traditional description in terms of *causality*, *necessity*, *functionality*.

All these conditions then suggest that a "free market" between Nations can be more appropriately modelled as a "Self-Organizing System". In fact, when "*The three-good two factor Problem*" is interpreted in the light of the MOP and its ITC [36], the Problem can be solved for an *arbitrary number of goods* (N_g), in the presence of *Three Productive Factors*: *Capital (K)*, *Labour (L) and Natural Resources (N)* [29].

12.3.4. Increase of Ordinality and Consciousness of a "Community" both in Its Internal and External Relationships

The analysis of this aspect, performed in the light of MOP and its correlative Formal Language (IDC), shows that:

- when a System of Nations (for example the European Community) has reached its Maximum Level of *Ordinality*;
- characterized by its corresponding internal *Stability*;
- and, consequently, a much deeper "Consciousness" of being an *effective Community*;
- all these aspects can have a *direct reflex* on *the improvement* of its Ordinal Relationships. Both *internal* to the single Nations and between them.

In addition, a further improved level of Relationships can manifest when such

Communities will establish New *Ordinal Relationships* with *other countries*.

The abovementioned aspects also suggest that, *in addition*, *any* considered Ordinal System of Nations could also play a *more decisive* role in *the respect of the Environment*, by assuming Strategic Decisions always in the light of the MOP and its IDC.

12.3.5. Relationships between Man and Environment. For Example: Climate Change Forecasts and the Sea Level Rise over the Period 1900-2000

In this case the "Empowering Communities" may manifest their increase of consciousness and their particular role with reference to *the Environment*, only in the respect, however, of the following conditions:

- The "prior condition" is that they have reached a sufficiently high level of Ordinality as Self-Organizing Systems, by adopting as a "*Reference Guide*" the Maximum Ordinality Principle and its correlative Formal Language (IDC).
- Afterwards, they know *very well* the Environmental Phenomenology pertaining to the aspects of specific interest.
- This means that they are able *to recognize* the "*Emerging Quality*" of the Processes they are going to deal with.
- And, as a fundamental aspect, they are systematically oriented at the research for a *possible Syntony* with such an "*Emerging Quality*".
- Finally, under these achieved conditions, even in the case of *potential adverse events*, they can *mitigate* the associated undesired "effects", always in adherence to the MOP *and its Formal Language (IDC)*.

In order to illustrate the importance of the previous conditions, we can recall, as an Ostensive example, the Sea Level Rise over the Period 1900-2000, as described in [29].

It is evident that such a process is difficult to contrast if its specific "origin" is not deeply known.

At a first glance, in fact, the process "seems to be *inexplicable*", because the correlative "causes" are still unknown [45].

However, this is simply due to the fact that the specific "causes" are systematically *researched for in* terms of *efficient causality*, *logical necessity*, *functional relationships*, that is they are researched for as the various processes involved were pure "mechanisms" (as illustrated at par. 2.1).

In reality the Process of Sea Level Rise can be analyzed in the light of the MOP by means of its associated Ordinal Simulator EQS (ib.), which operatively translates and faithfully represents the various *Harmony Relationships* between *all* the *different* Physical Systems involved in the process (sea, ice, hearth, sun, etc.).

Such Inter-Actions in fact, because of their *Ordinal Nature*, are precisely those that represent the real "*generative cause*" of that registered "unexpected" trend. Which, according to such an interpretation, is nothing but an "*Emerging Exit*" of *a Unique* "*Self-Organizing System*".

Consequently, "Empowering Communities" should correspondently modify

their way of "*Thinking*, *Decision Making*, and *Acting*" so as to research for the maximum Syntony with the "Emerging Quality" shown by the considered Processes, so as to minimize both present and future effects with respect to the Environment.

With specific reference to this latter respect, we will now consider a particular "*in-equivalence*" between the *Formal Languages* TDC and IDC.

12.3.6. A Particular "In-Equivalence" between the Formal Languages TDC and IDC, in the Case of Strategic Decisions with Reference to the Relationships between *Man* and *the Environment*

As anticipated in the title, the "in-equivalence" between the Formal languages TDC and IDC *becomes particularly evident when the General Methodology for Strategic Decisions involves the* Relationships between Man and the Environment *and*, *more specifically*, *when the latter are focused* in the Light *of the well-known categories of* "*Thinking*, *Decision Making*, and *Acting*".

In fact when the Methodology based on the Formal Language IDC, is compared, in this specific context, with other more *traditional* Strategic Decision Processes (such as, for instance, those that deal with *Environmental Accounting and Management* in prevailingly (or essentially) "*cardinal terms*"), the *Ordinal* Methodology reveals *its Maximum Significant Contribution* to the Radically New Perspective *offered to Modern Science*.

In fact, with *specific* reference to the *Verbal Enunciation* of the MOP, in which the term "System" now plays the Role of "Man" (or "Human System") and the term "Habitat" now plays the Role of the "Environment", it is possible to recognize that:

At the level of "Thinking":

Traditional Methodologies for Strategic Decisions, in spite of some specific *conceptual* progresses during the last decades, always intrinsically "*reflect*" the general idea that "every system is a *mechanism*". Consequently, at the level of "*Thinking*", the Environment is *nothing but a* Set of "Mechanisms", and consequently it is always considered as an "*object*", that can be *"used" as such*, in "*functional terms*", mainly to economic finalities.

The *New* Ordinal Perspective, on the contrary, is always orientated at describing the Environment as *One sole and Unique* "*Self-Organizing System*", even when considered in the specific contest of Economics. Consequently, and *as it usually happens*, The Ordinal Perspective always tends to *Maximize both the Ordinality* of *the "Economic" System* and that of *its correlative Environment*.

Such a *specific* difference, at the level of "*Thinking*", evidently leads to a correlative "*in-equivalence*" at the level of "Decision Making and Acting".

At the level of "Decision Making":

In the case of The Traditional Methodologies, the *powerful expressive capacity of the formal language adopted* (namely TDC), systematically "*forces*" the *Decision* toward the research for specific *"functional" solutions* and their subsequent *practical* implementation.

Whereas in the case of The Ordinal Perspective, the specific *capacity of its corresponding and completely different formal language adopted* (namely IDC), *is that which* systematically "*guides*" toward the research for specific *practical solutions*. *Which are now*, *and in all cases*, *of "Ordinal Nature"*, as well as their subsequent and "*adherent*" *actual implementation*. In this case with particular reference to the Environment, which always manifests, also in the Economic Field, its correlative, specific and *proper* "*Emerging Quality*".

At the level of Action:

It is exactly *where* the *profound differences* become particularly evident.

This is precisely because, in the case of The Traditional Perspectives, the corresponding *formal solutions*, which are of "*functional nature*", *become corresponding "necessary" consequential facts*.

Vice versa, the New Ordinal Approach, precisely because of its specific *Ordinal* Formal Language (IDC), always researches for the *actual* Maximum Ordinality (that is *the Maximum Synthony*) between the "System" (Man) and its surrounding "Habitat" (Environment).

In such a case, in fact, the corresponding *formal solutions*, which are always of "*Ordinal Nature*", *become perfectly and faithfully* adherent *consequential facts*, *that manifest the achieved researched Syntony*.

13. Conclusions

We have already shown the *Fundamental Role* of the "*Formal Languages*" TDC and IDC, respectively, and, at the same time, their *com-possibility* of adoption, at a Logical level.

This is because, in the *absence of any form of Perfect Induction*, *by nature* in both the two different typologies of "*subjacent" Logic* respectively adopted, they result as being always "*com-possible*", in the sense that they *do not exclude each other*. *Precisely because (as already said) none of the two has the property of the Perfect Induction*.

Consequently, *they cannot exclude each other*, *in an absolute sense*, *simply because they cannot do it*, *in any case whatsoever*. They simply *co-exist*.

At the same time, however, we have also seen their "in-equivalence" from an *operative* point of view.

This means that, precisely because of their "com-possibility" at a Logical Level, the "operative" conclusion would be that it is always possible to adopt *the one* or *the other* or, even better, *both together*. That is, *at the same time* (although *in parallel*), in order to possibly choose, in such a way, the *optimal operative solutions*, according to the validity (and confirmation) of their respective conclusions, when the latter are *assured* by their corresponding experimental results.

In such a case, however, it is fundamental to remember that, always in conformity to *the Absence of Perfect Induction*, which is precisely that *particular logical condition* which, in any case, allows such a *third possible option*:

- TDC is the *Formal Language* that faithfully "reflects" the presuppositions of the Traditional Scientific Approach, which is "*self-referential*"
- While IDC is the *Formal Language* that faithfully "reflects" the presuppositions of the New Scientific Approach, which is "*hetero-referential*".

The *Fundamental Reference* of the latter in fact is the "Emerging Quality" of Self-Organizing Systems. With respect to which, the New Ordinal Approach firstly modifies its *mental categories*, then adopts a faithfully corresponding *Formal Language*, which allows to formulate a General Reference Principle, the Maximum Ordinality Principle, in the light of which it always seeks for *the Best Synthony* with the "Emerging Quality" that *manifests itself* in the surrounding world.

A *Synthony* is *revealed* (*albeit* always *not less than*, as shown in [26]) by the Maximum Level of *Ordinality* each time *possibly attainable*. Obviously, because *always* in the *respectful and faithful adherence* to the *initial* conditions of the System as well as *those* of its surrounding Habitat.

Nonetheless, in reality there exists *another possible Option*, of *Higher Ordinality* with respect to the three options previously presented. An Option that, in Generative Logic, represents an *Over-Conclusion*.

Over-Conclusion

Such an *Option of Higher Ordinality*, in fact, is *actually possible*, because *the General Methodology for Strategic Decisions* previously shown has *a very general validity*. Consequently, it can be also adopted in an *even more general case* (with respect the ones previously considered): that is, it can *also* be adopted with Reference to the *Field* of "*Formal Languages*".

Consequently, there exists (at least) another *possible* Option, of *Higher Ordinality* with respect to the three options previously presented.

In fact, as already shown in the first Ostensive Example (par. 12.1) pertaining to EMA and MOP, it is always possible to adopt the Two Formal Languages *jointly*. However, not in "parallel" or in reciprocal "functional" terms, but *in Ordinal Terms*. In fact, in the *respectful adherence* to their specific *subjacent Logics*, the two Formal Languages TDC and IDC can *give origin* to a *Unique Ordinal Feed-Back Process* of *Generative Logical Nature*. That is, a *Logical* Self-Organizing *System* of Ordinality $\{\tilde{2}/\tilde{2}\}$.

In other terms, the *General Methodology for Strategic Decisions*, when adopted in the *Formal Field* of *Mathematical Logic*, clearly shows that *TDC and IDC are similar to "*Two Brothers*", which cooperate between them, also in the case of the successive phases of their reciprocal Inverse Priority*.

In such a case, in fact, *TDC will represent the* first step *and*, at the same time, the last step *of the considered* Unique Ordinal Feed-Back Process of Generative Nature. *In Formal Terms*:

$$\left\{\begin{pmatrix}\text{TDC}\\ \text{IDC}\end{pmatrix},\begin{pmatrix}\text{IDC}\\ \text{TDC}\end{pmatrix}\right\} \tag{15}$$

This consequently leads to the *following Over-Conclusion*:

The Maximum Ordinality Principle (MOP) and its Formal Language, *namely* The "*Incipient*" Differential Calculus (IDC), are really open, and at the same time *offer*, *a Very Radically New Perspective* to Modern Science because, when *the* General Methodology *for* Strategic Decisions, *deeply "rooted" in them*, *is adopted at such a* General Logical-Formal Level, it surprisingly *"reveals"*, *and at the same time "reflects"*, *at an operative level*, *that*:

TDC and IDC, *precisely because considered in the context of such a General Logical Structure*, *are "nothing but" a* unique "*Casket*" *of* "*Precious Pearls*"[3].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix 1. Process of Genesis of the Harmony Relationships

In this Appendix we want to point out that *the Harmony Relationships* represent an "*Emerging Solution*" with respect to the Solution to the First Fundamental Equation and, at the same time, a corresponding "*Exceeding*" Solution.

Let us then recall the basic elements that will enable us to show that the Harmony Relationships represent an "Emerging Extra" of *Generative Nature*, [23] [26] [27] [28].

The Solution to the First Fundamental Equation, in fact, allow us to write the following *topological* "Assignation Relationships"

$$\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{\alpha}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{for} \quad j=3,4,\cdots,N \quad \text{(A1.1)}$$

which, when re-written in terms of "Incipient" Derivatives, up to the order $N-1$, they represent the "*Diffusive Generativity*" inside the *Self-Organising System*, and assume the form

$$\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}} \overset{*}{=} \left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}} \quad \text{for} \quad k=1,2,\cdots,N-1 \qquad \text{(A1.2)}$$

in which, for simplicity of notation, the Ordinalities $\left\{\tilde{2}/\tilde{2}\right\}$, which appear in Equation (A1.1), are thought as being included in the symbols of the quantities to which they refer to, and $\tilde{\lambda}_{1j}$ represent $N-1$ associated Correlating Factors.

Equation (A1.2) cannot be interpreted as a "necessary consequence" of Equation (A1.1), because the latter are obtained on the basis of "Incipient" Derivatives. Consequently, they are all of *Generative Nature*.

In fact, if rewritten in the following form

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}} = \tilde{1} \quad \text{for} \quad k=1,2,\cdots,N-1 \qquad \text{(A1.3)}$$

they allow to assert that the considered System is already characterized by a proper and specific "*Interior Unit*", of *Generative Nature*, formally represented by the symbol "$\tilde{1}$".

Such a "Unity", however, is still in the form of "*Not Less Than*". This is because:

- in a Generative Contest, they are certainly not *the parts* that, through the Relationships "*between*" them, give "Origin" to the "Excess of Unity";
- because it is exactly true the opposite: in fact, it is the *Generative Unit* of the System that, with its *proper* "*Excess*", *Qualifies* the Relationships "*between*" *the parts*.

Consequently, the most Adherent Formulation of the Self-Organizing *Diffusive* Generative Process is that which can be obtained by re-proposing Equation

(A1.3) in the form

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}}\overset{*}{=}\left\{\tilde{1}\right\}\quad\text{for}\quad\forall k \tag{A1.4}$$

or better, even more properly, as follows

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}}\overset{*}{=}\left\{\tilde{1}\right\}^{\frac{1}{\left(N\tilde{-}1\right)}},\quad j=2,\cdots,N \tag{A1.5}$$

in which the symbol $\left\{\tilde{1}\right\}$ now formally represents the *Generative Whole*, which, at the same time, is *Self-Organizing* and of *Ordinal Nature*, while its *unique* and *sole* exponent $1\Big/\left(N\tilde{-}1\right)$ explicitly represents the fundamental concept previously anticipated, that is: it is the "Whole", with its *proper* Generativity "*Excess*", the one that properly "*Qualifies*" the Relationships "*Between*" *the parts*.

This is obviously true not in the sense of Relationships understood "two by two", but as the specific Reflex of an Ordinal Unit, which, in any case, represents an "Irreducible Excess" with respect to the simple "composition" of the single "parts".

Consequently, Relation (A1.5), can also be written in the form

$$\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{1j}\right\}\overset{*}{=}\left\{\tilde{1}\right\}^{\frac{1}{\left\{N\tilde{-}1\right\}}}\circ\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t)\oplus\overset{\circ}{\tilde{\lambda}}_{12}\right\}\quad\text{for}\quad j=2,3,\cdots,N \tag{A1.6}$$

which, when reinterpreted in terms of "*Progenitor Relationships*", finally leads to the formal expression of the Harmony Relationships. The latter, in fact, when written in the form

$$\left\{\tilde{\alpha}_{1,j+1}(t)\oplus\tilde{\lambda}_{1,j+1}(t)\right\}\overset{*}{=}\left(\sqrt[N\tilde{-}1]{\left\{\tilde{1}\right\}}\right)_{j}\circledR\left\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\right\}\quad\text{for}\quad j=1,2,\cdots,N-1 \tag{A1.7}$$

clearly show that the *Diffusive Generativity* "updates", by Assignation, the *same reference couple* "12".

Equation (A1.7) then show that all the elements of the Ordinal Matrix in **Figure 2**, can be obtained on the basis of *one sole couple* $\tilde{\alpha}_{12}(t)$ assumed as reference and $N-1$ associated Correlating Factors.

In this respect, it is also worth noting that condition (A1.2) is properly that which represent the fundamental presupposition of what could be termed as an *Intensive Whole*, precisely because of the "*consonance*" between all the generative derivatives up to the order $N-1$, which are the "*Reflex*" of the *Diffusive Generativity* inside the System.

This is the specific reason why, by means of the MOP, and its correlative

Harmony Relationships, it was possible to reconsider some "particular" problems that, in the Traditional Scientific Literature, are generally known as being "*unsolvable*", "*intractable*", "*with drift*". Whose solutions ended up by showing that the Maximum Ordinality Principle has an extremely general validity [46].

Appendix 2. The Ordinal Roots of Unity $\left\{\tilde{1}\right\}$

In this respect it is worth observing that Relationships (A1.7) are written in such a form only for reasons of clarity and exposition simplicity. In such a form, in fact, it could seem that the various elements that characterize the System are "still" related, "between" them, according to Relationships of the type "two by two".

In reality, if one makes explicit the term $\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j$ according to its more specific and proper meaning, that is as $\left\{\tilde{1}\right\}^{\frac{1}{\left\{\tilde{N-1}\right\}}} \equiv \left\{\tilde{1}\right\}^{\frac{1}{\left\{N-1,\left(\tilde{N-1}\right)\right\}}}$, in which $N-1$ refers to the cardinality, while $\left(\tilde{N-1}\right)$ refers to the Internal Ordinal "$(N-1)$-ary" Relationship, it is possible to more appropriately write (by also pointing out the Ordinalities $\left\{\tilde{2},\tilde{2}\right\}$, previously underwritten)

$$\left\{\tilde{\alpha}_{1j}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \oplus \left\{\tilde{\lambda}_{1j}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \overset{*}{=} \left\{\tilde{1}\right\}^{\frac{1}{\left\{N-1,\left(\tilde{N-1}\right)\right\}}} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \quad \text{(A2.1)}$$

that is, even more explicitly, in the form

$$\left\{\tilde{\alpha}_{i,j+1}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \oplus \left\{\tilde{\lambda}_{i,j+1}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \overset{*}{=} \begin{pmatrix} \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_1 \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_2 \\ \vdots \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_{N-1} \end{pmatrix} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\left\{\tilde{2}/\tilde{2}\right\}} \quad \text{(A2.2)}$$

from which it is possible to recognize that the single "cardinal" values that in Equation (A1.7) *appear* as they were "distinct", and, in addition, as being "separated", in reality they are the *Reflex of an Ordinal Unit* that transcends them, and it relates them in the form of an $(N-1)$-*ary* Relationship.

This is the aspect that (more than others) clearly manifests that the Harmony Relationships represent an "*Excess*" with respect the initial Assignation Relationships (A1.1) and (A1.2).

As far as the "explicit" meaning of the Ordinal Routs of Unity is concerned, previously synthetically indicated in the form

$$\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j \quad \text{for} \quad j=1,2,3,\cdots,N-1 \quad \text{(A2.3)}$$

it is worth expressly pointing out that the symbol $\left\{\tilde{1}\right\}$ represents the *Unity of*

the System (understood as a *Whole*), with specific reference to the *Unity of its Proper Space* (as well as its *Relational Space*).

Such a Fundamental Unit can be then expressed by the following Relationship

$$\left\{\tilde{1}\right\} = e^{\left\{\alpha \circledR \tilde{i} \oplus \beta \circledR \tilde{j} + \gamma \oplus \tilde{k}\right\}} \tag{A2.4}$$

Consequently, the Ordinal Roots $\left(\sqrt[N-1]{\left\{\tilde{1}\right\}}\right)_I$ will be represented in the following form

$$\left\{\tilde{1}\right\}_I = e^{\frac{\left\{\alpha \circledR \tilde{i} \oplus \beta \circledR \tilde{j} + \gamma \oplus \tilde{k}\right\}}{N-1}} \tag{A2.5}$$

where:

- $\tilde{i}, \tilde{j}, \tilde{k}$ are the fundamental *spinors* of the Relational Space [26] [38], which replace the traditional versors $\vec{i}, \vec{j}, \vec{k}$, understood in their more general sense, that is, as the specific foundation of any given System;
- α, β, γ are respectively equal to

$$\alpha = \varepsilon_1 + \frac{4\pi \cdot l}{N-1}, \quad \beta = \varepsilon_2 + \frac{2\pi \cdot l}{N-1} \quad \text{e} \quad \gamma = \varepsilon_3 + \frac{2\pi \cdot l}{N-1} \tag{A2.6}$$

- where the "periodicity" of the "spinor" $\tilde{i}$ is equal to 4π, because expressed in *steradians*;
- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radiants (each), because these spinors are always "orthogonal", both between them, and with respect to the spinor $\tilde{i}$ (an orthogonality that can be understood, inter alia, as a form of *reciprocal* "*irreducibility*");
- the quantities $\varepsilon_1, \varepsilon_2, \varepsilon_3$ represent specific "parameters" of the *Relational Space* each time considered, with specific reference to the "couple 12".

Sometimes (for example in the case of Protein Folding), for an easier "topological" representation Equation (A2.6) can also represented as

$$\frac{\alpha}{N-1} = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1}, \quad \frac{\beta}{N-1} = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1}, \quad \frac{\gamma}{N-1} = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1} \tag{A2.7}$$

which however can always re-proposed in the previous form (A2.6) through an appropriate choice of the parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$.

On the basis of the previous exposition, it should be even clearer that the Harmony Relationships represent an "*Irreducible Excess*", that is an "*Exceeding*" Manifestation of a *Generative System*, which, at the same time, is *Self-Organizing*, of *Ordinal Nature*, and, above all, is understood as *a Whole from the very beginning*, and not vice versa.