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Effect of Initial Stress on a Thermoelastic Functionally Graded Material with Energy Dissipation

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Abstract

In this paper we introduce the effect of initial stress on a magneto-thermoelastic functionally graded material (FGM) with Green Naghdi theory with energy dissipation. A system of PDE was obtained. The normal mode analysis method is used to convert these equations into ODE and get the analytical solution to write expressions for displacements, temperatures, stresses. Some comparisons carried out to view the initial stress influence on the field variables. Numerical results are graphed to view the influence of initial stress. Some particular cases are deduced in this study.

Keywords

Thermoelasticity, Functionally Graded, Normal Mode Method, Initial Stress, Energy Dissipation

1. Introduction

Thermoelasticity has many applications and one of them is thermoelastic stress analysis (TSA) which consists of qualities appraisal of a model of radar antennae support column subjected to the bending; the magneto elasticity has practical applications in geophysics, optics, plasma physics etc. The linear theory of elasticity is important for engineering when the structure or material is under external or internal loadings. When the structure or the material's temperature changes this implies a thermal loading. A thermal shock is a sudden variation in temperature from hot to cold or vice versa, when the interval of thermal shock is equal to the order of the lowest natural frequency of structure, this type of problems classified under thermoelasticity which assumes that temperature is expressed from the 1st law of thermodynamic. Biot [1] presented the uncoupled thermo-elasticity theory. There was a shortcoming in this theory and the temperature resulting from the equation of heat follows a parabolic behavior which means that there is an infinite speed of propagation for it, which has a paradox with the physical observations.

Two relaxations times were firstly introduced by Muller in generalized theory of thermoelasticity. With the aid of his entropy inequality he inserted a close approach by considering restrictions on a class off constitutive equations. Green and Naghdi [2] introduced three completely different models of thermoelasticity, which are called thermoelasticity of type I, thermoelasticity of type II and thermoelasticity of type III. Green and Naghdi created a concept in generalized thermoelasticity. He postulated that there is no wasted energy in thermo-elastic process *i.e.* thermo-elasticity without energy dissipation. This principal means that the heat flow does not consist of energy dissipation. When Fourier conductivity is dominant, then the temperature relation has undammed the solutions of thermal wave without wasted energy or energy dissipation. Many researchers devoted to carry out different articles in thermoelasticity, in the light of the GN theories of type II or/and of type III. Chandrasekharaiah [3] showed uniqueness theorems using energy method. Ezzat *et al.* [4] [5] studied the usage of GN theories a mathematical model of magneto-thermoelasticity. Kumar *et al.* [6] and Sharma *et al.* [7] studied several cases in GN theories.

Functionally graded materials was introduced first as a thermal barrier by a group of material scientists [8], other than homogenous materials (FGM) is different class of advances composite material which were developed in recent decades and used in many engineering applications. The properties of (FGM) is gradually change in material properties with respect to spatial coordinates and has the ability to reduce thermal stresses when a thermal shock occur. For these properties (FGM) has a widely applications in industry and engineering applications as we mentioned such as aerospace, nuclear reactors, pressure vessels and pipes. Kumar *et al.* discussed the influence of rotation, magnetic on (FGM) subjected to mechanical load [9], Mohamed H. Hendy discussed the application of fractional order theory to a functionally graded perfect conducting thermoelastic half space with variable Lame's Modulii [10], Vel and Batra, and Qian and Batra studied the three dimensional steady or transient thermal stress problems of functionally graded rectangular plate [11] "sadek", Javaheri and Eslami studied thermal bulking of functionally graded plates [12], Sherief and Abd El-Latief discussed modeling of variable Lamé's modulii for a FGM generalized thermoelastic half Space [13]. Abd-Alla *et al.* [14] discussed Propagation of Rayleigh waves in magneto-thermoelastic half-space of a homogenous orthotropic material under the effect of rotation, initial stress and gravity field. Abd-Alla *et al.* [15] studied Rayleigh waves in magneto elastic half-space of orthotropic material under influence of initial stress and gravity field. Abd-Alla *et al.* [16] studied Influence of rotation, magnetic field, initial stress, and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Othman *et al.* [17] studied the effect of initial stress and Hall current on a magneto-thermoelastic porous medium with microtemperatures. Kumar *et al.* [18] studied the effect of initial stress on the propagation characteristics of waves in fiber-reinforced transversely isotropic thermoelastic material under an inviscid liquid layer.

In our paper, we introduce the effect initial stress on a functionally graded thermoelastic solid. Normal mode technique is used to find the expressions for the variables considered. The comparison of different theories of thermo-elasticity, *i.e.* Green Naghdi type II, Green Lindsay, Chandrasekharaiah and Tzou (DPL) model is carried out. The distributions of variables are displayeded graphically, and to the best of our knowledge, this case not discussed before.

2. Basic Equations

The governing equation of an isotropic, homogenous elastic medium without heat source:

2.1. Equation of Motion

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{1}$$

2.2. Constitutive Relations

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e - \beta T)\delta_{ij} - p(w_{ij} + \delta_{ij}) \tag{2}$$

2.3. Equation of Heat

$$KT_{,ii} + K^* \dot{T}_{,ii} = \rho c_e \ddot{T} + \gamma T_0 \ddot{e} \tag{3}$$

2.4. Strain-Displacement Relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4}$$

$$w_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{5}$$

3. Formulation of the Problem

By using the Cartesian coordinates the thermoelastic isotropic medium is considered in half space ($x \geq 0$) and in the plane of *xy*, therefore $u_i = (u, v, 0)$.

In non-homogenous medium the parameters $\lambda, \mu, \beta, k, k^*, \rho$ are not constant and depend on the position so we can replace them by

$$\lambda_0 f(\vec{x}), \mu_0 f(\vec{x}), \beta_0 f(\vec{x}), k_0 f(\vec{x}), k_0^* f(\vec{x}), \rho_0 f(\vec{x})$$

where $\lambda_0, \mu_0, \beta_0, k_0, k_0^*, \rho_0$ are considered to be constants and $f(\vec{x})$ is a dimensionless function in space variable.

From Equations (2), (4) and (5) in (1) we get

$$\begin{aligned} \rho_0 f(x)\ddot{u} = {} & \frac{\partial f}{\partial x}\left[2\mu u_{,x} + \lambda_0 (u_{,x} + v_{,y}) - \beta_0 \theta\right] \\ & + f(x)\left[2\mu_0 u_{,xx} + \lambda_0 (u_{,xx} + v_{,xy}) - \beta_0 \theta_{,x}\right] - \frac{1}{2} p(v_{,xy} - u_{,yy}) \end{aligned} \tag{6}$$

$$\rho_0 f(x)\ddot{v} = \frac{\partial f}{\partial x}\mu_0\left(u_{,y}+v_{,x}\right) + f(x)\mu_0\left(u_{,xy}+v_{,xx}\right) - \frac{1}{2}p\left(v_{,xx}-u_{,xy}\right) + f(x)\left[2\mu_0 u_{,yy} + \lambda_0\left(u_{,xy}+v_{,yy}\right) - \beta_0\theta_{,y}\right] \tag{7}$$

$$\left(K\nabla^2 - k^*\nabla^2\frac{\partial}{\partial t}\right)T - \rho c_e\ddot{T} - \beta T_0\dot{e} = 0, \tag{8}$$

By assuming that: $f(x) = e^{-rx}$, where r is a dimensionless parameter. By using dimensionless variables

$$x_i' = \frac{w^*}{c_0}x_i,\ u_i' = \frac{\rho_0 c_0 w^*}{\gamma_0 T_0}u_i,\ t' = w^* t,\ w^* = \frac{\rho_0 c_e c_0^2}{k_0},$$
$$c_0^2 = \frac{\lambda_0 + 2\mu_0}{\rho_0},\ p' = \frac{p}{\lambda_0+\mu_0}$$

By using the dimensionless quantities the Equations (6) and (7) take the form

$$\ddot{u} = -r\left[E_{11}u_{,x} + E_{12}v_{,y} - E_{13}\theta\right] + u_{,xx} + E_{17}v_{,xy} - E_{15}\theta_{,x} + E_{18}u_{,yy} \tag{9}$$

$$\ddot{v} = -rE_{21}\left[u_{,y}+v_{,x}\right] + v_{,yy} + E_{22}v_{,xx} - E_{15}\theta_{,x} + E_{23}u_{,xy} \tag{10}$$

where $E_{11} = \frac{c_1}{w^*}$, $E_{12} = \frac{\lambda_0}{\rho_0 c_1 w^*}$, $E_{13} = \frac{\beta_0 c_1}{\gamma_0 w^*}$, $E_{14} = \frac{\lambda_0}{\rho_0 c_1^2}$, $E_{15} = \frac{\beta_0}{\gamma_0}$, $E_{16} = \frac{\mu_0}{\rho_0 c_1^2}$,

$$\nabla^2\theta + E_{31}\nabla^2\dot{\theta} - r\left(E_{11}\theta_{,x} + E_{32}\dot{\theta}_{,x}\right) = E_{33}\dot{\theta} + E_{34}\ddot{e}\ , \tag{11}$$

where $E_{18} = E_{16} + \frac{p}{2}$, $E_{21} = \frac{\mu_0}{\rho_0 c_1 w^*}$, $E_{22} = E_{16} - \frac{p}{2}$, $E_{23} = E_{16} + \frac{p}{2} + E_{14}$

Equation (3) after dimensionless become

$$\nabla^2\theta + E_{31}\nabla^2\dot{\theta} - r\left(E_{11}\theta_{,x} + E_{32}\dot{\theta}_{,x}\right) = E_{33}\dot{\theta} + E_{34}\ddot{e} \tag{12}$$

where $E_{31} = \frac{k_0 w^*}{k_0^*}$, $E_{31} = \frac{c_1 k_0 w^*}{k_0^*}$, $E_{33} = \frac{\rho_0 c_e c_1^2}{k_0^*}$, $E_{34} = \frac{\gamma_0\beta_0}{\rho_0 k_0^* c_1 w^*}$

We will use the normal mode analysis method to get the exact solution as

$$\left[u, v, \theta, \sigma_{ij}\right] = \left[u^*, v^*, \theta^*, \sigma_{ij}^*\right](y)e^{(wt+ibx)}, \tag{13}$$

where ω is the complex time constant (frequency), i is the imaginary unit, b is the wave number in the x-direction and $\left[u^*, v^*, e^*, \theta^*, \sigma_{ij}^*\right]$ are the amplitudes of the functions.

Applying Equation (16) in Equations (13)-(15) we get

$$\left(E_{18}D^2 - A_1\right)u^* + A_2 Dv^* + A_3\theta^* = 0, \tag{14}$$

$$\left(D^2 - A_4\right)v^* + A_5 Du^* - E_{15}D\theta^* = 0, \tag{15}$$

$$\left(A_6 D^2 - A_9\right)\theta^* - A_8 u^* - A_9 Dv^* = 0, \tag{16}$$

where

$$A_1 = b^2 + w^2 + irbE_{11},\ A_2 = igE_{17} - rE_{12},\ A_3 = rE_{13} - ibE_{15},$$
$$A_4 = w^2 + irE_{21}b + b^2E_{22},\ A_5 = -rE_{21} + ibE_{23},\ A_6 = 1 + E_{31},$$
$$A_7 = b^2 + E_{31}b^2w + ibwrE_{32} + irbE_{11} + E_{33}w^2,\ A_8 = iE_{34}w^2b,\ A_9 = E_{34}w^2$$

Eliminating u^*, v^*, θ^* from Equations (17)-(19) we obtain the sixth order differential equation

$$\left(D^6 - AD^4 + BD^2 - C\right)\left\{u^*(y), v^*(y), \theta^*(y)\right\} = 0, \tag{17}$$

where $A = \left(E_{18}A_7 + E_{18}A_4A_6 + A_9E_{15}E_{18} + A_5A_2A_6\right)/\left(E_{18}A_{16}\right)$,
$B = \left(A_4A_7E_{18} + A_1A_7 - A_3A_9A_5 + A_8A_2E_{15} + A_1A_4A_6 + A_9E_{15}A_1 + A_5A_2A_7\right)/\left(E_{18}A_{16}\right)$,
$C = \left(A_1A_4A_7 + A_3A_4A_8\right)/\left(E_{18}A_{16}\right)$

Equation (17) can be factored into

$$\left\{\prod_{n=1}^{3}\left(D^2 - k_n^2\right)u^*(y), v^*(y), \theta^*(y)\right\} = 0 \tag{18}$$

$$u = \sum_{n=1}^{3} M_n e^{wt + ibx - k_n y}, \tag{19}$$

$$v = \sum_{n=1}^{3} H_{1n} M_n e^{wt + ibx - k_n y}, \tag{20}$$

$$\theta = \sum_{n=1}^{3} H_{2n} M_n e^{wt + ibx - k_n y}, \tag{21}$$

where $M_n\ (n = 1, 2, 3)$ are some constants,

$$H_{1n} = \frac{k_nE_{15}\left(E_{18}k_n^2 - A_1\right) + A_3A_5k_n}{A_2E_{15}k_n^2 + A_3\left(k_n^2 - A_4\right)},\ H_{2n} = \frac{A_2A_5k_n^2 - \left(E_{18}k_n^2 - A_1\right)\left(k_n^2 - A_4\right)}{A_3\left(k_n^2 - A_4\right) + E_{15}A_2k_n^2}.$$

To get the solution of stresses, substitute from Equations (23) and (24) in the dimensionless of Equation (2) we get

$$\sigma_{xx} = e^{-rx}\sum_{n=1}^{3} H_{3n} M_n e^{wt + ibx - k_n y} - p, \tag{22}$$

$$\sigma_{yy} = e^{-rx}\sum_{n=1}^{3} H_{4n} M_n e^{wt + ibx - k_n y} - p, \tag{23}$$

$$\sigma_{xy} = \sum_{n=1}^{3} H_{5n} M_n e^{wt + ibx - k_n y}, \tag{24}$$

where $H_{3n} = ib - E_{14}k_nH_{1n} - H_{2n}$, $H_{4n} = -k_nH_{1n} + ibE_{14} - H_{2n}$,
$H_{5n} = E_{16}\left(-k_n + ibH_{1n}\right) - e^{rx}\frac{p}{2}\left(k_n + ibH_{1n}\right)$.

4. Applications

To determine the coefficients $M_n\ (n = 1, 2, 3)$ we consider the boundary conditions at $y = 0$;

1) The normal stress condition:

$$\sigma_{xx} = 0, \tag{25}$$

2) The tangential stress condition:

$$\sigma_{xy} = 0, \tag{26}$$

2) A thermal shock is applied at the boundary:

$$T = p_1 e^{wt+ibx}, \tag{27}$$

Apply these conditions we get

$$\left.\begin{aligned} H_{51}M_1 + H_{52}M_2 + H_{53}M_3 &= 0 \\ H_{71}M_1 + H_{72}M_2 + H_{73}M_3 &= 0 \\ H_{21}M_1 + H_{22}M_2 + H_{23}M_3 &= p_1 \end{aligned}\right\} \tag{28}$$

By using the inverse matrix or Cramer rule we can get M_1, M_2 and M_3.

5. Numerical Results and Discussion

For the purpose of numerical results we choose copper as a thermoelastic material for which we take the following values of the physical constants. Following Dhaliwal and Singh [19] magnesium material was chosen for this purpose.

$$\lambda = 9.4\times10^{10}\ \mathrm{N/m^2},\ \mu = 4\times10^{10}\ \mathrm{N/m^2},\ k = 1.7\times10^{2}\ \mathrm{N/s\cdot K},$$
$$T_0 = 298\ \mathrm{K},\ \alpha_t = 7.4033\times10^{-7}\ \mathrm{N/m^2},\ \rho = 1.74\times10^{3}\ \mathrm{kg/m^3},$$
$$c_e = 383.1\ \mathrm{J\cdot kg^{-1}\cdot K^{-1}},\ w = -0.7,\ x = 0.2,\ k^* = 297,\ p_1 = 10$$

The previous numerical technique was used for the distribution of the real part of the temperature T, the displacement, components u, v and the stresses components σ_{xx}, σ_{xy}, for the problem in the following figures.

Figures 1-4 compare the values in absence and existence of initial stress. The red line denote for the absence and the blue line denote for the existence of initial stress.

Figure 1 displays the horizontal displacement and y, the curves decrease first as $0 \le y \le 0.2$ than increase in $0.2 \le y \le 0.9$ than converges to zero after that.

Figure 2 shows the vertical displacement and y, the curves increase till reach the value 0.8 then converges to zero next.

Figure 3 describes σ_{yy} with y, the graph sharply increase intially till y is 0.3 followed by decreasing as $0.3 \le y \le 0.7$ then converges to zero as $y \ge 0.7$.

Figure 4 investigates σ_{xx} with y, the energy dissipation values of σ_{xx} rapidly increasing when $0 \le y \le 0.3$ then decreasing when $0.3 \le y \le 0.8$ and get closer to zero next, without energy dissipation the values of σ_{xx} have small increment as $0 \le y \le 0.3$ and decrease slowly and converges to zero as $y \ge 0.3$.

Figures 5-10 are carried out in comparing different values for the parameter r, the blue line represent the case $r = 0$, while the red line when $r = 0.5$

Figure 5 shows the horizontal displacement u with y, the curve decrease in the range $0 \le y \le 0.8$ then converges to zero.

Figure 6 depicts the vertical displacement v with y, the graph increase as $0 \le y \le 0.6$ and then it get closer to zero as $y \ge 0.6$.

Figure 7 depicts the temperature T with y, at $r = 0$ the graph increase very slowly in $0 < y < 0.75$ then slowly decrease then approach after that to zero, at $r = 0.5$ the temperature rapidly increase when $0 \le y \le 0.1$ then converges to zero

Figure 8 displays shear stress σ_{yy} with *y*. The values of shear stress decrease as $0 \le y \le 0.4$ then get closer to zero as *y* become greater.

Figure 9 illustrates tangential stress σ_{xy} with *y*. In this figure; the values of the tangential stress decrease in the range $0.1 \le y \le 0.4$ and increase when $0.4 \le y \le 1.5$ then it tends to zero.

Figure 10 illustrates normal stress σ_{xx} versus *y*, the curve decrease in the range $0 \le y \le 0.4$ and then converges to zero, and we notice the values of normal stress are almost equal in the two different values of *r*.

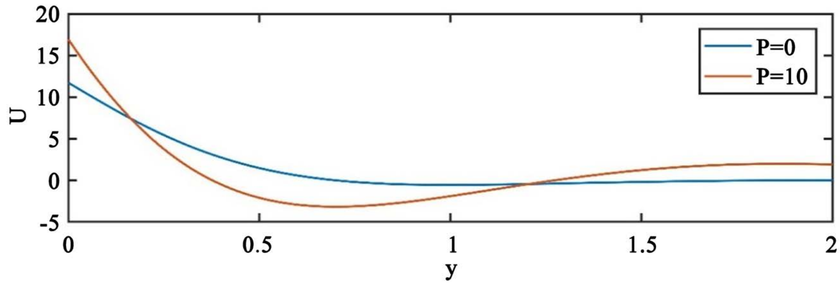

Figure 1. Horizontal displacement *u* with *y*.

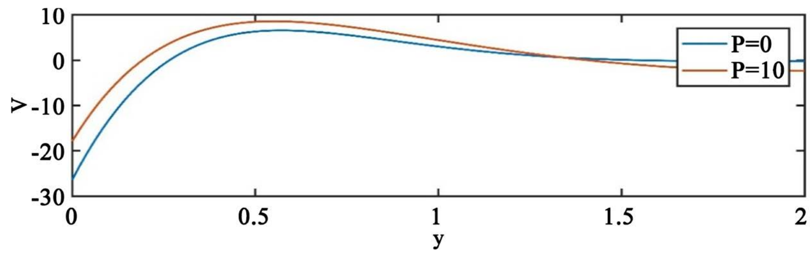

Figure 2. Vertical displacement *v* with *y*.

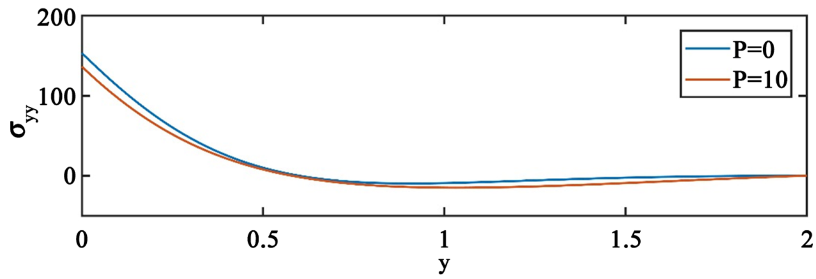

Figure 3. Shear stress σ_{yy} with *y*.

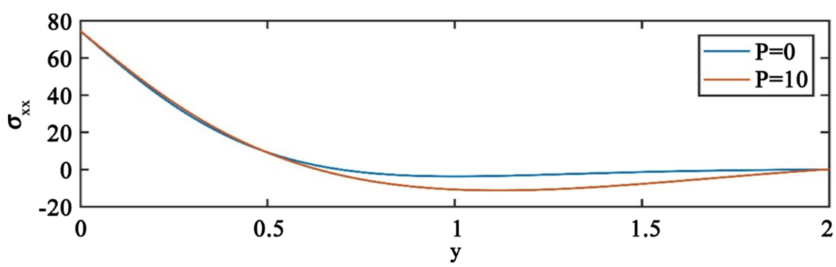

Figure 4. Normal stress σ_{xx} with *y*.

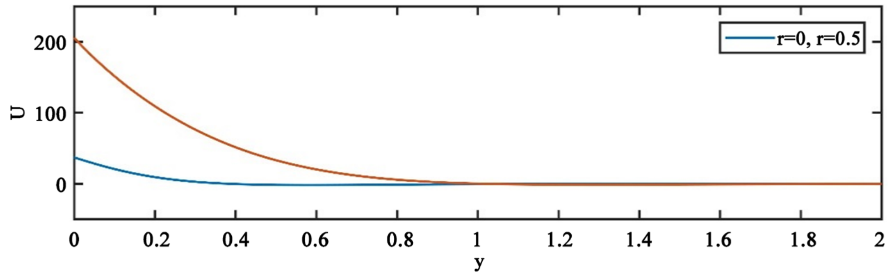

Figure 5. Horizontal displacement *u* with *y*.

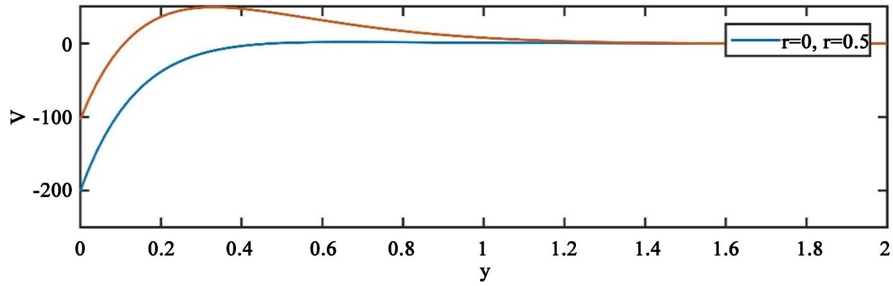

Figure 6. Vertical displacement *v* with *y*.

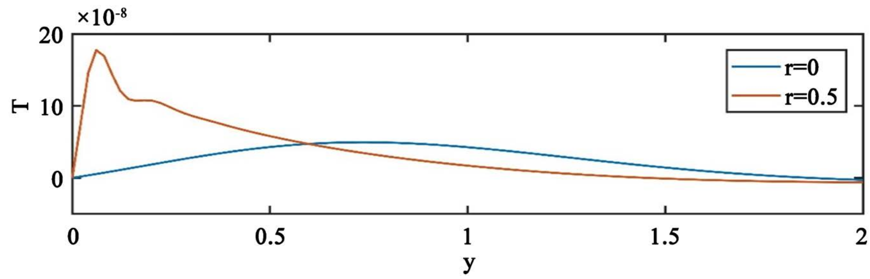

Figure 7. Temperature *T* with *y*.

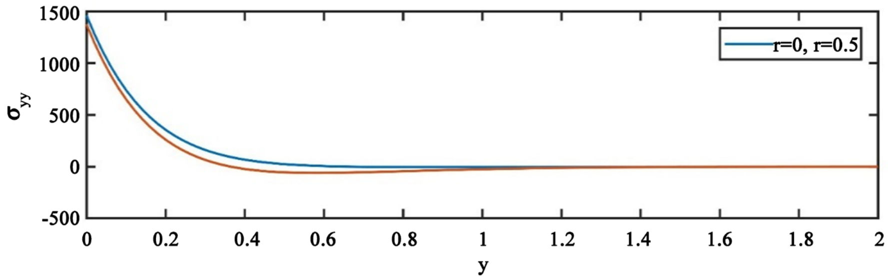

Figure 8. Shear stress σ_{yy} with *y*.

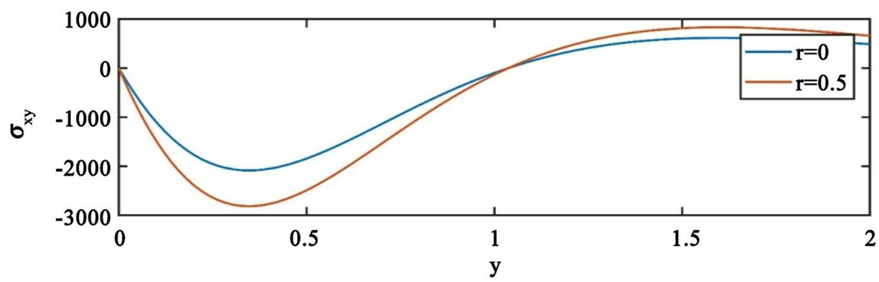

Figure 9. Tangential stress σ_{xy} with *y*.

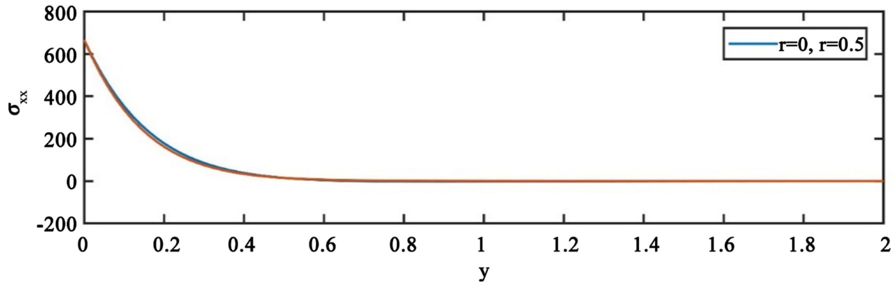

Figure 10. Normal stress σ_{xx} with *y*.

6. Conclusion

In this paper we studied the effect of initial stress on functionally graded material under the context of Green-Naghdi theory. We used Normal Mode Method to get an analytical solution and expressions for the distributions of stresses, strains, displacements, current density and temperature. It is observed that these distributions are affected by current and its magnetic field is more than its absence in other words the effect of hall current cannot be neglected. The temperature distribution is always positive and the phenomenon of infinite speed does not exist, according to different values of r the distributions differ.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Nomenclature

σ_{ij}	Components of stresses
μ_0	Magnetic permeability
ρ	Density
λ, μ	Lame's constants
u	Displacement vector
K	Thermal conductivity
T	Absolute temperature
c_e	Specific heat at constant strain