

A New Strong-Field Metric Closer to Observation in Gravity Theory

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Abstract

The Schwarzschild metric is the most commonly used metric solution that can be derived from Einstein's field equations. It is used to predict most gravitational phenomena that can be checked with observation, especially for weak field phenomena. However, in one of the final steps of deriving this metric, one relies on Newton's theory, which suggests that the standard and well-known Schwarzschild metric is valid for only weak fields. The Schwarzschild solution itself is an exact solution to Einstein's field equation; there is no doubt about that. However, it is the last calibration step of one of the parameters that makes the standard Schwarzschild solution likely valid only in weak fields. We present a new metric that is an exact solution to Einstein's field equation that provides the same predictions as the weak field Schwarzschild metric in weak gravitational fields, but gives significant new predictions in strong gravitational fields, particularly for black holes. We have derived this metric in a more formal way before, but this paper offers a new and interesting angle on the metric.

Keywords

General Relativity Theory, Schwarzschild Metric, Weak Field, Strong Fields, Black Holes

1. Background

Haug and Spavieri [1] have recently introduced a new exact metric solution to Einstein's field equation that they have coined *the Mass-Charge Metric in Curved Spacetime*. Their formal derivation of the new solution can be difficult to fully grasp without being a GR specialist. In this paper, we show a simpler and more informal way that leads to the same metric. For a full rigorous derivation, we rec-

commend our other paper [1], but this paper gives a new perspective suggesting that perhaps the Schwarzschild solution is in reality only a weak field solution mistaken as a strong field solution.

The gravitational field strength gradually increases as the escape velocity increases. A weak field approximation will only be a good approximation when $v_e \ll c$. So when $v_e \ll c$ one can still possibly distinguish a weak field approximation and a strong field solution by experiments with high precision measurement, for example in the gravity field of the sun, a point we will return to shortly. Close to the event horizon of a black hole v_e approaches c and a weak field approximation will give very different predictions than a strong field exact solution.

The Haug-Spavieri Mass-Charge metric is an exact vacuum solution around a spherical object in a spherical coordinate system. We will also focus on a spherical coordinate system in this paper. The solution presented in this paper corresponds to what we can call the minimal solution of the Haug-Spavieri metric (where the charge and magnetic moment are set to zero) and also it corresponds to the extremal solution of the Reissner-Nordström metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

In this paper, we will arrive at this metric from a different angle that in many ways is more intuitive, but again for a more rigorous derivation see our paper on the Mass-Charge metric.

2. The Standard Schwarzschild Metric

Einstein's [2] field equations are given by:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

Based on the assumption of a static, uncharged, spherical mass, as well as setting the cosmological constant to zero, Schwarzschild [3] [4] derived the following metric for spherical polar coordinates:

$$c^2 d\tau^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (3)$$

where α has to be determined before the metric can be used for any practical purposes. The standard methodology in the literature to determine the value of α is to rely on Newton's [5] theory; in short;

$$\frac{1}{2} m v_e^2 = \frac{GMm}{R} \Leftrightarrow \left(\frac{v_e^2}{c^2}\right)_{\text{Newton}} = \frac{2GM}{Rc^2}$$

$$\frac{v_e^2}{c^2} = 1 \rightarrow \alpha = R_s = \frac{2GM}{c^2}$$

This yields the standard well-known (textbook) Schwarzschild metric:

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 R}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (4)$$

We note that only a few books on general relativity, such as Hobson, Efstathiou, and Lasenby [6], specifically mention that this is a weak field limit. Other books, such as Weinberg's [7], Poisson and Will's [8], and the major work by Misner, Thorne, and Wheeler [9], do not point out specifically that the last step in deriving their final Schwarzschild metric is linked to calibrating it to Newton's weak field limit and that the solution is therefore likely valid for weak fields only.

If this is the case, the weak field Schwarzschild metric is not appropriate for making predictions related to strong gravitational fields, including studies of black holes at the centers of galaxies and quasars. For instance, it has been used to predict the escape velocity and the Schwarzschild radius in general relativity theory. We argue that such weak-field metric predictions are likely not valid in strong gravitational fields, which we define as fields where the escape velocity v_e is significant relative to c . We do not contest the existence of black holes but, as we will soon see, a new strong-field metric solution gives different and more logical predictions that even seem to be in line with observations.

To illustrate the weakness of the weak field limit, consider the escape velocity in Newton's theory, which is determined by solving the equation for v :

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \quad (5)$$

which gives $v = \sqrt{\frac{2GM}{R}}$. It is evident that this formula cannot be valid when v is close to the speed of light because the kinetic energy formula used in the derivation, $E_k \approx \frac{1}{2}mv^2$, is only the first term of a Taylor series expansion from Einstein's relativistic kinetic energy, $E_k = mc^2\gamma - mc^2$. However, it is well known that general relativity theory predicts the same escape velocity as Newton, as shown in [10]. For years, it has been a mystery to us why the escape velocity derived from general relativity theory is the same as that derived from Newton's theory, since the one derived from Newtonian theory is clearly valid only when $v \ll c$. We think we have found the reason: in general relativity theory, the final Schwarzschild metric is mistakenly assumed to be valid in strong gravitational fields, despite being "calibrated" to Newton theory for weak fields and then used also for strong gravitational fields predictions.

We claim that the general relativity escape velocity, which is identical to the one we get from pure Newton, is actually valid for a weak gravitational field only, because it is derived from the Schwarzschild metric, which, in its last step, is calibrated to the weak field Newtonian gravity theory.

So, nothing is wrong with the Schwarzschild metric itself; it is just that one needs to be aware that the last step used to turn it into a practical and useful formula relies on Newton and that the resulting solution is not necessarily of general validity and is likely valid for the special case of weak gravitational fields only. The

issue, therefore, is how to obtain a metric of general validity that also holds for the strongest gravitational fields and, as a special case, reduces to the Schwarzschild metric.

In the next section, we consider a generic metric of the Reissner-Nordström type [11] [12], which is an exact solution of Einstein's field equations and contains two constants to be determined. Then, we discuss two approaches for the calibration of this more general metric that led to an expression suitable for describing gravitation also in strong fields.

We are naturally well aware that the Schwarzschild metric, despite giving the same escape velocity and radius of a black hole as Newtonian theory, predicts strongly curved space-time when the escape velocity is close to c , while Newtonian theory predicts flat space-time. We think this is the reason many GR experts assume the Schwarzschild metric must be valid in a strong gravitational field. Still, we question whether it is valid to use an escape velocity that is identical to the Newtonian escape velocity, which is clearly only valid when $v_e \ll c$. Again, this comes from calibrating the Schwarzschild metric to Newton theory, which we will argue makes the Schwarzschild metric strictly valid only in weak gravitational fields.

3. The New Metric

Let us consider the following metric,

$$ds^2 = \left(1 - \frac{\alpha}{R} + \frac{u^2}{R^2}\right) c^2 dt^2 - \left(1 - \frac{\alpha}{R} + \frac{u^2}{R^2}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (6)$$

this is an exact solution to Einstein's field equations with the general form of the Reissner-Nordström metric [11] [12]. Also, the standard Schwarzschild metric is a special case of this solution when $u = 0$ and α is calibrated to the Newton weak field limit.

In our first approach, we suggest a heuristic way to calibrate our new metric by analogy with the standard calibration of the Schwarzschild metric. Then, in the more rigorous second approach of Section 3, we exploit the property that for Planck masses, the Newtonian gravitational force is equivalent to the Coulomb force between two Planck charges.

We will here suggest how to get a new strong field type of Schwarzschild metric but now based on considering the strong gravitational field. The Newton limit is given by

$$\begin{aligned} \frac{1}{2} m v_e^2 &= \frac{GMm}{R} \Leftrightarrow \left(\frac{v_e^2}{c^2}\right)_{\text{Newton}} = \frac{2GM}{Rc^2} \\ \frac{v_e^2}{c^2} &= 1 \rightarrow \alpha = R_s = \frac{2GM}{c^2} \end{aligned} \quad (7)$$

The corresponding special relativistic expression is (see Appendix A and B for a derivation of the same escape velocity directly from the extremal Reissner-Nordström metric and the minimal Haug-Spavieri metric solution),

$$\begin{aligned} \gamma mc^2 - mc^2 &= \frac{\gamma m GM}{R} \\ \frac{1}{\gamma^2} &= 1 - \frac{v_e^2}{c^2} = \left(1 - \frac{GM}{Rc^2}\right)^2 = 1 - 2\frac{GM}{Rc^2} + \frac{G^2 M^2}{R^2 c^2} \\ \left(\frac{v_e^2}{c^2}\right)_{relativity} &= 2\frac{GM}{Rc^2} - \frac{G^2 M^2}{R^2 c^2} \end{aligned} \quad (8)$$

where in Equation (8) the factor gamma appears consistently. For the expression of the kinetic energy and the interaction potential energy we have the correspondence,

$$\begin{aligned} \left(\frac{v_e^2}{c^2}\right)_{Newton} &\rightarrow \left(\frac{v_e^2}{c^2}\right)_{relativity} \\ \frac{2GM}{Rc^2} &\rightarrow \frac{2GM}{Rc^2} - \frac{G^2 M^2}{R^2 c^2} \end{aligned}$$

Then, in correspondence to the Schwarzschild component, for our metric we obtain the expression

$$\left(1 - \frac{\alpha}{R}\right) = \left(1 - \frac{2GM}{Rc^2}\right) \rightarrow \left(1 - \frac{2GM}{Rc^2} + \frac{G^2 M^2}{R^2 c^2}\right) = \left(1 - \frac{2GM}{Rc^2} + \frac{U}{R^2 c^2}\right)$$

where U is a physical quantity that turns out to be related to the gravitational potential in our generic metric of the Reissner-Nordström type [1] [11] [12]. In an alternative second approach described below, we determine U starting directly from the original Reissner-Nordström metric and assuming that in the special case of a Planck mass particle, the charge is considered to be the Planck charge. By means of our first approach, our full strong field metric is therefore given by

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right)^{-1} dR^2 \\ &\quad - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \end{aligned} \quad (9)$$

which represents an exact solution of Einstein's field equations that we propose for interpreting experimental observations. Let us point out briefly some of the relevant aspects of our new metric. In tensor matrix form we have

$$g_{\mu,\nu} = \begin{pmatrix} -\left(1 - \frac{2GM}{Rc^2} - \frac{G^2 M^2}{c^4 R^2}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{Rc^2} + \frac{G^2 M^2}{c^4 R^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & R^2 \sin^2 \theta \end{pmatrix} \quad (10)$$

4. Testability

For all weak field phenomena, the new metric gives predictions that are virtually identical to those of the standard (weak field) Schwarzschild metric. The first term

is identical to the weak field Schwarzschild metric approximation, which is the cornerstone of much of gravitational physics. For large values of R (weak field $R \gg \frac{GM}{c^2}$), the second term is almost negligible with respect to the first.

Most of the very solid tests of general relativity have been done in the weak field, such as gravitational time dilation, gravitational red-shift, and light deflection. In many weak field experiments, one can set up situations where one has direct observations in all parts of the experiment. For example, for gravitational time dilation, all one needs are basically two atomic clocks at different altitudes. For gravitational red-shift, one can send a light beam down from a tower and thereby have full control of the observations of the frequency sent out and received, as first published by Pound and Rebka [13] in 1959. There is no doubt that the weak field Schwarzschild metric approximation gives very accurate predictions for a long series of gravity phenomena observed in a weak gravitational field.

Observations from a strong gravitational field are much more difficult, as there is no strong gravitational field on Earth. However, it seems that even the Sun has a sufficiently strong gravitational field to set up experiments to distinguish whether the Schwarzschild metric is only a weak-field approximation, as suggested and discussed by Haug and Spavieri [14]. The gravitational time dilation in the new strong field metric is given by (see the appendix for how we derive the escape velocity):

$$\begin{aligned} t_1 &= t_2 \sqrt{1 - \frac{v_e^2}{c^2}} \\ t_1 &= t_2 \sqrt{1 - \frac{\sqrt{\frac{2GM}{R} - \frac{G^2 M^2}{c^2 R^2}}}{c^2}} \\ t_1 &= t_2 \left(1 - \frac{GM}{c^2 R} \right) \end{aligned} \quad (11)$$

This is in contrast to the Schwarzschild metric, which predicts

$$t_1 = t_2 \sqrt{1 - \frac{v_e^2}{c^2}} = t_2 \sqrt{1 - \frac{2GM}{c^2 R}}.$$

We should actually be able to test the difference even

on Earth using very accurate clocks. If we place one high-precision clock on the surface of the Earth and another one initially synchronized in orbit at 20,000 km, then over 30 days we should expect to observe about 5.91×10^{-13} s:

$$t \left(\frac{1 - \frac{GM}{c^2 R_2}}{\sqrt{1 - \frac{2GM}{c^2 R_2}}} - \frac{1 - \frac{GM}{c^2 R_1}}{\sqrt{1 - \frac{2GM}{c^2 R_1}}} \right) \approx 5.91 \times 10^{-13} \text{ s} \quad (12)$$

Since the gravitational field is much stronger around the Sun, one could place satellites in different orbits around the Sun with high-precision clocks; here even standard atomic clocks should suffice as suggested and discussed by Haug and Spavieri [14].

5. The Strong Field Limit Gives a Perfect Fit between the Planck Scale and the Planck Mass Micro Black Hole, While the Weak Field Schwarzschild Metric Cannot Do So

The Planck scale is considered important for the potential development of quantum gravity. Micro black holes mentioned in the literature are suggested to have a mass around the Planck mass. The possible closer link between the Planck scale and micro black holes could be of great importance in further understanding gravity and its potential connection to the Planck scale. **Table 1** presents various properties associated with the Planck scale (see also [15]).

Table 1. The table shows a series of properties linked to the Planck scale. The Planck charge is defined as per the pre-2019 standard. However, in older standards, the elementary charge and the Planck charge were defined differently from today.

Planck property	Formula:
Planck length	$l_p = \sqrt{\frac{G\hbar}{c^3}}$
Planck time	$t_p = \sqrt{\frac{G\hbar}{c^5}}$
Planck mass	$m_p = \sqrt{\frac{\hbar c}{G}}$
Planck energy	$E_p = \sqrt{\frac{\hbar c^5}{G}}$
Planck temperature	$T_p = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$
Planck surface area sphere	$A = 4\pi r^2 = 4\pi l_p^2$
Planck surface area cube	$A = 6l_p^2$
Planck volume sphere	$V_p = \frac{4}{3}\pi l_p^3$
Planck volume cube	$V_{p,c} = r^3 = l_p^3$
Planck acceleration	$a_p = \frac{Gm_p}{l_p^2}$
Planck density	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck power	$\frac{m_p c^2}{t_p} = \frac{c^4}{G}$
Planck pressure	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$
Planck speed	$\frac{l_p}{t_p} = c$
Planck charge	$q_p = \sqrt{4\pi\epsilon_0\hbar c} = \frac{e}{\alpha} = \sqrt{\frac{\hbar}{c}} \cdot 10^7$

Extensive analysis shows that the micro black holes in the standard Schwarzschild metric cannot match more than one or two aspects of the Planck scale with any mass candidate for a micro black hole. We have shown this in **Table 2**.

Table 2. The table shows a series of suggested mass candidates that have been suggested to be micro black holes. None of them when worked out from the Schwarzschild metric can match more than a few properties of the Planck scale.

	Fits at least one Planck property					No fit to the Planck scale				
Candidate number:	1	2	3	4	5	6	7	8	9	10
Reference first mentioned:	[16]	[17]	[18] [19]	[20]	This paper	[21]	[20]	[20]	[20]	[18] [22]
Micro black hole mass candidate:	m_p	$\frac{1}{2}m_p$	$\sqrt{\pi}m_p$	$\frac{1}{4}m_p$	$\frac{1}{\sqrt{8}}m_p$	$2m_p$	πm_p	$\frac{1}{\sqrt{\pi}}m_p$	$\sqrt{2}m_p$	$\frac{1}{\sqrt{2}}m_p$
Schwarzschild radius $R_s = \frac{2Gm}{c^2}$	$2l_p$	l_p	$2\sqrt{\pi}l_p$	$\frac{1}{2}l_p$	$\frac{1}{\sqrt{2}}l_p$	$4l_p$	$2\pi l_p$	$\frac{2}{\sqrt{\pi}}l_p$	$\sqrt{8}l_p$	$l_p\sqrt{2}$
Reduced Compton wavelength $\frac{\hbar}{mc}$	l_p	$2l_p$	$\frac{l_p}{\sqrt{\pi}}$	$4l_p$	$l_p\sqrt{8}$	$\frac{l_p}{2}$	$\frac{l_p}{\pi}$	$l_p\sqrt{\pi}$	$\frac{l_p}{\sqrt{2}}$	$l_p\sqrt{2}$
Compton wavelength $\frac{h}{mc}$	$2\pi l_p$	$4\pi l_p$	$2\sqrt{\pi}l_p$	$8\pi l_p$	$\sqrt{32}\pi l_p$	πl_p	$2l_p$	$2\sqrt{\pi^3}l_p$	$\sqrt{2}\pi l_p$	$\sqrt{8}\pi l_p$
Schwarzschild time $\frac{R_s}{c}$	$2t_p$	t_p	$2\sqrt{\pi}t_p$	$\frac{1}{2}t_p$	$\frac{1}{\sqrt{2}}t_p$	$4t_p$	$2\pi t_p$	$\frac{2}{\sqrt{\pi}}t_p$	$\sqrt{8}t_p$	$\sqrt{2}t_p$
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	$\frac{c}{\sqrt{2}}$	$c\sqrt{2\pi}$	$\frac{c}{\sqrt{8}}$	$c/2$	$c\sqrt{8}$	$c\pi\sqrt{2}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$2c$	c
Escape velocity at λ	$\frac{c}{\sqrt{\pi}}$	$\frac{c}{2\sqrt{\pi}}$	c	$\frac{c}{\sqrt{16\pi}}$	$\frac{c}{\sqrt{8\pi}}$	$\frac{2c}{\sqrt{\pi}}$	$c\sqrt{\pi}$	$\frac{c}{\pi}$	$\frac{c\sqrt{2}}{\sqrt{\pi}}$	$\frac{c}{\sqrt{2\pi}}$
Schwarzschild density $\frac{m}{\frac{4}{3}\pi R_s^3}$	$\frac{1}{8}\rho_p$	$\frac{1}{2}\rho_p$	$\frac{1}{8\pi}\rho_p$	$2\rho_p$	ρ_p	$\frac{1}{32}\rho_p$	$\frac{1}{8\pi^2}\rho_p$	$\frac{\pi}{8}\rho_p$	$\frac{1}{16}\rho_p$	$\frac{1}{4}\rho_p$
Planck energy mc^2	E_p	$\frac{E_p}{2}$	$\sqrt{\pi}E_p$	$\frac{E_p}{4}$	$\frac{E_p}{\sqrt{8}}$	$2E_p$	πE_p	$\frac{E_p}{\sqrt{\pi}}$	$\sqrt{2}E_p$	$\frac{E_p}{\sqrt{2}}$
Surface area acceleration $g = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	$\frac{a_p}{2}$	$\frac{a_p}{4\sqrt{\pi}}$	a_p	$\frac{a_p}{\sqrt{2}}$	$\frac{a_p}{8}$	$\frac{a_p}{4\pi}$	$\frac{a_p\sqrt{\pi}}{4}$	$\frac{a_p}{\sqrt{32}}$	$\frac{a_p}{\sqrt{8}}$
Number of Planck scale matches	3	2	1	1	1	0	0	0	0	0
Compton or reduced matches	0	0	1	0	0	0	0	0	0	1
Conflict Planck scale limits $R_s < l_p$	no	no	no	yes	yes	no	no	no	no	no

However, when we take into account that the black hole radius is given by $R_{s,2} = \frac{GM}{c^2}$ (the radius where $v_e = c$) in the strong field metric, then a Planck mass micro black hole fits all aspects of the Planck scale as seen in **Table 3**.

Table 3. The table shows predictions for a Planck mass micro black hole under the standard Schwarzschild metric (weak field) and alternatively as predicted from our strong field metric. Under the standard Schwarzschild, one sees the Planck mass micro black hole only matches a few aspects of the Planck scale, while when taking into account the predictions from our strong field metric it matches all the aspects of the Planck scale.

	Schwarzschild metric	Strong field metric
Micro black hole mass candidate:	m_p	m_p
Escape radius where $v = c$	$2l_p$	l_p
Reduced Compton wavelength $\frac{\hbar}{mc}$	l_p	l_p
Schwarzschild time R_s/c	$2t_p$	t_p
Escape velocity at $\bar{\lambda}$	$c\sqrt{2}$	c
Density $\frac{m}{\frac{4}{3}\pi R^3}$	$\frac{1}{8}\rho_p$	ρ_p
Planck energy mc^2	E_p	E_p
Surface area acceleration $\kappa = \frac{Gm}{R_s^2}$	$\frac{a_p}{4}$	a_p
Planck charge	No	Yes
Number of Planck scale matches	3	9 (all)
Reduced Compton wavelength match	no	yes

Similar results can be achieved with the extremal solution of the Reissner–Nordström metric, as demonstrated by Haug and Spavieri [23]. The strong gravitational field solution presented here is also mathematically identical to the extremal solution of the Reissner–Nordström metric. However, here we arrive at such a solution from a quite different perspective. Again, we ask whether the standard Schwarzschild metric is only valid in weak gravitational fields, despite most GR specialists claiming it is also valid in strong gravitational fields. That it is only valid in weak gravitational fields, we believe, largely stems from its calibration to Newtonian gravitational theory.

6. Conclusion

In this paper, we have presented a new metric solution, valid for weak and strong gravitational fields, as an alternative to the Schwarzschild solution. This is equivalent to the Mass-Charge metric given by Haug and Spavieri. Our solution provides more logical predictions, such as for micro black holes, where the Planck mass black hole now fits all aspects of the Planck scale. This is not possible to achieve with the weak field Schwarzschild metric. Haug and Spavieri have recently also suggested a verifiable test that can be performed in the Sun’s gravitational field to test whether the standard Schwarzschild metric is only a weak field metric

as we here suggest or not. Overall, we believe that our new metric represents an important step forward in understanding the behavior of strong gravitational fields, and we look forward to future research in this area.

Data Availability Statement

No data has been used for this study.

Conflicts of Interest

The authors declare no conflict of interest.

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Appendix

Appendix A: Escape Velocity Derivation

We start with the metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (13)$$

Define

$$f(R) = 1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2} = \left(1 - \frac{GM}{c^2 R}\right)^2. \quad (14)$$

Radial Motion

For radial motion ($d\theta = d\phi = 0$):

$$ds^2 = f(R) c^2 dt^2 - f(R)^{-1} dR^2. \quad (15)$$

For a massive particle:

$$ds^2 = c^2 d\tau^2. \quad (16)$$

Thus:

$$c^2 d\tau^2 = f(R) c^2 dt^2 - f(R)^{-1} dR^2. \quad (17)$$

Dividing by $d\tau^2$:

$$c^2 = f(R) c^2 \left(\frac{dt}{d\tau}\right)^2 - f(R)^{-1} \left(\frac{dR}{d\tau}\right)^2. \quad (18)$$

Conserved Energy

Define conserved energy per unit mass:

$$\varepsilon = \frac{E}{mc^2} = f(R) \frac{dt}{d\tau}. \quad (19)$$

Thus:

$$\frac{dt}{d\tau} = \frac{\varepsilon}{f(R)}. \quad (20)$$

Substitute:

$$c^2 = \frac{\varepsilon^2 c^2}{f(R)} - \frac{1}{f(R)} \left(\frac{dR}{d\tau}\right)^2. \quad (21)$$

Multiply by $f(R)$:

$$f(R) c^2 = \varepsilon^2 c^2 - \left(\frac{dR}{d\tau}\right)^2. \quad (22)$$

So:

$$\left(\frac{dR}{d\tau}\right)^2 = c^2 (\varepsilon^2 - f(R)). \quad (23)$$

Escape Condition

At infinity:

$$f(\infty) = 1. \quad (24)$$

For marginal escape:

$$\frac{dR}{d\tau} = 0 \Rightarrow \varepsilon = 1. \quad (25)$$

Local Velocity

For a static observer:

$$\varepsilon = \sqrt{f(R)}\gamma, \quad (26)$$

where

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}. \quad (27)$$

For escape:

$$1 = \sqrt{f(R)}\gamma. \quad (28)$$

Thus:

$$\gamma = \frac{1}{\sqrt{f(R)}}. \quad (29)$$

So:

$$1 - \frac{v_e^2}{c^2} = f(R). \quad (30)$$

Escape Velocity

$$v_e(R) = c\sqrt{1-f(R)}. \quad (31)$$

Substitute $f(R)$:

$$v_e(R) = c\sqrt{1 - \left(1 - \frac{2GM}{c^2R} + \frac{G^2M^2}{c^4R^2}\right)}. \quad (32)$$

Simplify:

$$v_e(R) = c\sqrt{\frac{2GM}{c^2R} - \frac{G^2M^2}{c^4R^2}}. \quad (33)$$

Horizon Check

For extremal case:

$$R_H = \frac{GM}{c^2}. \quad (34)$$

Then:

$$v_e(R_H) = c. \quad (35)$$

Appendix B: Escape Velocity from the Second-Order Geodesic Equation

Metric and Radial Motion

We begin with the extremal Reissner–Nordström metric or the minimal solution of the Haug–Spavieri metric (that also is the metric derived in this paper):

$$ds^2 = f(R)c^2 dt^2 - f(R)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{36}$$

where

$$f(R) = 1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2} = \left(1 - \frac{GM}{c^2 R}\right)^2. \tag{37}$$

For purely radial motion:

$$d\theta = d\phi = 0, \tag{38}$$

so

$$ds^2 = f(R)c^2 dt^2 - f(R)^{-1} dR^2. \tag{39}$$

Second-Order Geodesic Equation

The geodesic equation is:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{40}$$

For the radial coordinate R :

$$\frac{d^2 R}{d\tau^2} + \Gamma_{tt}^R \left(\frac{dt}{d\tau}\right)^2 + \Gamma_{RR}^R \left(\frac{dR}{d\tau}\right)^2 = 0. \tag{41}$$

Christoffel Symbols

Metric components:

$$g_{tt} = f(R)c^2, \quad g_{RR} = -f(R)^{-1}. \tag{42}$$

Inverse components:

$$g^{tt} = \frac{1}{f(R)c^2}, \quad g^{RR} = -f(R). \tag{43}$$

Using

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\nu} (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}), \tag{44}$$

we obtain:

(a) Γ_{tt}^R

$$\Gamma_{tt}^R = -\frac{1}{2} g^{RR} \partial_R g_{tt} = \frac{1}{2} c^2 f(R) f'(R). \tag{45}$$

(b) Γ_{RR}^R

$$\Gamma_{RR}^R = \frac{1}{2} g^{RR} \partial_R g_{RR} = -\frac{1}{2} \frac{f'(R)}{f(R)}. \tag{46}$$

Radial Geodesic Equation

Substituting:

$$\frac{d^2 R}{d\tau^2} + \frac{1}{2} c^2 f'(R) \left(\frac{dt}{d\tau}\right)^2 - \frac{1}{2} \frac{f'(R)}{f(R)} \left(\frac{dR}{d\tau}\right)^2 = 0. \tag{47}$$

Rewriting:

$$\frac{d^2 R}{d\tau^2} = -\frac{1}{2}c^2 f f' \left(\frac{dt}{d\tau}\right)^2 + \frac{1}{2} \frac{f'}{f} \left(\frac{dR}{d\tau}\right)^2. \quad (48)$$

Time Geodesic Equation

$$\frac{d^2 t}{d\tau^2} + 2\Gamma_{tR}^t \frac{dt}{d\tau} \frac{dR}{d\tau} = 0. \quad (49)$$

With

$$\Gamma_{tR}^t = \frac{1}{2} \frac{f'}{f}, \quad (50)$$

we obtain:

$$\frac{d^2 t}{d\tau^2} + \frac{f'}{f} \frac{dt}{d\tau} \frac{dR}{d\tau} = 0. \quad (51)$$

This integrates to:

$$\frac{d}{d\tau} \left(f(R) \frac{dt}{d\tau} \right) = 0. \quad (52)$$

Thus:

$$f(R) \frac{dt}{d\tau} = \varepsilon. \quad (53)$$

Substitute into Radial Equation

$$\frac{dt}{d\tau} = \frac{\varepsilon}{f(R)}. \quad (54)$$

Then:

$$\frac{d^2 R}{d\tau^2} = -\frac{1}{2}c^2 \varepsilon^2 \frac{f'}{f} + \frac{1}{2} \frac{f'}{f} \left(\frac{dR}{d\tau}\right)^2. \quad (55)$$

So:

$$\frac{d^2 R}{d\tau^2} = \frac{f'}{2f} \left[\left(\frac{dR}{d\tau}\right)^2 - c^2 \varepsilon^2 \right]. \quad (56)$$

Normalization Condition

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = c^2. \quad (57)$$

Thus:

$$f c^2 \left(\frac{dt}{d\tau}\right)^2 - f^{-1} \left(\frac{dR}{d\tau}\right)^2 = c^2. \quad (58)$$

Substitute:

$$c^2 \varepsilon^2 - \left(\frac{dR}{d\tau}\right)^2 = c^2 f. \quad (59)$$

Hence:

$$\left(\frac{dR}{d\tau}\right)^2 = c^2 (\varepsilon^2 - f). \quad (60)$$

Escape Condition

At infinity:

$$f(\infty) = 1. \quad (61)$$

For marginal escape:

$$\left. \frac{dR}{d\tau} \right|_{\infty} = 0. \quad (62)$$

Thus:

$$\varepsilon = 1. \quad (63)$$

Local Velocity

For a static observer:

$$\varepsilon = \sqrt{f(R)} \gamma, \quad (64)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (65)$$

Thus:

$$1 = \sqrt{f(R)} \gamma. \quad (66)$$

So:

$$1 - \frac{v_e^2}{c^2} = f(R). \quad (67)$$

Escape Velocity

$$v_e(R) = c \sqrt{1 - f(R)}. \quad (68)$$

Substitute:

$$v_e(R) = c \sqrt{1 - \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2} \right)}. \quad (69)$$

that can be rewritten as:

$$v_e(R) = c \sqrt{\frac{2GM}{c^2 R} - \frac{G^2 M^2}{c^4 R^2}} \quad (70)$$

Horizon Check

$$R_H = \frac{GM}{c^2}. \quad (71)$$

$$v_e(R_H) = c. \quad (72)$$