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A Geometric Model of Infinity with Empirical Justification through Analysis of the Deviation between Experimental Data and Quantum Theory for Hardy's Paradox

Douglas Chesley Gill

Independent Researcher, Midland Canada
Email: douggill921@gmail.com

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Abstract

The issue, examined across multiple disciplines, is that infinities cannot be represented as discrete collections. Incompleteness is unavoidable. The *1-D* geometric model examines the paradoxical relationship that develops when attempting to define infinite states as contained structures. Two representations of the right triangle are constructed within the unit circle. The first follows standard geometric principles. The second assigns a counter-rational unitary value to each of the triangle's vectors and incorporates emergence within its structure. Although the two formats of the unit circle have a null relationship, they are shown to share membership by calculating the cosine-square for the right triangle. The geometric structure serves as a model to understand the role of paradox in the representation of infinity. The model is applied to explain the mechanism of emergence in the collapse of the wavefunction, and to Hardy's paradox, offering a rationale for the discrepancy between its formal calculations and experimental results.

Keywords

Hardy's Paradox, Infinity, Emergence, Paradox, Russell's Paradox, Quantum Formalism, Null State, Empty State, Bell's Inequality, EPR Experiments, Nonlocality

1. The Paradoxical Structure of Infinity

Infinites take many forms; however, a general definition incorporating a boundary condition can be stated. Definition: The boundary of an infinity is the absolute limit beyond which the property of the state does *not* extend.

By inductive argument, a corollary follows from numerous studies of infinity in mathematics and philosophy. It will not be possible to complete the listing of the named elements or properties on a logical basis in a finite number of steps, and the state will remain null for conclusion on its boundary.

The above definition for infinity, along with the self-circular prohibition to conclusion, codifies what cannot be known instead of what can be determined in defining the property of a state as an infinity.

2. The Static and Dynamic Formats of Infinity

Infinities come in both "static" and dynamic formats. Linguistic statements intended to universally (infinitely) contain all logical reference to an argument's property devolve into self-circular regressions, prohibiting a conclusion. Apart from the circularity of infinite regression, they lack a dynamic framework and are static.

On the other hand, experiments on the collapse of the wavefunction demonstrate the dynamic change that occurs across the dimensional structures of correlated quantum and classical states.

3. The Static Infinity of the Russell Paradox

Russell's paradox is the set of all sets (R) that are not members of themselves [1].

Russell's paradox is the prime example of self-circular infinite regression in a logical argument. Another example having the same structural basis is the liar paradox [2]: "I am telling a lie."

In both cases, the logical *not* function negates the property that identifies each segment in the argument. In the liar paradox, the *not* function operates as the negation of a truth statement.

By including the logical *not* in the Russell paradox argument, the direct identification of the property that defines the elements is prohibited. Whatever property is named, the members of the set do not contain that property, and the property is *imaginary* on an analogous basis that the square root of minus one (i) is termed imaginary in mathematics.

There are two parts to the argument in Russell's paradox:

1) The *not* function prohibits the member-segments within (R) from having either discrete identities or relationships.

2) The set (R) shares the property of the membership category with its members and should qualify for membership within itself. If (R) is placed within itself, it is an error since, by the *not* function, it is not a member of itself. However, if it is not placed within itself, it is an error because it shares the property of its elements.

The only resolution to the logical paradox is to impose a restriction on the structure of sets under the Zermelo–Fraenkel (ZF) rule of set theory. Infinite sets are prohibited [3]. Nevertheless, if one conjectures that paradox is a mechanism in fundamental (universal) structures, then the ZF rule does not resolve the underlying paradox. Instead, it avoids it.

How Does the Russell Set Fit the Model's Definition of an Infinity?

Definition: The boundary of an infinity is the absolute limit beyond which the property of the state does *not* extend, and by its corollary, must remain null for conclusion.

The Russell set fits the above definition of an infinity. It's stated property is that it is an infinite collection of *all* sets that are not members of themselves.

The placement of (R) meets the corollary of the definition. It is *null* for a conclusion, since it has a paradoxical alternative placement outside the boundary to its own infinity.

4. The Dynamic Infinity in the Half-Silvered Mirror Experiment

In the half-silvered mirror experiment, a photon beam is directed at the first half-silvered mirror, which splits (quantum entangles) the wavefunction in a one-quarter phase shift between the orthogonal transmitted and reflected paths. Two fully silvered mirrors, positioned one on each path, redirect the entangled portions of the wavefunction to a second half-silvered mirror at the apparatus's exit, which reverses the phase entanglement.

Detection devices are positioned on each of the two orthogonal paths of exit beyond the final half-silvered mirror ([4], pp. 261-262).

Conservation of the Degree of Freedom

Before the first half-mirror, the wavefunction of the photon has a known (classical) path on the x -axis. This means it occupies one degree of freedom in a classical space with two degrees of freedom: the x and y axes.

When the wavefunction passes through the first half-mirror, it is down-converted to a quantum basis.

When the wavefunction exits the second half-mirror, it reenters the two-dimensional structure of the classical plane, which has two degrees of freedom. The photon projects on the same axis it entered the apparatus, the y-axis remains *null*, and the photon's one degree of freedom is conserved.

However, if a detection device exists within the path structure of the apparatus on either of the two parallel paths, the interference pattern between the two entangled phases of the photon instantaneously collapses. The photon will then have a 50:50 classical probability of being in either path of the apparatus, and when it passes through the second half-mirror, there is a 50:50 probability of occupying either orthogonal direction of the half-mirror.

The important takeaway is that the photon's one degree of freedom is conserved inside and outside of the apparatus. If the wavefunction is collapsed, only one of the two (real) orthogonal axes contributes to its propagation in a single event. On the inside, if the wavefunction is not collapsed, it is entangled on both axes, and only one orthogonal axis is real. The other is imaginary from a classical perspective.

The different dimensional complexities of the two spaces (quantum and classical) reveal the dynamic property of emergence across them. Both frameworks are null for conclusion in a single event based on the probability structure that applies to each.

The *null* property for the interior of the apparatus is created by the *iy*-axis. The interior is *null* for a detection-based conclusion.

The *null* property of the exterior, in the higher-dimensional complexity of classical space, includes event sequencing in *imaginary* time. The space's probability structure remains *null* for the conclusion on a single-event basis because it takes two probabilistic events in sequence to *fill* the space completely.

Finally, the relationship between the two fundamental orthogonal frameworks (quantum and classical) on the two-dimensional plane is paradoxical and incompatible with a logically consistent interpretation.

The structure meets the definition of infinity in Section 1. The quantum relationship structure of the interior is classically inconsistent in two frameworks for projecting the photon.

5. The *1-D* Geometric Model

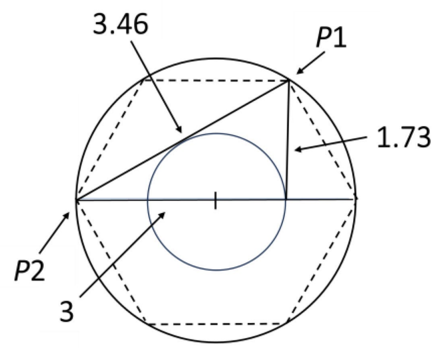

Figure 1. With geometrically consistent vectors.

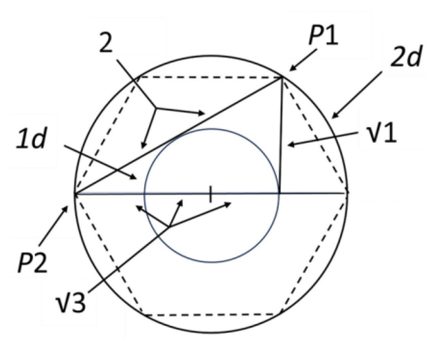

Figure 2. Formally inconsistent.

The *1-D* geometric model analyzes the structure of infinity through the relationship between the two forms of the unit circle in Figure 1 and Figure 2 ([5], p. 3711). The first format adheres to standard geometric principles, while the second assigns a counter-rational, formally nonsensical unitary value of "1" to the vectors on the circumference and the right triangle. Despite the null relationship between the two geometries, the cosine-square calculations for both triangles agree.

In Figure 1, the diameter of the outer circumference is 4, while the relevant segment for calculating the cosine-square value measures 3.

The sides of the 30-60-90 triangle are 3, 1.732, and 3.464. In Figure 2, each vector segment, given a value of "1", becomes a unitary state, having no external frame of reference for its property on a formal basis. The two geometric structures within the unit circle's boundary have a *null* relationship yet share common membership on a unitary basis through the calculation of the cosine-square.

6. The Nonformal Structure of Figure 2

The structure in Figure 2 has an emergent property that the inner and outer circumferences represent distinct dimensional levels, increasing outward from a one-dimensional level (on the inner circumference) to a two-dimensional level (on the outer circumference).

Vectors that eccentrically cross inward carry the square root, and consequently, do not extend in a consistent dimensional space. The hypotenuse consists of two vectors that start and end concentrically on the outer circumference, and together, the square roots cancel. The point of origin for the vectors is eccentric to the circumferences at the ninety-degree vertex of the right triangle.

The side of the right triangle on the x-axis has an eccentric transition to the ninety-degree vertex across three infinities. The square root is applied on a nonformal basis that groups the three infinities into a single state within the function.

6.1. Figure 1 Calculations (Formal)

$$P1 \text{ - } \mathrm{Cos}^2\,(60) = (1.73205/3.4641)^2 = 0.25 \tag{1}$$

$$P2 \text{ - } -\mathrm{Cos}^2\,(30) = -(3/3.4641)^2 = -0.75 \text{ (phase shift 180 degrees)} \tag{2}$$

6.2. Figure 2 Calculations (Nonformal)

$$P1 \text{ - } \mathrm{Cos}^2\,(60) = (\sqrt{1}/2)^2 = 0.25 \tag{3}$$

$$P2 \text{ -} -\mathrm{Cos}^2\,(30) = -(\sqrt{3}/2)^2 = -0.75 \text{ (phase shift 180 degrees)} \tag{4}$$

7. The Paradoxical Structure of Figure 1 and Figure 2

Figure 2 is a fundamentally closed nonformal structure (as a unitary state) with no logical frame of reference outside its boundary. Nevertheless, by computing the cosine square in each figure, Figure 2 shares a common, albeit formally hidden, relationship with Figure 1.

The best way to understand the hidden relationship between the two geometries and the paradox they present is to remove the values assigned to the vectors in both geometries.

What is left is a geometric figure with no stated mathematical framework. The assumption is that it has a *fixed-dimensional* basis on a *real* two-dimensional plane, defined by the orthogonal vectors of the (x, y) axes.

Figure 2 negates the *fixed-plane* assumption by attaching square-root values to the vectors, and the inner circumference is then dimensionally inconsistent with that in **Figure 1**.

A final step in justifying the relationship of the two geometries is the calculation of the cosine-square. The two geometries are paradoxically conjoined, having noncongruent dimensional frameworks, yet are congruent for the calculation.

The inner and outer circumferences in **Figure 2**, and the relationship between the two geometries, are emergent. In **Figure 2**, the two-dimensional complexity on the outer circumference emerges from the one-dimensional basis of the inner circumference.

Finally, the consistent *fixed* basis of the dimensional structure of **Figure 1** is emergent from the inconsistent and *nonfixed* dimensional basis in **Figure 2**.

The relationship of the two geometries meets the definition of infinity in Section 1. The unit circle contains (as an infinity) both geometries, and it is not possible to list the named elements or properties on a logical basis.

8. Opening the Quantum State in Degrees of Freedom

What determines the degree of freedom in the *1-D* geometric model? Backing up, classical states have discrete frameworks consisting of preexisting segment structures. The segments, their arrangement, and groupings can be selected on an observational basis, and establish the degree of freedom represented in the state.

Instead, in the *1-D* geometric model, just as the dimensional structure is emergent, in **Figure 2**, so is the degree of freedom when the quantum state is opened by detection through measurement. Before opening, the quantum state has null degrees of freedom because it lacks a discrete structure.

The *1-D* model analyzes the degree of freedom that develops in the sequenced, "emergent" entry into the quantum framework via measurement.

9. Observing the Quantum State without Collapse

Unlike the half-silvered mirror experiment, Bell's inequality and Hardy's paradox involve mechanisms that enable the gathering of information about a complex waveform without collapsing the quantum basis of the state.

Bell's inequality does this by examining the orthogonal relationship between entangled photons when measured using calcite crystals with angular misalignment. The orthogonal connection between the photons indicates that they form a quantum basis in classical separation.

Hardy's paradox uses weak measurement to directly examine the complex wavefunction of two entangled particles measured at two Dark Ports in a four-path quantum structure.

Both experiments open the native waveform rotation of the quantum state in two degrees of freedom (two calcite crystals in Bell's inequality and two Dark Ports in Hardy's paradox).

10. The Emergence of Degrees of Freedom in Figure 2

In **Figure 2**, the dimensional structure and degrees of freedom are depicted to emerge sequentially, as measured at the rotation points $P1$ and $P2$ in the waveform.

The emergent format contradicts the fundamental principles of classical logic and mathematics, in which the dimensionality of the classical state does not emerge but is preexistent and fixed.

The fixed basis is crucial for the consistency required in mathematical operations such as multiplication and exponentiation. In the geometry of the Cartesian plane, each axis contributes to the two-dimensional space on a similarly fixed basis.

11. Bell's Inequality

Bell's inequality considers the polarization correlation of entangled photons as they pass through separate calcite crystals when rotated relative to each other. The experimental verification of Bell's inequality by Alain Aspect *et al.* confirmed that no local theory can explain any phenomenon that is governed by quantum formalism ([6] p.p. 225-227). The observed probabilities are quantum, not classical.

Graphical representation of the disagreement between crystals illustrates the region in which Bell's inequality cannot be satisfied by a local theory ([4] p. 248).

The theoretical calculation of the polarization correlation, "... is an elementary exercise in quantum theory. This calculation predicts that $PC(\theta) = \cos^2(\theta)$ ([6] p. 224)."

The inequality is demonstrated by comparing the probability distributions, in quantum and classical settings, for two degrees of freedom. On a classical basis, for a structure that contains two (unweighted) degrees of freedom, the maximum disagreement between randomly generated outcomes is fifty percent.

For the flip of two coins, or equally the flip of one coin in sequenced events, the maximum disagreement for the outcome is fifty percent.

The Theoretical Calculation of Bell's Inequality

Bell's inequality is demonstrated by calculating the probability of disagreement when two calcite crystals have opposite rotations of 30 degrees. The maximum disagreement between the polarizations reaches 3 in 4, contradicting the expected outcome under classical probability, which, for comparison, would be at most 2 in 4.

Section 3.1. and 3.2. compare the pertinent calculations from **Figure 1** and **Figure 2**.

1) A 30-degree rotation of one crystal results in a PC equal to 1 in 4 (0.25).

2) A 30-degree rotation of the second crystal, resulting in a 60-degree separation

between them, results in a polarization disagreement of 3 in 4 (0.75).

The results for Bell's inequality are predicted by the analysis of the rotation of the wavefunction in Figure 2 and are further discussed in the following sections for Hardy's paradox.

12. Hardy's Paradox

Hardy's paradox is a thought experiment proposed by Lucien Hardy in which a particle and its antiparticle can interact without annihilating each other [7]. Aharonov *et al.* calculated the theoretical quantum-level probability structure [8], and Lundeen and Steinberg conducted an experiment using entangled photons [9].

There are four potential pathway combinations for two entangled particles in Hardy's paradox—two inner paths and two outer paths. The mixed states are when one particle occupies an inner path and the other an outer path (In/Out).

The analysis of interest to the geometric model, is the weak value of the interference, for the particles at the two Dark Ports when they have joint occupation on the inner paths (Both/In) ([8], p. 4) [calculation result (17)], and when they have joint occupation on the outer paths (Both/Out) ([8], p. 4) [calculation result (19)].

12.1. Theoretical Data

The theoretical calculations for weak values of joint cohabitation of entangled particles, recorded at the Dark Ports (as calculated by Aharonov *et al.*), are listed in the paper by Lunden & Steinberg ([9], p. 3).

1) Cohabitation on the inner paths, dark ports = 0.

2) Cohabitation on the outer paths, dark ports = −1.

12.2. Experimental Data

Experiment weak values by Lunden and Steinberg ([9], p. 3)

1) Cohabitation on the inner paths, dark ports = 0.245.

2) Cohabitation on the outer paths, dark ports = −0.759.

The two sources of experimental error are ([9], p.4),

1) Imperfect switch efficiency (85% ± 3%), and

2) Interaction free measurement probabilities (IFM) of [95% ± 3%] for (E) and [94% ± 4%] for (P).

The expected experimental errors do not appear to account for the divergence between the data and the theoretical calculations by Aharonov *et al.* Instead, the results are within one percent of the values calculated for Bell's inequality for similarly entangled particles and match the rotational analysis of the waveform at $P1$ and $P2$ in Figure 2.

13. The Divergence between Theory and Experiment in Hardy's Paradox

Figure 2 serves as the template to analyze the discrepancy between the theoretical calculations in Section 12.1 and the experimental results in Section 12.2 for Hardy's paradox.

The geometry illustrates the sequential emergence of degrees of freedom in the wavefunction, from null (in the absence of measurement before rotation) to values calculated at $P1$ and $P2$ on a linear basis, for two degrees of freedom at the quantum level.

Calculating the cosine square at $P1$ and $P2$ translates the angular relationship for the right triangle's vectors into a two-dimensional classical framework of discrete probabilities.

The pure phases at the Dark Ports as rotations on the circumference:

-One degree of freedom emerges at $P1$, in the first 60-degree rotation, and

-Two degrees of freedom emerge at $P2$ in the second 120-degree rotation.

The *1-D* model conjectures that the discrepancy between the calculated and experimental values in Hardy's paradox is due to the limitation imposed on the opening of the quantum state, in which degrees of freedom are sequentially emergent. The four-path waveform is opened in two emergent degrees of freedom at the Dark Ports.

14. Discussion—Modelling the Paradoxical Structure of Infinity

There are numerous conjectures that question the conclusions of experiments on the relationship between local and nonlocal phenomena and theorize new approaches.

They range widely, including among them the hidden variable model, string theory, and the role of consciousness itself. None can be ruled out, but at this point, they are conjectures in theoretical modelling without experimental validation.

In the final analysis, they are not relevant to the *1-D* geometric model's thesis, which adopts paradox as a fundamental mechanism. From the outset, all such arguments categorically rule out paradox as a mechanism.

Despite the seemingly nonsensical geometry of **Figure 2** on a classical basis, it provides a rationale for the discrepancy between theory and experimental results in Hardy's paradox. The argument is used to validate the *1-D* model's representation of infinity, emergence, and the role of paradox as a fundamental mechanism across boundaries representing infinities.

The *1-D* geometric model argues that when the quantum state is opened by detection mechanisms, the sequenced emergence of degrees of freedom is fundamentally incompatible with the consistent basis on which degrees of freedom are represented in the classically fixed basis of formal mathematics.

However, it is the only way to understand the conservation principle that applies between the dimensional structures of correlated quantum and classical states.

The Collapse of the Wavefunction

One of the fundamental mysteries of quantum phenomena dates back to the original discussions by Einstein, Podolsky, and Rosen about the collapse of the wavefunction. Bell's inequality highlights the issue because the collapse violates the rules of the classical framework of space and time, as well as our everyday experience.

The *1-D* geometric model offers a clear perspective on solving the issue, using the half-silvered mirror experiment as an example to illustrate the conservation of degrees of freedom between quantum and classical states. The "particle" of the quantum wavefunction moves across the orthogonal two-path structure of the experiment, within a one-dimensional framework (only the x-axis is real).

When the photon's state is collapsed by a disturbance, it enters a higher-dimensional framework and gains a second degree of freedom, enabling it to exhibit discrete properties and a classical probability for occupation on either path of the two-path structure.

Nevertheless, what was *imaginary* on the one-dimensional platform of the quantum structure (the *iy*-axis in the half-mirror experiment) transforms to *imaginary time*, and the phenomenon of change is introduced.

The sequenced probabilities, which the mechanism of time introduces, maintain the "incompleteness" framework of the photon's state. The degree of freedom in the classical format surpasses the degree of freedom that the photon exhibits in any single event.

The issue with interpreting the conservation mechanism on a consistent logical and mathematical basis, other than as a "collapse," is that it is emergent across boundaries having inconsistent relationships.

The paradoxical relationship between the structures in Figure 1 and Figure 2 offers a fundamental, visual, and broad representation of the connection between quantum structure—which prohibits discrete observation—and classical structure—that includes space, action in imaginary time, and gravity.

The two frameworks are inherently incompatible, linked through the mechanism of paradox.

The search to understand the relationship between quantum and classical structure at the largest scale includes the search for a theory of quantum gravity.

The *1-D* geometric toy model suggests a fundamental principle at the simplest level of complexity, which raises the question of whether it has significance for the complex models of the Universe.

The issue ultimately comes down to interpreting the Universe within a single framework, and the *1-D* geometry shows that in its own very small theory-of-everything framework (the unit circle), there are two equally valid but paradoxically linked geometries.

If the relationship suggests a fundamental principle, applicable at the largest of scales, then a theory of quantum gravity will not be possible.

The novel argument of the *1-D* model, that paradox is a fundamental mechanism in Nature, has applications across logic, mathematics, philosophy, and physics.

Conflicts of Interest

The author declares no conflicts of interest in the publication of this paper.

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