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The NUVO Equation: A Scalar Variational Law on Conformal Space

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Abstract

We formulate and analyze *The NUVO Equation* (TNE), a covariant scalar Euler-Lagrange equation for the field $\lambda(x)$ that determines the conformal metric $g_{\mu\nu} = \lambda^2\eta_{\mu\nu}$ on unit-constrained NUVO space. Starting from an invariant scalar action, we derive the field equation for λ, identify the associated Noether currents, and establish a rigorous functional framework for its stationary reductions. Within this scalar-geometric setting we show that, under clearly stated assumptions and limiting regimes, solutions of TNE admit effective equations that reproduce the standard governing equations of several physical domains, including the Newtonian Poisson equation, first post-Newtonian metric behavior, stationary Schrödinger transport, depletion-driven irreversibility, and the finite-mode resonance structure underlying three-generation constraints. These correspondences do not assert that TNE replaces existing theories in their full generality; rather, they demonstrate that a single scalar variational law provides a coherent mathematical backbone from which classical, relativistic, quantum, and depletion phenomena arise as sector-specific reductions. The resulting framework organizes previously developed NUVO results within a common scalar-conformal geometric structure. The resulting framework organizes previously developed NUVO results within a common scalar-conformal geometric structure. The framework is presented as a mathematical foundation result, establishing well-posedness, consistency, and admissible correspondence structure rather than advancing quantitative phenomenological predictions.

Keywords

NUVO Space, Scalar Geometry, Quantization, Coherence, Scalar Field Modulation, Loop Dynamics

1. Introduction and Motivation

Modern theoretical physics employs several distinct but internally consistent frameworks—classical mechanics, special and general relativity, quantum mechanics, and thermodynamics—each governed by its own variational principles and field equations. These formalisms coexist successfully but are usually treated as separate layers, with correspondence relations imposed *a posteriori* (for example, the nonrelativistic limit of relativistic wave equations, or the weak-field limit of Einstein's equations). It is therefore natural to ask whether, within a suitably chosen geometric setting, a single scalar variational law can be written whose admissible solutions reproduce the equations already governing these domains as appropriate limiting cases.

The Nuvo programme addresses this question by working with a scalar-conformal geometry based on a smooth, positive, dimensionless field $\lambda(x)$ that modulates a flat background metric according to

$$g_{\mu\nu} = \lambda^2(x)\eta_{\mu\nu}, \ \lambda(x) > 0. \tag{1}$$

The scalar field λ is interpreted as a unit-constraint field: it rescales local measures of length, time, and curvature, while the tensor $g_{\mu\nu}$ encodes the physical geometry experienced by observers. Within this framework, $\lambda(x)$ functions as a geometric regulator of admissible scalar-modulated configurations rather than as an independent dynamical observable.

Earlier peer-reviewed papers in the series—*NUVO Space I-II*: *Scalar Conformal Geometry and PN*$_1$ *Tests* [1] [2] and *NUVO Quantization I-III*: *Scalar Transport and Quantum Holonomy* [3]-[5]—established the scalar-geometric structure underlying gravitational, quantum, and transport phenomena in Nuvo space. In particular, they showed that:

- The connection, curvature, and post-Newtonian behaviour of $(M, g = \lambda^2\eta)$ admit the standard weak-field gravitational tests as admissible limits;
- Scalar transport and holonomy in this geometry yield stationary quantum-like equations under closure and regularity assumptions;
- Admissible scalar-modulated configurations naturally distinguish conservative from irreversible evolution through capacity and regularity constraints;
- High-curvature scalar configurations admit only a finite number of stable resonance modes under closure and saturation conditions;
- And scalar-conformal transport admits spinorial holonomy corrections in regimes where conformal connections are nontrivial.

The ontological interpretation of these structures—including the role of scalar availability, admissibility, and structural closure—has been consolidated separately and is assumed throughout the present work. Here we focus exclusively on the variational and analytical consequences of that framework.

Against this background, the present paper asks a more focused question: *does there exist a single scalar variational principle on* Nuvo *space whose Euler-Lagrange equation defines the class of admissible* λ *-configurations from which these sectoral laws emerge as consistent reductions*?

The main purpose of this work is to answer that question in the affirmative *at the level of scalar admissibility*. We introduce a covariant scalar action on the conformal manifold $(M, g = \lambda^2\eta)$, vary it with respect to λ, and obtain a scalar field equation that we refer to as *The NUVO Equation* (*TNE*). The equation is not proposed as a replacement for the Einstein or Schrödinger equations in their usual dynamical roles; rather, it defines a unifying admissibility constraint on scalar-modulated geometries whose closed, regular, and sector-compatible limits reproduce those equations under clearly stated assumptions.

From a geometric standpoint, TNE expresses an extremal condition for a scalar energy functional built from curvature and a self-potential for λ, coupled to matter through an effective Lagrangian. From a structural standpoint, it functions as a generator of admissible scalar configurations within the Nuvo framework. When analyzed in appropriate regimes, admissible solutions of TNE yield:

- The Newtonian potential and post-Newtonian corrections consistent with standard weak-field tests [1] [2];
- Stationary scalar-transport equations of Schrödinger type with a geometric effective potential arising from drift cancellation [3]-[5];
- Depletion-like coherence and bandwidth limitations arising from scalar capacity constraints;
- Curvature-saturation limits that enforce a finite number of admissible scalar resonance modes;
- And spin-holonomy corrections arising from conformal transport in scalar-modulated geometries.

The derivations of these correspondences are not repeated in full; instead, we show how the same scalar admissibility equation, when combined with sector-specific assumptions and closure conditions, recovers the structures established in those earlier works.

The structure of the paper is as follows. §2 summarizes the geometric framework of Nuvo space and the postulates assumed throughout. §3 introduces the scalar action and derives TNE in both geometric and background forms. §4 places the stationary reduction in a rigorous operator setting, providing self-adjointness and domain results needed later. §5 develops a hierarchy of limiting regimes and identifies how the familiar equations of Newtonian gravitation, special and general relativity, quantum mechanics, spin dynamics, and irreversible transport emerge as admissible reductions of TNE within the Nuvo context. Subsequent sections examine the energy and stability structure of the scalar field and discuss the interpretation and empirical status of the resulting framework.

Throughout we distinguish carefully between statements that hold at the level of the scalar variational law itself and statements that require additional modelling choices, such as the form of the scalar potential $V(\lambda)$, the choice of matter coupling in L_{matt}, or the imposition of stationarity or weak-field conditions. The present work does not assert that TNE uniquely determines physical observables; rather, it establishes the admissible scalar configurations upon which further accounting and observational classification may be imposed.

Although the NUVO equation reproduces familiar dimensionless constants—most notably the fine-structure constant—in appropriate limits, these constants are not introduced as fundamental couplings. Instead, they arise as effective observational residues of discrete closure structure implicit in the scalar-geometric framework, while the underlying admissibility is governed entirely by the variational law itself.

2. Geometric Framework and Postulates

We recall the geometric setting of Nuvo space and fix notation. Full technical details appear in the peer-reviewed NUVO Space papers [1] [2], which establish the conformal scalar geometry, associated connection, curvature identities, and variational framework used throughout this work. Ontological interpretation, admissibility structure (GRASP), and closure-accounting principles (CAS) are assumed as consolidated in the NUVO ontology synthesis and are not re-derived here.

2.1. Conformal Scalar Metric

Let M be a smooth n-dimensional manifold endowed with a flat background metric $\eta_{\mu\nu}$ and a positive scalar field $\lambda : M \to (0,\infty)$. The physical metric is taken to be conformally related:

$$g_{\mu\nu} = \lambda^2(x)\eta_{\mu\nu},\ g^{\mu\nu} = \lambda^{-2}(x)\eta^{\mu\nu},\ \lambda(x) > 0. \tag{2}$$

Here $\eta_{\mu\nu}$ serves as a reference structure for coordinates and comparisons between frames, while $g_{\mu\nu}$ carries the physical geometric content. The scalar field $\lambda(x)$ acts as a *unit-constraint field*, rescaling local measures of length, time, and curvature. Distinct pairs (λ,η) related by conformal rescaling correspond to different unit conventions describing the same physical geometry.

Because $g_{\mu\nu}$ is conformally flat by construction, all curvature information is encoded in derivatives of λ. This does not trivialize the geometry: λ carries a genuine scalar degree of freedom, and its admissible configurations are determined by the variational principle introduced in §3. The conformal structure separates physical geometry from the reference metric η while keeping curvature analysis analytically tractable.

2.2. Conformal Connection and Curvature

The Levi-Civita connection $\nabla^{(g)}$ associated with $g_{\mu\nu}$ has coefficients

$$\Gamma^{\rho}_{\mu\nu} = \delta^{\rho}_{\mu}\partial_{\nu}\ln\lambda + \delta^{\rho}_{\nu}\partial_{\mu}\ln\lambda - \eta_{\mu\nu}\eta^{\rho\sigma}\partial_{\sigma}\ln\lambda, \tag{3}$$

which are torsion-free and metric-compatible. In four dimensions, the scalar curvature satisfies the standard conformal identity (see, e.g., [6] [7])

$$R(g) = -6\lambda^{-3}\tilde{\Box}\lambda,\ \tilde{\Box} = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}, \tag{4}$$

as reviewed in *NUVO Space I* [1]. This identity provides the geometric bridge between the scalar field λ and the curvature scalar appearing in the scalar action, and it forms the starting point for weak-field and linearized analyses.

2.3. Unit-Constrained Frames and Conformal Gauge

Each local observer is described by a *unit-constrained frame* F in which $\lambda_F = 1$ at the frame origin. Transformations between such frames act by conformal rescaling

$$\left(\eta_{\mu\nu}, \lambda\right) \mapsto \left(\alpha^2 \eta_{\mu\nu}, \lambda/\alpha\right), \ \alpha > 0, \tag{5}$$

leaving the physical metric $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ invariant. This gauge freedom identifies physically equivalent choices of units while maintaining a fixed geometry. The scalar field λ is therefore not a pure gauge artifact: although only $g_{\mu\nu}$ is directly observable, λ provides the internal bookkeeping that tracks unit normalization and curvature.

2.4. Scalar Transport and Continuity

Scalar transport in Nuvo space is described by a conserved four-current

$$J^{\mu}_{\text{sin}} = \lambda \rho u^{\mu}, \tag{6}$$

where ρ is a scalar density and u^{μ} a local four-velocity. In the absence of depletion, this current satisfies the covariant continuity law

$$\nabla^{(g)}_{\mu} J^{\mu}_{\text{sin}} = 0. \tag{7}$$

This continuity relation underpins the scalar transport structure developed in the NUVO Quantization papers [3]-[5]. In the present work it functions as a structural constraint on admissible matter couplings but is not varied directly; instead, it reappears naturally through the Noether structure of the scalar action (Appendix B).

2.5. Postulates for the NUVO Equation

The NUVO Equation is derived under the following standing assumptions:

1) **Scalar conformality.** Physical geometry is determined by a conformal metric $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ on a smooth manifold M with flat reference metric $\eta_{\mu\nu}$.

2) **Unit normalization.** Each local frame satisfies $\lambda_F = 1$ at its origin, with frame transformations given by (5).

3) **Scalar continuity.** In the absence of depletion, scalar transport satisfies the covariant conservation law (7).

4) **Causality.** Disturbances of λ propagate with finite characteristic speed not exceeding c, consistent with the background light cone.

5) **Scalar variational principle.** Admissible configurations of λ are determined by extremization of a scalar action of the form introduced in §3, with suitable scalar potential $V(\lambda)$ and matter coupling L_{matt}.

GRASP (Geometric Admissibility Principle). GRASP is a structural admissibility criterion stating that not every formally writable configuration of fields or trajectories corresponds to a physically realizable state. Within the present framework, GRASP restricts attention to configurations that admit consistent transport, closure, and comparison across unit-constrained frames. Operationally, GRASP enforces that local geometric constructions must extend coherently to global configurations without violating conservation, causality, or frame consistency.

Structural admissibility conditions (GRASP), boundary behavior, and closure requirements are assumed as established in the NUVO ontology synthesis. Within that admissible domain, the Euler-Lagrange equation of the scalar action yields the NUVO Equation, which governs admissible scalar geometries on $\left(M, g=\lambda^2\eta\right)$. See Figure 1 for a visual outline of this paper.

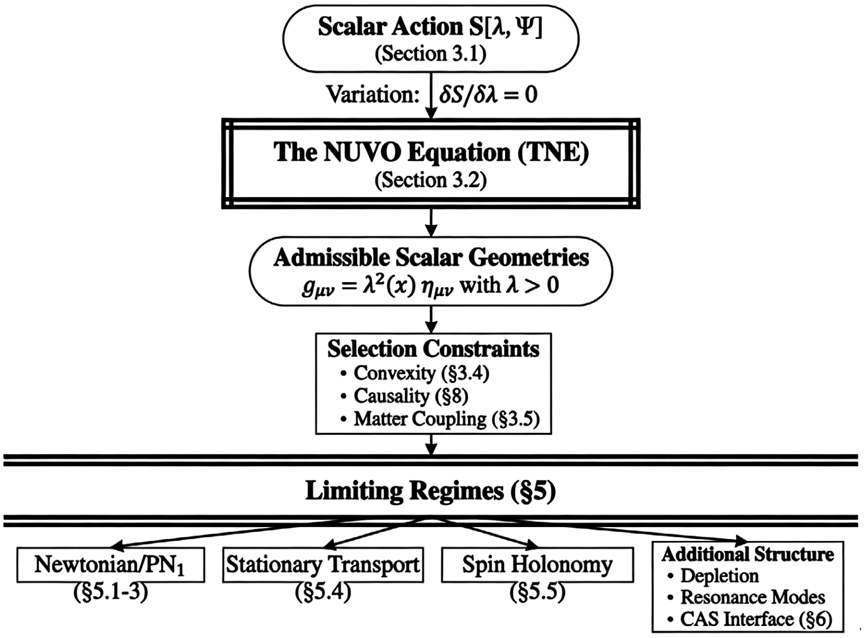

Figure 1. Structural hierarchy of the NUVO variational framework. The scalar action (8) yields the field equation TNE through variation with respect to λ. Selection constraints (§3.4) and regularity hypotheses (§4) define the admissible class of scalar geometries. Limiting regimes (§5) are obtained by imposing additional assumptions (stationarity, weak curvature, etc.) appropriate to each physical sector. Section numbers indicate where each element is developed.

3. Action Principle and Derivation of the NUVO Equation (TNE)

We now introduce a scalar action functional for the field λ and derive its Euler-Lagrange equation. This scalar field equation will be referred to as *The NUVO Equation* (*TNE*) within the Nuvo framework. In this section we focus on the variational structure and the associated conservation statements; issues of functional analytic setting and operator realizations are deferred to §4. Structural admissibility (GRASP), closure-accounting structure (CAS), and the consolidated ontological interpretation of scalar availability are assumed as established in the NUVO ontology synthesis and are not re-derived here.

3.1. Scalar Action Functional

Let $S[\lambda, \Psi]$ denote the total action of the scalar field λ coupled to a generic matter sector Ψ:

$$S[\lambda,\Psi]=\int_M \left[\frac{\kappa}{12}R(g)-V(\lambda)+L_{\text{matt}}(\Psi,\lambda)\right]\mathrm{d}V_g, \tag{8}$$

where $R(g)$ is the Ricci scalar of the conformal metric $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$, κ is a coupling constant relating curvature to scalar energy density, $V(\lambda)$ is a scalar self-potential, and L_{matt} is an effective matter Lagrangian through which the scalar field modulates masses, couplings, or other physical parameters. The conformal volume factor is

$$\mathrm{d}V_g=\sqrt{|g|}\,\mathrm{d}^4x=\lambda^4\mathrm{d}^4x.$$

Scalar-tensor actions of this type and their conformal properties have been studied extensively in the literature (see [8] [9]).

Using the conformal identity $R(g)=-6\lambda^{-3}\tilde{\Box}\lambda$ from (4) yields the background representation

$$S[\lambda,\Psi]=\int \lambda^4\left[-\frac{\kappa}{2}\lambda^{-4}(\nabla\lambda)^2-V(\lambda)+L_{\text{matt}}(\Psi,\lambda)\right]\mathrm{d}^4x, \tag{9}$$

where $(\nabla\lambda)^2=\eta^{\mu\nu}\partial_\mu\lambda\partial_\nu\lambda$. The two expressions (8) and (9) are equivalent in four dimensions (up to boundary terms) and may be used interchangeably.

Relation to standard conformal scalar actions. The action in Equation (9) is formally distinct from the Penrose-Chernikov-Tagirov conformally coupled scalar action and from scalar-tensor theories such as Brans-Dicke. In standard formulations, the scalar field is an independent dynamical degree of freedom coupled to curvature. Here, by contrast, the scalar field λ *defines* the conformal metric itself through $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$, and the variational principle is performed under this geometric constraint. The resulting Euler-Lagrange equation therefore governs admissible conformal geometries rather than an auxiliary matter field.

3.2. Variation with Respect to λ

Varying the action with respect to λ, while holding $\eta_{\mu\nu}$ and Ψ fixed and discarding boundary terms, gives

$$\frac{\delta S}{\delta\lambda}=-\frac{\kappa}{6}\lambda^3R(g)-4\lambda^3V(\lambda)-\lambda^4V'(\lambda)-\frac{\partial}{\partial\lambda}\left(\lambda^4L_{\text{matt}}\right)=0. \tag{10}$$

Equation (10) is the *geometric form* of the NUVO Equation: a scalar balance between curvature, potential, and matter contributions on the conformal manifold (M,g).

Using the curvature identity (4), this may be rewritten on the flat background as

$$\beta\tilde{\Box}\lambda=U'(\lambda)+\frac{\partial L_{\text{matt}}}{\partial\lambda},\ \beta=3\kappa,\ U'(\lambda)=\lambda^3\left(4V+\lambda V'\right), \tag{11}$$

with $\tilde{\Box}=\eta^{\mu\nu}\partial_\mu\partial_\nu$. This background form displays explicitly the propagation of scalar disturbances through the reference geometry. The scalar potential V and matter coupling L_{matt} remain unspecified at this level; their forms reflect physical modeling choices appropriate to specific applications.

3.3. Physical Interpretation and Scope

In the geometric form (10), the $R(g)$ term encodes curvature, while $V(\lambda)$ and L_{matt} supply, respectively, self-interaction and coupling to matter. Unlike Einstein's field equations, TNE is not a tensor equation and does not determine an energy-momentum tensor uniquely. Instead, it expresses an extremal condition for a scalar energy functional whose critical points describe admissible curvature profiles on conformal NUVO space.

Within the consolidated NUVO interpretation, this scalar balance is identified with admissible distributions of scalar availability and transport capacity, but the variational derivation itself is independent of that interpretation. When we compare TNE to governing equations of general relativity or quantum mechanics, we do so strictly in the sense of *limiting correspondence*: under additional assumptions (e.g. weak-field expansions, stationarity, and specific choices of V and L_{matt}), solutions of TNE generate metrics and effective potentials that reproduce known equations to the order considered. No claim is made that TNE replaces these theories outside the NUVO limits under which they are recovered.

3.4. Selection Constraints on the Scalar Potential $V(\lambda)$

The scalar variational law derived in §3 does not fix a unique functional form for the self-potential $V(\lambda)$. However, admissibility within the NUVO framework imposes a set of nontrivial *selection constraints* that sharply restrict the physically viable class of potentials. These constraints arise from consistency requirements of geometry, causality, stability, and correspondence with known physical limits.

- **Existence of a homogeneous vacuum.** $V(\lambda)$ must admit at least one stationary point at $\lambda = 1$,

$$V'(1) = 0,$$

corresponding to a homogeneous, curvature-free vacuum. This condition ensures that flat spacetime arises as an admissible closed configuration (§5.2) and that perturbative expansions about $\lambda = 1$ are well defined.

- **Local convexity near the vacuum.** The potential must be locally convex at $\lambda = 1$,

$$U''(1) > 0,$$

so that small fluctuations of λ satisfy a well-posed, causal linearized equation (§8). This condition guarantees local stability of the homogeneous background and excludes tachyonic or runaway curvature modes.

- **Bounded scalar energy.** The associated scalar energy functional

$$\mathcal{E}[\lambda] = \int \left[a \left|\nabla \ln \lambda\right|^2 + U(\lambda) \right] \mathrm{d}^3 x$$

must be bounded below. This excludes potentials with unbounded negative directions and ensures that admissible scalar configurations admit finite energy and meaningful variational extrema.

- **Controlled loss of convexity at high curvature.** If $V(\lambda)$ permits a high-curvature regime, any loss of convexity ($U''(\lambda)<0$) must occur only beyond a finite threshold λ_{sat} and over a bounded interval. This requirement ensures that instability, when present, signals the breakdown of stationary admissibility rather than catastrophic divergence, and permits a finite spectrum of metastable scalar configurations.

These constraints substantially narrow the admissible class of scalar potentials. While they do not uniquely determine $V(\lambda)$, they exclude arbitrary tuning and ensure that any admissible choice supports causal propagation, stability in the weak-field regime, and controlled departure from convexity at high curvature. Specific functional forms may be selected by additional physical input or phenomenological criteria, but the variational structure of TNE itself already enforces the above restrictions.

On uniqueness of the admissible potential. The selection constraints introduced in this section do not uniquely fix a single functional form for the scalar potential. Rather, they restrict the potential to a class of functions satisfying positivity, convexity near the homogeneous vacuum, and controlled growth at large λ. The saturation behavior discussed in Section 8 arises generically within this admissible class and does not rely on fine tuning of a specific functional form.

3.5. Illustrative Matter Coupling: Electromagnetic and Dirac Fields

To demonstrate that the scalar variational framework admits standard matter sectors without violating gauge or relativistic structure, we record an explicit example of a minimally coupled electromagnetic and Dirac field on the conformal NUVO background $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$. This example is intended to establish internal consistency and admissibility only; it is not proposed as a complete matter theory.

Electromagnetic sector. Consider the Maxwell action

$$S_{\text{EM}}=-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}\sqrt{|g|}\,\mathrm{d}^4x,\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu. \tag{12}$$

In four spacetime dimensions, the Maxwell action is conformally invariant: under $g_{\mu\nu}\mapsto\lambda^2 g_{\mu\nu}$, one has $\sqrt{|g|}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$ invariant. As a result, the electromagnetic field couples to the scalar-modulated geometry without introducing explicit λ-dependent gauge terms. Gauge invariance and the standard Maxwell equations are therefore preserved identically.

Dirac sector. Let ψ be a Dirac spinor minimally coupled to the electromagnetic field. Using a conformal tetrad $e^a{}_\mu=\lambda\delta^a{}_\mu$, the Dirac action takes the form

$$S_{\text{D}}=\int\bar{\psi}\left(i\gamma^\mu D_\mu-m\right)\psi\sqrt{|g|}\,\mathrm{d}^4x, \tag{13}$$

where

$$D_\mu=\partial_\mu+\Omega_\mu-iqA_\mu,\quad \Omega_\mu=\frac{1}{2}\omega_\mu^{ab}\Sigma_{ab},$$

is the spinor covariant derivative associated with the conformal spin connection. Writing $X = \ln \lambda$, the conformal contribution to Ω_μ introduces terms proportional to $\partial_\mu X$, while leaving the $U(1)$ gauge structure unchanged.

Effective matter Lagrangian. The combined matter contribution to the scalar action may therefore be written schematically as

$$L_{\text{matt}} = L_{\text{EM}}(g, A) + L_{\text{D}}(g, \psi, A), \tag{14}$$

with all λ-dependence entering through the conformal geometry and its associated connections. No explicit modification of gauge couplings or internal symmetry structure is required.

Scope and interpretation. This example demonstrates that standard electromagnetic and fermionic matter sectors can be embedded consistently within the scalar-conformal NUVO framework while preserving gauge invariance and relativistic covariance. The resulting λ-dependence in the matter sector arises geometrically, not through ad hoc coupling prescriptions. A detailed analysis of spin transport and magnetic-moment corrections induced by the conformal spin connection is deferred to dedicated work and is not required for the validity of the scalar variational law derived here.

3.6. Noether Currents and Conservation Structure

Because the action (8) is diffeomorphism-invariant on (M, g), Noether's theorem yields a covariantly conserved stress-energy tensor for the combined scalar-matter system. In the background representation (9), the scalar contribution may be written in the standard form (modulo signature conventions)

$$T^{\mu\nu}_{(\lambda)} = \frac{\kappa}{6}\left(\partial^\mu \lambda \partial^\nu \lambda - \frac{1}{2}\eta^{\mu\nu}(\nabla\lambda)^2\right) - \eta^{\mu\nu}\lambda^4 V(\lambda), \tag{15}$$

with additional terms arising from L_{matt} that reproduce the usual matter stress-energy contributions in the corresponding limits.

A second conserved quantity arises from internal rescaling symmetries of λ under suitable conditions and is closely related to the scalar transport current (6). The full derivations are given in Appendix B. Their role here is to confirm that the scalar action produces well-defined energy and flux currents consistent with the conservation postulates of §2.

3.7. Summary

Equations (10) and (11) provide the two equivalent forms of the NUVO Equation: a scalar Euler-Lagrange equation governing the field $\lambda(x)$ on conformal NUVO space. The remainder of the paper studies this equation from three complementary perspectives:

1) Its stationary reduction and operator-theoretic properties (§4);

2) Its limiting forms in weak-field, stationary, and high-curvature regimes (§5);

3) And its energy, stability, and depletion structure as encoded by admissibility and closure constraints within the consolidated NUVO framework.

In each case we will be explicit about the additional assumptions required and the sense in which correspondence with standard physical equations is obtained.

4. Functional Setting and Operator Framework

The NUVO Equation in its background form,

$$\beta \tilde{\Box} \lambda = U'(\lambda) + \frac{\partial L_{\text{matt}}}{\partial \lambda},$$

is a nonlinear scalar field equation. In many physical applications of interest—most notably the stationary or quasi-stationary regimes used in the NUVO quantization papers [3]-[5]—the field λ is treated as time-independent (or slowly varying), and the matter sector is governed by an effective Hamiltonian depending parametrically on λ.

To place these reductions on a mathematically controlled footing, this section develops the functional-analytic framework needed for the stationary scalar-transport equation and its associated Hamiltonian. Our goal is technical: under broad regularity assumptions on λ and on admissible physical potentials, the stationary reduction yields a Schrödinger-type operator that is self-adjoint and semibounded. This ensures that the stationary reduction is well posed in the usual operator-theoretic sense.

No claim is made that TNE *derives* quantum mechanics as a physical postulate. Rather, the scalar geometry provides a consistent variational setting in which Schrödinger-type operator structures arise from the stationary reduction and admit rigorous control.

4.1. Field Regularity and Admissible Potential Classes

For technical convenience we set $X = \ln \lambda$, so that $g = \mathrm{e}^{2X}\eta$ and, in the stationary reduction, the geometric effective potential takes the form

$$V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m}\left(|\nabla X|^2 - \Delta X\right).$$

The following hypotheses are assumed throughout this section. They are consistent with the admissibility conditions of §3.4 and with the scalar profiles used in the NUVO quantization literature.

(H1) Regularity of the scalar field. $X \in H^2_{\text{loc}}(\mathbb{R}^3) \cap L^\infty_{\text{loc}}(\mathbb{R}^3)$ and $\nabla X \in L^3_{\text{loc}}(\mathbb{R}^3)$. (*Ensures* V_{eff} *lies in natural form-bounded classes.*)

(H2) Physical potentials. $V_{\text{phys}} \in K_{3,\text{loc}} + L^\infty_{\text{loc}}$. (*Includes Coulomb potentials and short-range interactions.*)

(H3) Boundary conditions. On bounded domains $\Omega \subset \mathbb{R}^3$, either Dirichlet or Neumann conditions are imposed, consistent with standard elliptic theory.

(H4) Stationarity. The stationary reduction assumes $\partial_t X = 0$ and $\lambda > 0$ almost everywhere.

These represent mild regularity assumptions. They are satisfied by the analytic scalar profiles used in NUVO hydrogen modelling, oscillator models, and general weak-field solutions obtained from the scalar variational law.

4.2. Drift Cancellation in the Stationary Scalar Transport Equation

In the stationary setting, the scalar-transport equation developed in the NUVO quantization literature reduces to the following identity between ψ and a rescaled field ϕ:

$$-\frac{\hbar^2}{2m}\Delta\psi + V_{\text{phys}}\psi = \frac{1}{\lambda}\left[-\frac{\hbar^2}{2m}\Delta\phi + \left(V_{\text{phys}} + V_{\text{eff}}\left[\lambda\right]\right)\phi\right],\ \psi = \phi/\lambda.$$

A key technical point is the cancellation of first-order drift terms involving $\nabla\lambda$. The following lemma states the identity precisely; it may be combined with Hardy-type inequalities [10] and Kato's perturbation framework [11] [12] to control the resulting operator.

Lemma 1 (Drift cancellation). Let $X = \ln\lambda$ satisfy **(H1)** and define $\psi = \phi/\lambda$. Then for smooth compactly supported ϕ,

$$-\frac{\hbar^2}{2m}\Delta\psi + V_{\text{phys}}\psi = \frac{1}{\lambda}\left[-\frac{\hbar^2}{2m}\Delta\phi + \left(V_{\text{phys}} + \frac{\hbar^2}{2m}\left(\left|\nabla X\right|^2 - \Delta X\right)\right)\phi\right].$$

Thus all first-order drift terms cancel identically, and the stationary scalar transport reduces to a Schrödinger equation with geometric potential $V_{\text{eff}}\left[\lambda\right]$.

Proof. Write $\psi = \lambda^{-1}\phi$ and expand $\Delta\left(\lambda^{-1}\phi\right)$. Use $\nabla\lambda^{-1} = -\lambda^{-1}\nabla X$ and $\Delta\lambda^{-1} = \lambda^{-1}\left(\left|\nabla X\right|^2 - \Delta X\right)$. □

This cancellation is the mathematical reason the stationary scalar-transport equation takes Schrödinger form rather than a drift-diffusion form. Structurally, it reflects the compatibility between conformal geometry and unit-constrained normalization in the stationary reduction.

4.3. Self-Adjointness and the KLMN/Hardy Framework

Define the stationary Hamiltonian on $\mathbb{R}^3$ by

$$H := -\frac{\hbar^2}{2m}\Delta + V_{\text{phys}} + V_{\text{eff}}\left[\lambda\right].$$

The natural realization is as the KLMN form sum on the form domain $H^1\left(\mathbb{R}^3\right)$; when the potentials satisfy the hypotheses below, elliptic regularity yields the operator domain $D\left(H\right) = H^2\left(\mathbb{R}^3\right)$.

Theorem 2 (Self-adjointness of the stationary NUVO Hamiltonian). *Assume* (***H1***) - (***H2***) *and, in addition,* $X \in H^2\left(\mathbb{R}^3\right) \cap L^\infty\left(\mathbb{R}^3\right)$ *with* $\Delta X \in L^{3/2}\left(\mathbb{R}^3\right) + L^\infty\left(\mathbb{R}^3\right)$. *Then*:

1) The geometric potential

$$V_{\text{eff}}\left[\lambda\right] = \frac{\hbar^2}{2m}\left(\left|\nabla X\right|^2 - \Delta X\right)$$

lies in $K_3 + L^\infty$ and is Δ-form bounded with relative bound < 1.

2) The operator H is self-adjoint and bounded below on its natural domain, and the associated quadratic form is closed and semibounded on $H^1\left(\mathbb{R}^3\right)$.

3) H is the KLMN form sum of $-\frac{\hbar^2}{2m}\Delta$ and $V_{\text{phys}} + V_{\text{eff}}$.

Proof sketch. V_{phys} is Δ-form bounded under the Kato-class hypothesis. Under **(H1)**, the terms $|\nabla X|^2$ and ΔX lie in $L^{3/2}+L^{\infty}$, implying that V_{eff} is form-bounded with arbitrarily small relative bound. The KLMN theorem then yields a closed, semibounded quadratic form with domain H^1; elliptic regularity provides the stated operator realization under the additional global assumptions on X.

Corollary 1 (Lower bound for the geometric potential). There exists $C \in \mathbb{R}$, depending on $\|\nabla X\|_{L^3}^2 + \|\Delta X\|_{L^{3/2}+L^{\infty}}$, such that $V_{\text{eff}}(x) \geq -C$ almost everywhere.

This ensures that the geometric potential generated by a stationary λ does not yield runaway spectral instabilities. No special form of λ is assumed: any scalar profile with regularity consistent with the hypotheses above produces a well-defined self-adjoint Schrödinger operator.

4.4. Bounded Domains and Boundary Conditions

If $\Omega \subset \mathbb{R}^3$ is a bounded Lipschitz domain, the same analysis applies with

$$\begin{aligned} D(H) &= H^2(\Omega) \cap H_0^1(\Omega), && (\text{Dirichlet}), \\ D(H) &= \left\{u \in H^2(\Omega) : \partial_n u|_{\partial\Omega} = 0\right\}, && (\text{Neumann}). \end{aligned}$$

Standard elliptic theory (Lax-Milgram, Sobolev embeddings, and Kato-class bounds) applies verbatim. These operator domains will be used later when analyzing confined scalar resonances and curvature-saturated modes.

4.5. Summary of Section 4

Under mild and physically natural assumptions we have shown that:

1) The stationary reduction yields a Schrödinger operator with geometric potential $V_{\text{eff}}[\lambda]$;

2) All first-order drift terms cancel identically due to the conformal structure;

3) The induced Hamiltonian is self-adjoint and semibounded on natural domains;

4) The resulting framework matches the operator-theoretic structures used in the NUVO quantization papers.

Thus, the operator-theoretic foundations of stationary NUVO transport arise directly from the scalar geometry, providing a rigorous basis for the correspondence results developed in §5.

5. Hierarchy of Limits and Physical Correspondence

Remark. Several of the correspondences discussed in this section—most notably the stationary Schrödinger reduction and the weak-field post-Newtonian regime—were established in earlier peer-reviewed NUVO papers and are not rederived here. Our purpose is to show that they arise as *admissible limiting regimes* of the scalar variational equation derived in §3. Additional sectors sometimes considered in the broader NUVO program (e.g. depletion-driven irreversibility, conformal spin transport, and high-curvature resonance saturation) are mentioned only at the level of structural placement and are not used as prerequisites in the present paper.

The NUVO Equation derived in §3 defines an admissibility condition on scalar configurations $\lambda(x)$ on the conformal manifold $\left(M, g=\lambda^2\eta\right)$. To compare this scalar geometry with familiar physical equations, we develop a hierarchy of limiting regimes in which additional simplifying assumptions are imposed. Each regime is understood as follows:

> Under specified assumptions (weak curvature, stationarity, slow modulation, high curvature, etc.), solutions of TNE that satisfy the corresponding regularity and closure conditions induce effective equations for observables whose standard forms match those used in classical mechanics, relativity, and quantum mechanics to the order considered. These correspondences hold only within the stated admissible regimes and do not assert dynamical equivalence outside them.

Each subsection specifies the assumptions on λ and on the matter sector under which the correspondence is meaningful.

5.1. Classical Mechanics and the Newtonian Limit

We begin with the static, weak-curvature regime. Write

$$\lambda(x)=1+\varphi(x),\ |\varphi|\ll 1,$$

and assume:

1) $\partial_t\lambda=0$ (static configuration);
2) $|\nabla\varphi|\ll 1$ (weak curvature);
3) V and L_{matt} admit Taylor expansions about $\lambda=1$.

Linearizing (11) gives

$$\tilde{\Box}\varphi\simeq\frac{1}{\beta}\left[U''(1)\varphi+\partial_\lambda L_{\text{matt}}\big|_{\lambda=1}\right]. \tag{16}$$

Neglecting time derivatives yields a Poisson-type equation,

$$\nabla^2\varphi=4\pi G_{\text{eff}}\rho_{\text{eff}},\ G_{\text{eff}}=\frac{1}{4\pi\beta}, \tag{17}$$

where ρ_{eff} is generated by $-\partial_\lambda L_{\text{matt}}\big|_{\lambda=1}$.

Interpretation. In this regime φ functions as an effective Newtonian potential generated by admissible scalar profiles. Equation (17) reproduces the weak-field gravitational sector identified in *NUVO Space I* [1] as an admissible classical limit.

5.2. Special Relativity (Flat Limit)

Flat spacetime arises when the scalar field is homogeneous:

$$\lambda\equiv 1.$$

Assume:

1) $\lambda = \text{const.}$;

2) $V'(\lambda) = 0$ at $\lambda = 1$;

3) $\partial_\lambda L_{\text{matt}}\big|_{\lambda=1} = 0$.

Then (11) is identically satisfied and $g_{\mu\nu} = \eta_{\mu\nu}$. This configuration represents a homogeneous admissible vacuum of TNE, corresponding to special relativity in the conformal gauge.

5.3. General Relativity Correspondence (First Post-Newtonian Order)

Next consider weak but non-negligible curvature. Assume:

1) $\lambda = 1 + \Phi/c^2$ with $|\Phi|/c^2 \ll 1$;

2) $\partial_t \lambda$ small;

3) spatial gradients of Φ consistent with PN_1 scaling;

4) $V'(1) = 0$.

Using $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$,

$$g_{00} = -\lambda^2 = -\left(1 + 2\Phi/c^2\right) + O\left(\Phi^2\right),$$

which matches the PN_1 metric in isotropic coordinates.

Linearization of (10) yields

$$\frac{\kappa}{\lambda}\tilde{\Box}\lambda = 6\left[V(\lambda) + \frac{\lambda}{4}V'(\lambda)\right] + \frac{\partial L_{\text{matt}}}{\partial \lambda}. \tag{18}$$

Within this admissible regime one recovers first-order weak-field gravitational effects such as perihelion advance, gravitational redshift, Shapiro delay, and weak-field light bending, as verified in *NUVO Space II* [2]. No claims are made beyond PN_1 in the present work.

5.4. Stationary Transport Limit (Schrödinger-Type Reduction)

This reduction concerns stationary scalar transport on conformal NUVO space and is not a derivation of quantum measurement or probability structure. The stationary transport sector arises under:

1) $\partial_t \lambda = 0$;

2) Slow spatial modulation of λ on the characteristic matter length scales;

3) Matter dynamics governed by a quadratic kinetic operator in the stationary limit;

4) Applicability of the drift-cancellation identity (Lemma 1).

Setting $\psi = \phi / \lambda$, the stationary reduction yields

$$-\frac{\hbar^2}{2m}\nabla^2\phi + \left[V_{\text{phys}}(r) + V_{\text{eff}}[\lambda](r)\right]\phi = E\phi, \tag{19}$$

with geometric potential

$$V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m}\left(|\nabla \ln \lambda|^2 - \nabla^2 \ln \lambda\right). \tag{20}$$

Interpretation. In this regime V_{eff} encodes curvature contributions of admissible scalar geometry. Under the regularity hypotheses of §4, the induced operator is self-adjoint and semibounded, so the stationary transport sector is mathematically well posed and matches the operator structures used in the NUVO quantization papers [3]-[5]. This correspondence is a stationary reduction within the NUVO framework and is not asserted as a foundational replacement of quantum mechanics.

Scope of the quantum correspondence. The derivation in this section establishes a formal correspondence between a stationary reduction of the scalar transport equation and the mathematical form of a Schrödinger-type eigenvalue problem. It does not address the foundational postulates of quantum theory, including probabilistic interpretation, state vector collapse, or entanglement. Accordingly, the present result should be understood as a structural correspondence at the level of differential equations, not as a complete derivation of quantum mechanics as a physical theory.

5.5. Additional Regimes (Outline Only)

Beyond the weak-field and stationary limits emphasized above, the NUVO programme considers further admissible regimes in which scalar availability, transport constraints, and closure conditions impose additional structure. Examples include:

- **Conformal spin transport.** Spinor fields coupled to $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ acquire additional connection terms proportional to $\partial_\mu \ln\lambda$ in the corresponding tetrad formulation, yielding small corrections in appropriate weak-gradient limits.
- **Irreversible and depletion-driven evolution.** Time-dependent admissible configurations $\lambda(x,t)$ can be associated with effective dissipation or depletion terms in transport balances, with the precise form depending on modeling choices for the nonconservative sector.
- **High-curvature saturation and finite-mode structure.** When admissible solutions approach curvature saturation scales, closure and stability constraints can restrict the number of metastable resonance modes supported by the scalar geometry.

These sectors are not required for the derivations in the present paper and are therefore not developed here; they are included only to indicate where additional NUVO analyses fit within the hierarchy of admissible reductions of TNE.

5.6. Summary of Correspondences

The limiting regimes above organize the primary correspondences used in this paper:

- Static weak curvature $\Rightarrow$ Poisson/Newtonian limit;
- Homogeneous $\lambda \Rightarrow$ flat (special relativistic) limit;
- PN_1 weak-field scaling $\Rightarrow$ agreement with GR to first post-Newtonian order;
- Stationary reduction $\Rightarrow$ Schrödinger-type operator with geometric potential.

Each reduction from TNE requires explicit assumptions on λ, on V, and on L_{matt} that ensure regularity and closure in the stated regime. Together they show that, within the NUVO scalar-geometric framework, a single scalar variational law organizes classical, relativistic, and stationary quantum-transport structures as a hierarchy of admissible limits.

6. Interface to Closure Accounting Structure (CAS)

The preceding sections establish TNE as a scalar variational law on Nuvo space and show how classical, relativistic, and stationary transport equations arise as admissible limiting regimes under explicit assumptions. However, a scalar admissibility law alone does not uniquely determine which admissible configurations correspond to *observable* physical states, nor does it supply an intrinsic bookkeeping semantics for transitions between such states. In the consolidated NUVO framework, these questions are handled by the *Closure Accounting Structure* (CAS), which is treated as a separate, non-dynamical layer and is assumed here as established elsewhere. The Closure Accounting Structure (CAS) is not introduced here as a new dynamical principle. Rather, it is a previously established structural framework developed in [13], where its axioms, scope, and limitations are stated in full. The present section records only the minimal interface required to connect the scalar variational law derived here to that accounting framework.

CAS (Closure Accounting Structure). The Closure Accounting Structure is a formal bookkeeping framework used to track which geometric or dynamical configurations are admissible, observable, or closed under the allowed operations of the theory. CAS does not introduce new dynamics; rather, it provides a systematic way to distinguish configurations that are internally consistent and operationally realizable from those that are merely formal solutions of the equations.

6.1. Admissibility versus Observability

Let $\mathcal{C}$ denote the configuration space of sufficiently regular scalar fields λ (and associated matter fields, when present) on $\left(M, g=\lambda^2\eta\right)$, subject to the baseline regularity and boundary conditions assumed throughout this paper. The NUVO Equation determines an admissible subset $\mathcal{A}\subset\mathcal{C}$ consisting of configurations satisfying the scalar Euler-Lagrange equation and, when invoked, the hypotheses defining the relevant regime (stationarity, weak-field scaling, boundedness, etc.).

CAS introduces a further selection

$$\mathcal{O}\subset\mathcal{A},$$

where $\mathcal{O}$ is the set of *observable* (*ledger-admissible*) *configurations*. Membership in $\mathcal{O}$ is not determined by TNE alone: it is defined by closure criteria expressing that the configuration can be globally and consistently entered into an accounting ledger (in particular, that the relevant conserved measures and boundary fluxes admit a consistent closure semantics under admissible transitions).

Thus the division of labor is:

- **Admissibility (TNE).** Determines which configurations lie in $\mathcal{A}$ by scalar variational stationarity (and regime hypotheses when imposed).
- **Observability (CAS).** Selects $\mathcal{O} \subset \mathcal{A}$ by closure constraints and assigns accounting semantics to transitions among observable configurations.

6.2. Ledger Closure and Conserved Currents

A principal point of contact between TNE and CAS is the conservation structure present in the scalar variational framework. Noether currents and continuity statements derived from the action furnish canonical closure quantities for ledger accounting. In CAS language, a conservation law is interpreted as a *ledger closure condition*: net change in a quantity across an admissible process is accounted for by boundary flux, or by explicitly modeled exchange terms when present.

Concretely, in closed regimes a continuity statement of the form

$$\nabla_\mu^{(g)} J^\mu = 0$$

implies that for any compact region $\Omega \subset M$ with boundary $\partial\Omega$,

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_\Omega J^0 \mathrm{d}V_g = -\int_{\partial\Omega} J^i n_i \mathrm{d}S_g ,$$

so changes are fully captured by boundary flux. In open or nonconservative regimes, source terms such as

$$\nabla_\mu^{(g)} J^\mu = -\Gamma$$

are treated as explicit ledger line-items rather than as violations: closure is maintained by extending the ledger to include the modeled exchange term Γ. In this way, the conservation structure derived from TNE supplies the canonical set of quantities that CAS tracks across admissible transitions.

6.3. What TNE Does Not Encode

The interface is deliberately asymmetric. TNE determines admissibility in the sense of variational stationarity, but it does *not* encode the additional combinatorial and ordering structure required for a complete accounting of observable physics. In particular, TNE does not by itself specify:

- An adjacency relation on $\mathcal{O}$ (which observable configurations may be connected by a single admissible transition);
- An ordering/selection principle when multiple ledger-admissible outcomes exist;
- A classification of admissible but non-observable configurations in $\mathcal{A} \setminus \mathcal{O}$;
- Event-level bookkeeping rules by which discrete closure, exchange, or resonance constraints are recorded as ledger updates.

These elements are supplied by CAS as a separate structural layer.

6.4. Interface Map

The operational interface may be summarized schematically as

$$(\lambda, \text{matter}) \in \mathcal{C} \xrightarrow{\text{TNE}} (\lambda, \text{matter}) \in \mathcal{A} \xrightarrow{\text{CAS}} (\lambda, \text{matter}) \in \mathcal{O}$$
$$\xrightarrow{\text{ledger}} \text{observable entries and transitions.}$$

In this workflow, TNE supplies the admissible scalar geometry and associated conservation structure, while CAS supplies the observability criteria and the accounting semantics needed to interpret admissible solutions as physically meaningful states and transitions.

Positioning. Including this interface does not elevate CAS to a modification of the field equations. It states only how the scalar variational law developed here functions as an admissibility backbone for a closure-based accounting programme defined independently of the dynamics.

7. Unified Interpretation of Energy, Mass, and Curvature

The NUVO Equation constrains admissible scalar geometries through a variational principle, but it does not by itself prescribe a unique physical interpretation of energy, mass, or lifetime. Nevertheless, the scalar-geometric framework provides a natural organizational language in which curvature, inertial response, and coherence may be related under additional modeling assumptions. This section summarizes how such interpretations arise *compatibly* with TNE, while maintaining a clear separation between variational necessity and phenomenological identification.

Throughout, statements about mass scaling, lifetimes, or resonance structure are understood as *interpretive overlays* consistent with the scalar dynamics, not as consequences derived uniquely from TNE in isolation.

7.1. Scalar Curvature Energy

The scalar contribution to the stress-energy tensor, obtained in §3 from the action (8), is

$$T^{\mu\nu}_{(\lambda)} = \frac{\kappa}{6}\left(\partial^\mu\lambda\partial^\nu\lambda - \frac{1}{2}\eta^{\mu\nu}(\nabla\lambda)^2\right) - \eta^{\mu\nu}\lambda^4 V(\lambda), \tag{21}$$

with $(\nabla\lambda)^2 = \eta^{\mu\nu}\partial_\mu\lambda\partial_\nu\lambda$. In a local unit-constrained frame with negligible time dependence of λ, the scalar contribution to the energy density is approximately

$$W_\lambda := T^{00}_{(\lambda)} \simeq \frac{\kappa}{12} g^{\mu\nu}\partial_\mu\lambda\partial_\nu\lambda = \frac{\kappa}{12}\lambda^{-2}\left|\nabla_\eta\lambda\right|^2. \tag{22}$$

The corresponding scalar energy functional is

$$\mathcal{E}_\lambda = \frac{\kappa}{12}\int \lambda^{-2}\left|\nabla_\eta\lambda\right|^2 \mathrm{d}^3x. \tag{23}$$

These terms represent the kinetic component of the scalar variational problem. Within NUVO, they quantify curvature stored in the scalar geometry rather than a separate material field.

7.2. Capacity Interpretation and Sinertia Flux

In the consolidated NUVO ontology, the scalar field $\lambda(x)$ may be interpreted as parameterizing the local *capacity* of the substrate to support curvature. A commonly used phenomenological parametrization introduces a capacity (or sinertia) density $\Sigma(x)$ satisfying

$$\Sigma(x) \propto \lambda^{-1}(x), \tag{24}$$

so that large λ corresponds to reduced local capacity and hence stronger curvature. This relation is *not* imposed by the scalar variational principle; it is an interpretive identification used in depletion and coherence analyses.

Combined with the continuity structure

$$\nabla_{\mu}^{(g)} J_{\sin}^{\mu} = 0 \ \left(\text{closed regimes}\right),$$

or with explicit source terms in non-closed regimes, this identification links the geometric evolution of λ to capacity transport without altering the field equation itself.

7.3. Mass as a Curvature Scale

Interpretive status. The mass-curvature scaling discussed in this subsection is presented as a heuristic interpretation that organizes the behavior of solutions to the field equations. It is not derived directly from the variational principle itself, but rather serves as an interpretive bridge between the geometric results obtained earlier and familiar physical language. All rigorously derived results are contained in the preceding field equations and stability analysis.

The scalar variational law does not uniquely define inertial mass; mass enters through the matter Lagrangian L_{matt}. Nevertheless, it is often convenient to associate localized curvature excitations with an effective mass scale by comparing their characteristic scalar length to the relativistic length

$$\ell_m = \frac{Gm}{c^2}.$$

If one assumes that the characteristic spatial scale of a localized scalar configuration grows linearly with the amplitude of λ, one arrives at the heuristic correspondence

$$m \propto \lambda, \ \ell_m = \ell_0 \lambda, \tag{25}$$

for some reference length ℓ_0. This correspondence is not required for the validity of TNE itself and is invoked here solely as an interpretive identification when relating localized scalar curvature excitations to effective inertial mass. This identification should be understood as an *interpretive mapping* compatible with the scalar geometry, analogous to how $E = mc^2$ relates mass and energy without being a variational law. The NUVO Equation permits such an interpretation but does not require it.

7.4. Coherence, Depletion, and Lifetime Scaling

Time-dependent scalar configurations admit non-closed regimes in which capacity is depleted. In such cases the continuity law takes the schematic form

$$\nabla_{\mu}^{(g)} J_{\text{sin}}^{\mu} = -\Gamma_{\text{dep}}(\lambda)\rho, \tag{26}$$

where Γ_{dep} models effective loss of capacity. Introducing a coherence quality factor

$$Q(\lambda) = \frac{\lambda\omega_0}{2\Gamma_{\text{dep}}(\lambda)}, \quad \omega_0 = \frac{mc^2}{\hbar},$$

one may, under additional assumptions on Γ_{dep} and the mass-curvature identification (25), obtain lifetime scalings of the schematic form

$$\tau^{-1} \propto m^5. \tag{27}$$

Such relations are not consequences of TNE alone; they arise only when specific depletion models and coherence assumptions are supplied. In the present paper, they serve solely to illustrate the range of phenomenological structures that can be organized consistently within the scalar-geometric framework.

7.5. Conservation Structure

Noether's theorem applied to the scalar action yields conservation of the total energy-momentum tensor,

$$\nabla_{\mu}^{(g)}\left(T_{(\lambda)}^{\mu\nu} + T_{\text{matt}}^{\mu\nu}\right) = 0. \tag{28}$$

In addition, internal rescaling symmetries of the scalar field give rise to the sinertia current J_{sin}^{μ}, which satisfies

$$\nabla_{\mu}^{(g)} J_{\text{sin}}^{\mu} = 0 \ (\text{closed regimes}), \quad \nabla_{\mu}^{(g)} J_{\text{sin}}^{\mu} = -\Gamma_{\text{dep}}(\lambda)\rho \ (\text{non-closed regimes}). \tag{29}$$

These relations express complementary aspects of the same geometric structure: covariance of the scalar action and admissible rescaling of units. When depletion is modeled, the conservation identities are extended rather than violated, in accordance with the closure-accounting interpretation.

7.6. Summary

This section has organized the energetic and interpretive content associated with the scalar field λ without extending the variational law itself. The NUVO Equation constrains admissible scalar geometries; within that admissible set, curvature energy, capacity transport, effective mass scales, and coherence lifetimes may be introduced as consistent interpretive layers. Their detailed phenomenology depends on additional modeling choices and is developed elsewhere. Here they serve to demonstrate the breadth of physical interpretation compatible with the scalar variational backbone provided by TNE.

8. Analytical Structure and Stability

The NUVO Equation (TNE) defines a nonlinear scalar field theory on a conformal background. Its linearized dynamics, dispersion properties, and stability criteria follow directly from the scalar variational structure introduced in §3. This section develops those properties systematically, making a sharp distinction between:

1) Results that follow *intrinsically* from TNE as a scalar Euler-Lagrange equation;

2) Additional structures that arise only when specific depletion or resonance models are supplied.

The purpose is to establish the mathematical stability properties of admissible scalar configurations without importing phenomenology beyond what is explicitly stated.

8.1. Linearization about the Homogeneous Vacuum

Let

$$\lambda(x) = 1 + \epsilon\phi(x), \; |\epsilon| \ll 1,$$

and consider the background form of TNE,

$$\beta\tilde{\Box}\lambda = U'(\lambda) + \frac{\partial L_{\text{matt}}}{\partial\lambda}. \tag{30}$$

Assume that $\lambda = 1$ is a stationary point of the scalar potential,

$$U'(1) = 0,$$

and neglect $\partial_\lambda L_{\text{matt}}\big|_{\lambda=1}$, which would otherwise act as an external source term. Linearizing to first order in ϵ yields

$$\beta\tilde{\Box}\phi - U''(1)\phi = 0. \tag{31}$$

Equation (31) is a Klein-Gordon-type equation with causal propagation speed c. Provided $U''(1) \ge 0$, the Cauchy problem is well posed. Plane-wave solutions $\phi \sim \mathrm{e}^{\mathrm{i}(k\cdot x - \omega t)}$ satisfy

$$\omega^2 = c^2k^2 + \frac{U''(1)}{\beta}\frac{c^4}{\hbar^2}, \tag{32}$$

which may be written in terms of an effective stiffness parameter

$$m_{\text{eff}}^2 = \frac{U''(1)}{\beta}. \tag{33}$$

Interpretation. The parameter m_{eff} characterizes the linear response of the scalar field to small curvature perturbations. It does *not* represent a physical particle mass, but rather a measure of the local convexity of the scalar potential around the homogeneous vacuum. This is a generic feature of scalar variational theories and requires no additional NUVO-specific assumptions.

8.2. Energy Functional and Convexity

Neglecting matter contributions for clarity, the spatial scalar energy functional associated with TNE may be written as

$$\mathcal{E}[\lambda] = \int\left[a|\nabla\ln\lambda|^2 + U(\lambda)\right]\mathrm{d}^3x, \tag{34}$$

with $a > 0$ determined by the curvature term in the action. Variation of (34) reproduces the stationary part of the NUVO Equation.

The second variation takes the form

$$\frac{\delta^2 \mathcal{E}}{\delta\lambda^2} = 2a\lambda^{-2}\left|\nabla \ln \lambda\right|^2 + U''(\lambda). \tag{35}$$

Thus, the energy functional is locally convex whenever $U''(\lambda) > 0$. Loss of convexity occurs when $U''(\lambda)$ crosses zero, signaling the onset of instability or bifurcation.

Interpretation. Equation (35) shows that nonlinear stability of admissible scalar configurations is governed primarily by the curvature of the scalar potential $U(\lambda)$. Convexity therefore provides a clean and model-independent stability criterion intrinsic to the variational structure.

8.3. Saturation and Instability Threshold

Define the *saturation value* λ_{sat} as the smallest positive solution of

$$U''(\lambda_{\text{sat}}) = 0. \tag{36}$$

For $\lambda < \lambda_{\text{sat}}$, the scalar energy functional is convex and stationary solutions are linearly stable. For $\lambda > \lambda_{\text{sat}}$, convexity is lost and no small perturbation remains bounded.

This definition is purely mathematical and follows directly from TNE. Any physical interpretation of λ_{sat} requires additional modeling input.

8.4. Coherence Bounds and Saturation Behavior

The scalar variational structure of TNE implies intrinsic limits on the amplitude and stability of admissible scalar configurations. As shown in §8, linearized fluctuations propagate causally with finite speed, and nonlinear stability is controlled by the convexity of the scalar potential through the condition $U''(\lambda) > 0$.

When this convexity condition fails, stationary solutions cease to be energetically stable, defining a saturation threshold λ_{sat} beyond which small perturbations grow. This transition is a direct mathematical consequence of the second variation of the scalar energy functional and does not rely on additional physical modeling.

The physical interpretation of such saturation behavior—including coherence loss, irreversibility, or finite resonance structure—requires supplementary assumptions about transport, dissipation, or microscopic structure that lie beyond the scope of the present variational analysis. Various interpretive frameworks compatible with TNE have been explored elsewhere, but are not required for the mathematical conclusions established here.

Accordingly, within this work λ_{sat} is treated as a purely geometric stability threshold arising from the scalar variational law itself. Any further identification of this threshold with phenomenological or particle-level structure is deferred to future investigations. An illustrative example of a scalar potential exhibiting a homogeneous vacuum, a convex stability region, and loss of convexity at a saturation threshold is shown in Figure 2.

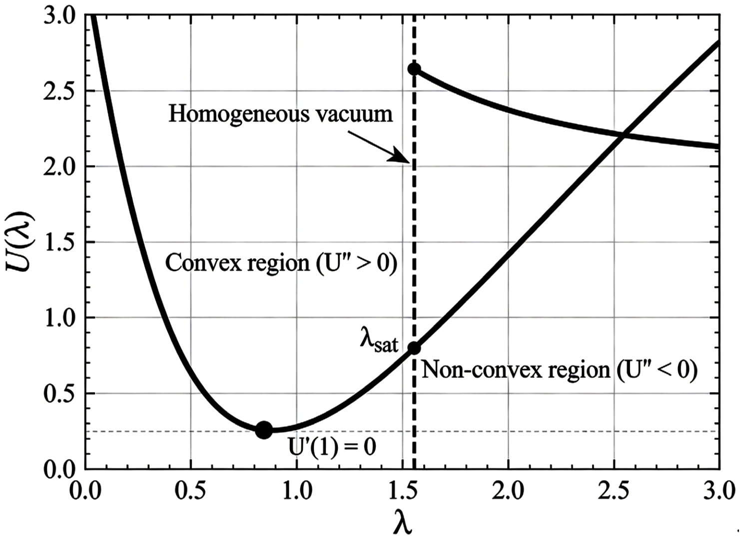

Figure 2. Illustrative scalar potential $U(\lambda)$ exhibiting a homogeneous vacuum at $\lambda = 1$ with $U'(1) = 0$, a convex regime ($U'' > 0$) supporting stable scalar configurations, and loss of convexity at a saturation threshold λ_{sat} where $U''(\lambda) = 0$. The specific functional form and parameter values are schematic and serve only to illustrate the convexity structure discussed in Section 8.

8.5. Resonance Modes and Finite Spectra

Within the convex regime defined by $U''(\lambda) > 0$, the stationary reduction of TNE admits localized oscillatory solutions of standing-wave type. This follows from standard variational arguments for scalar fields confined by an effective potential and subject to regularity and boundary conditions.

As an illustrative model, consider radial configurations satisfying

$$\nabla^2 \lambda + k^2 \lambda = 0, \tag{37}$$

with regularity at the origin and boundary conditions imposed at a finite radius associated with the onset of nonconvexity or loss of stability. Such conditions lead to a discrete spectrum of admissible modes, indexed by integers through the allowed values of k.

More generally, for scalar variational problems with a convexity threshold at $\lambda = \lambda_{\text{sat}}$, only those stationary configurations whose amplitude remains entirely within the convex region are dynamically stable. Modes that probe regions where $U''(\lambda) \le 0$ lie beyond the domain of local stability and are excluded by the second-variation criterion. As a consequence, the admissible stationary spectrum is necessarily finite whenever the scalar potential possesses a finite convex domain.

The precise number of admissible modes depends on the detailed form of the scalar potential $U(\lambda)$, the geometry of the configuration, and the imposed boundary conditions. While specific models may yield particularly small finite spectra, such results require additional physical input beyond the scalar variational law itself and are therefore not asserted here. The present analysis establishes only the general and model-independent fact that *finite resonance spectra* *arise naturally from convexity and stability constraints in the scalar variational framework.*

8.6. Summary of Section 8

The analytical structure of the NUVO Equation establishes:

1) *Causal linear dynamics*: Small perturbations satisfy a Klein-Gordon equation with subluminal propagation.

2) *Convexity-based stability*: Nonlinear stability is governed by the sign of $U''(\lambda)$ through the second variation of the scalar energy functional.

3) *Saturation threshold*: Loss of convexity defines a critical amplitude λ_{sat} intrinsic to the variational structure.

4) *Imported coherence and resonance results*: Bounds on coherence times and finite resonance spectra arise only when additional depletion and transport models are supplied.

Thus, TNE itself provides a mathematically complete account of causality and stability for admissible scalar geometries, while phenomenological refinements such as depletion and generational structure may be layered on consistently without altering the underlying variational law.

9. Discussion, Conclusions, and Outlook

The preceding sections establish *The NUVO Equation* (*TNE*) as a covariant scalar Euler-Lagrange equation on the conformal manifold $\left(M, g=\lambda^2\eta\right)$, derived from a single variational principle. The central result is structural: TNE defines a mathematically controlled admissibility law for scalar modulations $\lambda(x)$ that is rich enough to reproduce familiar gravitational and stationary transport structures as limiting reductions under explicit hypotheses.

The discussion below synthesizes what is obtained *directly* from TNE and what lies outside the scope of the present paper.

9.1. What Is Established Intrinsically by TNE

At the level of geometry, the NUVO framework restricts the physical metric to a conformal class and encodes curvature through a single scalar degree of freedom $\lambda(x)$. This restriction does not trivialize curvature: in four dimensions the scalar curvature of $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$ is determined by derivatives of λ via the conformal identity (4), and the action principle of §3 supplies a covariant field equation for admissible configurations.

From the variational structure alone, the paper establishes:

1) Causality and well-posed linear dynamics. Linearization about a homogeneous background yields a Klein-Gordon-type equation (31) with causal propagation and a stability criterion controlled by $U''(1)$.

2) Convexity-based nonlinear stability. The stationary energy functional admits a second-variation criterion (35) showing that local stability of scalar configurations is governed by the sign of $U''(\lambda)$.

3) A precise separation between admissibility and interpretation. TNE determines the admissible set of scalar configurations, while any additional selection of observable states or transition bookkeeping belongs to a distinct accounting layer (cf. the CAS interface in §6).

These statements are independent of any particular choice of scalar potential $V(\lambda)$ or matter coupling L_{matt} beyond the regularity assumptions stated in §3 and §4.

9.2. Correspondence Results as Regime-Specific Reductions

The principal physical correspondences developed in §5 are obtained only after imposing additional regime hypotheses.

- **Weak-field gravity (Newtonian and PN$_1$).** In the static weak-curvature limit, linearization of λ around $\lambda = 1$ yields a Poisson-type equation for the scalar perturbation and recovers the standard first post-Newtonian structure within the conformal gauge. These correspondences are established at PN$_1$ order and no claim is made here beyond that regime.
- **Stationary transport and Schrödinger form.** In stationary regimes, drift cancellation yields a Schrödinger-type operator with a geometric effective potential $V_{\text{eff}}[\lambda]$, and §4 provides a self-adjointness and semiboundedness framework under mild regularity assumptions.

In each case, the role of TNE is to supply admissible scalar geometries and the associated curvature and transport structure. The reductions are not claimed as universal replacements for Einstein's or Schrödinger's equations; rather, they demonstrate that those structures arise naturally as admissible limits within the scalar-conformal setting.

9.3. Scope Limitations

The NUVO Equation is a scalar admissibility law. It does not, by itself, uniquely determine: 1) the physical form of $V(\lambda)$, 2) the detailed coupling encoded in L_{matt}, or 3) an observability/selection rule when multiple admissible solutions exist. Accordingly, the present paper does not assert unique quantitative predictions without specifying these modeling inputs.

Likewise, extensions sometimes considered in the broader NUVO research program—such as non-closed transport regimes, depletion-driven irreversibility, conformal spin transport effects, or high-curvature resonance classification—are not required for the core results proved here and are not used as prerequisites. They should be regarded as optional overlays that may be developed consistently on top of the scalar variational backbone, provided they do not modify the action principle or its Euler-Lagrange equation.

9.4. Relation to Scalar-Tensor and Conformal Gravity Frameworks

Scalar-tensor and conformal gravity theories have a long history, beginning with the Brans-Dicke framework and extending through modern conformal and Weyl-invariant models. Because the NUVO Equation also employs a scalar field and a conformal metric structure, it is important to clarify how the present framework differs in both formulation and intent.

Comparison with Brans-Dicke theory. In Brans-Dicke and related scalar-tensor theories, the scalar field acts as a dynamical replacement for the Newtonian gravitational constant, coupling directly to the Ricci scalar through an action of the form

$$S_{\mathrm{BD}} = \int \left(\phi R - \frac{\omega}{\phi} (\nabla \phi)^2 + L_{\mathrm{matt}} \right) \sqrt{|g|}\, \mathrm{d}^4 x.$$

The scalar ϕ is itself a dynamical gravitational degree of freedom, and the theory modifies Einstein's equations at the tensorial level. Observable deviations from general relativity arise through scalar-mediated forces and are constrained by post-Newtonian experiments.

By contrast, the NUVO framework does not introduce an independent tensorial gravitational dynamics beyond the conformal metric $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$. All curvature is encoded in a single scalar field λ, and the variational principle governs admissible scalar configurations rather than modifying Einstein's field equations. The NUVO Equation is therefore not a scalar-tensor theory in the Brans-Dicke sense: it does not posit an additional gravitational force carrier, nor does it replace the Einstein tensor with a scalar-modified field equation.

Comparison with conformal and Weyl gravity. Conformal gravity models, including Weyl gravity, enforce local conformal invariance and typically employ higher-derivative actions constructed from the Weyl tensor, such as

$$S_{\mathrm{Weyl}} = \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{|g|}\, \mathrm{d}^4 x.$$

These theories modify gravitational dynamics through fourth-order field equations and introduce new propagating degrees of freedom, often accompanied by ghost or stability issues.

NUVO differs fundamentally in both construction and scope. The conformal factor λ is not a gauge redundancy to be fixed or eliminated, nor does the theory impose full conformal invariance at the level of the action. Instead, λ functions as a unit-constraint field whose variation determines admissible scalar-modulated geometries. The resulting NUVO Equation is second order, scalar, and variationally well posed, avoiding the higher-derivative instabilities characteristic of Weyl-type models.

Conceptual distinction. The key distinction is structural. Brans-Dicke theory extends general relativity by adding a scalar gravitational degree of freedom. Conformal gravity replaces Einstein dynamics with a conformally invariant tensor theory. NUVO instead adopts a scalar-conformal geometry from the outset and asks a different question: *which scalar-modulated geometries are admissible under a single variational law, and how do familiar physical equations emerge as limiting reductions of that admissibility constraint*?

In this sense, NUVO is not a competing gravitational theory but a scalar-geometric framework within which gravitational, quantum, and irreversible behaviors can be organized as sector-specific reductions. The comparison with scalar-tensor and conformal gravity theories clarifies that NUVO occupies a distinct conceptual niche rather than duplicating existing approaches.

9.5. Falsifiability and Observational Access Channels

The present work is concerned primarily with the formulation and analysis of a scalar variational law governing admissible conformal geometries. As such, it does not aim to provide a catalogue of precision predictions or parameter fits. Nevertheless, the NUVO Equation defines a framework that is, in principle, falsifiable through multiple observational channels once specific modeling choices are made.

Falsifiability within NUVO operates at two distinct levels.

Structural falsifiability. At the structural level, the NUVO framework would be falsified if no admissible choice of scalar potential $V(\lambda)$ and matter coupling L_{matt} could simultaneously satisfy the selection constraints of §3.4 and reproduce established weak-field, stationary, and causal behavior. In particular, failure to admit a stable homogeneous vacuum, causal scalar propagation, or bounded scalar energy would invalidate the variational framework itself. These criteria are independent of detailed phenomenology and provide a sharp internal test of consistency.

Phenomenological falsifiability. At the phenomenological level, NUVO admits multiple channels through which deviations from standard theories may arise when specific potentials and couplings are chosen. These include, but are not limited to:

- Geometric corrections to effective quantum potentials in stationary transport regimes;
- Departures from general relativity beyond first post-Newtonian order;
- Conformal spin-connection effects in fermionic transport;
- Coherence loss or saturation phenomena associated with time-dependent scalar modulation.

Any concrete realization of the framework that predicts effects incompatible with existing experimental bounds would be ruled out, thereby constraining or excluding the corresponding model within the NUVO class.

Discrete resonance structure as a falsifiability channel. A particularly sharp potential falsifiability channel arises in the high-curvature, stationary sector of the scalar variational law. For admissible scalar potentials satisfying the selection constraints of §, the NUVO framework permits the possibility that only a finite number of metastable scalar resonance modes exist below a saturation threshold λ_{sat}. If future analysis were to show that no admissible potential consistent with weak-field and causal requirements yields such a finite bound, this class of NUVO models would be ruled out.

Conversely, the existence of a robust upper bound on admissible stationary modes would constitute a distinctive structural signature of the scalar-geometric framework. A detailed investigation of this question, including conditions under which a fourth stable mode is excluded, is deferred to subsequent work.

Scope of the present work. The absence of numerical predictions in this paper reflects a deliberate separation between *admissibility structure* and *model instantiation*. The NUVO Equation establishes the former. Quantitative predictions necessarily depend on the latter and are deferred to subsequent work focused on specific sectors and observational regimes. This separation parallels standard practice in effective field theory and mathematical physics, where foundational structure is established prior to phenomenological specialization.

In this sense, the NUVO framework is testable without being prematurely overconstrained. It delineates a finite and structured space of admissible scalar-geometric models, each of which may be confronted with experiment once fully specified.

9.6. Outlook and Falsifiable Directions

The scalar-conformal variational framework suggests several concrete directions for further analysis and empirical constraint, each of which requires specifying $V(\lambda)$ and L_{matt}:

1) Higher-order post-Newtonian analysis. Extending the correspondence beyond PN_1 (e.g. to 2PN order) would provide a sharper comparison with precision gravitational tests and would clarify the extent to which conformal restriction remains viable.

2) Spectroscopic and bound-state constraints. The geometric effective potential $V_{\text{eff}}[\lambda]$ in stationary regimes provides a direct pathway to quantitative constraints from atomic and molecular spectra once a physically motivated scalar profile is fixed.

3) Time-dependent solutions and stability near loss of convexity. Numerical study of fully time-dependent solutions of TNE, especially near any convexity threshold $U''(\lambda)=0$, would test whether stable evolutions persist under physically reasonable boundary and regularity conditions.

4) Accounting/observability interface. Formal development of closure-based observability criteria (CAS) can be pursued independently of the field equation, clarifying which admissible solutions correspond to observable states and how transition bookkeeping is organized.

9.7. Concluding Statement

In summary, TNE provides a compact scalar variational law on conformal NUVO space that is mathematically well posed, causally propagating, and equipped with a clean convexity-based stability criterion. Within explicitly stated limiting regimes, it reproduces weak-field gravitational structure and a stationary Schrödinger-type transport operator with geometric potential. The framework thereby serves as a unified *admissibility backbone* for scalar-modulated geometries, while leaving model-specific phenomenology to the choice of potential, matter coupling, and any external accounting or observability structures layered on top.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendices

A. Derivation Details for TNE

We record the Euler-Lagrange derivation of TNE from the scalar action

$$S[\lambda,\Psi]=\int_M\left[\frac{\kappa}{12}R(g)-V(\lambda)+L_{\text{matt}}(\Psi,\lambda)\right]\mathrm{d}V_g,\ \mathrm{d}V_g=\sqrt{|g|}\,\mathrm{d}^4x=\lambda^4\mathrm{d}^4x,$$

where $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$, and the variation is taken with respect to λ while holding $\eta_{\mu\nu}$ and Ψ fixed. Boundary terms are discarded under the admissible boundary conditions stated in the main text.

1) Curvature term

In four dimensions, the scalar curvature of the conformal metric satisfies

$$R(g)=-6\lambda^{-3}\tilde{\Box}\lambda,\ \tilde{\Box}=\eta^{\mu\nu}\partial_\mu\partial_\nu.$$

Consider

$$S_R=\int\frac{\kappa}{12}R(g)\mathrm{d}V_g=\int\frac{\kappa}{12}\lambda^4R(g)\mathrm{d}^4x.$$

Varying gives

$$\delta\left(\lambda^4R\right)=4\lambda^3\delta\lambda R+\lambda^4\delta R.$$

Using $R(g)=-6\lambda^{-3}\tilde{\Box}\lambda$,

$$\delta R=-6\delta\left(\lambda^{-3}\tilde{\Box}\lambda\right)=-6\left(-3\lambda^{-4}\delta\lambda\tilde{\Box}\lambda+\lambda^{-3}\tilde{\Box}\delta\lambda\right).$$

Substituting and integrating by parts the $\tilde{\Box}\delta\lambda$ term (discarding boundary terms) yields the compact functional derivative

$$\frac{\delta S_R}{\delta\lambda}=-\frac{\kappa}{6}\lambda^3R(g).$$

2) Potential and matter terms

For the potential term

$$S_V=-\int\lambda^4V(\lambda)\mathrm{d}^4x,$$

one finds

$$\frac{\delta S_V}{\delta\lambda}=-\left(4\lambda^3V(\lambda)+\lambda^4V'(\lambda)\right).$$

For the matter term

$$S_{\text{matt}}=\int\lambda^4L_{\text{matt}}(\Psi,\lambda)\mathrm{d}^4x,$$

the variation yields

$$\frac{\delta S_{\text{matt}}}{\delta\lambda}=\frac{\partial}{\partial\lambda}\left(\lambda^4L_{\text{matt}}\right)\equiv4\lambda^3L_{\text{matt}}+\lambda^4\frac{\partial L_{\text{matt}}}{\partial\lambda}.$$

3) Euler-Lagrange equation

Setting $\delta S/\delta\lambda=0$ gives the geometric form of TNE:

$$-\frac{\kappa}{6}\lambda^3R(g)-\left(4\lambda^3V(\lambda)+\lambda^4V'(\lambda)\right)+\frac{\partial}{\partial\lambda}\left(\lambda^4L_{\text{matt}}\right)=0.$$

Equivalently,

$$-\frac{\kappa}{6}\lambda^3 R(g)=4\lambda^3 V(\lambda)+\lambda^4 V'(\lambda)-\frac{\partial}{\partial\lambda}\left(\lambda^4 L_{\text{matt}}\right).$$

Substituting $R(g)=-6\lambda^{-3}\tilde{\Box}\lambda$ yields the background form

$$\beta\tilde{\Box}\lambda=U'(\lambda)-\frac{\partial}{\partial\lambda}\left(\lambda^4 L_{\text{matt}}\right)\lambda^{-4},\ \beta=3\kappa,\ U'(\lambda)=\lambda^3\left(4V(\lambda)+\lambda V(\lambda)\right).$$

In the common case where the λ-dependence of matter enters through $L_{\text{matt}}(\Psi,\lambda)$ without additional powers from the measure (or after absorbing measure factors into the definition of the effective matter coupling), this is written schematically as

$$\beta\tilde{\Box}\lambda=U'(\lambda)+\frac{\partial L_{\text{matt}}}{\partial\lambda},$$

as used in the main text.

This completes the Euler-Lagrange derivation of TNE.

B. Noether Currents and Conservation Structure

This appendix records the Noether currents associated with the scalar variational formulation of TNE in its flat-background representation and states explicitly the conditions under which additional internal currents exist. No conservation law beyond stress-energy conservation is assumed unless the corresponding symmetry conditions are satisfied.

1) Translation invariance and stress-energy conservation

Work in the background form of the action,

$$S[\lambda,\Psi]=\int\mathcal{L}(\lambda,\partial\lambda;\Psi)\,\mathrm{d}^4x,$$

with $\eta_{\mu\nu}$ fixed. If the Lagrangian density $\mathcal{L}$ has no explicit dependence on the coordinates x^μ, Noether's theorem yields the canonical stress-energy tensor

$$T^{\mu}{}_{\nu}=\frac{\partial\mathcal{L}}{\partial(\partial_\mu\lambda)}\partial_\nu\lambda-\delta^{\mu}{}_{\nu}\mathcal{L}\ \ (+\text{ matter contributions}),$$

which satisfies

$$\partial_\mu T^{\mu}{}_{\nu}=0\quad\text{on solutions of the Euler-Lagrange equation for }\lambda.$$

Rewriting this statement in geometric variables yields covariant conservation of the total stress-energy tensor with respect to the Levi-Civita connection of $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$:

$$\nabla^{(g)}_{\mu}\left(T^{\mu\nu}_{(\lambda)}+T^{\mu\nu}_{\text{matt}}\right)=0.$$

This conservation law follows directly from spacetime translation invariance and holds independently of any additional modeling assumptions.

2) Internal rescaling currents (conditional)

An internal rescaling of the scalar field,

$$\lambda\mapsto(1+\epsilon)\lambda,$$

generates a Noether current *only if* the action is invariant under this transformation, possibly up to a total divergence. Such invariance requires specific homogeneity properties of the scalar potential $V(\lambda)$ and of the matter coupling $L_{\text{matt}}(\Psi,\lambda)$ and is therefore *not* generic.

When this symmetry is present, Noether's theorem yields a conserved current of the form

$$J^{\mu}_{\text{scale}} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\lambda)}\lambda \; (\text{up to standard improvement terms}), \; \partial_{\mu}J^{\mu}_{\text{scale}} = 0.$$

If the rescaling symmetry is explicitly broken—by the choice of $V(\lambda)$, by matter coupling, or by time-dependent boundary conditions—the divergence of J^{μ}_{scale} acquires a source term determined by the symmetry-breaking contribution. In such cases the associated continuity relation is replaced by a balance law rather than a conservation law.

Accordingly, any continuity equation for internal scalar currents appearing in the main text is to be understood as a *closure-consistent conservation statement* valid only under the stated symmetry or modeling assumptions. No internal current conservation is implied by the NUVO Equation itself in the absence of those conditions.

C. Spinor Transport on Admissible Scalar-Conformal Geometry

This appendix records the minimal geometric structure required to analyze Dirac spinor transport on scalar-conformal geometries admissible under The NUVO Equation (TNE). No independent dynamical equations for spinors are introduced, and no back-reaction on the scalar variational law is assumed. Spin degrees of freedom are treated purely as *probes* of scalar-modulated geometry.

The purpose of this appendix is twofold: 1) to demonstrate that standard spin transport is well-defined on NUVO scalar-conformal space, and 2) to establish the geometric origin of small spin-holonomy corrections that appear in later NUVO analyses. A full dynamical treatment of spinor fields is deferred to subsequent work.

C.1. Dirac Operator on Scalar-Conformal Backgrounds

Let the admissible spacetime geometry be defined by the conformal metric

$$g_{\mu\nu} = \lambda^2(x)\eta_{\mu\nu},$$

with $\lambda(x) > 0$ satisfying the scalar Euler-Lagrange equation TNE. Introduce an orthonormal frame

$$e^{a}{}_{\mu} = \lambda\delta^{a}{}_{\mu}, \; e_{a}{}^{\mu} = \lambda^{-1}\delta_{a}{}^{\mu}.$$

The associated spin connection is

$$\omega_{\mu}{}^{ab} = \delta^{a}{}_{\mu}\partial^{b}\ln\lambda - \delta^{b}{}_{\mu}\partial^{a}\ln\lambda, \tag{38}$$

and the spinor covariant derivative takes the standard form

$$D_{\mu} = \partial_{\mu} + \frac{1}{4}\omega_{\mu}{}^{ab}\gamma_{a}\gamma_{b} - iqA_{\mu}.$$

Writing $X = \ln\lambda$, the Dirac operator becomes

$$iD = i\partial + \frac{3}{2}\partial X - qA. \tag{39}$$

This expression is exact for any admissible scalar field $\lambda(x)$ and involves no additional assumptions beyond those already imposed by TNE.

C.2. Foldy-Wouthuysen Reduction and Spin Holonomy

To assess observable consequences in the nonrelativistic regime, one may perform a standard Foldy-Wouthuysen (FW) transformation of the Dirac Hamiltonian associated with (39). Writing

$$H = \beta m + \mathcal{O} + \mathcal{E},$$

with

$$\mathcal{O} = \boldsymbol{\alpha}\cdot(\boldsymbol{p} - q\boldsymbol{A}) + \frac{3}{2}\boldsymbol{\alpha}\cdot\nabla X,\ \mathcal{E} = q\Phi,$$

the FW expansion to order $1/m^2$ yields the usual Pauli Hamiltonian plus additional terms proportional to gradients of X.

In particular, isolating the magnetic interaction produces

$$H_{\text{FW}} \supset -\frac{q}{2m}(1+\delta a_\ell)\boldsymbol{S}\cdot\boldsymbol{B}, \tag{40}$$

where δa_ℓ arises entirely from the conformal spin connection and depends on $\nabla X = \nabla \ln \lambda$.

Under mild regularity assumptions on admissible scalar profiles (e.g. $\nabla X \in L^3_{\text{loc}}$), the additional term is finite and perturbative. Its detailed evaluation depends on the scalar environment and is not fixed by TNE alone.

C.3. Scope and Interpretive Limits

The analysis above establishes that:

- Dirac spinor transport is well-defined on any scalar-conformal geometry admissible under TNE.
- Conformal modulation of the metric induces additional spin-holonomy terms through the spin connection, without introducing new degrees of freedom.
- These corrections arise kinematically from geometry and do not modify the scalar variational law or its conservation structure.

No claim is made that spin dynamics are derived from TNE, nor that spinor fields play any role in determining admissible scalar configurations. The results recorded here serve only to establish the geometric compatibility between the scalar variational framework and standard spinor transport.

A full dynamical treatment of spinor fields, including phenomenological applications and environment-dependent corrections, is deferred to a separate work building explicitly on the admissible geometries classified by The NUVO Equation.

D. Linearised Operator and Retarded Green Function

Linearizing TNE about $\lambda = 1$ yields, after setting $\lambda = 1 + \epsilon\phi$ and keeping terms

$O(\epsilon)$,

$$\left(\tilde{\Box}-\mu^2\right)\phi=0,\ \ \mu^2=\frac{U''(1)}{\beta},$$

where $\tilde{\Box}=\eta^{\mu\nu}\partial_\mu\partial_\nu$. Writing dimensions explicitly, the corresponding Klein-Gordon mass parameter is

$$m_{\text{eff}}=\frac{\hbar}{c}\mu.$$

The retarded Green function G_{ret} satisfies

$$\left(\tilde{\Box}-\mu^2\right)G_{\text{ret}}\left(x-x'\right)=-\delta^{(4)}\left(x-x'\right),\ \ G_{\text{ret}}\left(t<0,\boldsymbol{x}\right)=0.$$

In $3+1$ dimensions the standard closed form is

$$G_{\text{ret}}\left(t,r\right)=\theta\left(t\right)\left[\frac{\delta\left(ct-r\right)}{4\pi r}-\frac{\mu}{4\pi}\theta\left(ct-r\right)\frac{J_1\left(\mu\sqrt{c^2t^2-r^2}\right)}{\sqrt{c^2t^2-r^2}}\right],\ \ r=\left|\boldsymbol{x}-\boldsymbol{x}'\right|,$$

where J_1 is the Bessel function of the first kind. Thus, when $\mu\neq 0$ the retarded response has support *on and inside* the light cone, while remaining strictly causal.

E. Saturation Scale λ_{sat}

This appendix illustrates the convexity-loss definition of λ_{sat} using a simple model potential. It is an example only; no parameter values are asserted as canonical.

Let $X=\ln\lambda$ and consider a quartic model

$$U\left(X\right)=\frac{\gamma}{2}X^2-\frac{\sigma}{3}X^3+\frac{\delta}{4}X^4,\ \ \gamma>0,\sigma>0,\delta>0.$$

Then

$$U''\left(X\right)=\gamma-2\sigma X+3\delta X^2.$$

Loss of convexity occurs when $U''\left(X_{\text{sat}}\right)=0$, giving

$$X_{\text{sat}}=\frac{\sigma\pm\sqrt{\sigma^2-3\gamma\delta}}{3\delta},\ \ \lambda_{\text{sat}}=\exp\left(X_{\text{sat}}\right),$$

provided $\sigma^2\geq 3\gamma\delta$. This exhibits explicitly how the saturation threshold depends on the potential parameters and clarifies that λ_{sat} is a stability feature of U rather than an additional postulate.