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On Some Properties of Mannheim Curves in a Strict Walker 3-Manifolds

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Abstract

In this paper we study the geometry of Mannheim curves in a strict Walker 3-manifold and we obtain explicit parametric equations for Mannheim curves and timelike Mannheim curves, respectively. We determine the distance between two corresponding points of the Mannheim pair of curves and show that the distance depends on the curvature. We discuss the relationship between the curvature and torsion of a pair of Mannheim curves in a strict Walker Manifold. We finish with an example of Mannheim pair curves to illustrate the result.

Keywords

Mannheim Curve, Curvature, Torsion, Mannheim Partner, Walker Manifolds

1. Introduction

In the study of the fundamental theory and the characterizations of space curves, the corresponding relations between the curves are a very interesting and important problem [1]. The well-known Bertrand curve is characterized as a kind of corresponding relation between the two curves. For the Bertrand curve α, it shares the normal lines with another curve β, called the Bertrand mate or Bertrand partner curve of α. In this paper, we are concerned with another kind of associated curves, called the Mannheim curve and Mannheim mate (partner curve) in the history of differential geometry. In this work, we call them simply the Mannheim pair.

From elementary differential geometry, we know clearly about the characterizations of the Bertrand pair. But there are rather few works on the Mannheim pair. According to [2], it is known that a space curve in $\mathbb{R}^3$ is a Mannheim curve if and only if its curvature κ and torsion κ satisfy the formula $\kappa = \lambda\left(\kappa^2 + \tau^2\right)$, where λ is a nonzero constant. In [3], B. Y. Chen characterizes the curve which satisfies $\frac{\tau}{\kappa} = as + b$, $a \neq 0$. In [2], the authors give the necessary and sufficient conditions for a curve in 3 Euclidean space to be a Mannheim partner of a given curve. They also show that the Mannheim curve of generalized helix is a straight line. In [4], the authors proved that the distance between corresponding points of the Mannheim partner curves in the three-dimensional Heisenberg group is constant.

Motivated by the above works, in this paper, we study the Mannheim partner curves in a three-dimensional Walker manifold M^3. We will give the necessary and sufficient conditions for a curve to be a Mannheim partner curve of another curve in the three Walker Manifolds and show that, in contrast to the Euclidean case, the distance between two corresponding points is constant if and only if its curvature is constant. The paper is organized as follows: Apart from the introduction in Section 2, we give some preliminary tools about Mannheim curves Walker 3-dimensional space. In Section 3, we study Mannheim curves in a strict Walker 3-manifolds and the last section talks about the Mannheim partner of helices.

2. Preliminaries

2.1. Mannheim Partner Curves

Definition 2.1. [2] *Let* $\mathbb{R}^3$ *be the* 3*-dimensional Euclidean space with the standard inner product. If there exists a corresponding relationship between the space curves* α *and* β *such that, at the corresponding points of the curves, the principal normal lines of* α *coincides with the binormal lines of* β, *then* α *is called a Mannheim curve, and* β *a Mannheim partner curve of* α. *The pair* $\{\alpha, \beta\}$ *is said to be a Mannheim pair.*

The curve $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ in 3-dimensional Euclidean space is parametrized by the arc-length parameter s and from the definition above the Mannheim partner curve of α is given by $\beta : J \subset \mathbb{R} \to \mathbb{R}^3$ in 3-dimensional Euclidean space $\mathbb{R}^3$ with the help of figure 1 [4] such that

$$\beta(s) = \alpha(s) + \lambda(s) B(s); s \in I$$

where λ is a smooth function on I and B is the binormal vector field of α. We should remark that the parameter s generally is not an arc-length parameter of β.

2.2. The Geometry of Walker Manifold

A Walker n-manifold is a pseudo-Riemannian manifold, which admits a field of null parallel r-planes, with $r \leq \frac{n}{2}$. The canonical forms of the metrics were investigated by A. G. Walker ([5]). Walker has derived adapted coordinates to a parallel plan field. Hence, the metric of a three-dimensional Walker manifold $\left(M, g_f^{\epsilon}\right)$ with coordinates (x, y, z) is expressed as

$$g_f^{\epsilon} = dx \circ dz + \epsilon dy^2 + f(x,y,z) dz^2 \tag{2.1}$$

and its matrix form as

$$g_f^{\epsilon} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \epsilon & 0 \\ 1 & 0 & f \end{pmatrix} \text{ with inverse } \left(g_f^{\epsilon}\right)^{-1} = \begin{pmatrix} -f & 0 & 1 \\ 0 & \epsilon & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

for some function $f(x,y,z)$, where $\epsilon = \pm 1$ and thus $D = \mathrm{Span}\,\partial_x$ as the parallel degenerate line field. Notice that when $\epsilon = 1$ and $\epsilon = -1$ the Walker manifold has signature $(2,1)$ and $(1,2)$ respectively, and therefore is Lorentzian in both cases.

It follows after a straightforward calculation that the Levi-Civita connection of any metric (2.1) is given by [6]:

$$\begin{aligned} &\nabla_{\partial_x}\partial z = \frac{1}{2} f_x \partial_x, \quad \nabla_{\partial_y}\partial z = \frac{1}{2} f_y \partial_x, \\ &\nabla_{\partial_z}\partial z = \frac{1}{2}\left(f f_x + f_z\right)\partial_x + \frac{1}{2} f_y \partial_y - \frac{1}{2} f_x \partial_z \end{aligned} \tag{2.2}$$

where ∂_x, ∂_y and ∂_z are the coordinate vector fields $\frac{\partial}{\partial_x}$, $\frac{\partial}{\partial_y}$ and $\frac{\partial}{\partial_z}$, respectively. Hence, if $\left(M, g_f^{\epsilon}\right)$ is a strict Walker manifolds *i.e.*, $f(x,y,z) = f(y,z)$, then the associated Levi-Civita connection satisfies [6]

$$\nabla_{\partial_y}\partial z = \frac{1}{2} f_y \partial_x, \quad \nabla_{\partial_z}\partial z = \frac{1}{2} f_z \partial_x - \frac{\epsilon}{2} f_y \partial_y. \tag{2.3}$$

Note that the existence of a null parallel vector field (*i.e.* $f = f(y,z)$) simplifies the non-zero components of the Christoffel symbols and the curvature tensor of the metric g_f^{ϵ} as follows:

$$\Gamma_{23}^1 = \Gamma_{32}^1 = \frac{1}{2} f_y, \; \Gamma_{33}^1 = \frac{1}{2} f_z, \; \Gamma_{33}^2 = -\frac{\epsilon}{2} f_y \tag{2.4}$$

Proposition 2.2. *Starting from local coordinates* (x,y,z) *for which* (2.1) *holds, Let*

$$e_1 = \partial_y, \; e_2 = \frac{2-f}{2\sqrt{2}}\partial_x + \frac{1}{\sqrt{2}}\partial_z, \; e_3 = \frac{2+f}{2\sqrt{2}}\partial_x - \frac{1}{\sqrt{2}}\partial_z \tag{2.5}$$

Then they formed a local pseudo-orthonormal frame fields on $\left(M, g_f^{\epsilon}\right)$.

Proof. Indeed, we get $g_f^{\epsilon}(e_1,e_1) = \epsilon$, $g_f^{\epsilon}(e_2,e_2) = 1$ and $g_f^{\epsilon}(e_3,e_3) = -1$. □

Let now u and v be two vectors in M. Denoted by $\left(\vec{i}, \vec{j}, \vec{k}\right)$ the canonical frame in $\mathbb{R}^3$.

Proposition 2.3. *For the covariant derivatives of the Levi-Civita connection of the left-invariant metric* g *defined above, we have*

$$\nabla_{e_i} e_j = \begin{bmatrix} 0 & \frac{1}{4} f_y (e_2 + e_3) & -\frac{1}{4} f_y (e_2 + e_3) \\ \frac{1}{4} f_y (e_2 + e_3) & -\frac{\epsilon}{4} f_y e_1 & \frac{\epsilon}{4} f_y e_1 \\ -\frac{1}{4} f_y (e_2 + e_3) & \frac{\epsilon}{4} f_y e_1 & -\frac{\epsilon}{4} f_y e_1 \end{bmatrix}. \tag{2.6}$$

Proof. The curvature tensor field of ∇ is given by

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

where $X, Y, Z \in \Gamma(M)$. If we denote by

$$R_{ijk} = R(e_i, e_j)e_k,$$

where the indices i, j, k take the values $1,2,3$. Then the non-zero components of the curvature tensor field are [6]

$$\begin{aligned} R_{121} &= -R_{131} = -\frac{1}{4} f_{yy}(e_2 + e_3), \\ R_{122} &= -R_{123} = -R_{132} = R_{133} = \frac{\epsilon}{4} f_{yy} e_1. \end{aligned} \tag{2.7}$$

□

The vector product of u and v in (M, g_f^ϵ) with respect to the metric g_f^ϵ is the vector denoted by $u \times_f v$ in M defined by

$$g_f^\epsilon(u \times_f v, w) = \det(u, v, w) \tag{2.8}$$

for all vector w in M, where $\det(u,v,w)$ is the determinant function associated to the canonical basis of $\mathbb{R}^3$.

Proposition 2.4. *If* $u = (u_1, u_2, u_3)$ *and* $v = (v_1, v_2, v_3)$ *are two vectors in* $\mathbb{R}^3$ *then by using* (2.8), *we have*:

$$u \times_f v = \left(\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix} - f \begin{vmatrix} u_2 & v_2 \\ u_3 & v_3 \end{vmatrix} \right) \vec{i} - \epsilon \begin{vmatrix} u_1 & v_1 \\ u_3 & v_3 \end{vmatrix} \vec{j} + \begin{vmatrix} u_2 & v_2 \\ u_3 & v_3 \end{vmatrix} \vec{k} \tag{2.9}$$

Proof. We develop the two members of equation and after a simple calculation using the determinant function we get the resultants. □

Proposition 2.5. [6] *The Walker cross product in* M *has the following properties*:

1) The Walker cross product is bilinear and anti-symmetric.

2) $X \times_f Y$ is perpendicular both of X and Y.

3) The frame defined in (2.5) verify the following: $e_1 \times_f e_2 = -e_3$, $e_2 \times_f e_3 = -e_1$ and $e_3 \times_f e_1 = e_2$.

Proof. We use the definition of cross product and compute. □

3. Mannheim Curves in Strict Walker 3-Manifold

Let $\alpha : I \subset \mathbb{R} \to (M, g_f^\epsilon)$ be a curve parametrized by its arc-length s. We can define the Mannheim curve as in the case of Euclidean space.

Definition 3.1. *A curve* $\alpha : I \subset \mathbb{R} \to (M, g_f^\epsilon)$ *is said to be a Mannheim curve if there exists an other curve* $\beta : J \subset \mathbb{R} \to (M, g_f^\epsilon)$ *such that* $\beta(f(s)) = \alpha(s) + \lambda(s) B(s)$ *where* $\lambda : I \to \mathbb{R}$ *is a smooth function and* $f : I \to J$ *is a function such that* $f(s)$ *is arc-length parametrization of* β.

Note that as the principal normal vector of the spacelike curve α is timelike, the Mannheim mate curve β can be a timelike curve or a spacelike curve. The

Frenet frame of α is formed by the vectors T, N and B along α where T is the tangent, N the principal normal and B the binormal vector.

Theorem 3.2. [6] *They satisfied the Frenet formulas*

$$\begin{cases} \nabla_T T(s) = \epsilon_2 \kappa(s) N(s) \\ \nabla_T N(s) = -\epsilon_1 \kappa T(s) - \epsilon_3 \tau B(s) \\ \nabla_T B(s) = \epsilon_2 \tau(s) N(s) \end{cases} \tag{3.1}$$

where κ *and* τ *are respectively the curvature and the torsion of the curve* α, *with* $\epsilon_1 = g_f(T;T)$; $\epsilon_2 = g_f(N;N)$ *and* $\epsilon_3 = g_f(B,B)$.

Proof. We can consider the unit speed normal which is opposite of the principal normal vector. □

Theorem 3.3. [4] *For a Mannheim curve* α *there exists a Mannheim partner* β *such that* $\{\alpha, \beta\}$ *is a pair of Mannheim curves.*

Proof. As N and B^* are linearly dependents,

$$\begin{aligned} \beta &= \alpha - \lambda B^* \\ &= \alpha - \lambda k N. \end{aligned} \tag{3.2}$$

□

Theorem 3.4. *Let* (α, β) *be a Mannheim pair in Walker manifold* M. *The distance between corresponding points of the Mannheim partner curves in* M *is constant if and only if the curvature of* α *is a constant.*

Proof. Let $\{\alpha, \beta\}$ be a couple of Mannheim curves in a strict Walker 3-manifold. We note $\{T, N, B\}$ and $\{T^\star, N^\star, B^\star\}$ the Frenet frames of the curves α and β respectively. According to figure 1 [4], we can write:

$$\alpha(s) = \beta(s^\star) + \lambda(s^\star) B^\star(s^\star) \tag{3.3}$$

By derivation of Equation (3.3) we have:

$$\frac{\mathrm{d}\alpha(s)}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}s^\star} = \frac{\mathrm{d}\beta(s^\star)}{\mathrm{d}s^\star} + \frac{\mathrm{d}\lambda(s^\star)}{\mathrm{d}s^\star} B^\star(s^\star) + \lambda(s^\star)\frac{\mathrm{d}B^\star(s^\star)}{\mathrm{d}s^\star} \tag{3.4}$$

Using the above Equation (3.4) and the fact of N et B^* coincide, and $B^* = kN$ we have:

$$T(s)\frac{\mathrm{d}s}{\mathrm{d}s^\star} = T^\star(s^\star) + \lambda'(s^\star) k N(s) + \lambda(s^\star)\varepsilon_2^* \tau^\star(s^\star) N^\star(s^\star) \tag{3.5}$$

Applying the Walker metric g on the two members of equation above and computing the scalar product with N, we have

$$\begin{aligned} \frac{\mathrm{d}s}{\mathrm{d}s^\star} g(T(s), N(s)) &= g(T^\star(s^\star), N(s)) + \lambda'(s^\star) k g(N(s), N(s)) \\ &\quad + \lambda(s^\star)\varepsilon_2^* \tau^\star(s^\star) g(N^\star(s^\star), N(s)) \end{aligned} \tag{3.6}$$

As N and $B^\star$ are linearly dependent, we get

$$g(T,N) = 0,\ g(T^*, N) = g(T^\star, B^\star) = 0,$$
$$g(N,N) = \pm 1,\ g(N^*, N) = g(N^*, B^*) = 0.$$

So $\lambda'(s^\star) = 0$ and in that case λ is a non zero constant. On the other hand, according to the definition of distance function between two points we get:

$$\begin{aligned} d\left(\beta\left(s^{\star}\right),\alpha\left(s\right)\right) &= \left\|\alpha\left(s\right)-\beta\left(s^{\star}\right)\right\| \\ &= \left|\lambda\left(s^{\star}\right)\right|\left\|B^{\star}\left(s^{\star}\right)\right\| \\ &= \left|\lambda\right|\sqrt{\left\langle B^{\star},B^{\star}\right\rangle} \\ &= \left|\lambda\right|\sqrt{\left\langle kN\left(s\right),kN\left(s\right)\right\rangle} \\ &= \left|\lambda\right|\left|k\right|\sqrt{\left|g_f^{\varepsilon}\left(N,N\right)\right|} \\ &= \left|\lambda\right|\left|k\right|\sqrt{\left|\varepsilon_2\right|}. \end{aligned}$$

So the distance $d\left(\beta\left(s^{\star}\right),\alpha\left(s\right)\right)$ is constant if k is constant. □

We establish now the relation between curvatures and torsions of α and β at the correspondents' points.

Theorem 3.5. *Let* $\{\alpha,\beta\}$ *be a pair of Mannheim in Walker* 3*-manifold. Then the torsion of* β *is obtained as*

$$\tau^{*} = \frac{\varepsilon_1}{\varepsilon_3\varepsilon_2^{*}}\cdot\frac{\kappa}{\lambda\tau}$$

Proof. According to the relation $T\dfrac{\mathrm{d}s}{\mathrm{d}s^{*}} = T^{*} + \varepsilon_2^{*}\lambda\tau^{*}N^{*}$. we get

$$T = \frac{\mathrm{d}s^{*}}{\mathrm{d}s}T^{*} + \varepsilon_2^{*}\lambda\tau^{*}\frac{\mathrm{d}s^{*}}{\mathrm{d}s}N^{*}. \tag{3.7}$$

And we have

$$\begin{cases} T = \cos\theta T^{*} + \sin\theta N^{*} \\ B = -\sin\theta T^{*} + \cos\theta N^{*} \end{cases} \tag{3.8}$$

where θ is the angle between T and T^{*} at the corresponding points of α and β respectively.

From (3.7) et (3.8), we have

$$\cos\theta = \frac{\mathrm{d}s^{*}}{\mathrm{d}s}, \tag{3.9}$$

$$\sin\theta = \varepsilon_2^{*}\lambda\tau^{*}\frac{\mathrm{d}s^{*}}{\mathrm{d}s}. \tag{3.10}$$

By derivation of (3.2), we obtain:

$$\frac{\mathrm{d}\beta\left(s^{*}\right)}{\mathrm{d}s^{*}} = \frac{\mathrm{d}\alpha\left(s\right)}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}s^{*}} - k\lambda\frac{\mathrm{d}N}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}s^{*}}$$

$$\Rightarrow T^{*} = T\frac{\mathrm{d}s}{\mathrm{d}s^{*}} + k\varepsilon_1\lambda\kappa T\frac{\mathrm{d}s}{\mathrm{d}s^{*}} + k\varepsilon_3\lambda\tau B\frac{\mathrm{d}s}{\mathrm{d}s^{*}},$$

$$T^{*} = \left(1+k\varepsilon_1\lambda\kappa\right)\frac{\mathrm{d}s}{\mathrm{d}s^{*}}T + k\varepsilon_3\lambda\tau\frac{\mathrm{d}s}{\mathrm{d}s^{*}}B. \tag{3.11}$$

The Equation (3.8) give

$$\begin{cases} \cos\theta T^{*} = T - \sin\theta N^{*} \\ \sin\theta T^{*} = -B + \cos\theta N^{*} \end{cases}$$

$$\Rightarrow \begin{cases} T^* = \dfrac{T - \sin\theta N^*}{\cos\theta} \\ T^* = \dfrac{-B + \cos\theta N^*}{\sin\theta}. \end{cases} \tag{3.12}$$

The equation

$$(3.12) \Rightarrow \frac{T - \sin\theta N^*}{\cos\theta} = \frac{-B + \cos\theta N^*}{\sin\theta}$$

$$\Rightarrow \sin\theta\left(T - \sin\theta N^*\right) = \cos\theta\left(-B + \cos\theta N^*\right)$$

$$\Rightarrow \sin\theta T - \sin^2\theta N^* = -B\cos\theta + \cos^2\theta N^*$$

$$\Rightarrow \sin\theta T + B\cos\theta = \left(\cos^2\theta + \sin^2\theta\right)N^*.$$

According to $\cos^2\theta + \sin^2\theta = 1$, we get the equation:

$$N^* = \sin\theta T + B\cos\theta,$$

$$\Rightarrow N^* = \sin\theta T + \cos\theta B.$$

And from the Equation (3.8), we obtain:

$$\begin{cases} \sin\theta N^* = T - \cos\theta T^* \\ \cos\theta N^* = B + \sin\theta T^* \end{cases}$$

$$\Rightarrow \begin{cases} N^* = \dfrac{T - \cos\theta T^*}{\sin\theta} \\ N^* = \dfrac{B + \sin\theta N^*}{\cos\theta} \end{cases} \tag{3.13}$$

$$(3.13) \Rightarrow \frac{T - \cos\theta T^*}{\sin\theta} = \frac{B + \sin\theta N^*}{\cos\theta}$$

$$\Rightarrow \cos\theta\left(T - \cos\theta T^*\right) = \sin\theta\left(B + \sin\theta T^*\right)$$

$$\Rightarrow \cos\theta T - \cos^2\theta T^* = \sin\theta B + \sin^2\theta T^*$$

$$\Rightarrow \cos\theta T - \sin\theta B = \left(\sin^2\theta + \cos^2\theta\right)T^*.$$

And we have

$$T^* = \cos\theta T - \sin\theta B. \tag{3.14}$$

So

$$\begin{cases} T^* = \cos\theta T - \sin\theta B \\ N^* = \sin\theta T + \cos\theta B. \end{cases} \tag{3.15}$$

From (3.11) and (3.14), we obtain:

$$\cos\theta = \left(1 + k\varepsilon_1\lambda\kappa\right)\frac{\mathrm{d}s}{\mathrm{d}s^*}, \tag{3.16}$$

$$\sin\theta = -k\varepsilon_3\lambda\tau\frac{\mathrm{d}s}{\mathrm{d}s^*}. \tag{3.17}$$

By multiplication of the two Equation (3.9) and (3.16), and (3.10) and (3.17) respectively we get

$$\begin{aligned} \cos^2\theta &= 1 + k\varepsilon_1\lambda\kappa, \\ \sin^2\theta &= -k\varepsilon_3\varepsilon_2^*\lambda^2\tau\tau^*. \end{aligned} \tag{3.18}$$

Adding the Equation (3.18) we have

$$\cos^2\theta + \sin^2\theta = 1 + k\varepsilon_1\lambda\kappa - k\varepsilon_3\varepsilon_2^*\lambda^2\tau\tau^*.$$

And from $\cos^2\theta + \sin^2\theta = 1$ we have

$$\begin{aligned} &1 + k\varepsilon_1\lambda\kappa - k\varepsilon_3\varepsilon_2^*\lambda^2\tau\tau^* = 1 \\ &\Rightarrow -k\varepsilon_3\varepsilon_2^*\lambda^2\tau\tau^* = 1 - 1 - k\varepsilon_1\lambda\kappa \\ &\Rightarrow \tau^* = \frac{\varepsilon_1\lambda\kappa}{\varepsilon_3\varepsilon_2^*\lambda^2\tau} \\ &\Rightarrow \tau^* = \frac{\varepsilon_1\kappa}{\varepsilon_3\varepsilon_2^*\lambda\tau}, \end{aligned}$$

So we have $\tau^* = \frac{\varepsilon_1}{\varepsilon_3\varepsilon_2^*} \cdot \frac{\kappa}{\lambda\tau}$ □

Theorem 3.6. *Let* $\{\alpha, \beta\}$ *be a pair of Mannheim in Walker manifold. We have*

$$\varepsilon_3\mu\tau - \varepsilon_1\lambda\kappa = \frac{1}{k},$$

where λ *and* μ *are nonzero real numbers.*

Proof. We use the fact that

$$\cos\theta = (1 + k\varepsilon_1\lambda\kappa)\frac{\mathrm{ds}}{\mathrm{ds}^*}$$

et

$$\sin\theta = -k\varepsilon_3\lambda\tau\frac{\mathrm{ds}}{\mathrm{ds}^*}$$

$$\Rightarrow \frac{\cos\theta}{1 + k\varepsilon_1\lambda\kappa} = \frac{\mathrm{ds}}{\mathrm{ds}^*}, \tag{3.19}$$

and

$$-\frac{\sin\theta}{k\varepsilon_3\lambda\tau} = \frac{\mathrm{ds}}{\mathrm{ds}^*}. \tag{3.20}$$

Adding the relations (3.19) and (3.20); and after calculation we get the result. □

Example 3.7. *To illustrate our main result, we give an example of a pair of Mannheim curves. Let*

$$\alpha(s) = \left(\cos\frac{s}{\sqrt{2}}; \sin\frac{s}{\sqrt{2}}; \frac{s}{\sqrt{2}}\right)$$

be a Mannheim curve with arc-length parameter s. *Then the Mannheim partner of* α *is the curve* $\beta(s) = \left(0; 0; \frac{s}{\sqrt{2}}\right)$. *Indeed, the principal normal vector of* α *is* $N(s) = \left(-\cos\frac{s}{\sqrt{2}}; -\sin\frac{s}{\sqrt{2}}; 0\right)$ *and we consider* $\beta(s) = \alpha(s) + N(s)$. *In this* *example, the distance between two corresponding points is constant because the curvature of* α *is constant.*

This example matches the result obtained by Fan-Wang [2] in Euclidean space whereby the Mannheim partner of helix is a straight line.

4. Conclusion

In this paper we study the geometry of Mannheim curves in a strict Walker 3-manifold. For the first time, we introduced the geometric elements of the strict Walker 3-manifold by calculation of the Christoffel symbols, the Levi-Civita connection, curvature and the cross product. The second concerned our results. In this paper, two main results are obtained. The first one is that, in contrast to the case of Euclidean, the distance between two corresponding points is constant if its curvature is constant. In the second result, we established the relation between the torsion of the partner and the curvature and torsion of the Mannheim curve. This paper shows that some results in Euclidean space can be generalized to the Walker manifold. We finish with an example of a Mannheim pair of curves. In the future, we can extend our study to the Mannheim curves in the Walker 4-manifolds.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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