Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(1), 466-479
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

# An Exact CMB Photon Radiation Density $\Omega_\gamma$ of the Universe Derived from $R_{H_t} = ct$ Cosmology

**Espen Gaarder Haug**

Norwegian University of Life Sciences, Christian Magnus Falsensvei 18, Aas, Norway Ås, Norway
Email: espenhaug@mac.com

**How to cite this paper:** Haug, E.G. (2026) An Exact CMB Photon Radiation Density $\Omega_\gamma$ of the Universe Derived from $R_{H_t} = ct$ Cosmology. *Journal of Applied Mathematics and Physics*, **14**, 466-479.
https://doi.org/10.4236/jamp.2026.141024

**Received:** November 27, 2025
**Accepted:** January 27, 2026
**Published:** January 30, 2026



## Abstract

We will demonstrate that the photon energy density parameter $\Omega_\gamma$ of the universe can be derived exactly within the $R_{H_t} = ct$ cosmology. We find that it must be precisely: $\Omega_\gamma = \frac{1}{5760\pi} \approx 5.526\times10^{-5}$, which lies well within the 95% confidence interval for the photon radiation density reported by the Particle Data Group (PDG). This exact result implies that there is no uncertainty in the radiation density—at least within certain subclasses of the $R_{H_t} = ct$ cosmology—and that such a model is consistent with observations. Furthermore, in the standard model, the number density of CMB photons cannot be predicted from the Hubble parameter, but only from the CMB temperature. For the first time, we derive a new equation for the number density of CMB photons that allows prediction based solely on the Hubble parameter and the Planck length. This leads to a predicted number density of CMB photons equal to $n_\gamma = 410.71\pm0.26$ photons per cm$^3$, which is remarkably close to the value reported by the PDG when we use the $H_0$ value recently proposed by Haug and Tatum, $H_0 = 66.8943\pm0.0287$ km/s/Mpc. However, when using the value $H_0 = 73.30\pm1.04$ km/s/Mpc predicted by Riess *et al.*, the photon number density prediction falls far outside even the five-sigma confidence interval reported by the PDG. As we briefly discussed, this discrepancy is related to the Hubble tension observed in the ΛCDM model.

## Keywords

Photon Energy Density Parameter, Friedmann Equation, Critical Density, Number Density CMB Photons, Thermodynamical Friedmann Equation

## 1. Introduction

We will study the CMB radiation density, the CMB photon number density, and the CMB radiation density parameter within a subclass of the $R_{H_t} = ct$ cosmology. We will demonstrate that the CMB photon density parameter appears to be exactly $\Omega_\gamma = \frac{\rho_\gamma}{\rho_{cr}} = \frac{1}{5760\pi}$. Furthermore, we show that it is possible to predict both the CMB radiation density and the photon number density without knowing the CMB temperature—only the Hubble parameter is required. To our knowledge, this is not possible in the standard Λ-CDM model. In the standard model, the photon number density can only be calculated from the CMB temperature.

Although the $R_{H_t} = ct$ cosmology is much less well-known and less frequently discussed than the ΛCDM model, there remains an active debate surrounding it among multiple researchers; see [1]-[4]. In the $R_{H_t} = ct$ cosmology, the universe expands at the speed of light—or, equivalently, no information, including gravity, can travel faster than the expansion. Melia [5] has demonstrated that recent observations from JWST related to early, well-formed galaxies fit well with $R_H = ct$ cosmology, but are more challenging to explain in the Λ-CDM model. Melia [6] has also summarized 18 types of observational tests comparing the Λ-CDM model with $R_H = ct$, pointing out that most of these tests seem to favor $R_H = ct$.

A notable subclass of the $R_{H_t} = ct$ cosmology is black hole cosmology. One such model, proposed by Haug and Tatum [7], appears to fit the full SN Ia distance ladder perfectly. This model is based on a thermodynamic Friedmann-type equation [8]. The idea of black hole cosmology dates back at least to 1972 with Pathria [9] and continues to be actively explored today [10]-[15], even though the Λ-CDM model currently dominates mainstream cosmology. The critical mass within the Hubble sphere is given by:

$$M_{cr} = \frac{c^2 R_H}{2G} \tag{1}$$

Solving the Schwarzschild radius formula $R_s = \frac{2GM}{c^2}$ for the mass of a black hole yields $M = \frac{c^2 R_s}{2G}$, indicating a striking mathematical correspondence between black holes and the universe as a whole. Christillin. [16] points out that we in a black hole universe must have

$$R = \frac{2GM_u}{c^2 R_u} = 1 \tag{2}$$

and that the Schwarzschild radius is determined by the Hubble constant, see also Stuckey [17].

Recent findings from JWST have also made black hole cosmology an interesting alternative to the Λ-CDM according to Sharmir [18], related to spinning black holes (the Kerr [19] and Kerr-Newman [20] metric). We will here not get into spinning black hole universes, but stick to non-spinning black holes, but that could be a further extension to look into in relation to the work we will present.

What we discuss in this paper is consistent with a growing black hole, where the radius evolves as $R_{H_t} = ct$. However, it is also compatible with a steady-state black hole cosmology, where the general relativistic metric implies that the density inside the black hole varies with distance from the center, as discussed in [21]. Additionally, this holds in the extremal universe scenario, where the density inside the black hole's Hubble sphere is non-uniform, as indicated in [22].

## 2. Deriving the Photon Energy Density Parameter $\Omega_\gamma$

The photon energy density, $\rho_\gamma$, is typically calculated via the following integral (see, for example, Weinberg [23]):

$$\rho_{\gamma,0}c^2 = \int_0^\infty h\nu n(\nu)\,\mathrm{d}\nu = a_b T_0^4 \tag{3}$$

which implies:

$$\rho_{\gamma,0} = \frac{a_b T_0^4}{c^2} \tag{4}$$

Here, $c$ is the speed of light, $T_0$ is the present-day blackbody temperature (i.e., the CMB temperature), and the radiation density constant $a_b$ is given by:

$$a_b = \frac{8\pi^5 k_b^4}{15c^3h^3} = \frac{4}{c}\sigma \tag{5}$$

where $\sigma$ is the Stefan–Boltzmann constant, $\sigma = \frac{2\pi^5 k_b^4}{15c^2h^3} = \frac{\pi^2 k_b^4}{60c^2\hbar^3}$, and $k_b$ is the Boltzmann constant.

In 1978, Emslie and Green [24] (see also Weinberg [25]) expressed the photon energy density relative to the critical Friedmann [26] energy density as:

$$\Omega_{\gamma,0} = \frac{\rho_{\gamma,0}}{\rho_{cr,0}}$$

$$\Omega_{\gamma,0} = \frac{\frac{a_b T_0^4}{c^2}}{\frac{3H_0^2}{8\pi G}} \tag{6}$$

Haug and Tatum [8] have recently shown that the Friedmann equation can be expressed in thermodynamic form, leading to a critical density given by:

$$\rho_{cr,0} = \frac{3H_0}{8\pi G} = T_0^4 \frac{23040\pi}{c^3}\sigma \tag{7}$$

Substituting this into equation (6), we obtain:

$$\Omega_{\gamma,0} = \frac{\frac{a_b T_0^4}{c^2}}{T_0^4 \frac{23040\pi}{c^3}\sigma}$$

$$\Omega_{\gamma,0} = \frac{4\sigma}{23040\pi\sigma}$$

$$\Omega_{\gamma,0} = \frac{1}{5760\pi} \approx 5.52621330180192 \times 10^{-5} \tag{8}$$

This value lies well within the 95% confidence interval for the photon radiation density reported by the Particle Data Group (PDG)[1]. They give a 95% confidence interval: $5.08 \times 10^{-5}$ to $5.68 \times 10^{-5}$ ($5.35 \pm 0.15 \times 10^{-5}$ for $1\sigma$, the 68.3% confidence interval). Refer to **Appendix A** for a demonstration that the CMB photon energy density parameter remains constant for times other than $\boldsymbol{t_0}$.

The Haug and Tatum [7] model appears to potentially also resolve the Hubble tension and seems to outperform the Λ-CDM model in several key respects, as recently discussed in [27]. While further investigation is certainly warranted, it is high time that the astrophysics community more seriously consider alternatives to the Λ-CDM model.

## 3. Radiation Density from Black Hole Universe

We will show one more way to arrive at the same result as above, but from a different angle. The Schwarzschild radius is given by $R_s = \frac{2GM}{c^2}$, which solved for $M$ gives $M = \frac{c^2 R_s}{2G}$, and the total energy of the black hole is then:

$$E = Mc^2 = \frac{c^4 R_s}{2G} \tag{9}$$

The energy density of a black hole is then given by:

$$\rho_{BH} = \frac{E}{\frac{4}{3}\pi R_s^3}$$

$$\rho_{BH} = \frac{\frac{c^4 R_s}{2G}}{\frac{4}{3}\pi R_s^3}$$

$$\rho_{BH} = \hbar \frac{c}{l_p} \frac{1}{\frac{8}{3}\pi R_s^2 l_p}$$

$$\rho_{BH} = \frac{\frac{1}{2}E_p}{\frac{4}{3}\pi R_s^2 l_p} \tag{10}$$

where $E_p = \sqrt{\frac{\hbar c^5}{G}} = \hbar \frac{c}{l_p}$ is the Planck energy. It is interesting to see that the energy density of the whole black hole $\frac{E}{\frac{4}{3}\pi R_s^3}$ is identical to $\frac{\frac{1}{2}E_p}{\frac{4}{3}\pi R_s^2 l_p}$.

[1] https://pdg.lbl.gov/2023/reviews/rpp2023-rev-astrophysical-constants.pdf.

Next, the Hawking [28] temperature of a black hole is given by:

$$T_H = \frac{\hbar c}{k_b 4\pi R_s} \tag{11}$$

The smallest possible black hole is considered to be a Planck mass black hole. It has a Schwarzschild radius of

$$R_s = \frac{2Gm_p}{c^2} = 2l_p \tag{12}$$

The Hawking temperature of the Planck mass black hole is therefore

$$T_{H,\max} = \frac{\hbar c}{k_b 4\pi R_s} = \frac{\hbar c}{k_b 8\pi l_p} \tag{13}$$

We call this the maximum Hawking temperature, as the Hawking radiation increases inversely proportional to the radius, and this is likely the smallest possible black hole. The Planck mass black hole has been suggested to be related to the most important elementary particle; see Motz and Epstein [29]. They assumed this particle existed at the beginning of the universe and then radiated into the particles we know today. Haug [30] has discussed how such a Planck mass particle could be the building block of today's particles, despite the Planck mass at first glance appearing far too large. Our aim is not to explore in depth why and exactly how Planck mass particles are important; we will simply ask the reader to assume they could be important.

In a large black hole, the minimum Hawking radiation will be:

$$T_{H,\min} = \frac{\hbar c}{k_b 4\pi R_s} \tag{14}$$

It has been suggested by multiple authors [31]-[35] that black holes could be related to Carnot's [36] heat engine theory. In an ideal Carnot engine operating at optimal (most efficient) conditions, there is an equilibrium temperature given by $T = \sqrt{T_{\max} T_{\min}}$. This is the geometric mean temperature of the lowest and highest possible temperatures in the engine. The geometric mean Hawking temperature is then given by:

$$T_{Hg} = \sqrt{T_{H,\max} T_{H,\min}} \tag{15}$$

We assume that the Hubble sphere is an ideal black-hole Carnot engine; see [36].

Next, the radiation energy density is given by the following integral:

$$\rho_\gamma = \frac{1}{c^2}\int_0^\infty h\nu n(\nu)\,\mathrm{d}\nu = \frac{aT_t^4}{c^2} \tag{16}$$

where $a = \frac{4}{c}\sigma = \frac{\pi^2 k_b^4}{15\hbar^3 c^3}$ is the radiation constant, $\sigma$ is the Stefan–Boltzmann constant, $\nu$ is the frequency of the radiation of interest, and the number density of photons per unit frequency is given by (as normally derived from Planck's law):

$$n(\nu) = \frac{8\pi\nu^2}{c^3}\frac{1}{\mathrm{e}^{\frac{h\nu}{k_b T}} - 1} \tag{17}$$

We next apply this to the geometric mean Hawking temperature and obtain:

$$\rho_\gamma = aT_g^4$$

$$\rho_\gamma = a\sqrt{T_{H,\max}T_{H,\min}}^2$$

$$\rho_\gamma = aT_{\max}^2 T_{\min}^2$$

$$\rho_\gamma = \frac{\pi^2 k_b^4}{15c^3\hbar^3}\left(\hbar\frac{c}{k_b 8\pi l_p}\right)^2\left(\hbar\frac{c}{k_b 4\pi R_s}\right)^2$$

$$\rho_\gamma = \hbar\frac{c}{15360\pi^2 R_s^2 l_p^2}$$

$$\rho_\gamma = \hbar\frac{c}{l_p}\frac{1}{15360\pi^2 R_s^2 l_p}$$

$$\rho_\gamma = \frac{E_p}{15360\pi^2 R_s^2 l_p} \tag{18}$$

We now calculate the radiation density parameter of a black hole and find that it must be given by:

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_{BH}} = \frac{1}{5760\pi} \approx 5.5262\times10^{-5} \tag{19}$$

This is again the same as the radiation density of the universe as we suggested in the sections above. It is also important to note here that the CMB temperature is indeed given by the geometric mean temperature of the maximum and minimum possible Hawking temperatures within the Hubble sphere (see [37]):

$$T_{cmb,0} = \sqrt{T_{\max}T_{\min,0}} \approx 2.725\ \text{K} \tag{20}$$

where $T_{\max}$ is as defined above and $T_{\min,0}$ is as defined before, but now with $R_s = R_{H_t}$.

So we have now demonstrated that this seems to support the idea that the Hubble sphere could indeed be a black hole universe. Haug [38] has discussed in detail how this likely indicates that the Hubble sphere is an extremal black hole Carnot engine, related to the extremal solution of the Reissner–Nordström metric. As he has demonstrated, this is not in conflict with using the standard Hawking temperature.

## 4. Consistent with $\rho_{\gamma,t} = \rho_0(1+z)^4$

It is well known from standard cosmology that we have $\rho_\gamma(t) = \rho_0(1+z)^4$, where $z$ is the cosmological redshift. We also have the well-known observationally based relation:

$$T_t = T_0(1+z) \tag{21}$$

Given $\rho_{\gamma,t} = \frac{a_b T_t^4}{c^2}$, substituting $T_t = T_0(1+z)$ yields:

$$\rho_{\gamma,t} = \frac{a_b T_t^4}{c^2}$$

$$\rho_{\gamma,t} = \frac{a_b T_0^4 (1+z)^4}{c^2}$$

$$\rho_{\gamma,t} = \rho_0 (1+z)^4 \tag{22}$$

which is the well-known result.

Interestingly, Haug and Tatum [39] have also demonstrated that in their $R_{H_t} = ct$ model, one finds:

$$\rho_{cr,t} = \rho_{cr,0} (1+z)^4 \tag{23}$$

where $\rho_{cr,t} = \frac{3H_t}{8\pi G}$ is the critical Friedmann density at time $t$. This result is inconsistent with the Λ-CDM model but is fully consistent with the $R_{H_t} = ct$ cosmology. Although the photon radiation density changes over time, the critical density changes proportionally. This explains why the photon radiation density parameter remains exact and constant in our model: both densities vary proportionally as $\propto \frac{1}{R_{H_t}^2}$.

Figure 1 shows both the CMB radiation energy density and the critical energy density as functions of cosmic time. We see that both decline rapidly and are proportional to the inverse square of the Hubble radius: $\rho_\gamma \propto \rho_{cr} \propto \frac{1}{R_{H_t}^2}$, with the Hubble radius following the $R_{H_t} = ct$ principle. Therefore, the CMB radiation density parameter in this $R_{H_t} = ct$ model remains constant at all times, as derived in **Appendix A**.

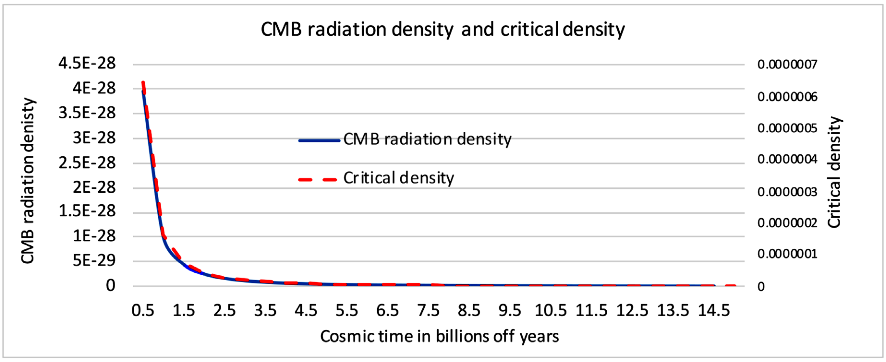


**Figure 1.** The figure shows the CMB radiation energy density and the critical density over time in our $R_{H_t} = ct$ cosmology.

From Figure 1, it looks like the CMB density and the critical density both decrease to zero after about 5 billion years and remain there. However, this is not correct. In Figure 2, we have separated the data from 5 billion years to 14.5 billion years, and we see that the density still continues to fall. The physical intuition behind the much faster drop in energy density at the beginning of the universe is simply related to spherical geometry. For example, from 1 billion years to 3 billion years, the radius of the sphere increased by 3 times, the volume by $3^3 = 27$ times, and the mass increased linearly with $R_{H_t}$. On the other hand, from 5 billion to 10 billion years, the radius increased by only 2 times, the volume by $2^3 = 8$ times, and still, the mass increased linearly. So, this is simply related to the fact that the density is proportional to the inverse square of the radius.

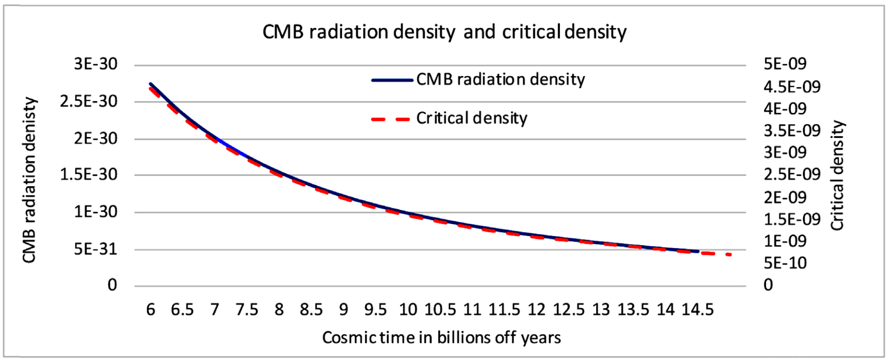


**Figure 2.** The figure shows the CMB radiation energy density and the critical density over time from 5 billion to 14.5 billion years in our $R_{H_t} = ct$ cosmology. This can be seen as a zoomed-in view of **Figure 1**, focusing on the period from 5 to 14.5 billion years.

## 5. The Number Density of CMB Photons Can Be Predicted from $H_0$ Instead of $T_0$

In this section, we derive a new equation that predicts the number density of CMB photons from the Hubble constant $H_0$, rather than from the CMB temperature $T_0$. We begin with the standard expression for the CMB photon number density (see Weinberg [23]):

$$n_{\gamma 0} = \int_0^{\infty} \frac{8\pi^2 \nu^2 \mathrm{d}\nu}{\mathrm{e}^{\frac{h\nu}{k_b T}} - 1} = \frac{30\zeta(3)}{\pi^4} \times \frac{a_b T_0^3}{k_b} \tag{24}$$

Here, $a_b = \frac{\pi^2 k_b^4}{15c^3\hbar^3}$, and in our $R_{H_t} = ct$ model variant, we use:

$$T_0 = \frac{\hbar c}{k_b 4\pi\sqrt{R_{H_0} 2l_p}} \tag{25}$$

where $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ is the Riemann zeta function ($\zeta(3) \approx 1.2021$). Substituting into equation (24), we obtain:

$$n_{\gamma,0} = \frac{30\zeta(3)}{\pi^4} \frac{\frac{\pi^2 k_b^4}{15c^3\hbar^3} \frac{\hbar^3 c^3}{k_b^3 \pi^3 128 R_{H_t} l_p \sqrt{R_{H_t} 2l_p}}}{k_b}$$

$$n_{\gamma,0} = \frac{30\zeta(3)}{\pi^4} \frac{1}{\pi 1920 R_{H_t} l_p \sqrt{R_{H_t} 2l_p}} \tag{26}$$

This demonstrates that the number density of CMB photons can be predicted without knowing the CMB temperature, as equation (26) depends only on the Hubble constant and the Planck length. The dimensional consistency is also evident: the denominator contains $R_{H_t} l_p \sqrt{R_{H_t} 2l_p}$, which has the dimension of a volume, as expected for number density.

To evaluate the accuracy of this prediction, we use the value $\frac{30\zeta(3)}{\pi^4} = \frac{30\sum_{n=1}^{\infty}\frac{1}{n^3}}{\pi^4} \approx 0.3702$ and substitute it into equation (26):

$$n_{\gamma,0} = 0.3702 \times \frac{1}{\pi 1920 \frac{c}{H_0} l_p \sqrt{\frac{c}{H_0} \times 2l_p}} = 410.71 \pm 0.26 \text{ photons/cm}^3 \tag{27}$$

when using $H_0 = 66.8943 \pm 0.0287$ km/s/Mpc, a value determined by matching the full SN Ia distance ladder from the UnionPlusSH0ES database, as described by Haug and Tatum [7]. This result is remarkably close to and fully consistent with the value reported by the Particle Data Group (PDG): 410.73 ± 0.27 photons/cm³. (See also Weinberg, who reports 410 photons/cm³ on page 107 using $T = 2.725$ K, though without a confidence interval, as his book is primarily pedagogical.)

However, if we use the Hubble constant from Riess *et al.* [40], $H_0 = 73.30 \pm 1.04$ km/s/Mpc, we obtain a significantly different prediction:

$$n_{\gamma,0} = 0.3702 \times \frac{1}{\pi 1920 \frac{c}{H_0} l_p \sqrt{\frac{c}{H_0} \times 2l_p}} = 470.40 \pm 10 \text{ photons/cm}^3 \tag{28}$$

This value lies far outside even the five-sigma confidence interval reported by the PDG (410.73 ± 5 × 0.27 photons/cm³). We can therefore conclude that the $H_0$ value reported by Riess *et al.* is not consistent with the observed number density of CMB photons.

Previously, it was not possible to predict the number density of CMB photons from $H_0$ alone, as no such equation existed—at least not within the ΛCDM framework. We argue that this discrepancy is closely related to the Hubble tension observed in Λ-CDM, and that this photon density-based prediction is simply a new and independent way of detecting that tension. We might call this a CMB photon number density tension.

In contrast, within the $R_{H_t} = ct$ model of Haug and Tatum [7], no such Hubble tension arises: both CMB and SN Ia data yield the same precise value, $H_0 = 66.8943 \pm 0.0287$ km/s/Mpc. We have now also shown that this is consistent with the observed CMB photon number density.

An in-depth analysis of the Hubble tension is beyond the scope of this paper, but a good starting point is the recently published paper cited above. Our findings here lend additional support to the $R_{H_t} = ct$ model variant proposed by Haug and Tatum.

## 6. Conclusions

Derivations based on the Haug and Tatum thermodynamic version of the

Friedmann equation demonstrate that the predicted CMB photon radiation density in their model is exact and given by $\Omega_{\gamma,t} = \frac{1}{5760\pi} \approx 5.52621330180192 \times 10^{-5}$ across all epochs of the $R_{H_t} = ct$ universe. This does not imply that the photon radiation energy density itself remains constant over time; rather, the ratio of the photon radiation density to the time-varying critical Friedmann density remains constant and exact. This prediction lies well within the 95% confidence interval for the CMB radiation density reported by the Particle Data Group (PDG), which spans from $5.08 \times 10^{-5}$ to $5.68 \times 10^{-5}$ ($5.35 \pm 0.15 \times 10^{-5}$ for $1\sigma$).

As expected, this exact radiation density is not consistent with the predictions of the Λ-CDM model at earlier cosmic epochs, since it is rooted in the $R_{H_t} = ct$ cosmology framework. Nonetheless, recent comparative studies [5] [27] suggest that $R_{H_t} = ct$ cosmology is gaining increasing support.

In addition, we have derived a new equation for the number density of CMB photons that requires only the Hubble parameter and the Planck length as input. This equation predicts a CMB photon number density of $n_\gamma = 410.71 \pm 0.26$ photons/cm$^3$, which closely matches the value reported by the PDG. Notably, in this framework, one can choose either $T_0$ or $H_0$ to calculate the photon number density—while the standard model requires the CMB temperature as input. We consider this a significant theoretical advancement.

When using the value of $H_0$ predicted by Haug and Tatum—obtained by calibrating their model to the full SN Ia distance ladder—we recover a photon number density fully consistent with PDG data and, by extension, the observed CMB temperature. However, using the $H_0$ value estimated by Riess *et al.*, $H_0 = 73.30 \pm 1.04$ km/s/Mpc, yields a predicted CMB photon number density that lies more than six standard deviations away from the PDG reported value.

This provides yet another line of evidence supporting the conclusion that the Λ-CDM model suffers from a persistent Hubble tension problem—while the recently proposed Haug and Tatum $R_{H_t} = ct$ model appears to avoid this issue entirely.

## Data Availability Statements

No data was used for this study except from in references clearly given in the paper. That is we have compared our predictions with the ones given by Particle Data Group PDG
https://pdg.lbl.gov/2023/reviews/rpp2023-rev-astrophysical-constants.pdf.

## Conflicts of Interest

The author declares no conflict of interest.

## References

[1] John, M.V. (2019) $R_h = ct$ and the Eternal Coasting Cosmological Model. *Monthly Notices of the Royal Astronomical Society: Letters*, **484**, L35-L37. https://doi.org/10.1093/mnrasl/sly243

[2] Melia, F. and Shevchuk, A.S.H. (2011) The $R_h = ct$ Universe. *Monthly Notices of the Royal Astronomical Society*, **419**, 2579-2586. https://doi.org/10.1111/j.1365-2966.2011.19906.x

[3] Melia, F. (2021) Thermodynamics of the $R_h = ct$ Universe: A Simplification of Cosmic Entropy. *The European Physical Journal C*, **81**, Article No. 234. https://doi.org/10.1140/epjc/s10052-021-09028-5

[4] Melia, F. (2023) A Resolution of the Monopole Problem in the $R_h = ct$ Universe. *Physics of the Dark Universe*, **42**, Article ID: 101329. https://doi.org/10.1016/j.dark.2023.101329

[5] Melia, F. (2024) Strong Observational Support for the $R_h = ct$ Timeline in the Early Universe. *Physics of the Dark Universe*, **46**, Article ID: 101587. https://doi.org/10.1016/j.dark.2024.101587

[6] Melia, F. (2016) The Linear Growth of Structure in the $R_h = ct$ Universe. *Monthly Notices of the Royal Astronomical Society*, **464**, 1966-1976. https://doi.org/10.1093/mnras/stw2493

[7] Haug, E.G. and Tatum, E.T. (2025) Solving the Hubble Tension Using the PantheonPlusSH0ES Supernova Database. *Journal of Applied Mathematics and Physics*, **13**, 593-622. https://doi.org/10.4236/jamp.2025.132033

[8] Haug, E.G. and Tatum, E.T. (2025) Friedmann Type Equations in Thermodynamic Form Lead to Much Tighter Constraints on the Critical Density of the Universe. *Discover Space*, **129**, Article No. 6. https://doi.org/10.1007/s11038-025-09566-y

[9] Pathria, R.K. (1972) The Universe as a Black Hole. *Nature*, **240**, 298-299. https://doi.org/10.1038/240298a0

[10] Zhang, T.X. and Frederick, C. (2013) Acceleration of Black Hole Universe. *Astrophysics and Space Science*, **349**, 567-573. https://doi.org/10.1007/s10509-013-1644-6

[11] Zhang, T.X. (2018) The Principles and Laws of Black Hole Universe. *Journal of Modern Physics*, **9**, 1838-1865. https://doi.org/10.4236/jmp.2018.99117

[12] Easson, D.A. and Brandenberger, R.H. (2001) Universe Generation from Black Hole Interiors. *Journal of High Energy Physics*, **2001**, Article 24. https://doi.org/10.1088/1126-6708/2001/06/024

[13] Gaztanaga, E. (2022) The Black Hole Universe, Part I. *Symmetry*, **14**, Article 1849. https://doi.org/10.3390/sym14091849

[14] Roupas, Z. (2022) Detectable Universes Inside Regular Black Holes. *The European Physical Journal C*, **82**, Article No. 255. https://doi.org/10.1140/epjc/s10052-022-10202-6

[15] Lineweaver, C.H. and Patel, V.M. (2023) All Objects and Some Questions. *American Journal of Physics*, **91**, 819-825. https://doi.org/10.1119/5.0150209

[16] Christillin, P. (2014) The Machian Origin of Linear Inertial Forces from Our Gravitationally Radiating Black Hole Universe. *The European Physical Journal Plus*, **129**, Article No. 175. https://doi.org/10.1140/epjp/i2014-14175-2

[17] Stuckey, W.M. (1994) The Observable Universe Inside a Black Hole. *American Journal of Physics*, **62**, 788-795. https://doi.org/10.1119/1.17460

[18] Shamir, L. (2025) The Distribution of Galaxy Rotation in *JWST* Advanced Deep Extragalactic Survey. *Monthly Notices of the Royal Astronomical Society*, **538**, 76-91. https://doi.org/10.1093/mnras/staf292

[19] Kerr, R.P. (1963) Gravitational Field of a Spinning Mass as an Example of Algebrai-

cally Special Metrics. *Physical Review Letters*, **11**, 237-238. https://doi.org/10.1103/physrevlett.11.237

[20] Newman, E.T. and Janis, A.I. (1965) Note on the Kerr Spinning-Particle Metric. *Journal of Mathematical Physics*, **6**, 915-917. https://doi.org/10.1063/1.1704350

[21] Haug, E.G. and Spavieri, G. (2023) Mass-Charge Metric in Curved Spacetime. *International Journal of Theoretical Physics*, **62**, Article No. 248. https://doi.org/10.1007/s10773-023-05503-9

[22] Haug, E.G. (2024) The Extremal Universe Exact Solution from Einstein's Field Equation Gives the Cosmological Constant Directly. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 386-397. https://doi.org/10.4236/jhepgc.2024.101027

[23] Weinberg, S. (2008) Cosmology. Oxford University Press.

[24] Emslie, A.G. and Green, R.M. (1978) Can the Einstein-De Sitter Model Adequately Describe the Universe at Any Epoch? *Astrophysics and Space Science*, **58**, 181-188. https://doi.org/10.1007/bf00645385

[25] Weinberg, S. (2008) Gravitation and Cosmology. Wiley.

[26] Friedman, A. (1922) Über die Krümmung des Raumes. *Zeitschrift für Physik*, **10**, 377-386. https://doi.org/10.1007/bf01332580

[27] Haug, E.G. and Tatum, E.T. (2024) How a New Type of $R_h = ct$ Cosmological Model Out-Performs the $\Lambda$-CDM Model in Numerous Categories and Resolves the Hubble Tension.

[28] Hawking, S.W. (1974) Black Hole Explosions? *Nature*, **248**, 30-31. https://doi.org/10.1038/248030a0

[29] Motz, L. and Epstein, J. (1979) The Gravitational Charge $1/2\sqrt{\hbar c}$ as a Unifying Principle in Physicsas a Unifying Principle in Physics. *Il Nuovo Cimento A*, **51**, 88-113. https://doi.org/10.1007/bf02822327

[30] Haug, E.G. (2020) Collision-Space-Time: Unified Quantum Gravity. *Physics Essays*, **33**, 46-78. https://doi.org/10.4006/0836-1398-33.1.46

[31] Opatrný, T. and Richterek, L. (2011) Black Hole Heat Engine. *American Journal of Physics*, **80**, 66-71. https://doi.org/10.1119/1.3633692

[32] Hendi, S.H., Eslam Panah, B., Panahiyan, S., Liu, H. and Meng, X. (2018) Black Holes in Massive Gravity as Heat Engines. *Physics Letters B*, **781**, 40-47. https://doi.org/10.1016/j.physletb.2018.03.072

[33] Wei, S. and Liu, Y. (2019) Charged Ads Black Hole Heat Engines. *Nuclear Physics B*, **946**, 114700. https://doi.org/10.1016/j.nuclphysb.2019.114700

[34] DiMarco, M.C., Jess, S.L., Hennigar, R.A. and Mann, R.B. (2023) Universality for Black Hole Heat Engines near Critical Points. *Physical Review D*, **107**, Article ID: 044001. https://doi.org/10.1103/physrevd.107.044001

[35] Kruglov, S.I. (2025) The Heat Engine of Magnetic Black Holes in Ads Space with Rational Nonlinear Electrodynamics. *Canadian Journal of Physics*, **103**, 448-454. https://doi.org/10.1139/cjp-2024-0146

[36] Carnot, S. (1924) Réflexions sur la puissance motrice du feu et sur les machines propres á développer cette puissance. *Annales scientifiques de l'É.N.S.*, No. 1, 393-457.

[37] Haug, E.G. (2025) The CMB Temperature Is Simply the Geometric Mean: $T_{cmb} = \sqrt{T_{min} T_{max}}$ of the Minimum and Maximum Temperature in the Hubble

Sphere. *Journal of Applied Mathematics and Physics*, **13**, 1085-1096. https://doi.org/10.4236/jamp.2025.134056

[38] Haug, E.G. (2025) The Hubble Sphere as and Extremal Reissner-Nordstrom Black Hole Carnot Engine Operating at the CMB Temperature Off: $T_{cmb} = \sqrt{T_{max} T_{min}} \approx 2.725k$. Cambridge University Press. https://doi.org/10.33774/coe-2025-w2dxp

[39] Haug, E.G. and Tatum, E.T. (2025) A Newly-Derived Cosmological Redshift Formula Which Solves the Hubble Tension and Yet Maintains Consistency with $T_t = T_0(1+z)$, the $R_h = ct$ Principle and the Stefan-Boltzmann Law. *European Journal of Applied Physics*, **7**, 48-50. https://doi.org/10.24018/ejphysics.2025.7.1.368

[40] Riess, A.G., Yuan, W., Macri, L.M., Scolnic, D., Brout, D., Casertano, S., *et al.* (2022) A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km $S^{-1}$ $Mpc^{-1}$ Uncertainty from the Hubble Space Telescope and the SH0ES Team. *The Astrophysical Journal Letters*, **934**, L7. https://doi.org/10.3847/2041-8213/ac5c5b

[41] de Martino, I., Atrio-Barandela, F., da Silva, A., Ebeling, H., Kashlinsky, A., Kocevski, D., *et al.* (2012) Measuring the Redshift Dependence of the Cosmic Microwave Background Monopole Temperature with Planck Data. *The Astrophysical Journal*, **757**, 144. https://doi.org/10.1088/0004-637x/757/2/144

[42] Li, Y.Y, Hincks, A.D., Amodeo, S., Battistelli, E.S., Bond, J.R., Calabrese, E., *et al.* (2021) Constraining Cosmic Microwave Background Temperature Evolution with Sunyaev-Zel'Dovich Galaxy Clusters from the Atacama Cosmology Telescope. *The Astrophysical Journal*, **922**, Article 136. https://doi.org/10.3847/1538-4357/ac26b6

[43] Riechers, D.A., Weiss, A., Walter, F., Carilli, C.L., Cox, P., Decarli, R., *et al.* (2022) Microwave Background Temperature at a Redshift of 6.34 from $H_2O$ Absorption. *Nature*, **602**, 58-62. https://doi.org/10.1038/s41586-021-04294-5

## Appendix A

This result, given in equation 8: $\Omega_{\gamma,0}=\frac{1}{5760\pi}$, is valid at the present time ( $t_0$ ). We will here demonstrate that it is valid at any time $t$. This constancy is consistent with at least two types of $R_{H_t}=ct$ cosmological models. We use the well-known relation [41]-[43]:

$$T_t=T_0\left(1+z\right) \tag{29}$$

Thus, in $R_{H_t}=ct$ black hole cosmology, the photon energy density at earlier times in the universe must be:

$$\rho_{\gamma,t}=\frac{a_bT_0^4\left(1+z\right)^4}{c^2}=\frac{a_bT_t^4}{c^2} \tag{30}$$

Furthermore, the critical density, as shown by Haug and Tatum [8], must be:

$$\rho_{cr,t}=\frac{3H_t}{8\pi G}=T_0^4\left(1+z\right)^4\frac{23040\pi}{c^3}\sigma=T_t^4\frac{23040\pi}{c^3}\sigma \tag{31}$$

Substituting these into the following expression, we get:

$$\Omega_{\gamma,t}=\frac{\rho_{\gamma,t}}{\rho_{cr,t}}$$

$$\Omega_{\gamma,t}=\frac{\frac{a_bT_t^4}{c^2}}{T_t^4\frac{23040\pi}{c^3}\sigma}$$

$$\Omega_{\gamma,t}=\frac{4\sigma}{23040\pi\sigma}$$

$$\Omega_{\gamma,t}=\frac{1}{5760\pi}\approx 5.52621330180192\times 10^{-5} \tag{32}$$

In a growing black hole $R_{H_t}=ct$ cosmology, the photon radiation density ratio remains constant and exact throughout the entire cosmic epoch. This is inconsistent with the predictions of the ΛCDM model in earlier epochs, though our prediction should still hold for the present epoch in that model.