

Geometric Origin of Quantization: Deriving the Schrödinger Framework from NUVO Scalar Coherence

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How to cite this paper: Austin, R.W.

(2026) Geometric Origin of Quantization: Deriving the Schrödinger Framework from NUVO Scalar Coherence. *Journal of Applied Mathematics and Physics*, 14, 61-75. <https://doi.org/10.4236/jamp.2026.141004>

Received: November 14, 2025

Accepted: December 28, 2025

Published: December 31, 2025

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Abstract

We prove that the stationary NUVO scalar transport equation

$\lambda^2 \nabla_\eta^2 \psi + (2 \nabla_\eta \ln \lambda \cdot \nabla_\eta + \nabla_\eta^2 \ln \lambda) \psi = 0$ is gauge-equivalent, under

$\psi = \phi / \lambda$, to a Schrödinger eigenproblem $\left(-\frac{\hbar^2}{2m} \Delta + V_{\text{phys}} + V_{\text{eff}}[\lambda] \right) \phi = E \phi$,

with geometric potential $V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m} (|\nabla \ln \lambda|^2 - \Delta \ln \lambda)$. Under mild decay

bounds on $\nabla \ln \lambda$ and $\Delta \ln \lambda$, H_λ is semibounded and self-adjoint via the Friedrichs extension; for Kato-class V_{phys} the operator domain equals

$H^2(\mathbb{R}^3)$. If additionally $\lambda, \lambda^{-1} \in L^\infty$, the map $\phi \mapsto \phi / \lambda$ is a bounded simi-

larity on L^2 . We give solvable profiles, a Birman-Schwinger/Lippmann-Schwinger representation, and a coupled partial-wave system for anisotropic λ . Consistency checks (constant λ , hydrogenic expectations) and spin terms from a conformal Dirac reduction confirm the construction. In combination with prior results of *Quantization III*, this paper establishes the Schrödinger framework as a scalar-geometric limit of NUVO, with $V_{\text{eff}}[\lambda]$ playing the role of a geometric quantum potential.

Keywords

NUVO Space, Scalar Geometry, Quantization, Coherence, Scalar Field Modulation, Loop Dynamics

1. Introduction and Motivation

Quantization is traditionally introduced axiomatically through linear operators,

probability amplitudes, and postulates that stand apart from the geometry of space. Alternative programs—such as Weyl’s scale geometry [1], Madelung’s hydrodynamic interpretation [2], geometric quantization [3], and pilot-wave formulations [4] [5]—have long sought a deeper bridge between spatial structure and quantum behavior. The NUVO framework follows this geometric direction by treating the physical metric as a conformal scalar modulation

$$g_{\mu\nu} = \lambda^2 \eta_{\mu\nu},$$

where the scalar field $\lambda(\mathbf{r})$ encodes the local availability of an underlying substrate quantity (*sinertia*) and determines curvature, scale, and effective dynamics.

Relation to curved spacetime. Although the present work develops the Schrödinger framework in a conformally flat background, the NUVO scalar geometry admits a fully general-relativistic extension. In earlier gravitational analyses [6] [7], the same scalar field λ was shown to reproduce standard weak-field predictions (light bending, perihelion shift, redshift) when coupled to a dynamical metric. Thus the conformal structure used here should be viewed as the single-particle, stationary limit of a broader scalar-geometric framework that already interfaces consistently with curved spacetime.

Foundational aspects of this scalar geometry were developed in NUVO Space I-II [8] [9], which introduced the unit-constrained frame bundle, the conformal scalar structure, the NUVO Laplacian, scalar current conservation, and the variational and analytical framework for fields evolving on NUVO space. Building on this geometric foundation, the NUVO Quantization I-III program [10]-[12] established that scalar phase continuity and coherence constraints lead naturally to Bohr-Sommerfeld-type quantization rules and semiclassical orbit conditions.

The goal of the present work is to show that the stationary NUVO transport law governing λ -modulated scalar motion can be reduced, under a gauge transformation, to the standard Schrödinger eigenvalue equation with an additional geometric potential. This establishes the Schrödinger framework as a scalar-geometric limit of NUVO theory: quantum behavior arises not from postulated operator rules but from the geometry of scalar coherence encoded in λ .

Scope of the present work. The analysis developed here concerns the single-particle sector of the Schrödinger framework that emerges from NUVO scalar geometry. A full multi-particle treatment would require a configuration-space scalar field $\lambda(r_1, \dots, r_N)$ and an associated continuity structure, which lie beyond the scope of the present manuscript. The derivation presented here should therefore be viewed as the one-particle limit of a broader scalar framework that will be extended to multi-particle and entangled systems in subsequent work.

Section 2 derives the gauge reduction and the effective potential; Section 3 presents probability, current, and the scalar measure; Section 4 provides consistency checks and the semiclassical interpretation; Sections 5-9 give solvable and topological examples; Section 10 establishes self-adjointness of the Schrödinger operator; and Sections 11-13 address the spectral, coupled, and time-dependent regimes. A final discussion (Section 15) compares this construction with earlier ge-

ometric approaches to quantum mechanics.

2. From the NUVO Transport Law to the Schrödinger Form

Notation and Standing Assumptions

We work on (\mathbb{R}^3, η) with flat metric η and write $\nabla \equiv \nabla_\eta$, $\Delta \equiv \nabla \cdot \nabla$. Let $\lambda : \mathbb{R}^3 \rightarrow (0, \infty)$, $\lambda \in C^2$, satisfy

$$|\nabla \ln \lambda(\mathbf{r})| \leq \frac{C_1}{1+r}, \quad |\Delta \ln \lambda(\mathbf{r})| \leq \frac{C_2}{1+r^2}. \quad (1)$$

Physical justification of the decay assumptions. The bounds in (1) express that the scalar modulation $\lambda(r)$ is generated by a localized geometric or physical structure whose influence relaxes at large distance. The conditions $|\nabla \ln \lambda(r)| \leq C_1/(1+r)$ and $|\Delta \ln \lambda(r)| \leq C_2/(1+r^2)$ ensure that the associated curvature density $|\nabla \ln \lambda|^2$ is integrable, so that a finite amount of “scalar curvature energy” is stored in the coherence halo surrounding the source. These falloff conditions are the direct analogue of the short-range or Coulomb-decay hypotheses commonly imposed on external potentials in nonrelativistic quantum mechanics, and they parallel the asymptotic-flatness conditions used in weak-field gravitational settings: outside a finite coherence region, the scalar field must relax smoothly toward a constant background value to avoid unphysical infinite curvature or energy.

Let $V_{\text{phys}} \in L^2_{\text{loc}}(\mathbb{R}^3)$ be infinitesimally form-bounded with respect to $-\Delta$ (e.g. Coulomb or harmonic potentials). All symbols are defined at first appearance. Throughout, eigenfunctions ϕ of H_λ are understood in the operator sense as elements of $H^2(\mathbb{R}^3)$ when the Kato-class hypotheses on V_{phys} are imposed, and the corresponding NUVO fields $\psi = \phi/\lambda$ live in the weighted space L^2_λ introduced in Section 3.

(U) Optional boundedness for similarity. When explicitly stated, we additionally assume $\lambda, \lambda^{-1} \in L^\infty(\mathbb{R}^3)$, which makes $M_{1/\lambda}$ a boundedly invertible map on L^2 ; in that case the NUVO transport equation and the Schrödinger equation are related by a bounded similarity on L^2 .

Notation summary

For convenience, we collect here some of the main symbols used throughout the paper:

Symbol	Meaning
η	flat Euclidean metric on \mathbb{R}^3
$\lambda(\mathbf{r})$	scalar coherence factor (conformal modulation)
ψ	NUVO scalar wavefunction in $L^2_\lambda(\mathbb{R}^3)$
ϕ	Schrödinger-frame wavefunction in $L^2(\mathbb{R}^3)$
V_{phys}	physical (external) potential
$V_{\text{eff}}[\lambda]$	geometric effective potential from λ

Continued

H_λ	Schrödinger operator $-\frac{\hbar^2}{2m}\Delta + V_{\text{phys}} + V_{\text{eff}}[\lambda]$
$\langle \cdot, \cdot \rangle_\lambda$	L^2_λ inner product from $d\mu_\lambda = \lambda^{-3}d^3x$

Theorem 1 (Gauge reduction to Schrodinger form) Under (1), the NUVO transport operator

$$\mathcal{T}_\lambda \psi := \lambda^2 \Delta \psi + (2\nabla \ln \lambda \cdot \nabla + \Delta \ln \lambda) \psi \tag{2}$$

satisfies

$$\mathcal{T}_\lambda (\phi/\lambda) = 0 \Leftrightarrow \left(-\frac{\hbar^2}{2m} \Delta + V_{\text{phys}} + V_{\text{eff}}[\lambda] \right) \phi = E\phi, \tag{3}$$

with

$$V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m} (|\nabla \ln \lambda|^2 - \Delta \ln \lambda). \tag{4}$$

Moreover the corresponding Schrödinger operator

$$H_\lambda := -\frac{\hbar^2}{2m} \Delta + V_{\text{phys}} + V_{\text{eff}}[\lambda]$$

is semibounded and self-adjoint on $L^2(\mathbb{R}^3)$ via the Friedrichs extension; for Kato-class V_{phys} the operator domain equals $H^2(\mathbb{R}^3)$.

Proof. See **Appendix** for a detailed proof. The key step is the drift-cancellation (Doob/ground-state) identity (Lemma 2), which reduces the normalized transport operator $(\Delta + 2\nabla \ln \lambda \cdot \nabla + \Delta \ln \lambda)$ to a Schrödinger operator with geometric potential $V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m} (|\nabla \ln \lambda|^2 - \Delta \ln \lambda)$ under the gauge map $\psi = \phi/\lambda$. Self-adjointness and semiboundedness then follow from the inequalities of Hardy and Kato combined with the KLMN theorem; see Proposition 4. \square

Bound-state regularity. Write $\phi(\mathbf{r}) = \sum_{\ell m} u_\ell(r) Y_{\ell m}(\Omega)/r$. Then $u_\ell(r) \sim r^{\ell+1}$ as $r \rightarrow 0$ and $u_\ell(r) \rightarrow 0$ as $r \rightarrow \infty$.

3. Probability, Current, and Scalar Measure

3.1. Probability Current and Continuity Equation

The time-dependent scalar wave $\psi(t, \mathbf{r})$ obeys

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \left[\lambda^2 \nabla^2 \psi + (2\nabla \ln \lambda \cdot \nabla + \Delta \ln \lambda) \psi \right]. \tag{5}$$

Multiplying by ψ^* and subtracting the conjugate yields

$$\partial_t |\psi|^2 + \nabla \cdot \mathbf{J}_\lambda = 0, \quad \mathbf{J}_\lambda = \frac{\hbar}{m} \Im(\psi^* \lambda^2 \nabla \psi). \tag{6}$$

Thus \mathbf{J}_λ is the conserved scalar (sinertia) probability current. For $\lambda \rightarrow 1$, \mathbf{J}_λ reduces to the standard quantum current.

3.2. Scalar Measure and Inner Product

The scalar geometry induces the volume form $d\mu_\lambda = \lambda^{-3}d^3x$ and the inner product

$$\langle \psi_1, \psi_2 \rangle_\lambda = \int \psi_1^*(\mathbf{r}) \psi_2(\mathbf{r}) \lambda^{-3} d^3x. \quad (7)$$

Under the gauge map $\psi = \phi/\lambda$, one checks $\langle \psi, \psi \rangle_\lambda = \int |\phi|^2 d^3x$, so the physical norm is preserved.

4. Consistency Checks and Semi-Classical Limit

Constant λ .

If $\lambda \equiv \lambda_0$, then $V_{\text{eff}} = 0$ and $\psi = \phi/\lambda_0$ trivially recovers the Schrödinger equation.

Scaling.

For $\lambda(\mathbf{r}) = \lambda_0 \tilde{\lambda}(\mu\mathbf{r})$ with $V_{\text{phys}} \equiv 0$,

$$V_{\text{eff}}[\lambda](\mathbf{r}) = \frac{\hbar^2 \mu^2}{2m} \left(|\nabla \ln \tilde{\lambda}|^2 - \Delta \ln \tilde{\lambda} \right) (\mu\mathbf{r}), \quad (8)$$

consistent with the energy scaling $E \mapsto \mu^2 E$ of the pure geometric problem.

Semi-Classical Limit and Emergent \hbar

Let $\lambda = e^{S/(mc^2)}$. Then $\psi = \phi/\lambda$ implies a total phase $S_{\text{tot}} = S_{\text{geom}} + S_{\text{dyn}}$. With $\phi = Re^{iS/\hbar}$ and $\nabla S = m\mathbf{v}$,

$$\frac{1}{\hbar} \oint \nabla S \cdot d\mathbf{l} = 2\pi n, \quad (9)$$

so \hbar plays the role of the action quantum associated with one closed scalar-coherence cycle, connecting directly with the Bohr-Sommerfeld rule used in *Quantization III* [12]. This semiclassical identification is interpretive rather than a derivation of the numerical value of \hbar : the constant is fixed empirically (for example, from hydrogenic spectra), while NUVO provides a geometric interpretation of its role once measured.

5. Examples of λ Profiles

(a) Pure Geometric Binding

$\lambda(r) = \exp(-\alpha r^2/2)$ ¹ gives

$$V_{\text{eff}}(r) = \frac{\hbar^2}{2m} (\alpha^2 r^2 - 3\alpha) = \frac{1}{2} m\omega^2 r^2 - \frac{3}{2} \hbar\omega, \quad \omega = \frac{\hbar\alpha}{m}. \quad (10)$$

(b) Coulomb Halo

For $\lambda(r) = 1 + a/r$,

¹Strictly speaking, this profile does not satisfy the global decay assumption (1), since $|\nabla \ln \lambda| = \alpha r$ grows linearly in r . The example is included to illustrate how a harmonic-oscillator potential arises from a simple Gaussian λ ; self-adjointness and spectral properties in this case follow from the standard harmonic-oscillator theory. The decay assumptions in Section 2 can be relaxed to allow linear growth at infinity without changing the qualitative conclusions, but we keep the simpler hypotheses there for clarity.

$$V_{\text{eff}}(r) = \frac{\hbar^2 a^2}{mr^2(r+a)^2}. \tag{11}$$

Combined with $V_{\text{phys}} = -e^2/(4\pi\epsilon_0 r)$, this adds a small geometric correction.

6. Holonomy and Quantization

For $\phi = Re^{iS/\hbar}$ and $\psi = \phi/\lambda$, single-valuedness of ψ around any loop γ gives²

$$\frac{1}{\hbar} \oint_{\gamma} \nabla S \cdot d\mathbf{l} - \oint_{\gamma} \nabla \ln \lambda \cdot d\mathbf{l} = 2\pi n. \tag{12}$$

For spherically symmetric $\lambda(r)$, $\ln \lambda$ is single-valued so the second term vanishes:

$$\oint p_r dr = 2\pi\hbar \left(n_r + \ell + \frac{1}{2} \right), \tag{13}$$

where the $+\frac{1}{2}$ is the Maslov index for two turning points.

7. Bound-State Quantization via Scalar Phase Closure

Scalar coherence requires that the total phase $\Phi = \arg(\phi) - \ln \lambda$ be single-valued on closed orbits:

$$\oint \nabla \Phi \cdot d\mathbf{l} = 2\pi n. \tag{14}$$

For stationary spherically symmetric λ this reduces to the standard Bohr-Sommerfeld condition, identifying orbital quantization in *Quantization III* with scalar phase closure.

8. Hydrogenic Correction and Lamb-Shift Scale

For $\lambda = 1 + a/r$ and Coulomb $V_{\text{phys}} = -e^2/(4\pi\epsilon_0 r)$, the first-order geometric shift is

$$\Delta E_{n\ell} = \frac{\hbar^2}{ma_0^2} \frac{a^2}{n^3 \left(\ell + \frac{1}{2} \right)}. \tag{15}$$

Taking $a = r_e = 2.82 \times 10^{-15}$ m and evaluating the hydrogenic expectation $\langle r^{-2} \rangle_{2S}$ yields a geometric energy shift of order

$$\Delta E_{2S} \sim 10^{-6} E_{2S}.$$

This is numerically comparable to the magnitude of the Lamb shift [13]. We emphasize that this agreement is at the level of order-of-magnitude only: the Lamb shift itself is a QED radiative effect arising from vacuum fluctuations and electron self-energy, whereas the present term is a classical geometric correction

²Locally, $\nabla \ln \lambda$ is an exact one-form and therefore integrates to zero on closed loops contained in simply connected regions where λ is smooth and single-valued. Nontrivial contributions to the phase-closure condition arise only in global sectors where $\ln \lambda$ fails to be single-valued—such as in the presence of defects, conical singularities, or nontrivial bundle topology. All holonomy-based quantization statements in Sections 6 - 9 are to be understood in this global sense.

from the NUVO scalar field λ . No claim is made here that NUVO reproduces or replaces the full QED Lamb-shift mechanism.

Uniqueness of the geometric profile.

The map $\lambda \mapsto V_{\text{eff}}[\lambda]$ is not injective: multiplying λ by any nonzero constant leaves V_{eff} unchanged, and more generally distinct λ -profiles can produce the same effective potential when boundary conditions differ. Recovering λ from V_{eff} requires solving a nonlinear elliptic equation and is therefore not unique without additional geometric or boundary data. This nonuniqueness is analogous to gauge freedom in other geometric formulations.

Physical interpretation of this nonuniqueness. Because the Schrödinger-frame dynamics depend only on $V_{\text{eff}}[\lambda]$, distinct scalar geometries can lead to indistinguishable quantum behavior whenever they produce the same effective potential. In the NUVO framework, λ encodes geometric structure that is generally not observable independently of the induced dynamics, so the projection $\lambda \mapsto V_{\text{eff}}[\lambda]$ naturally discards information about the underlying scalar geometry. This is analogous to gauge redundancy: different λ -profiles may represent the same physical state from the perspective of a Schrödinger observer. Recovering a unique λ therefore requires additional geometric or boundary data, reflecting the fact that standard quantum mechanics does not retain the full geometric content of the NUVO scalar field.

9 Anisotropic and Topological Sectors

(a) Smooth Anisotropy

For $\lambda(\rho, \phi) = \lambda_0(\rho)[1 + \varepsilon f(\phi)]$ with $\int_0^{2\pi} f(\phi) d\phi = 0$,

$$\delta V_{\text{eff}}(\rho, \phi) = \frac{\hbar^2}{2m\rho^2} [\varepsilon^2 f'^2 - \varepsilon f''] + O(\varepsilon^3). \quad (16)$$

Choosing $f(\phi) = \cos k\phi$ couples $m \leftrightarrow m \pm k$ and splits angular degeneracies; the coupling bandwidth follows SO(3) triangle rules via Gaunt coefficients [14].

(b) Conical Defect

A wedge deficit $2\pi\delta$ yields single-valuedness in $\tilde{\phi} \in [0, 2\pi(1-\delta))$ and replacement $m \mapsto m/(1-\delta)$, giving

$$\Delta E_{n\ell m} \approx \frac{\hbar^2}{ma_0^2} \frac{\delta m^2}{n^3 \left(\ell + \frac{1}{2}\right)}. \quad (17)$$

This applies to the globally flat cone (curvature concentrated at the tip).

10. Self-Adjointness and Domain Summary

A detailed proof of the self-adjointness and semiboundedness of H_λ appears in Proposition 4 in **Appendix**. Here we summarize the key ingredients. Assume $\lambda > 0$, $\lambda \in C^2$, and $|\nabla \ln \lambda| \leq C_1/(1+r)$, $|\nabla^2 \ln \lambda| \leq C_2/(1+r^2)$. Then $V_{\text{eff}} \in L_{\text{loc}}^2$ and $|V_{\text{eff}}| \leq C/r^2$. Near the origin the bound (1) implies that $V_{\text{eff}}(r)$ behaves no worse than c/r^2 , so the Hardy inequality

$$\int_{\mathbb{R}^3} \frac{|u(\mathbf{r})|^2}{r^2} d^3r \leq 4 \int_{\mathbb{R}^3} |\nabla u(\mathbf{r})|^2 d^3r, \quad u \in H^1(\mathbb{R}^3),$$

controls the associated quadratic form with arbitrarily small relative bound. Away from the origin, V_{eff} is bounded and hence trivially form-bounded. Combining this with the assumed infinitesimal form-boundedness of V_{phys} and the Kato-Lax-Milgram-Nelson (KLMN) theorem yields a unique lower-bounded self-adjoint realization of H_λ on $L^2(\mathbb{R}^3)$. By the Hardy and Kato inequalities [15]-[17], V_{eff} and typical V_{phys} are infinitesimally form-bounded relative to $-\Delta$; thus H_λ is self-adjoint via the Friedrichs extension and bounded below; for Kato-class V_{phys} the operator domain equals $H^2(\mathbb{R}^3)$.

11. Integral Representation and Spectral Equation

Let $W = V_{\text{phys}} + V_{\text{eff}}[\lambda]$. For resolvent parameter z ,

$$(H_\lambda - z)G_\lambda(\mathbf{r}, \mathbf{r}'; z) = \delta(\mathbf{r} - \mathbf{r}'), \tag{18}$$

and

$$G_\lambda = G_0 + G_0 W G_\lambda, \quad G_0(\mathbf{r}, \mathbf{r}'; E + i0) = -\frac{m}{2\pi\hbar^2} \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \tag{19}$$

Bound states satisfy $\psi = G_0(E)W\psi$, equivalently $\det(I - G_0(E)W) = 0$ (Fredholm scheme).

Birman-Schwinger principle. For $E < 0$ set

$K_E := |W|^{1/2} (-\Delta - \kappa^2)^{-1} \text{sgn}(W) |W|^{1/2}$ with $\kappa^2 = -2mE/\hbar^2$. Then E is an eigenvalue of H_λ iff 1 is an eigenvalue of K_E ; K_E is compact under (1) together with Kato-class V_{phys} .

12. Partial-Wave Coupled System

Expand

$$W(\mathbf{r}) = \sum_{\ell m, \ell' m'} W_{\ell m, \ell' m'}(r) Y_{\ell m}(\Omega) Y_{\ell' m'}^*(\Omega), \tag{20}$$

giving coupled radial equations

$$-\frac{\hbar^2}{2m} u_{\ell m}'' + \frac{\hbar^2 \ell(\ell+1)}{2mr^2} u_{\ell m} + \sum_{\ell' m'} W_{\ell m, \ell' m'}(r) u_{\ell' m'} = E u_{\ell m}, \tag{21}$$

with $u_{\ell m}(0) = u_{\ell m}(\infty) = 0$.

13. Time-Dependent $\lambda(t, \mathbf{r})$

For slowly varying λ , set $S = mc^2(\lambda - 1)$:

$$H(t) = \frac{\mathbf{p}^2}{2m} + S + V_{\text{eff}}[\lambda] + \frac{\hbar}{4m^2 c^2} \boldsymbol{\sigma} \cdot (\nabla S \times \mathbf{p}) - \frac{\hbar^2}{8m^2 c^2} \nabla^2 S. \tag{22}$$

Well-posedness

If $t \mapsto \lambda(t, \cdot)$ is (piecewise) Lipschitz in L^∞ with uniform bounds on $\nabla \ln \lambda$ and $\Delta \ln \lambda$, then the closed quadratic forms of $H(t)$ share a common domain

$H^1(\mathbb{R}^3)$ and vary Lipschitz in t . Equivalently, the associated forms

$$Q_t[\phi] := \langle \phi, H(t)\phi \rangle$$

satisfy $|Q_t[\phi] - Q_s[\phi]| \leq C|t-s|\|\phi\|_{H^1}^2$ for some constant C and all $\phi \in H^1(\mathbb{R}^3)$. By Kato's theorem on time-dependent self-adjoint forms, there exists a unique unitary propagator $U(t,s)$ on L^2 with $i\hbar\partial_t U(t,s)\phi = H(t)U(t,s)\phi$ for all $\phi \in H^1$ [18] [19].

Adiabatic Loop Example

For $\lambda(r;t) = 1 + \frac{a(t)}{r}$ with $a(t) = a_0 + \delta a \cos \Omega t$,

$$E_{n\ell jm}(t) = E_n^{(0)} + \frac{\hbar^2 a(t)^2}{ma_0^2 n^3 \left(\ell + \frac{1}{2}\right)} + \text{spin-orbit and Darwin terms.} \quad (23)$$

Cycle-averaged phase shifts are proportional to $(\delta a)^2/2$; nontrivial Berry phases require at least two modulated parameters.

14. Path-Integral and Lagrangian Formulation

A scalar Lagrangian density yielding the NUVO transport law is

$$\mathcal{L}_\lambda = \frac{\hbar^2}{2m} \lambda^2 |\nabla \psi|^2 + \frac{\hbar^2}{2m} (\nabla \ln \lambda)^2 |\psi|^2 + V_{\text{phys}} |\psi|^2. \quad (24)$$

Variation with respect to ψ^* recovers the normalized transport equation. Thus λ serves as a conformal coupling to kinetic density, providing the field-theoretic foundation of NUVO scalar [9] motion.

15. Relation to Earlier Approaches

Programs linking quantum dynamics to geometry include Weyl's scale-invariant gravity [1] [20], Madelung's hydrodynamic form of Schrödinger's equation [2] [21], and geometric quantization [3]. Pilot-wave and stochastic models [4] [5] reintroduce hidden variables to explain the quantum potential. In contrast, NUVO employs a single scalar λ that determines both curvature and wave behavior. The effective potential $V_{\text{eff}}[\lambda]$ emerges directly from geometry, replacing the ad-hoc quantum potential of earlier formulations.

Where NUVO overlaps with earlier ideas is in the appearance of a "quantum potential"-like term: when $R \propto \lambda^{-1}$, $V_{\text{eff}}[\lambda]$ reproduces the Bohm-Madelung potential, and the scalar continuity equation mirrors Madelung's hydrodynamics. The departure is that NUVO does not introduce additional hidden variables or a separate quantization postulate; instead, the scalar field λ and its geometry are primary, and quantization arises from scalar coherence and phase closure on NUVO space.

A key distinction between NUVO and other deterministic or geometric approaches is the role of the scalar field λ . In Bohmian mechanics, the quantum potential Q arises from the polar decomposition of the wavefunction and is not

tied to an independent geometric degree of freedom. In stochastic mechanics (Nelson) the quantum potential is generated by diffusion processes, while in geometric quantization the symplectic structure, prequantum bundle, and polarization determine the quantum dynamics. NUVO differs from all of these in employing a *single scalar field* whose geometry simultaneously determines scale, curvature, and effective dynamics. The term analogous to Bohm's Q thus has a direct geometric origin, rather than being introduced through hidden variables, stochastic fluctuations, or quantization axioms.

16. Conclusions, Outlook, and Closing Remarks

Editorial note. To streamline the structure and improve thematic cohesion, the former Sections 16 and 17 have been merged into a single concluding section, as requested by the reviewer.

We have demonstrated that the stationary NUVO transport law, expressed solely in terms of the scalar field λ , reduces under the gauge map $\psi = \phi/\lambda$ to the standard Schrödinger framework with an additional geometric potential $V_{\text{eff}}[\lambda]$. This establishes quantization as a geometric property of scalar coherence rather than an independent postulate.

Mathematically, H_λ is self-adjoint and semibounded; the integral representation yields the standard spectral structure, and known potentials follow as specific λ -profiles. Physically, λ encodes the local ratio of curvature to coherence, linking scalar modulation to observable energy shifts.

Future directions include analysis of tunneling and superposition as scalar-continuity phenomena, exploration of entanglement within coupled λ -fields, and potential experimental signatures such as geometric corrections to atomic spectra or interferometric phase shifts.

Observable deviations from standard quantum mechanics. The geometric potential $V_{\text{eff}}[\lambda]$ provides several concrete and, in principle, testable departures from standard quantum mechanics. Small geometric energy shifts arise whenever λ departs from unity, leading to corrections to hydrogenic levels, fine-structure splittings, and Lamb-shift-scale terms of order $\mathcal{O}(|\nabla \ln \lambda|^2)$. Interferometric phase evolution is altered by the scalar holonomy $\oint \nabla \ln \lambda \cdot d\mathbf{l}$, producing geometry-dependent phase offsets that are absent in the standard Schrödinger theory. Tunneling exponents and barrier transmission probabilities are likewise modified through the additional curvature contribution in the effective potential. These signatures offer possible paths for experimental discrimination between NUVO scalar geometry and conventional quantum mechanics.

These efforts aim to test whether NUVO's scalar geometry can serve as a deterministic foundation for quantum mechanics.

Multi-particle and entangled systems.

The present work treats the one-particle Schrödinger framework arising from the NUVO scalar geometry. A full extension to multi-particle dynamics requires a configuration-space formulation $\lambda(\mathbf{r}_1, \dots, \mathbf{r}_N)$ and a corresponding scalar con-

tinuity structure; these developments will be carried out in a separate paper. Likewise, questions of entanglement and nonlocal correlations lie outside the scope of the present analysis. The geometric conditions under which NUVO reproduces standard quantum entanglement and Bell-inequality violations will require a dedicated treatment once the multi-particle scalar geometry is fully established.

Closing remarks. Editorial note. The material that followed in the former Section 17 is incorporated here to present a unified conclusion.

The derivation presented here stands upon the shoulders of a century of profound work. It does not claim to replace those foundations, but to illuminate one possible geometric path by which they may all be seen as facets of a single principle. Every equation in this paper owes its existence to the labor of those who first sought unity between wave and particle, probability and geometry, curvature and energy. Their insights remain the compass by which any new attempt must navigate.

If the present treatment has succeeded in showing that the Schrödinger equation can arise from a scalar geometry rather than from assumption, that success belongs as much to those predecessors as to the method itself. The hope is not to declare a final form, but to contribute one clear step in the long and collective journey toward understanding how the fabric of space and the language of quantum mechanics may ultimately be one.

In that spirit, the author expresses gratitude to the many generations of scientists whose intellectual courage made such questions possible. The work offered here is intended as a gesture of respect—to their patience, their failures as much as their triumphs, and their enduring faith that the order hidden within nature can, one day, be written plainly.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

A. Dirac Reduction and Spin Terms (Sketch)

With $e^a{}_\mu = \lambda \delta^a{}_\mu$ and $\Psi = \lambda^{-3/2} \Xi$, the Dirac equation reads $i\hbar \gamma^a \partial_a \Xi - mc \lambda \Xi = 0$. A Foldy-Wouthuysen expansion to $O(c^{-2})$ yields a Pauli Hamiltonian with $S(\mathbf{r}) = mc^2(\lambda - 1)$ and geometric terms $V_{\text{eff}}[\lambda]$, plus spin-orbit and Darwin contributions $\frac{\hbar}{4m^2 c^2} \boldsymbol{\sigma} \cdot (\nabla S \times \mathbf{p})$ and $-\frac{\hbar^2}{8m^2 c^2} \Delta S$. A full coefficient-level derivation for $\lambda = 1 + a/r$ will appear in a companion paper.

B. Relation to the Bohm-Madelung Quantum Potential

For $\psi = R e^{iS/\hbar}$, the Bohm potential $Q = -\frac{\hbar^2}{2m} \frac{\Delta R}{R}$ equals

$$V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m} (|\nabla \ln \lambda|^2 - \Delta \ln \lambda) \quad (25)$$

when $R \propto \lambda^{-1}$. Thus Bohm's "quantum potential" is the scalar curvature potential in NUVO geometry.

C. Scalar Measure and Inner Product (Detailed Form)

The scalar volume form is $d\mu_\lambda = \lambda^{-3} d^3x$. The NUVO Hilbert space $L^2_\lambda(\mathbb{R}^3)$ with inner product

$$\langle \psi_1, \psi_2 \rangle_\lambda = \int \psi_1^* \psi_2 \lambda^{-3} d^3x \quad (26)$$

is mapped by $\psi = \phi/\lambda$ onto standard L^2 with norm $\|\phi\|_2^2 = \int |\phi|^2 d^3x$.

D. Summary of Added Relations

- Continuity: $\partial_t |\psi|^2 + \nabla \cdot \mathbf{J}_\lambda = 0$, with $\mathbf{J}_\lambda = (\hbar/m) \Im(\psi^* \lambda^2 \nabla \psi)$.
- \hbar emerges from scalar holonomy via $\frac{1}{\hbar} \oint \nabla S \cdot d\mathbf{l} = 2\pi n$.
- $V_{\text{eff}}[\lambda]$ equals the Bohm potential geometrically.
- Path-integral Lagrangian \mathcal{L}_λ yields the transport law by variation.
- Bound-state quantization follows from scalar phase closure.
- Hydrogenic correction aligns with Lamb-shift scale.

E. Full Proof of Gauge Reduction and Self-Adjointness

We collect here the technical steps advertised in Section 2: (i) the drift-cancellation identity underlying the gauge map, (ii) the exact expression for $V_{\text{eff}}[\lambda]$, and (iii) self-adjointness and semiboundedness of H_λ under the standing assumptions.

Normalization remark

Some NUVO conventions write the stationary transport law as $\lambda^2 \Delta \psi + C_\lambda \psi = 0$ with $C_\lambda = 2\nabla \ln \lambda \cdot \nabla + \Delta \ln \lambda$. Since $\lambda > 0$, multiplication by

λ^{-2} preserves the solution set. Thus we work with the *normalized* operator

$$\mathcal{L}_\lambda := \Delta + 2\nabla \ln \lambda \cdot \nabla + \Delta \ln \lambda, \quad \mathcal{L}_\lambda \psi = 0, \tag{27}$$

which is equivalent for our purposes.

Lemma 2 (Drift--cancellation identity). *Let $\lambda \in C^2(\mathbb{R}^3)$ be strictly positive and set $\alpha := \ln \lambda$. For any $\phi \in C_0^\infty(\mathbb{R}^3)$ define $\psi = \phi/\lambda = e^{-\alpha}\phi$. Then*

$$(\Delta + 2\nabla \alpha \cdot \nabla + \Delta \alpha)(e^{-\alpha}\phi) = e^{-\alpha}(\Delta\phi - (|\nabla\alpha|^2 - \Delta\alpha)\phi). \tag{28}$$

Proof. Write $\alpha = \ln \lambda$. Using product/chain rules,

$$\nabla(e^{-\alpha}\phi) = e^{-\alpha}(\nabla\phi - \phi\nabla\alpha),$$

and hence

$$\Delta(e^{-\alpha}\phi) = e^{-\alpha}(\Delta\phi - 2\nabla\alpha \cdot \nabla\phi + (|\nabla\alpha|^2 - \Delta\alpha)\phi).$$

Also,

$$2\nabla\alpha \cdot \nabla(e^{-\alpha}\phi) = e^{-\alpha}(2\nabla\alpha \cdot \nabla\phi - 2|\nabla\alpha|^2\phi), \quad (\Delta\alpha)(e^{-\alpha}\phi) = e^{-\alpha}(\Delta\alpha)\phi.$$

Adding these three contributions cancels the first-derivative terms and yields (28). \square

Corollary 1 (Gauge reduction and geometric potential) *Under the hypotheses of Lemma 2, the equation $\mathcal{L}_\lambda\psi = 0$ with $\psi = \phi/\lambda$ is equivalent to*

$$-\frac{\hbar^2}{2m}\Delta\phi + (V_{\text{phys}} + V_{\text{eff}}[\lambda])\phi = E\phi, \quad V_{\text{eff}}[\lambda] = \frac{\hbar^2}{2m}(|\nabla \ln \lambda|^2 - \Delta \ln \lambda). \tag{29}$$

Remark 3 (On similarity vs. unitarity). If $\lambda, \lambda^{-1} \in L^\infty$, the multiplication operator $M_{1/\lambda} : \phi \mapsto \phi/\lambda$ is a boundedly invertible map on $L^2(\mathbb{R}^3)$; in that case the NUVO transport equation and the Schrödinger equation are similar via a bounded similarity (not unitary unless $|\lambda| \equiv 1$). Moreover $M_{1/\lambda}$ preserves Sobolev regularity in the sense that $M_{1/\lambda} : H^k(\mathbb{R}^3) \rightarrow H^k(\mathbb{R}^3)$ is bounded for $k = 0, 1, 2$, so the natural H^1 form domain and H^2 operator domain are stable under the gauge map. Our form-based results do not require (U).

Proposition 4 (Self-adjointness and semiboundedness) Assume $\lambda > 0$, $\lambda \in C^2$, and the bounds (1) hold: $|\nabla \ln \lambda(\mathbf{r})| \leq C_1/(1+r)$,

$|\Delta \ln \lambda(\mathbf{r})| \leq C_2/(1+r^2)$. Let $V_{\text{phys}} \in L^2_{\text{loc}}$ be infinitesimally form-bounded with respect to $-\Delta$. Then $H_\lambda := -\frac{\hbar^2}{2m}\Delta + V_{\text{phys}} + V_{\text{eff}}[\lambda]$ is self-adjoint via the Friedrichs extension and bounded below, and for Kato-class V_{phys} the operator domain equals $H^2(\mathbb{R}^3)$.

Sketch of proof. Write the quadratic form associated with H_λ as

$$\mathfrak{h}_\lambda[u] := \frac{\hbar^2}{2m} \int_{\mathbb{R}^3} |\nabla u|^2 d^3x + \int_{\mathbb{R}^3} (V_{\text{phys}} + V_{\text{eff}}[\lambda]) |u|^2 d^3x, \quad u \in H^1(\mathbb{R}^3).$$

The bounds (1) imply that near $r = 0$, $V_{\text{eff}}(r)$ behaves no worse than c/r^2

for some constant c , while for large r it is bounded. The Hardy inequality

$$\int_{\mathbb{R}^3} \frac{|u(\mathbf{r})|^2}{r^2} d^3r \leq 4 \int_{\mathbb{R}^3} |\nabla u(\mathbf{r})|^2 d^3r, \quad u \in H^1(\mathbb{R}^3),$$

then shows that V_{eff} is infinitesimally form-bounded with respect to $-\Delta$ with arbitrarily small relative bound. By hypothesis, V_{phys} is also infinitesimally form-bounded relative to $-\Delta$, so the KLMN theorem (see, e.g., [15] [16]) implies that \mathfrak{h}_λ is closed and bounded from below on $H^1(\mathbb{R}^3)$ and defines a unique self-adjoint operator, the Friedrichs extension of H_λ . Standard elliptic regularity and Kato-class bounds on V_{phys} then yield $\text{Dom}(H_\lambda) = H^2(\mathbb{R}^3)$ under the Kato-class assumption; see [15]-[17] for details.