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A Link Merges Classical Mechanics to Quantum Theory (Part II)

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Abstract

This article is a continuation of Part(I) to demonstrate more aspects for a successful merge of classical mechanics to the quantum theory. The article starts first with the deduction of the general form of the complimentary energy, next a robust firm mathematical proof is given indicates that the complimentary energy has a wave nature and it is a potential energy, then the radiation condition has been determined, a vector formula expresses the corresponding force was deduced and this force was called adhesion force. A mathematical convergence process has proved the duality of the electromagnetic wave parameters and particle parameters, which results in the existence of a "wave-particle" object. While finding the relationship between the electromagnetic wave parameters and particle parameters, a mathematical relationship was found indicating that not every moving charged mass generates a magnetic field. This relationship for the mass was linked to the effective mass tensor, The geometric representation of the constant energy surface and its relationship to the energy levels of both the orbital model (Bohr model) and the cloud model for different geometric 3D shapes were investigated as a sample of the possible geometric interpretations of the energy equation, that is by employing the triangle method, it is possible to create a geometric representation of the constant energy surfaces. The relation between the energy levels, quantum numbers, and band width is represented geometrically. As a result of proving the relationship between the motion of an electromagnetic wave and the motion of a particle associated with the wave, two new, unprecedented mathematical formulas were developed. The first formula specifies a moving charged mass and its relationship to the resulting wavelength. Thus, the radius of the charged mass is obtained in terms of its wavelength and mass density. The second formula specifies the kinetic energy of a photon. The grand potentials are the potential energy of the Newton's Law of Universal Gravitation, Coulomb's Law, and Inverse Square Law of Electromagnetic Waves Intensity. From the potential energy of Newton's law of universal gravitation, the wavelength of the gravitational wave, as well as the wavelength of quantum gravity were deduced. A mathematical relationship was then deduced that determines the lengths of the functions of conic sections, and in particular the length of perimeter of ellipses. From the potential energy of Coulomb's law of attraction, a mathematical formulation of the energy levels in general for any atom of an element was obtained. This mathematical formulation was compared with the results obtained by Bohr and Schrödinger for the hydrogen atom. As for the inverse square law of electromagnetic radiation intensity, several innovative mathematical equations have been deduced that link different parameters together. Following the contents of this article requires to be quite familiar with the contents of the first part with the same title, as well as a complete understanding of the concepts of intermediate parameters derived and used in the various equations. Otherwise, it will be extremely difficult to follow the contents of this article.

Keywords

Complementary Energy, Adhesion Force, Effective Mass, Constant Energy Surface, Photon's Energy, Gravity Waves, Quantum Gravity, Hydrogen Atom, Electromagnetic Waves Intensity

1. Introduction

In 1864, James Clerk Maxwell published his four equations, which summarized the laws and results of electrical and magnetic experiments known at the time. One of the most important achievements of these equations was the prediction of the existence of electromagnetic waves. Maxwell's equations continued to be used to explain the properties of light waves and all types of electromagnetic waves until some physical phenomena emerged that physicists at the time could not explain using Maxwell's equations. Physicists therefore resorted to developing another approach to explaining physical phenomena: quantum theory. This theory was established upon the interpretation of data collected from various experiments conducted between 1900 and 1925 on thermal radiation, particle-to-particle collisions, and particle-to-wave collisions, in addition to attempts to explain the spectra of the elements of the periodic table and their relationship to the structure of their atoms.

The beginning of quantum theory was a turning point in physics and a breaking point for classical mechanics. This research demonstrates the mathematical continuation and compatibility of both quantum theory and classical mechanics, contrary to what many physicists expected, which has positive implications for a better understanding of physics. For instance, it became clear that there is no conflict between the theory of electromagnetic waves and quantum theory, and that the orbital model of the atom, known as the Bohr model, and the cloud model of the atom are compatible. Chemists still rely on the orbital model (Bohr model) to design and understand the properties of various chemical molecules, while the cloud model is also used to understand and study semiconductors and crystalline properties, meaning that both models have validity and applications. A proposal was made for a model that combines the advantages of the two models using a method called the triangle method, which is the torus model, has been illustrated in a geometrical manner.

A successful merge of classical mechanics to the quantum theory enables us to derive new and unprecedented mathematical relationships that could not have been deduced before, leading to a deeper understanding and better comprehension of physical phenomena, in a purpose to study these phenomena in a comprehensive, multifaceted manner that has a positive impact on theoretical, experimental, and practical levels. In this article a group of unprecedented mathematical relationships are deduced.

Quantum theory has become a part of modern physics. There is another theory that is considered another component of modern physics, which is the theory of general relativity, the prevailing assumption that the study of gravitational waves is limited to the theory of general relativity is incorrect. This research presents a mathematical relationship that explains gravitational waves and quantum gravity waves without addressing the assumptions of general relativity. It will thus provide researchers with a new dimension and a new addition to the available information for studying gravitational waves and understanding their nature from a different perspective, contributing to the explanation of many cosmic phenomena affecting the surface of planet Earth.

2. Applied Methodology

2.1. The General Format of the Complementary Energy

From Part (I) for particle j we have $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ are related to the particle total energy $\mathcal{E}_j$ and the total energy of the system of particles $\mathcal{E}$ by [1]

$$\mathcal{E}_j = \tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j, \quad j = 1, \cdots, N \tag{1}$$

in which $\sum_{j=1}^{N} \tilde{\alpha}_j = 1$, $\sum_{j=1}^{N} \tilde{\beta}_j = 0$, $m_j v_j \neq 0$, $j = 1, \cdots, N$

From Part (I) we have [1]

$$v_{j,1,2} = \tilde{v}_j \pm d_j, \quad j = 1, \cdots, N \tag{2}$$

$$\text{in which } d_j = \sqrt{\tilde{v}_j^2 + \tilde{v}_{\perp j}^2}, \quad \tilde{v}_{\perp j} = \sqrt{\frac{2\mathcal{E}_j}{m_j}}, \quad j = 1, \cdots, N \tag{3}$$

Substituting from relation (1) into Equation (3) we get

$$d_j^2 = \tilde{v}_j^2 + \frac{2\mathcal{E}_j}{m_j} = \tilde{v}_j^2 + \frac{2}{m_j}\left(\tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j\right), \quad j = 1, \cdots, N$$

Rewriting the above equation gives

$$d_j^2 = \tilde{v}_j^2 + \left(\sqrt{\frac{2}{m_j}\tilde{\alpha}_j \boldsymbol{\mathcal{E}}}\right)^2 + \frac{2}{m_j}\tilde{\beta}_j, \quad j = 1, \cdots, N \tag{4}$$

For the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ where

$\vec{c} = \vec{a} + \vec{b} \rightarrow \vec{c} \cdot \vec{c} = \left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right) = c^2 = a^2 + b^2 + 2ab\cos\tilde{\theta}$

Comparing the above equality with Equation (4) we get

$$\frac{2}{m_j}\tilde{\beta}_j = 2\tilde{v}_j \sqrt{\frac{2}{m_j}\tilde{\alpha}_j \boldsymbol{\mathcal{E}}} \cos\tilde{\theta}_j \rightarrow \tilde{\beta}_j = \tilde{v}_j \sqrt{2m_j \tilde{\alpha}_j \boldsymbol{\mathcal{E}}} \cos\tilde{\theta}_j, \quad j = 1, \cdots, N \tag{5}$$

The following steps is a proof that the complementary energy is potential energy Squaring both sides of relation (5) and rewrite

$$\tilde{\alpha}_j \boldsymbol{\mathcal{E}} = \frac{1}{2m_j \tilde{v}_j^2 \cos^2\theta'_j}\tilde{\beta}_j^2, \quad j = 1, \cdots, N \tag{5.1}$$

Substituting from relation (5.1) into Equation (1) we get

$$\boldsymbol{\mathcal{E}}_j = \frac{1}{2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j}\tilde{\beta}_j^2 + \tilde{\beta}_j \rightarrow \tilde{\beta}_j^2 + 2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j \tilde{\beta}_j - 2m_j \tilde{v}_j^2 \cos^2\tilde{\theta} \boldsymbol{\mathcal{E}}_j = 0,$$
$$j = 1, \cdots, N \tag{6}$$

The roots of Equation (6) are $\tilde{\beta}_{j,1,2}$ given by

$$\tilde{\beta}_{j,1,2} = \frac{-2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j \pm \sqrt{\left(2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j\right)^2 + 4 \times 2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j \boldsymbol{\mathcal{E}}_j}}{2}, \quad j = 1, \cdots, N$$

$$\tilde{\beta}_{j,1,2} = -m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j \pm \sqrt{\left(m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j\right)^2 + 2m_j \tilde{v}_j^2 \cos^2\tilde{\theta}_j \boldsymbol{\mathcal{E}}_j}, \quad j = 1, \cdots, N$$

Substitute $\theta'_j = 0 \rightarrow \cos\theta'_j = 1$ in order to get the value of $\tilde{\beta}_{0j,1,2}$ as follows

$$\tilde{\beta}_{0j,1,2} = -m_j \tilde{v}_j^2 \pm \sqrt{\left(m_j \tilde{v}_j^2\right)^2 + 2m_j \tilde{v}_j^2 \boldsymbol{\mathcal{E}}_j}, \quad j = 1, \cdots, N$$

Rewrite the above relation we get

$$\tilde{\beta}_{0j,1,2} = -m_j \tilde{v}_j \left(\tilde{v}_j \pm \sqrt{\tilde{v}_j^2 + \frac{2\boldsymbol{\mathcal{E}}_j}{m_j}}\right) = -m_j \tilde{v}_j v_j, \quad j = 1, \cdots, N \tag{5.2}$$

Equation (5.2) has been obtained by the substitution of the value of v_j from Equation (2).

The above relation is particle-wave equation used in Part(I) to deduce De Broglie's relations [1].

From the $\tilde{\beta}_{0j,1,2}$ format obtained in relation (5.2) it is obvious it is a potential energy as proved in Part(I) according to the general format of any potential energy.

Rewrite Equation (5) gives

$$\tilde{\beta}_j = \tilde{v}_j \sqrt{2m_j \tilde{\alpha}_j \boldsymbol{\mathcal{E}}} \cos\tilde{\theta}_j = \tilde{\beta}_{0j} \cos\tilde{\theta}_j, \quad j = 1, \cdots, N \tag{5.3}$$

From relation (5.2) we get

$$\tilde{\beta}_j = -m_j \tilde{v}_j v_j \cos\tilde{\theta}_j, \quad j = 1, \cdots, N \tag{5.4}$$

Equations (5.3) and (5.4) are the general format of the complementary energy $\lim_{v_j \to c} \tilde{\beta}_j = \tilde{\beta}_{0j}$ The limt given above is called radiation limt at that limt the wave will be able to be released from the material and the substance radiates energy and the particle-wave object switch to be wave-particle object with energy given by $\tilde{\beta}_{0j}$ and $\tilde{\theta}_j = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \cdots$ the angel $\tilde{\theta}_j$ is called the radiation angel notice that the angle $\tilde{\theta}_j$ is different than the angle θ_j used in Part(I) to be the deviation angle from mean velocity $\tilde{v}_j$.

2.2. The Nature of the Complementary Energy

Let $\tilde{\theta}_j = \sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}$ where $\sqrt{\gamma_j} = \frac{v_{2j}}{v_{1j}} \to v_{2j}^2 = v_{1j}^2 \gamma_j$ and $\tilde{\theta}_{0j}$ is constant,

$$j = 1, \cdots, N \tag{7}$$

Substituting with the value of $\tilde{\theta}_j$ given above in relation (5.4) results

$$\tilde{\beta}_j = -m_j \tilde{v}_j v_j \cos\tilde{\theta}_j = -m_j \tilde{v}_j v_j \cos\left(\sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}\right), \quad j = 1, \cdots, N \tag{8}$$

The partial derivative of Equation (8) twice with respect to x_j gives

$$\frac{\partial^2 \tilde{\beta}_j}{\partial x_j^2} = -\gamma_j \tilde{\beta}_j \to \tilde{\beta}_j = -\frac{1}{\gamma_j} \frac{\partial^2 \tilde{\beta}_j}{\partial x_j^2}, \quad j = 1, \cdots, N \tag{9}$$

The partial derivative of Equation (8) twice with respect to t and substituting from relation (7) gives the following

$$\frac{\partial^2 \tilde{\beta}_j}{\partial t^2} = -v_{2j}^2 \tilde{\beta}_j \to \tilde{\beta}_j = -\frac{1}{v_{2j}^2} \frac{\partial^2 \tilde{\beta}_j}{\partial t^2} = -\frac{1}{v_{1j}^2 \gamma_j} \frac{\partial^2 \tilde{\beta}_j}{\partial t^2}, \quad j = 1, \cdots, N \tag{10}$$

Equating Equations (9) and (10) gives

$$\frac{1}{\gamma_j} \frac{\partial^2 \tilde{\beta}_j}{\partial x_j^2} = \frac{1}{v_{1j}^2 \gamma_j} \frac{\partial^2 \tilde{\beta}_j}{\partial t^2} \to \frac{\partial^2 \tilde{\beta}_j}{\partial x_j^2} = \frac{1}{v_{1j}^2} \frac{\partial^2 \tilde{\beta}_j}{\partial t^2}, \quad j = 1, \cdots, N \tag{11}$$

Equation (11) is a wave equation some times called D'Alembert equation which represents a plane wave propagating with speed v_{1j}.

2.3. Adhesion Force (F_{adj})

In section (2.1) a proof is given that $\tilde{\beta}_j$ is some sort of a potential energy, since $\tilde{\beta}_j$ is a potential energy then the corresponding force F_{adj} is given by the grad $\tilde{\beta}_j$ written as follows

$$grad\,\tilde{\beta}_j = F_{adj} = \sqrt{\gamma_j} m_j \tilde{v}_j v_j \sin\left(\sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}\right), \quad j = 1, \cdots, N \tag{12}$$

The relation (12) is interpreted as the magnitude of the cross product of the two vectors $\vec{v}_j$ and $\vec{\tilde{v}}_j$ in which the magnitude of the velocity of the moving particle j and the magnitude of the mean velocity of the moving particle j respectively and the angel between the two vectors is $\tilde{\theta}_j$ given by $\tilde{\theta}_j = \left(\sqrt{\gamma_j} x_j - v_j t + \tilde{\theta}_{0j}\right)$.

Relation (12) is written in a vectors representation as follows

$$\left|\vec{F}_{adj}\right| = \sqrt{\gamma_j} m_j \tilde{v}_j v_j \sin\left(\sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}\right) = \left|\sqrt{\gamma_j} m_j \vec{v}_j \times \vec{\tilde{v}}_j\right|, \quad j = 1, \cdots, N$$

$$\vec{F}_{adj} = \sqrt{\gamma_j} m_j \left(\vec{v}_j \times \vec{\tilde{v}}_j\right), \quad j = 1, \cdots, N \tag{13}$$

The force vector $\vec{F}_{adj}$ given by the vector relation (13) is called the adhesion force vector which represents the necessary force to adhere the moving particle with the moving wave. Forces proportional to the speed of the moving objects are called dissipating forces such as damped harmonic oscillator, resistance force, viscosity force, and friction all the previous forces are acting in the opposite direction of the velocity of the moving particle. From the properties of the cross product of vectors the force $\vec{F}_{adj}$ is perpendicular on the plane where both the vectors $\vec{v}_j$ and $\vec{\tilde{v}}_j$ fall

Since $\sum_{j=1}^{N} \tilde{\beta}_j = 0$ then $\sum_{j=1}^{N} grad\,\tilde{\beta}_j = 0$ and this results the following vector equation

$$\sum_{j=1}^{N} \vec{F}_{adj} = 0 \,.\ i.e.\ \ \vec{F}_{adj} = -\sum_{j=1}^{N-1} \vec{F}_{adj}\,, \quad j = 1, \cdots, N \tag{14}$$

Equation (14) indicates that the adhesion force acting upon particle *j* is the resultant of the forces of rest *N-1* particles in the opposite direction, notice the adhesion force vanishes if the wave radiation condition occurs.

2.4. Restricted Potential energy (V_j)

From Part(I) the potential energy V_j of particle *j* is given by the relation [1]

$$V_j = -m_j \tilde{v}_j v_j\,, \quad j = 1, \cdots, N$$

Now we derive the restrict form of the potential energy by writing Equation (2) as follows

$$v_{j,1,2} = \tilde{v}_j \pm d_j = \tilde{v}_j \left(1 \pm \frac{d_j}{\tilde{v}_j}\right) = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{15}$$

Substituting from the above relation (15) into the potential energy relation gives

$$V_j = -m_j \tilde{v}_j v_j = -m_j \tilde{v}_j v_{j,1,2} = -m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{16}$$

Rewrite relation (15) we get

$$\frac{\mathrm{d}s_j}{\mathrm{d}t} = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right) \to \mathrm{d}s_j = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right)\mathrm{d}t\,, \quad j = 1, \cdots, N \tag{15.1}$$

Now assume the function $\left(1 \pm \frac{1}{\cos\theta_j}\right)$ is a constant function its value is $C_{1,2j}$

Substituting with $C_{1,2j}$ value in Equation (15.1) and integrating both sides we write

$$\mathrm{d}s_j = \tilde{\upsilon}_j C_{1,2j}\mathrm{d}t \rightarrow \int \mathrm{d}s_j = \tilde{\upsilon}_j C_{1,2j}\int \mathrm{d}t \rightarrow s_j = \tilde{\upsilon}_j C_{1,2j} t\,, \quad j = 1,\cdots,N \tag{17}$$

The integration constant is considered zero

Rewriting Equation (17) gives

$$\tilde{\upsilon}_j = s_j \frac{1}{C_{1,2j}t} \rightarrow \tilde{\upsilon}_j^2 = s_j^2 \frac{1}{C_{1,2j}^2 t^2}\,, \quad j = 1,\cdots,N \tag{17.1}$$

Substituting with $C_{1,2j}$ value in Equation (16) we get

$$V_j = -m_j\tilde{\upsilon}_j^2\left(1 \pm \frac{1}{\cos\theta_j}\right) = -m_j\tilde{\upsilon}_j^2 C_{1,2j}\,, \quad j = 1,\cdots,N \tag{16.1}$$

Substituting with $\tilde{\upsilon}_j^2$ value obtained from Equation (17.1) into Equation (16.1) we get

$$V_j = -m_j C_{1,2j}\tilde{\upsilon}_j^2 = -m_j C_{1,2j} s_j^2 \frac{1}{C_{1,2j}^2 t^2} = -\frac{m_j}{C_{1,2j}t^2} s_j^2\,, \quad j = 1,\cdots,N \tag{18}$$

$\phi_j'^2 = \dfrac{1}{\tau_j^2}$ where τ is the periodical time

The frequency of the wave is ν_j is related to the angular frequency ϕ_j' with the relation $\phi_j' = 2\pi\nu_j$

Substituting with $\phi_j' = 2\pi\nu$ into Equation (18) we get

$$V_j = -\frac{m_j}{C_{1,2j}t^2} s_j^2 = -\frac{1}{C_{1,2j}} m_j 4\pi^2\nu_j^2 s_j^2\,, \quad j = 1,\cdots,N \tag{18.1}$$

Comparing the above equation with the potential energy of the linear oscillator given by

$$V = -\frac{1}{2}4\pi^2 m\nu^2 s^2 = -\frac{1}{2}K_j s^2 \qquad \text{where} \quad K_j = 4\pi^2 m_j \nu_j^2$$

The comparison results $C_{1,2j} = -2 \rightarrow \left(1 \pm \dfrac{1}{\cos\theta_j}\right) = -2 \rightarrow \dfrac{1}{\cos\theta_j} = 3$

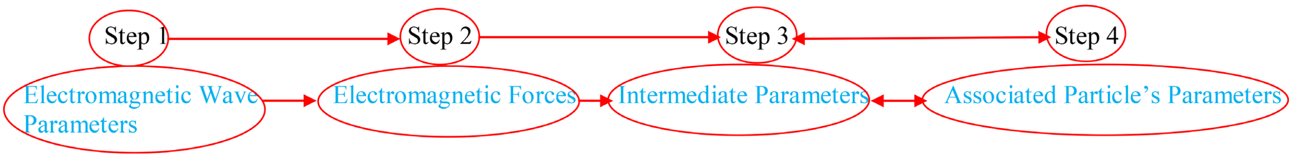
Substitute with the value of $K_{1,2j}$ into Equation (18.1) to get the restricted potential energy V_j for particle *j* as follows

$$V_j = -\frac{1}{2}K_{1,2j}s_j^2\,, \quad j = 1,\cdots,N \tag{18.2}$$

The deduced potential energy given by Equation (18.2) is the a potential energy of simple harmonic motion some time called linear oscillator [2]-[4], the properties of an assembly of linear oscillators form the basis of the theory of the specific heat capacity of polyatomic gases and solids [3]. The simple harmonic motion is the most favorite assumed motion description for modeling the motion of tiny particles resulting a good agreement with experimental data.

2.5. Electromagnetic Wave—Wave-Particle Object

In this section a convergence of the electromagnetic wave parameters and the wave-particle object parameters will be deduced, below is a diagram illustrates the derivation process sequence scheme.



Step 1: Electromagnetic wave parameters

The electromagnetic wave parameters are two vectors, the electric field E vector and the magnetic field H vector both satisfying the following wave equations derived from Maxwell's equations [5] [6]

$$\nabla^2 E = \frac{\mu\kappa}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \frac{\mu\kappa}{c^2}\frac{\partial^2 H}{\partial t^2}$$

where

c is the speed of light.

μ is defined as permeability of free space.

κ is defined as vacuum permittivity or dielectric constant.

Step 2: Electromagnetic forces

The electromagnetic forces $\vec{F}_{E,H}$ generated by the electric field E and the magnetic field H are given by Lorentz's force equation written as follows [5]-[8]

$$\vec{F}_{E,H} = e\vec{E} + \frac{e_j}{c}\vec{v}_j \times \vec{H}, \quad j = 1,\cdots,N$$

where

e_j is the electric charge of particle j, c the speed of light, and $\vec{v}_j$ is the velocity of the moving particle j.

$\vec{F}_H$ is the force acting on particle j with charge e and moving with velocity $\vec{v}_j$ due to the magnetic field $\vec{H}$ is given as follows

$$\vec{F}_H = \frac{e_j}{c}\vec{v}_j \times \vec{H}, \quad j = 1,\cdots,N \tag{19}$$

$$\vec{F}_E = e_j\vec{E}$$

Step 3: Intermediate Parameters

Mean velocity $\tilde{v}_j$ and complimentary energy $\tilde{\beta}_j$ are the intermediate parameters used in the derivation of the adhesion force vector $\vec{F}_{adj}$ given by the vector relation (13) is written

$$\vec{F}_{adj} = \sqrt{\gamma_j}m_j\left(\vec{v}_j \times \vec{\tilde{v}}_j\right), \quad j = 1,\cdots,N \tag{13}$$

The above relation is the general format of any adhesion force, where $\vec{F}_H$ given by relation (19) is an adhesion force of a magnetic field $\vec{H}$, so we equate the vectors relations (13) and (19) and writes

$$\vec{F}_H = \vec{F}_{adj} \to \frac{e_j}{c}\vec{v}_j \times \vec{H} = \sqrt{\gamma_j}m_j\left(\vec{v}_j \times \vec{\tilde{v}}_j\right), \quad j = 1,\cdots,N \tag{20}$$

A result from comparing both sides of Equation (20) we get the following vector equation which correlates the magnetic field vector $\vec{H}$ with the mean velocity vector $\vec{\tilde{v}}_j$ is written

$$\frac{e_j}{c}\vec{H} = \sqrt{\gamma_j} m_j \vec{\tilde{v}}_j, \quad j = 1, \cdots, N \tag{21}$$

The above Equation (21) is a special interpretation of Equation (20), but it gives a hit about the shape of the general format and will guide the derivation targeting the general format for Equation (21).

The general format for Equation (21) will be obtained by rewriting the vector Equation (20) as follows

$$\frac{e_j}{c}\vec{v}_j \times \vec{H} - \sqrt{\gamma_j} m_j \left(\vec{v}_j \times \vec{\tilde{v}}_j\right) = 0 \to \vec{v}_j \times \left(\frac{e_j}{c}\vec{H} - \sqrt{\gamma_j} m_j \vec{\tilde{v}}_j\right) = \vec{v}_j \times \vec{v}_j, \quad j = 1, \cdots, N \tag{20.1}$$

From Equation (20.1) we get

$$\vec{v}_j = \frac{e_j}{c}\vec{H} - \sqrt{\gamma_j} m_j \vec{\tilde{v}}_j \to \frac{e_j}{c}\vec{H} = \vec{v}_j + \sqrt{\gamma_j} m_j \vec{\tilde{v}}_j, \quad j = 1, \cdots, N \tag{22}$$

The corresponding scalar equation for the above vector equation is

$$\frac{e_j^2}{c^2}H^2 = v_j^2 + \left(\sqrt{\gamma_j} m_j \tilde{v}_j\right)^2 + 2\sqrt{\gamma_j} m_j \tilde{v}_j v_j \cos\tilde{\theta}_j, \quad j = 1, \cdots, N \tag{22.1}$$

From section 2.4 we have Equation (15) written

$$v_j = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right) \to v_j = \tilde{v}_j C_{1,2j}, \quad j = 1, \cdots, N \tag{15}$$

Substitute from Equation (15) given above into Equation (22.1) we get

$$\frac{e_j^2}{c^2}H^2 = \left(\tilde{v}_j C_{1,2j}\right)^2 + \left(\sqrt{\gamma_j} m_j \tilde{v}_j\right)^2 + 2\sqrt{\gamma_j} m_j \tilde{v}_j \tilde{v}_j C_{1,2j} \cos\tilde{\theta}_j, \quad j = 1, \cdots, N$$

Rewrite the above equation gets

$$\frac{e_j^2}{c^2}H^2 = \left(\sqrt{\gamma_j} m_j \tilde{v}_j \cos\tilde{\theta}_j\right)^2 \left(\frac{C_{1,2j}^2}{\left(\sqrt{\gamma_j} m_j \cos\tilde{\theta}_j\right)^2} + \frac{1}{\cos^2\tilde{\theta}_j} + 2\frac{C_{1,2j}}{\sqrt{\gamma_j} m_j \cos\tilde{\theta}_j}\right), \quad j = 1, \cdots, N \tag{23}$$

Using the vector Equation (21) as guide we get the following result

$$\text{The term } \frac{C_{1,2j}^2}{\left(\sqrt{\gamma_j} m_j \cos\tilde{\theta}_j\right)^2} + \frac{1}{\cos^2\tilde{\theta}_j} + 2\frac{C_{1,2j}}{\sqrt{\gamma_j} m_j \cos\tilde{\theta}_j} = \delta_{1,2j} + 1 = \tilde{\delta}_{1,2j}, \quad j = 1, \cdots, N \tag{24}$$

Substituting into Equation (23) from relation (24) we get

$$\frac{e_j^2}{c^2}H^2 = \left(\sqrt{\gamma_j} m_j \tilde{v}_j \cos\tilde{\theta}_j\right)^2 \tilde{\delta}_{1,2j}, \quad j = 1, \cdots, N \tag{25}$$

$$\text{Now let } \eta_{i,j} = \frac{C_{i,j}}{\sqrt{\gamma_j} m_j \cos\tilde{\theta}_j}, \quad i = 1, 2, \quad j = 1, \cdots, N \tag{26}$$

Substitute with $\eta_{i,j}$ value into the relation (24) gives

$$\eta_{i,j}^2 + 2\eta_{i,j} + \left(\frac{1}{\cos^2\tilde{\theta}_j} - 1 - \delta_{i,j}\right) = 0$$
$$\rightarrow \eta_{i,j}^2 + 2\eta_{i,j} + \frac{\sin^2\tilde{\theta}_j}{\cos^2\tilde{\theta}_j} - \delta_{i,j} = \eta_{i,j}^2 + 2\eta_{i,j} + \left(\tan^2\tilde{\theta}_j - \delta_{i,j}\right) = 0 \quad (23.1)$$

Solutions of the quadratic Equation (23.1) are

$$\eta_{1,2,i,j} = \frac{-2 \pm \sqrt{4 + 4\left(\tan^2\tilde{\theta}_j - \delta_{i,j}\right)}}{2} = -1 \pm \sqrt{1 + \left(\tan^2\tilde{\theta}_j - \delta_{i,j}\right)}, \quad j = 1, \cdots, N \quad (27)$$

From equations (26) and (27) we get the permitted values of the mass m_j

$$m_{i,k,j} = \frac{C_{i,j}}{\sqrt{\gamma_j}\,\eta_{k,i,j}\cos\tilde{\theta}_j}, \text{ where, } k, i = 1, 2, \quad j = 1, \cdots, N \quad (28)$$

The above relation indicates that there are specific values for the moving charged mass in order to its motion result a magnetic field, hence not any moving charged mass cause a magnetic field. A complete detailed analysis for Equation (28) will be given in Section 2.7.

Step 4: Associated Particle's Parameters

We take the square root of both sides of Equation (25) we get the magnitude of the magnetic field H as follows

$$\frac{e_j}{c}H = \sqrt{\tilde{\delta}_{1,2j}\gamma_j}\, m_j\tilde{v}_j\cos\tilde{\theta}_j \rightarrow H = \frac{1}{e_j}\sqrt{\tilde{\delta}_{1,2j}\gamma_j}\, m_j\tilde{v}_j c\cos\tilde{\theta}, \quad j = 1, \cdots, N \quad (25.1)$$

Now we take the limit value of the magnetic field H when the speed v_{1j} tends to the speed of light c the radiation limit of $\cos\tilde{\theta}$ becomes ±1, So Equation (25.1) is rewritten as follows

$$H = \pm\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j} m_j\tilde{v}_j c, \quad j = 1, \cdots, N \quad (25.2)$$

The right hand side of relation (25.2) is a complementary energy of the associated particle *j* with momentum $p_j = m_j\tilde{v}_j$ and speed *c* multiplied by $\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j}$ which is considered a constant [1].

Substituting into the relation (25.2) above with $c = \lambda_j\nu_j$ where λ is the wave length and ν is the frequency of the wave we get

$$H = \frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j} m_j\tilde{v}_j\lambda\nu = \frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j} h\nu, \quad j = 1, \cdots, N \quad (29)$$

where $p_j\lambda_j = h \rightarrow p_j = \frac{h}{\lambda_j}$ and h is Planck's constant, $j = 1, \cdots, N$ (29.1)

Note that the mass used to determine the momentum p_j is the available mass m_j given by the relation (28)

A proposed values for $\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j}$ to be $\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j}=1$ or $\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j}=\frac{1}{2\pi}$

Experimental results are the deciding factor regarding the correct value of $\frac{\sqrt{\tilde{\delta}_{1,2j}\gamma_j}}{e_j}$.

Relation (29) is rewritten as follows

$$H = h\nu_j \quad \text{or} \quad H = \hbar\nu_j \quad \text{where} \quad \hbar = \frac{h}{2\pi}, \quad j = 1,\cdots,N \tag{29.2}$$

The right hand side of relation (29.2) shows that the complementary energy of a particle-wave object obtained in Part(I) as one of De Broglie relations and the particle-wave object has a momentum p_j given by the relation (29.1) [1].

On the other hand, the left hand side the relation (29.2) is the magnetic field of a electromagnetic wave satisfies Maxwell's wave equation, That is relation (29.2) express the energy of the wave-particle object which is a bi-natural object a wave associated with a charged particle, so the connection between a wave and particle is established.

From Maxwell's wave equation, we can express the electric field of an electromagnetic wave in terms of the magnetic field of the wave using the following equation

$$H = \sqrt{\mu\kappa}E$$

2.6. Mass & Radius of a Moving Charge—Photon's Kinetic Energy (KE_c)

Reviewing the results obtained from the previous steps, we find that the magnetic field H satisfies two relations at same time the first one corresponds Maxwell's wave equation [5] [6] given in step1 and the second one is obtained due to the derivation steps.

The first relation is given as

$$H = H_0\cos\left(\frac{1}{\sqrt{\mu\kappa}}x - ct + \tilde{\theta}_0\right), \quad j = 1,\cdots,N \tag{30}$$

The second relation is Equation (25.1) is written as follows

$$H = \frac{1}{e_j}\sqrt{\tilde{\delta}_{1,2j}\gamma_j}\,m_j\tilde{v}_jc\cos\left(\sqrt{\gamma_j}x_j - v_{2j}t + \tilde{\theta}_{0j}\right), \quad j = 1,\cdots,N \tag{25.3}$$

The arguments variables of both cos functions in relation (30) and relation (25.3) must be equal we write

$$\left(\sqrt{\gamma_j}x_j - v_{2j}t + \tilde{\theta}_{0j}\right) = \left(\frac{1}{\sqrt{\mu\kappa}}x - ct + \tilde{\theta}_0\right)$$

Using the relation $\sqrt{\gamma_j} = \frac{v_{2j}}{v_{1j}} \to v_{2j} = v_{1j}\sqrt{\gamma_j}$ we get

$$\sqrt{\gamma_j} = \frac{1}{\sqrt{\mu\kappa}} \text{ and } v_{1j}\sqrt{\gamma_j} = c \to v_{1j} = c\sqrt{\mu\kappa}, \quad j = 1,\cdots,N \tag{31}$$

Consequently $v_{2j} = v_{1j}\sqrt{\gamma_j} \to v_{2j} = c$, $j = 1,\cdots,N$ (31.1)

From the relations (31) and (31.1) we can determine the value of the mean velocity $\tilde{v}_j$ as follows [1]

$$\tilde{v}_j = \frac{v_{1j} + v_{2j}}{2} = \frac{c\sqrt{\mu\kappa} + c}{2} = \frac{c}{2}\left(\sqrt{\mu\kappa} + 1\right), \quad j = 1,\cdots,N \tag{32}$$

Substituting into relation (29.1) from (32) and writes

$$p_j = m_j\tilde{v}_j = \frac{m_j c}{2}\left(\sqrt{\mu\kappa} + 1\right) = \frac{h}{\lambda_j} \to m_j = \frac{2h}{\lambda_j c\left(\sqrt{\mu\kappa} + 1\right)}, \quad j = 1,\cdots,N \tag{33}$$

Relation (33) gives the mass of a moving charge m_{ej} which inversely proportional to the wave length and the proportional constant is $\tilde{\tau} = \frac{2h}{c\left(\sqrt{\mu\kappa} + 1\right)}$.

If we consider the charged moving particle has a spherical shape so the volume of the moving charge is given by $V_{me} = \frac{4}{3}\pi r_{me}^3$, and the density of the charge is ρ

$$m_j = \frac{\tilde{\tau}}{\lambda_j} = \frac{4}{3}\pi r_{me}^3 \rho \to r_{me}^3 = \frac{3\tilde{\tau}}{4\pi\rho}\frac{1}{\lambda_j} \to r_{me} = \left(\frac{3\tilde{\tau}}{4\pi\rho}\frac{1}{\lambda_j}\right)^{\frac{1}{3}}, \quad j = 1,\cdots,N \tag{33.1}$$

Relation (33.1) gives the radius r_{me} of the moving charge as a function in wave length λ_j

$$c = \lambda_j \nu_j \to \lambda_j = \frac{c}{\nu_j}, \quad j = 1,\cdots,N$$

Substituting from the above relation into formula (33) we get

$$m_j = \frac{2h}{\lambda_j c\left(\sqrt{\mu\kappa} + 1\right)} = \frac{2\nu_j h}{c^2\left(\sqrt{\mu\kappa} + 1\right)}, \quad j = 1,\cdots,N \tag{33.2}$$

The kinetic energy of a particle j with mass m_j moving with the speed of light c is obtained by rewriting relation (33.2) as follows

$$KE_{cj} = \frac{1}{2}m_j c^2 = \frac{1}{\sqrt{\mu\kappa} + 1}\nu_j h = \frac{1}{\sqrt{\mu\kappa} + 1}\tilde{\beta}_{0j}, \quad j = 1,\cdots,N \tag{34}$$

Equation (34) indicates that a charged particle moving with the speed of light generates a wave its energy linearly proportional to De Broglie wave energy [1] and the proportional constant is equal $\frac{1}{\sqrt{\mu\kappa} + 1}$ which is a particle-wave object could be regarded as photon and the kinetic energy KE_{cj} is the kinetic energy of the photon. The previous deduced result negates the prevailing information that a mass moving at the speed of light goes to zero.

2.7. Charged Masses Tensor—Constant Energy Surface

Relation (28) given in section 2.5 represents the permitted values for the a moving charged mass in order to result its motion a magnetic field the relation is written

$$m_{i,k,j} = \frac{C_{i,j}}{\sqrt{\gamma_j}\,\eta_{i,k,j}\cos\tilde{\theta}_j}\text{, where, } k,i = 1,2\text{, } j = 1,\cdots,N \tag{28}$$

We can notice that left side of Equation (28) has the unit of mass, while the right side of the equation has no unit.

This indicates that this Equation (28) specifies the smallest mass of a moving charged particle that can generate a magnetic field. That is, any value for a moving charged mass greater or less than the values given by Equation (28) does not generate a magnetic field.

Now we get into the values contributing in to Equation (28)

$C_{i,j}$ has two values $C_{1,j}$ and $C_{2,j}$ given by the relation $\left(1 \pm \frac{1}{\cos\theta_j}\right)$

$\eta_{i,k,j}$ has two values $\eta_{1,k,j}$ and $\eta_{2,k,j}$ for each value of *k*,so $\eta_{i,k,j}$ has four values given by Equation (27) written

$$\eta_{1,2,k,j} = -1 \pm \sqrt{1 + \left(\tan^2\tilde{\theta}_j - \delta_{1,2,j}\right)}\text{, } j = 1,\cdots,N \tag{27}$$

Consequently $m_{i,k,j}$ has four values such values could be arranged in a matrix format as follows

$$m_j = \begin{bmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{bmatrix} \text{ like that } \eta_j = \begin{bmatrix} \eta_{1,1} & \eta_{1,2} \\ \eta_{2,1} & \eta_{2,2} \end{bmatrix}\text{, } j = 1,\cdots,N \tag{28.1}$$

$$\eta_{1,2,k,j} = -1 \pm \sqrt{1 + \left(\tan^2\tilde{\theta}_j - \delta_{1,2,j}\right)}\text{, } j = 1,\cdots,N$$

In case $\left|\delta_{1,2,j}\right| \le \tan^2\tilde{\theta}_j$ the values of the matrix η_j are real numbers.

In case $\left|\delta_{1,2,j}\right| > \tan^2\tilde{\theta}_j$ and $\left|\tan^2\tilde{\theta}_j - \delta_{1,2,j}\right| > 1$ the values of the matrix η_j are complex numbers in this case a wave function exists.

At speed of light $\tan^2\tilde{\theta}_j = 0$ the values of the matrix η_j are given by

$$\eta_{1,2,k,j} = -1 \pm \sqrt{1 - \delta_{1,2,j}}\text{, } j = 1,\cdots,N$$

In case $\left|\delta_{1,2,j}\right| \le 1$ the values of the matrix η_j are real numbers.

In case $\left|\delta_{1,2,j}\right| > 1$ the values of the matrix η_j are complex numbers in this case a wave function exists.

From Part(I) the total energy $\mathcal{E}_j$ of a particle *j* is given by the following equation [1]

$$\mathcal{E}_j\left(v_j\right) = \frac{1}{2}m_j\left(v_j - \tilde{v}_j\right)^2 - \frac{1}{2}m_j\tilde{v}_j^2\text{, } j = 1,\cdots,N$$

$$\mathcal{E}_j\left(v_j\right) = \frac{1}{2}\frac{m_j^2}{m_j}\left(v_j - \tilde{v}_j\right)^2 - \frac{1}{2}\frac{m_j^2}{m_j}\tilde{v}_j^2\text{, } j = 1,\cdots,N$$

$$\begin{aligned}\mathcal{E}_j\left(v_j\right) &= \frac{1}{2m_j}\left(m_j v_j - m_j \tilde{v}_j\right)^2 - \frac{1}{2m_j} m_j^2 \tilde{v}_j^2 \\ &= \mathcal{E}_j\left(P_j\right) = \frac{1}{2m_j}\left(P_j - p_j\right)^2 - \frac{1}{2m_j} p_j^2, \quad j = 1, \cdots, N\end{aligned} \tag{35}$$

Let us expand $\mathcal{E}_j\left(\vec{P}_j\right)$ into the Taylor series around one of the extremal points a vector $\vec{P}_{0j}$.

We already know from Equation (35) that we will consider the first three terms only from Taylor series, since the expansion is performed around an extremal point the first derivative $\left.\frac{\mathrm{d}\mathcal{E}_j}{\mathrm{d}\vec{P}_j}\right|_{\vec{P}_{0j}} = 0$ and consequently the second term in the expansion series is zero. The total energy $\mathcal{E}_j$ of the particle *j* is written

$$\mathcal{E}_j\left(P_j\right) = \mathcal{E}_j\left(\vec{P}_{0j}\right) + \frac{1}{2}\frac{\mathrm{d}^2\mathcal{E}_j}{\mathrm{d}\vec{P}_j^2}\left(\vec{P}_j - \vec{P}_{0j}\right)^2, \quad j = 1, \cdots, N$$

$$\frac{\mathrm{d}^2\mathcal{E}_j}{\mathrm{d}\vec{P}_j^2} = m_j^{*-1}, \quad j = 1, \cdots, N$$

Now we have $\frac{\mathrm{d}\mathcal{E}_j}{\mathrm{d}\vec{P}_j} = \left(\frac{\partial\mathcal{E}_j}{\partial P_{xj}}, \frac{\partial\mathcal{E}_j}{\partial P_{yj}}, \frac{\partial\mathcal{E}_j}{\partial P_{zj}}\right), \quad j = 1, \cdots, N$

$$\frac{\mathrm{d}^2\mathcal{E}_j}{\mathrm{d}\vec{P}_j^2} = \left(\frac{\mathrm{d}}{\mathrm{d}\vec{P}_j}\frac{\partial\mathcal{E}_j}{\partial P_{xj}}, \frac{\mathrm{d}}{\mathrm{d}\vec{P}_j}\frac{\partial\mathcal{E}_j}{\partial P_{yj}}, \frac{\mathrm{d}}{\mathrm{d}\vec{P}_j}\frac{\partial\mathcal{E}_j}{\partial P_{zj}}\right) = \begin{bmatrix} \frac{\partial^2\mathcal{E}_j}{\partial P_{xj}^2} & \frac{\partial^2\mathcal{E}_j}{\partial P_{yj}\partial P_{xj}} & \frac{\partial^2\mathcal{E}_j}{\partial P_{zj}\partial P_{xj}} \\ \frac{\partial^2\mathcal{E}_j}{\partial P_{xj}\partial P_{yj}} & \frac{\partial^2\mathcal{E}_j}{\partial P_{yj}^2} & \frac{\partial^2\mathcal{E}_j}{\partial P_{zj}\partial P_{yj}} \\ \frac{\partial^2\mathcal{E}_j}{\partial P_{xj}\partial P_{zj}} & \frac{\partial^2\mathcal{E}_j}{\partial P_{yj}\partial P_{zj}} & \frac{\partial^2\mathcal{E}_j}{\partial P_{zj}^2} \end{bmatrix}$$

The elements of the tensor m_j^{*-1} are $m_{i,kj}^{*-1} = \frac{\partial^2\mathcal{E}_j}{\partial P_{ij}\partial P_{kj}}, \quad j = 1, \cdots, N$.

The reciprocal effective mass tensor [2] [7] [8] its elements are the reciprocal mass, and it's 3 × 3 tensor and is defined as above.

The matrix given in Equation (28.1) is termed the effective mass tensor its elements are masses [2] [7] [8], the charged mass tensor given above is the effective mass tensor, so *the effective mass is positive in the minimum and negative in the maximum of the energy.*

The effective mass tensor in diagonal form is written

$$m_j = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

If the matrix m_j is not in a diagonal form it could be converted to the diagonal form by performing mathematical operations on rows and columns of the matrix, the resulting diagonal matrix elements are the eigenvalues of the original matrix.

The general equation for the constant-energy surface is

$$\mathcal{E}_j\left(P_j\right)=\mathcal{E}_{0j}+\frac{\left(P_{xj}-\vec{P}_{0xj}\right)^2}{2m_1}+\frac{\left(P_{yj}-\vec{P}_{0yj}\right)^2}{2m_2}+\frac{\left(P_{zj}-\vec{P}_{0zj}\right)^2}{2m_3}=\text{constant}\,,\quad j=1,\cdots,N$$

$$\frac{\left(P_{xj}-\vec{P}_{0xj}\right)^2}{a^2}+\frac{\left(P_{yj}-\vec{P}_{0yj}\right)^2}{b^2}+\frac{\left(P_{zj}-\vec{P}_{0zj}\right)^2}{c^2}=1\,,\quad j=1,\cdots,N \tag{36}$$

where $a^2=2m_1\left(\mathcal{E}_j-\mathcal{E}_{0j}\right)$, $b^2=2m_2\left(\mathcal{E}_j-\mathcal{E}_{0j}\right)$, $c^2=2m_3\left(\mathcal{E}_j-\mathcal{E}_{0j}\right)$, $j=1,\cdots,N$

Equation (36) is ellipsoid equation with semi-axes of the ellipsoid *a*, *b*, *c*, so the constant-energy surface has an ellipsoid shape. *The form of constant-energy surfaces determines many properties of semiconductors crystals* [8].

2.8. Geometric Representation of Constant Energy Surface-Wave Function

Energy levels—Band width

The importance of determining the constant energy surface [2] lies in the fact that it enables us to determine the different energy levels. There are two models for energy levels: the first is the orbital model, known as the Bohr model, which successfully explains the spectrum of the hydrogen atom. The second is the cloud model, which is based on solving the Schrodinger's equation. This section presents geometric examples of both models, accompanied by diagrams of surfaces such as the sphere, cone, prism, and cylinder that achieve energy constancy.

From Part(I) the total energy $\mathcal{E}_j$ of a particle *j* is given by the following equation [1]

$$\mathcal{E}_j\left(v_j\right)=\frac{1}{2}m_j\left(v_j-\tilde{v}_j\right)^2-\frac{1}{2}m_j\tilde{v}_j^2\,,\quad j=1,\cdots,N \tag{37}$$

From Part (I) we have [1]

$$v_{j,1,2}=\tilde{v}_j\pm d_j\rightarrow d_j^2=\left(v_{j,1,2}-\tilde{v}_j\right)^2\,,\quad j=1,\cdots,N \tag{2}$$

$$\text{in which}\quad d_j=\sqrt{\tilde{v}_j^2+\tilde{v}_{\perp j}^2}\,,\quad \tilde{v}_{\perp j}=\sqrt{\frac{2\mathcal{E}_j}{m_j}}\,,\quad j=1,\cdots,N \tag{3}$$

Rewriting Equation (37) in terms of the deviation velocity d_j^2 and the mean velocity $\tilde{v}_j$ becomes

$$\mathcal{E}_j\left(d_j\right)=\frac{1}{2}m_jd_j^2-\frac{1}{2}m_j\tilde{v}_j^2\,,\quad j=1,\cdots,N \tag{37.1}$$

The geometrical relation between d_j^2 and $\tilde{v}_j^2$ is a right triangle in the **Figure 1**.

The triangle $\Delta P_0P_1P_2$ in **Figure 1** was the main tool to deduce the wave vector and wave function solution of Schrodinger's equation in Part(I) [1]. Another advantage of the triangle method will be demonstrated in this section for determining the constant energy surface of a system of moving particles.

Rotating the triangle $\Delta P_0P_1P_2$ around the center "*O*" creates a circle with constant energy along its circumference, which can represent a wave front associated with a particle moving through space.

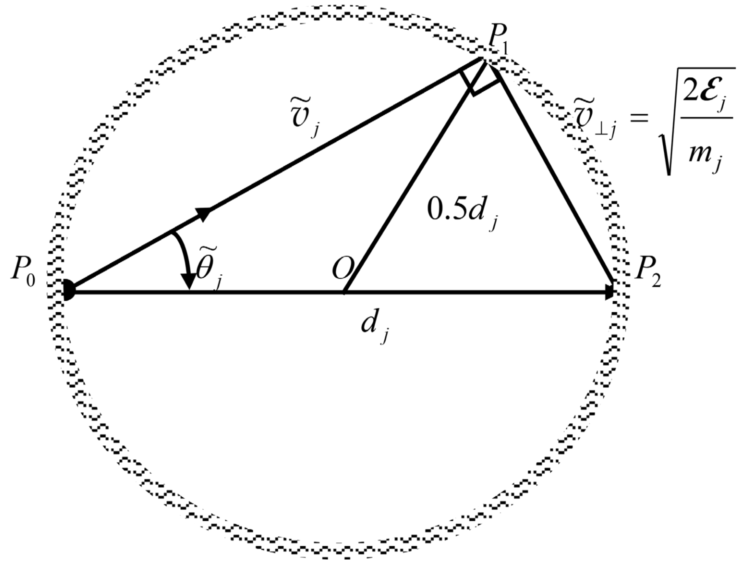


Figure 1. Geometrical representation of Equation (3).

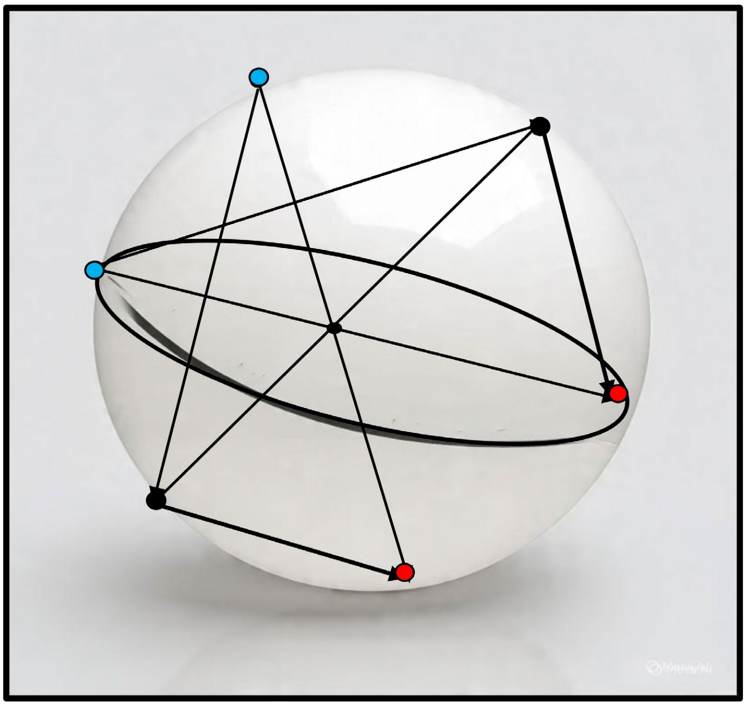


Figure 2. Three-dimensional geometrical representation of Equation (3).

In Figure 2 rotating the circle obtained from rotating the triangle $\Delta P_0P_1P_2$ in three-dimensional space around its diameter P_0P_2 creates a sphere with center "O" and diameter d_j where energy is constant across the entire surface of the sphere. so there is only one energy level which is the entire surface and no band width is this case because band width requires group of energy levels. The energy level is this case *nondegenerate* [3].

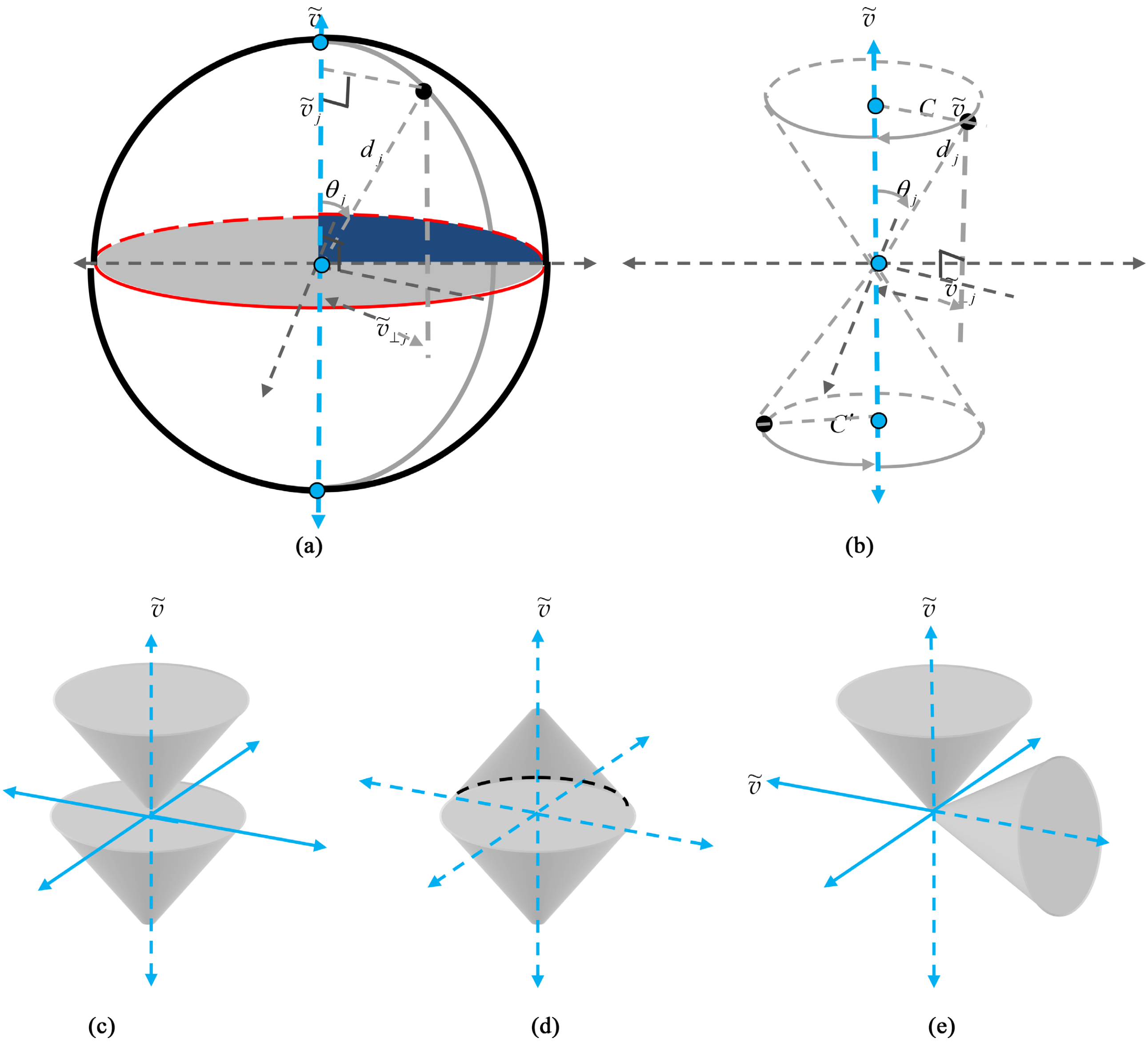


Figure 3. Three-dimensional representation of constant energy surfaces.

The fulfillment of Equation (3) implies that the moving particles are tied to each other. **Figure 3** is a 3-D representation of Equation (3) that illustrates two kinds of ties, constant deviation velocity tie sphere-shaped where d_j is constant for all the particles and it is the radius of the sphere, the other tie is the constant angular deviation tie cone -shaped where θ_j is constant for all the particles and it is the generation angle of the cone.

1) The sphere-shaped tie in **Figure 3(a)** shows the particles are constraint to the surface of a sphere with radius d_j which is the deviation velocity. The particles motion maintain a constant deviation velocity d_j Intersecting the sphere by a horizontal or vertical plane produces a circular orbit corresponding to an energy level where the energy is constant along the circular orbit. The particles which are positioned on the circle equator, where the mean velocity vanishes ($\tilde{v} = 0$), have their total energy reduced to kinetic energy only. It is suggested that these particles may be free electrons or free holes. The particles located at the center of the sphere where $\tilde{v}=0$ and $d_j=0$ are not moving because substituting in Equation (2) gives $v_j=0$ which suggests that they would be lattice ions. For every energy level in the upper semi-sphere exists a corresponding an energy level with same energy in the lower semi-sphere and for every energy level in the right semi-sphere exists a corresponding an energy level with same energy in the left semi-sphere, therefore for a specific value of energy there are four corresponding energy levels

2) The cone-shaped tie in Figure 3(b) shows the particles are constraint to the surface of a double-napped cone with a generating angle θ_j. The particles motion maintains a constant θ_j, the angular deviation from their mean velocity direction. The circles C and C' can be considered to be the trajectory of moving particles and energy levels. The particles located on the vertex of the cone where $\tilde{v}=0$ and $d_j=0$ are not moving because substituting in Equation (2) gives $v_j=0$.

A spin around the $\tilde{v}$ axis with angular speed ω_s is possible in both tie kinds that does not violate Equation (3). We have another angular speed ω_p which is the angular speed of the particle in the orbit. Also Figures 3(c)-(e) illustrates other possible configurations for the cone-shaped tie. For both ties the total energy for any particle lies on the $\tilde{v}$ axis is zero. *i.e.* $\mathcal{E}_j=0$

Important remark 1: The equatorial orbit of a spherical shape is the great circle perpendicular to the axis of rotation. Determining this orbit is essential for determining how it interacts with other shapes, given the presence of free particles in this orbit to form molecules. In the figure, the axis of rotation is in the z-axis, and thus the equatorial orbit is in the x-y plane. If the axis of rotation is in the y-axis, the equatorial orbit is in the x-z plane. In the case of a conical shape, the interaction with other shapes is through contact with those shapes.

Important remark 2: All particles in the cone-shaped tie have the same wave function, which is a solution to the Schrödinger equation. This function is constant for all particles that make up the cloud. In the sphere-shaped tie, only particles that share the same energy level have the same wave function, forming an orbital energy level.

Figure 4 illustrates two energy levels and the energy level number which is the quantum number [3] determined by the number of the antinodes. The amplitude of the wave determines the band width. Figure 4 is the geometrical representation of the equations obtained in section 2.6 in Part(I) refer to it [1].

Figure 5 represents special case when

$$\theta_j=\pm n\pi \quad \text{or} \quad \theta_j=\frac{\pi}{2}\pm n\pi, \quad n=0,1,2,\cdots,\tilde{N}_j \quad \text{(S)}$$

where $\tilde{N}_j$ is the maximum number of values for θ_j.

In this case the mean velocity $\tilde{v}_j$ and deviation velocity d_j for moving particles are parallel or perpendicular to each other, Consequently both the sphere-shaped tie and the cone-shaped tie are transformed to cylindrical shapes or straight line shapes where the particles move along the length of the cylinder to satisfy condition (S) as shown in **Figure 5**. The deviation velocities of the particles acquire a unified direction *i.e.* becomes converted to oscillating velocities along the length of the cylinder or the straight line.

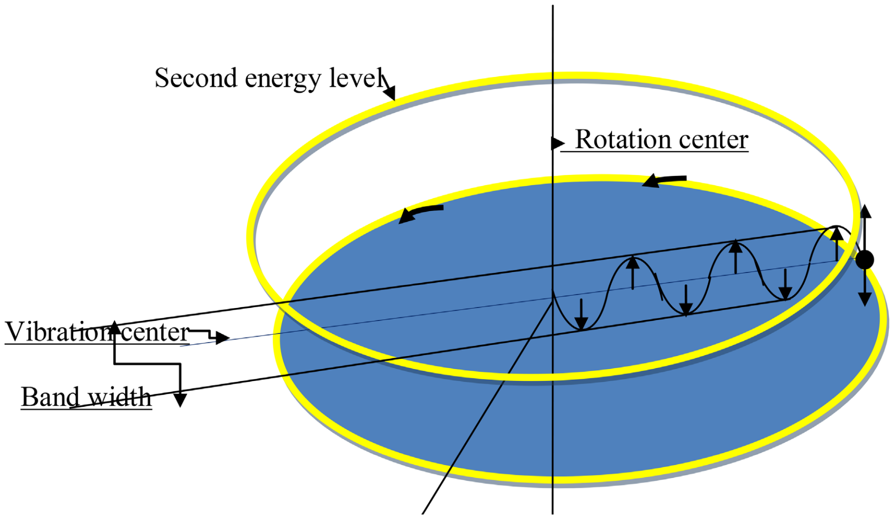


Figure 4. Geometrical representation for energy levels and band width.

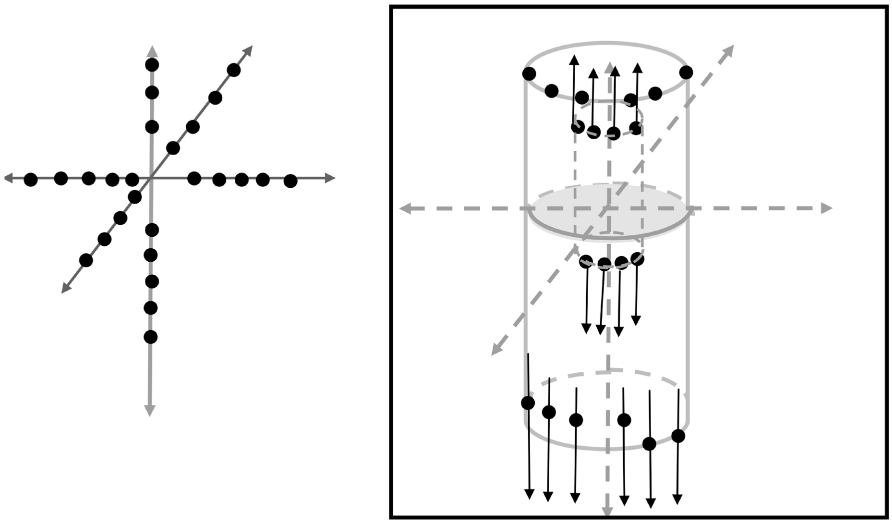


Figure 5. Special three-dimensional representation of Equation (3).

Figure 6 represents prism-shape results due to moving the triangle $\Delta P_0P_1P_2$ parallel to it self

- The red particles represents, the mean velocity vanishes ($\tilde{v}=0$), have their total energy reduced to kinetic energy only, It is suggested that these particles may be free electrons or free holes move along the edges.
- The blue particles represents $\tilde{v}=0$ and $d_j=0$ are not moving because substituting in Equation (2) gives $v_j=0$., which suggests that they would be lattice ions

The black particles it is suggested that these particles may be electrons or holes move along the edges.

Using the triangle method enable us to compose any crystal shape geometrically, since the triangle is the smallest geometrical unit a sample for a cubical crystal is given in Figure 6.

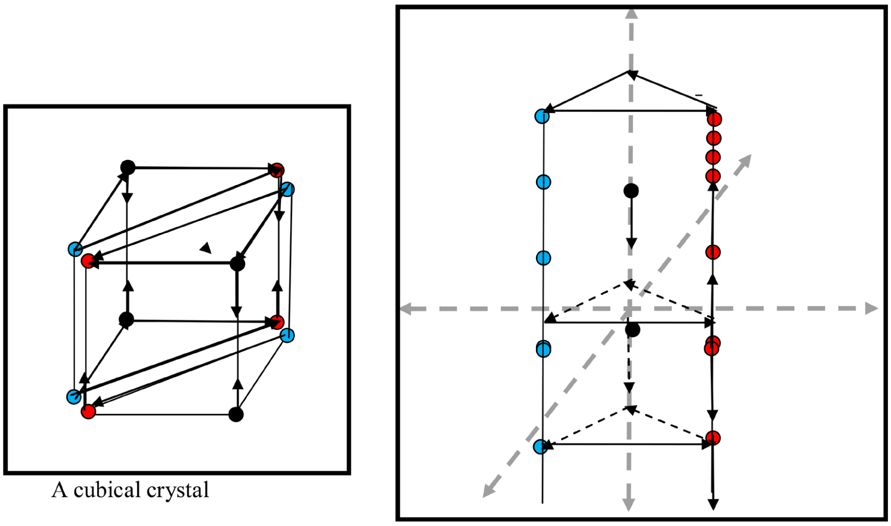


Figure 6. Three-dimensional represents translation of the triangle $\Delta P_0P_1P_2$.

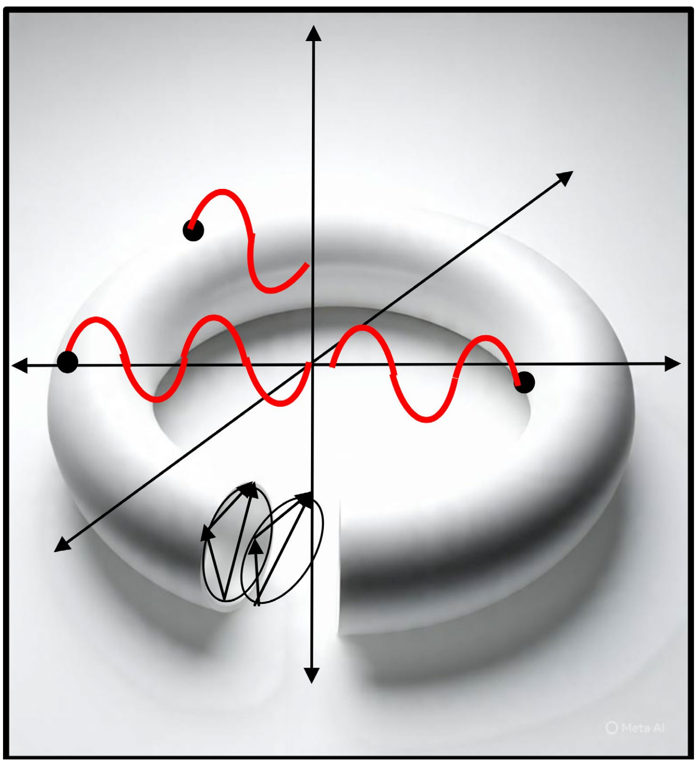


Figure 7. Three-dimensional shape represents translation of the circle from Figure 1 in a close loop.

Figure 7 illustrates a translation of the circle generated in Figure 1 along a closed loop generating a torus-shaped constant energy surface, the entire torus surface correspond to one energy level with the same wave function solution of Schrödinger equation [1] and the band width is the thickness of the torus. Because each energy level corresponds to one torus, so each level has a separate angular speed ω_s which is the angular speed of the torus.

2.9. Grand Potentials Energies—Gravity Wave—Quantum Gravity

In this section, the inverse laws namely Newton's Law of Universal Gravitation, Coulomb's Law, and Inverse Square Law of Electromagnetic Waves Intensity [4]-[6] will be compared with the derived laws obtained from the general format of the potential energy deduced in Part(I)

The steps for deriving all of these laws are very similar, as we will see in the following lines.

Starting we have from Section 2.4 Equation (15.1) is given as follows

$$\frac{\mathrm{d}s_j}{\mathrm{d}t} = \tilde{v}_j\left(1 \pm \frac{1}{\cos\theta_j}\right) \to \mathrm{d}s_j = \tilde{v}_j\left(1 \pm \frac{1}{\cos\theta_j}\right)\mathrm{d}t\,, \quad j = 1,\cdots,N \tag{15.1}$$

Now let $f_j = \left(1 \pm \frac{1}{\cos\theta_j}\right)$ and the function F_j is related to f_j by the following relation

$$f_j = F_j' \to \int f_j \mathrm{d}t = F_j\,, \quad j = 1,\cdots,N \tag{38}$$

Using the relation (38), Equation (15.1) is rewritten as follows

$$v_j = \tilde{v}_j F_j'\,, \quad j = 1,\cdots,N \tag{15.2}$$

Substituting the above relations into Equation (15.1) we get

$$s_j = \int \mathrm{d}s_j = \tilde{v}_j \int f_j \mathrm{d}t = \tilde{v}_j F_j \to \tilde{v}_j^2 = \frac{s_j^2}{F_j^2}\,, \quad j = 1,\cdots,N \tag{15.3}$$

From section 2.4 Equation (16) gives the potential energy V_j for a particle j is written

$$V_j = -m_j \tilde{v}_j^2\left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1,\cdots,N \tag{16}$$

Using the relation (38) by Substituting in Equation (16) above gives

$$V_j = -m_j \tilde{v}_j^2 f_j = -m_j \tilde{v}_j^2 F_j'\,, \quad j = 1,\cdots,N \tag{16.1}$$

Substituting in Equation (16.1) from relation (15.3) we get

$$V_j = -m_j \frac{s_j^2}{F_j^2} F_j' = -m_j s_j^2 \frac{F_j'}{F_j^2}\,, \quad j = 1,\cdots,N \tag{16.2}$$

Now we will determine the value of the ratio $\frac{F_j'}{F_j^2}$ by the following procedures

Let us consider the polar coordinates R_j and ϕ_j for particle *j* where R_j is the distance between the origin and the location of particle *j* and ϕ_j is the deviation angel from the horizontal direction, consequently ϕ_j' is the angular velocity of particle *j* we write Equation (15.2) as follows

$$v_j = \tilde{v}_j F_j' \to R_j \phi_j' = \tilde{v}_j F_j' \quad i.e. \quad j = 1, \cdots, N, \quad F_j' = \frac{R_j \phi_j'}{\tilde{v}_j} \tag{39}$$

R_j is a function in time. *i.e.* $R_j(t)$ it is not function in ϕ_j so we can perform the integration of both sides of Equation (39) as follows

$$\int R_j \mathrm{d}\phi_j = \int \tilde{v}_j \mathrm{d}F \to R_j \phi_j = \tilde{v}_j F_j \to F_j = \frac{R_j \phi_j}{\tilde{v}_j}, \quad j = 1, \cdots, N \tag{40}$$

Using the value of F_j' obtained from Equation (39) and the value of F_j obtained from Equation (40) we compose the ratio $\frac{F_j'}{F_j^2}$ given as follows

$$\frac{F_j'}{F_j^2} = \frac{R_j \phi_j'}{\tilde{v}_j} \frac{\tilde{v}_j^2}{R_j^2 \phi_j^2} = \frac{\varphi_j' \tilde{v}_j}{\phi_j^2} \frac{1}{R_j}, \quad j = 1, \cdots, N \tag{41}$$

By substituting with the ratio given by the relation (41) into Equation (16.2) to get the potential energy V_j as follows

$$V_j = -m_j s_j^2 \frac{F_j'}{F_j^2} = -\frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2} \frac{m_j}{R_j}, \quad j = 1, \cdots, N \tag{16.3}$$

Newton's Law of Universal Gravitation

The potential energy for Newton's Universal Gravitation is given as [4] [6]

$$V_j = G \frac{M_j m_j}{R_j}, \quad j = 1, \cdots, N \tag{42}$$

where G is a universal constant of the gravity and M_j is the mass at the origin and R_j is the distance between M_j and m_j.

Comparing Equation (16.3) with Equation (42) we get

$$-\frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2} = M_j G, \quad j = 1, \cdots, N \tag{43}$$

The displacement s_j of particle *j* in terms of polar coordinates is given by the relation $s_j = R_j \phi_j$. *i.e.*

$$s_j^2 = R_j^2 \phi_j^2, \quad j = 1, \cdots, N$$

Substitute from the above relation into the term $\frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2}$ given by Equation (43) we get

$$-\frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2} = -\frac{\phi_j' \tilde{v}_j R_j^2 \phi_j^2}{\phi_j^2} = -\phi_j' \tilde{v}_j R_j^2, \quad j = 1, \cdots, N$$

Let $\tilde{\phi}_j' = -\phi_j', \quad j = 1, \cdots, N$

Equation (43) becomes

$$\frac{\tilde{\phi}_j' \tilde{v}_j s_j^2}{\phi_j^2} = \frac{\tilde{\phi}_j' \tilde{v}_j R_j^2 \phi_j^2}{\phi_j^2} = M_j G \to M_j G = \tilde{\phi}_j' \tilde{v}_j R_j^2, \quad j = 1, \cdots, N \tag{43.1}$$

From Part(I) the momentum p_j for a moving particle j is connected to a wave length λ_j by the relation

$$p_j = m_j \tilde{v}_j = \frac{h_G}{\lambda_{Gj}}, \quad j = 1, \cdots, N \tag{44}$$

where h_G is a constant describes the motion for big objects, corresponding to Planck's constant h which is appropriate to describe the motion of tiny particles.

Consider the following equalities

$$2R_j = n_{Gj}\lambda_{Gj} \to 4R_j^2 = n_{Gj}^2\lambda_{Gj}^2 \to R_j^2 = \frac{n_{Gj}^2}{4}\lambda_{Gj}^2, \quad j = 1, \cdots, N \tag{45}$$

$$\text{Let } M_j = N_{m_j} m_j, \quad j = 1, \cdots, N \tag{46}$$

If $M_j \ge m_j$ then $N_{m_j} \ge 1$ and if $M_j \le m_j$ then $N_{m_j} \le 1$

Substituting from relation (44) and equalities (45) and (46) into Equation (43.1) gives

$$M_j G = \tilde{\phi}_j' N_{m_j} m_j \tilde{v}_j R_j^2 = \tilde{\phi}_j' N_{m_j} \frac{h_G}{\lambda_G} \frac{n_{Gj}^2}{4} \lambda_{Gj}^2 = N_{m_j} \frac{n_{Gj}^2}{4} \tilde{\phi}_j' h_G \lambda_{Gj}, \quad j = 1, \cdots, N \tag{43.2}$$

From equality (46) $m_j = \frac{M_j}{N_{m_j}}$ using this value for m_j we rewrite Equation (43.2) as follows

$$\lambda_{Gj} = \frac{4m_j}{\tilde{\phi}_j' n_{Gj}^2 h_G} G, \quad j = 1, \cdots, N \tag{47}$$

Relation (47) indicates that the wave length of the gravity wave is linearly proportional to G the universal constant of the gravity and the proportional constant is $\frac{4m_j}{\tilde{\phi}_j' n_{Gj}^2 h_G}$, m_j is the mass of a big object for instance (the earth, the moon, a planet).

A suggested values for n_{Gj} associated to the solar system planets where m_j is the mass of the planet.

In the following list the number next to the planet's name is the suggested value for n_{Gj} *i.e.* $n_{Gj} = 1, 2, \cdots, 8$ or multiples of the suggested value.

1) Mercury

2) Venus

3) Earth

4) Mars

5) Jupiter

6) Saturn

7) Uranus

8) Neptune

The suggested values for the quantum numbers to planets in the solar system are proposed as an energy levels numbers arranged according to their distance order from the sun which is the center of rotation of the entire solar system analogous to Bohr's orbital model for atom, and since the derivation origin for both of them is the same the same principle could be adopted. These proposed values are hint to start with. The experimental results are the deciding factor regarding the correct values of the quantum numbers for the solar system.

In case m_j is the mass of the moon $n_{Gj} = 1$.

For quantum gravity relation (47) becomes

$$\lambda_j = \frac{4m_j}{\tilde{\phi}'_j n_j^2 h} G, \quad j = 1, \cdots, N \tag{48}$$

In relation (48) m_j is the mass of a tiny particle for instance a O_2, CO_2, N_2, H_2O, NaCl, + Mg_2... molecules or ions already existing in the planet's atmosphere or the oceans

From Equation (43.1) we get the force F_{Mm} acting on the mass m_j written

$$\frac{M_j G}{R_j^2} = -\phi'_j \tilde{v}_j \to \phi'_j m_j \tilde{v}_j = -\frac{M_j m_j G}{R_j^2} = F_{Mm}, \quad j = 1, \cdots, N \tag{49}$$

<u>From Equation (49) some useful relationships will be deduced</u>

From relation (39) we have

$$v_j = R_j \phi'_j \to \phi'_j = \frac{v_j}{R_j}, \quad j = 1, \cdots, N \tag{39.1}$$

Substituting from the relation (39.1) into Equation (49) we get

$$\frac{M_j G}{R_j^2} = -\frac{v_j}{R_j} \tilde{v}_j \to \frac{M_j G}{R_j} = -\tilde{v}_j v_j, \quad j = 1, \cdots, N \tag{49.1}$$

$$v_j = \tilde{v}_j F'_j, \quad j = 1, \cdots, N \tag{15.2}$$

Substituting from the relation (15.2) into Equation (49.1) we get

$$\frac{M_j G}{R_j} = -\tilde{v}_j v_j = -\tilde{v}_j^2 F'_j \to R_j = -\frac{M_j G}{\tilde{v}_j^2 F'_j}, \quad j = 1, \cdots, N \tag{49.2}$$

R_j is a solution of Kepler's problem, given by the following formula [4] [6]

$$R_j = \frac{\tilde{e}_j \tilde{d}_j}{1 + \tilde{e}_j \cos\left(\phi'_j t\right)}, \quad j = 1, \cdots, N \tag{50}$$

Equating formula (50) with Equation (49.2) and rewrite we obtain

$$R_j = \frac{\tilde{e}_j \tilde{d}_j}{1 + \tilde{e}_j \cos\left(\phi'_j t\right)} = -\frac{M_j G}{\tilde{v}_j^2 F'_j} \to F'_j = -\frac{M_j G}{\tilde{v}_j^2 \tilde{e}_j \tilde{d}_j}\left(1 + \tilde{e}_j \cos\left(\phi'_j t\right)\right), \quad j = 1, \cdots, N \tag{51}$$

Integrating both sides of (51) gives

$$F_j = -\frac{M_j G}{\tilde{v}_j^2 \tilde{e}_j \tilde{d}_j}\left(t + \frac{1}{\phi'} \tilde{e}_j \sin\left(\phi'_j t\right)\right) + F_{0j}, \quad j = 1, \cdots, N \tag{52}$$

Now we have Equation (15.3) obtained previously as

$$s_j = \int \mathrm{d}s_j = \tilde{v}_j \int f_j \mathrm{d}t = \tilde{v}_j F_j, \quad j = 1, \cdots, N \tag{15.3}$$

From Equation (15.3) and Equation (52) we get

$$s_j = \tilde{v}_j F_j = -\frac{M_j G}{\tilde{v}_j \tilde{e}_j \tilde{d}_j}\left(t + \frac{1}{\phi'}\tilde{e}_j \sin\left(\phi_j' t\right)\right) + \tilde{v}_j F_{0j}, \quad j = 1, \cdots, N \tag{53}$$

Rewriting Equation (53) above we obtain the following

$$\left(t + \frac{1}{\phi'}\tilde{e}_j \sin\left(\phi_j' t\right)\right) = \frac{\tilde{v}_j \tilde{e}_j \tilde{d}_j}{M_j G}\left(\tilde{v}_j F_{0j} - s_j\right), \quad j = 1, \cdots, N \tag{53.1}$$

The Equations (51)-(53) are useful equations for farther other investigation researches.

We can rewrite Equation (49.1) which expresses the gravitational force as follows

$$\frac{M_j G}{R_j^2} = -\frac{v_j}{R_j}\tilde{v}_j \to F_{Mm} = -\frac{\tilde{v}_j}{R_j} m_j v_j, \quad j = 1, \cdots, N \tag{54}$$

Since $F_{Mm} = m_j \frac{\mathrm{d}v_j}{\mathrm{d}t}$ we can rewrite Equation (54) as follows

$$F_{Mm} = m_j \frac{\mathrm{d}v_j}{\mathrm{d}t} = -\frac{\tilde{v}_j}{R_j} m_j v_j \to \frac{1}{v_j}\mathrm{d}v_j = -\frac{\tilde{v}_j}{R_j}\mathrm{d}t, \quad j = 1, \cdots, N \tag{55}$$

Substituting from formula (50) which gives the value of R_j into Equation (55) we get

$$\frac{1}{v_j}\mathrm{d}v_j = -\frac{\tilde{v}_j}{\tilde{e}_j \tilde{d}_j}\left(1 + \tilde{e}_j \cos\left(\phi_j' t\right)\right)\mathrm{d}t, \quad j = 1, \cdots, N \tag{56}$$

Integrating both sides of Equation (56) we obtain

$$\ln v_j = -\frac{\tilde{v}_j}{\tilde{e}_j \tilde{d}_j}\left(t + \frac{1}{\phi'}\tilde{e}_j \sin\left(\phi_j' t\right)\right) + \tilde{s}_{0j}, \quad j = 1, \cdots, N \tag{57}$$

Substituting from Equation (53.1) into Equation (57) above we get

$$\ln v_j = -\frac{\tilde{v}_j}{\tilde{e}_j \tilde{d}_j}\frac{\tilde{v}_j \tilde{e}_j \tilde{d}_j}{M_j G}\left(\tilde{v}_j F_{0j} - s_j\right) + \tilde{s}_{0j}, \quad j = 1, \cdots, N \tag{57.1}$$

Now consider the following equalities

$$\tilde{\gamma}_{1j} = -\frac{\tilde{v}_j^2}{M_j G} \quad \text{and} \quad \tilde{\gamma}_{2j} = -\frac{\tilde{v}_j^3}{M_j G}F_{0j}, \quad j = 1, \cdots, N \tag{58}$$

Using the equalities (58) in Equation (57.1) and rewritten (57.1) as follows

$$\ln v_j = \left(\tilde{\gamma}_{2j} - \tilde{\gamma}_{1j} s_j\right) + \tilde{s}_{0j} = -\tilde{\gamma}_{1j} s_j + \left(\tilde{s}_{0j} + \tilde{\gamma}_{2j}\right), \quad j = 1, \cdots, N \tag{57.2}$$

Rewrite Equation (57.2) we get

$$v_j = \mathrm{e}^{\sigma_j - \tilde{\gamma}_{1j} s_j}, \quad \sigma_j = \left(\tilde{s}_{0j} + \tilde{\gamma}_{2j}\right), \quad j = 1, \cdots, N \tag{59}$$

$$\mathrm{d}s_j = \mathrm{e}^{\sigma_j - \tilde{\gamma}_{1j} s_j}\mathrm{d}t, \quad j = 1, \cdots, N \tag{59.1}$$

Rearranging (59.1) we get

$$e^{\tilde{\gamma}_{1j}s_j-\sigma_j}\,\mathrm{d}s_j=\mathrm{d}t\,,\quad j=1,\cdots,N \tag{60}$$

Integrating both sides of (60) results

$$\frac{1}{\tilde{\gamma}_{1j}}e^{\tilde{\gamma}_{1j}s_j-\sigma_j}=t+\tilde{t}_0\,,\quad j=1,\cdots,N \tag{61}$$

Rearranging (61) we get

$$\left(\tilde{\gamma}_{1j}s_j-\sigma_j\right)=\ln\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right),\quad j=1,\cdots,N \tag{61.1}$$

$$s_j=\frac{1}{\tilde{\gamma}_{1j}}\left(\ln\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right)+\sigma_j\right),\quad j=1,\cdots,N \tag{62}$$

Equation (62) can be used to determine the length of the ellipse perimeter.

From (62) The argument of function ln must be positive because that function is not defined for negative domain values. *i.e.*

$$\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right)\ge 0\,,\quad j=1,\cdots,N \tag{63}$$

We already know that $\tilde{\gamma}_{1j}<0$ so $\left(t+\tilde{t}_0\right)<0$ in order to keep satisfying the inequality (63) and this leads to the validity condition for the formula (62) given by the following inequality

$$t<-\tilde{t}_0\,,\quad j=1,\cdots,N \tag{64}$$

Since s_j is a length so it must be positive. *i.e.* the right hand side of the formula (62) is positive either and this results that the term $\ln\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right)+\sigma_j$ is negative because $\tilde{\gamma}_{1j}<0$ too, consequently

$$\ln\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right)+\sigma_j<0\rightarrow\ln\tilde{\gamma}_{1j}\left(t+\tilde{t}_0\right)<-\sigma_j$$

Coulomb's Law and atom's spectrum

The potential energy for Coulomb's Law is given as [2] [8]

$$V_j=\frac{1}{4\pi\kappa}\frac{q_{1j}q_{2j}}{R_j}\,,\quad j=1,\cdots,N \tag{56}$$

where κ is defined as vacuum permittivity or dielectric constant which is a universal constant and q_{1j} is the charge at the origin and R_j is the distance between q_{1j} and q_{2j}.

Comparing Equation (16.3) with Equation (56) we get

$$-\frac{m_j\phi_j'\tilde{v}_js_j^2}{\phi_j^2}=\frac{q_{1j}q_{2j}}{4\pi\kappa}\,,\quad j=1,\cdots,N \tag{66}$$

Coulomb's Law represents an electric force field has the same form as a gravitational field both of them is an inverse square field. Therefore, we can perform the same sequence of derivation steps as preformed before with the gravitational field and thus obtain the following result:

$$\frac{q_{1j}q_{2j}}{4\pi\kappa}=m_j\tilde{\phi}_j'\tilde{v}_jR_j^2\,,\quad j=1,\cdots,N \tag{67}$$

From Part(I) the momentum p_j for a moving particle j is connected to a wave length λ_j by the relation

$$p_j = m_j \tilde{v}_j = \frac{h}{\lambda_j}, \quad j = 1, \cdots, N \tag{68}$$

where h is Planck's constant which is appropriate to describe the motion of tiny particles.

For an atom the quantity $q_{1j} q_{2j}$ is replaced by $Ze.e.$ *i.e.* $q_{1j} q_{2j} = Ze_j^2$ where Z is the atomic number of the chemical element and Ze is the charge of the electron [2] [8]. Equation (67) becomes

$$\frac{Ze_j^2}{4\pi\kappa} = m_j \tilde{\phi}_j' \tilde{v}_j R_j^2, \quad j = 1, \cdots, N \tag{67.1}$$

Consider the following equality

$$2R_j = n_j \lambda_j \to 4R_j^2 = n_j^2 \lambda_j^2 \to R_j^2 = \frac{n_j^2}{4} \lambda_j^2, \quad j = 1, \cdots, N \tag{69}$$

Substituting from relation (68) and equality (69) into Equation (67.1) gives

$$\frac{Ze_j^2}{4\pi\kappa} = \tilde{\phi}_j' m_j \tilde{v}_j R_j^2 = \tilde{\phi}_j' \frac{h}{\lambda_j} \frac{n_j^2}{4} \lambda_j^2 = \frac{n_j^2}{4} \tilde{\phi}_j' h \lambda_j, \quad j = 1, \cdots, N \tag{67.2}$$

Rewrite Equation (67.2) above as follows

$$\lambda_j = \frac{Ze_j^2}{n_j^2 \pi \tilde{\phi}_j' h \kappa}, \quad j = 1, \cdots, N \tag{70}$$

In order to compare the result obtained in Equation (70) with the result obtained by Bohr's method and Schrodinger's equation we will determine the corresponding energy levels to λ_j given by Equation (70) in order to do that the following steps are implemented.

Using the relations (39.1) and the relation $v_{2j} = 2\tilde{v}_j - v_{1j}$ we express $\tilde{\phi}'$ as follows

$$\tilde{\phi}' = \frac{v_{2j}}{R_j} = \frac{2\tilde{v}_j - v_{1j}}{R_j} = \frac{2\left(2\tilde{v}_j - v_{1j}\right)}{n_j \lambda_j} = \frac{2\tilde{v}_j \left(2 - \frac{v_{1j}}{\tilde{v}_j}\right)}{n_j \lambda_j}, \quad j = 1, \cdots, N \tag{71}$$

Consider the following equalities

$$\tilde{\gamma}_{0j} = \frac{v_{1j}}{\tilde{v}_j}, \quad \tilde{\gamma}_{1j} = \left(1 - \tilde{\gamma}_{0j}\right), \text{ and } \tilde{\gamma}_{2j} = \left(\tilde{\gamma}_{1j} + 1\right) = \left(2 - \tilde{\gamma}_{0j}\right), \quad j = 1, \cdots, N \tag{72}$$

Substituting from equalities (72) into the relation (71) we get

$$\tilde{\phi}' = \frac{2\tilde{v}_j \left(2 - \frac{v_{1j}}{\tilde{v}_j}\right)}{n_j \lambda_j} = \frac{2\tilde{\gamma}_{2j} \tilde{v}_j}{n_j \lambda_j}, \quad j = 1, \cdots, N \tag{71.1}$$

The equation of the total energy of the particle j is written as

$$\begin{aligned} \mathcal{E}_j\left(v_j\right) &= \frac{1}{2} m_j \left(v_{2j} - \tilde{v}_j\right)^2 - \frac{1}{2} m_j \tilde{v}_j^2 \\ &= \frac{1}{2} m_j \left(2\tilde{v}_j - v_{1j} - \tilde{v}_j\right)^2 - \frac{1}{2} m_j \tilde{v}_j^2, \quad j = 1, \cdots, N \end{aligned}$$

$$\begin{aligned}\mathcal{E}_j\left(v_j\right) &= \frac{1}{2}m_j\left(\tilde{v}_j - v_{1j}\right)^2 - \frac{1}{2}m_j\tilde{v}_j^2 \\ &= \frac{1}{2}m_j\tilde{v}_j^2\left(1-\frac{v_{1j}}{\tilde{v}_j}\right)^2 - \frac{1}{2}m_j\tilde{v}_j^2 \\ &= \frac{1}{2}m_j\tilde{v}_j^2\left(\tilde{\gamma}_{1j}^2 - 1\right),\quad j=1,\cdots,N\end{aligned}$$

$$\mathcal{E}_j\left(p_j\right) = \frac{p_j^2}{2m_j}\left(\tilde{\gamma}_{1j}^2-1\right) = \frac{p_j^2}{2m_j}\left(\tilde{\gamma}_{1j}-1\right)\left(\tilde{\gamma}_{1j}+1\right),\quad j=1,\cdots,N$$

$$\mathcal{E}_j\left(p_j\right) = -\frac{v_{1j}}{\tilde{v}_j}\frac{p_j^2}{2m_j}\left(2-\frac{v_{1j}}{\tilde{v}_j}\right) = -\tilde{\gamma}_{0j}\tilde{\gamma}_{2j}\frac{p_j^2}{2m_j},\quad j=1,\cdots,N \tag{73}$$

Now we go back to the Equation (70) and use relation (71.1) we get

$$\lambda_j = \frac{Ze_j^2}{n_j^2\pi\tilde{\phi}_j' h\kappa} \to \lambda_j\tilde{\phi}_j' = \frac{Ze_j^2}{n_j^2\pi h\kappa} = \frac{2\tilde{v}_j}{n_j}\tilde{\gamma}_{2j},\quad j=1,\cdots,N \tag{70.1}$$

Multiply both sides of (70.1) by m_j we get

$$\frac{Ze_j^2}{n_j\pi h\kappa}m_j = 2m_j\tilde{v}_j\tilde{\gamma}_{2j} = 2p_j\tilde{\gamma}_{2j},\quad j=1,\cdots,N \tag{70.2}$$

Square both sides of (70.2) and write

$$\begin{aligned}\left(\frac{Ze_j^2}{\pi\kappa}\right)^2\frac{m_j}{h^2}\frac{1}{n_j^2} &= 4\frac{p_j^2}{m_j}\left(\tilde{\gamma}_{2j}\right)^2 = 8\left(\tilde{\gamma}_{2j}\right)\left(\tilde{\gamma}_{2j}\right)\frac{p_j^2}{2m_j} \\ \to \tilde{\gamma}_{2j}\frac{p_j^2}{2m_j} &= \frac{1}{8\tilde{\gamma}_{2j}}\left(\frac{Ze_j^2}{\pi\kappa}\right)^2\frac{m_j}{h^2}\frac{1}{n_j^2},\quad j=1,\cdots,N\end{aligned} \tag{74}$$

Substituting from Equation (74) into Equation (73) we get

$$\mathcal{E}_{nj} = -\tilde{\gamma}_{0j}\tilde{\gamma}_{2j}\frac{p_j^2}{2m_j} = -\frac{\tilde{\gamma}_{0j}}{8\tilde{\gamma}_{2j}}\left(\frac{Ze_j^2}{\pi\kappa}\right)^2\frac{m_j}{h^2}\frac{1}{n_j^2} = -\frac{\tilde{\gamma}_{0j}}{\tilde{\gamma}_{2j}\pi^2}\frac{Z^2m_je^4}{8\kappa^2h^2}\frac{1}{n_j^2},\quad j=1,\cdots,N \tag{75}$$

The results obtained by Bohr's method which is based on equating the available acting forces which are the electrostatic of attraction versus the centripetal force and use the postulate $mvR_n = n\hbar$, and Schrodinger's method which is based on solving Schrodinger's equation, both methods result the same formula expressing the energy levels for an atom with single electron and single proton the formula is given as follows [2]

$$\mathcal{E}_n = -\left(\frac{Ze^2}{4\pi\kappa}\right)^2\frac{m_j}{2\hbar^2}\frac{1}{n_j^2} = -\frac{Z^2m_je^4}{8\kappa^2h^2}\frac{1}{n_j^2} \tag{76}$$

Bohr's and Schrodinger's formula (76) [2] is a special case for formula (75) obtained due the successful merger of classical mechanics to the quantum theory the formula given by Equation (75) can fit N particles atoms not just one atom with single electron.

$$\text{let the ratio } S_{aj} = \frac{\tilde{\gamma}_{0j}}{\tilde{\gamma}_{2j}\pi^2}, \quad j = 1,\cdots,N \tag{77}$$

Let $S_{aj} = 1$ then the energy given by the relation (75) becomes the same obtained by Bohr's and Schrodinger's method formula (76) for hydrogen atom with single electron and $Z = 1$.

The ratio S_{aj} given by the relation (77) is called Spectra Positioning Ratio (SPR) can be used to determine the wave length of the emission lines spectra for various elements considering the hydrogen atom energy as a referring point for the rest atoms in the periodical table.

We make comparison between the permitted radii obtained by Bohr's method and the permitted radii result form the implemented method in this article.

We use the relation $m_j\tilde{\upsilon}_j = \frac{h}{\lambda_j} \to \frac{\tilde{\upsilon}_j}{\lambda_j} = \frac{h}{m_j\lambda_j^2}$ to substitute in relation (71.1) and write

$$\tilde{\phi}' = \frac{2\tilde{\gamma}_{2j}\tilde{\upsilon}_j}{n_j\lambda_j} = \frac{2h}{n_j m_j \lambda_j^2}\tilde{\gamma}_{2j}, \quad j = 1,\cdots,N \tag{71.1}$$

Now substitute into relation (70) from relation (71.1) given above we get

$$\begin{aligned} \lambda_j &= \frac{Ze_j^2}{n_j^2\pi\tilde{\phi}_j' h\kappa} = \frac{Zm_je_j^2}{2n_j\pi h^2\kappa}\frac{\lambda_j^2}{\tilde{\gamma}_{2j}} \to \frac{1}{\lambda_j} = \frac{Zm_je_j^2}{2\tilde{\gamma}_{2j}\pi h^2\kappa}\frac{1}{n_j} \\ &\to \lambda_j = \frac{2\tilde{\gamma}_{2j}\pi h^2\kappa}{Zm_je_j^2}n_j, \quad j = 1,\cdots,N \end{aligned} \tag{71.3}$$

In order to determine the permitted radii R_{n_j}, multiply both sides of (71.3) by $\frac{n_j}{2}$ we get

$$R_{n_j} = \frac{n_j}{2}\lambda_j = \frac{\tilde{\gamma}_{2j}\pi h^2\kappa}{Zm_je_j^2}n_j^2, \quad j = 1,\cdots,N \tag{71.4}$$

Now we calculate the value of $\tilde{\gamma}_{2j}$ for hydrogen atom where $S_{aj} = 1$, so relation (77) becomes $S_{aj} = \frac{\tilde{\gamma}_{0j}}{\tilde{\gamma}_{2j}\pi^2} = 1$ and use the relation $\tilde{\gamma}_{2j} = \left(2 - \tilde{\gamma}_{0j}\right)$ we get

$$S_{aj} = \frac{\tilde{\gamma}_{0j}}{\left(2 - \tilde{\gamma}_{0j}\right)\pi^2} = 1 \to \tilde{\gamma}_{0j} = \pi^2\left(2 - \tilde{\gamma}_{0j}\right) = 2\pi^2 - \pi^2\tilde{\gamma}_{0j} \to \tilde{\gamma}_{0j} = \frac{2\pi^2}{1+\pi^2}$$

$$\tilde{\gamma}_{2j} = 2 - \frac{2\pi^2}{1+\pi^2} = \frac{2 + 2\pi^2 - 2\pi^2}{1+\pi^2} = \frac{2}{1+\pi^2}$$

Substitute with the value of $\tilde{\gamma}_{2j}$ into relation (71.4) we get

$$R_{n_j} = \frac{n_j}{2}\lambda_j = \frac{\tilde{\gamma}_{2j}\pi h^2\kappa}{Zm_je_j^2}n_j^2 = \frac{2\pi h^2\kappa}{Zm_je_j^2}\frac{n_j^2}{1+\pi^2} = \frac{1}{1+\pi^2}\frac{2\pi\hbar^2\kappa}{Zm_je_j^2}n_j^2, \quad j = 1,\cdots,N \tag{71.5}$$

The relation (71.5) gives the permitted radii for hydrogen atom using the method developed in this article

The permitted radii for hydrogen atom using Bohr's method is $R_n = \frac{4\pi\hbar^2\kappa}{Zm_j e^2} n^2$ there is a factor $\frac{1}{2\left(1+\pi^2\right)}$ which requires explanation.

Suggested explanations are:

1) The term n^2 its exact value is $\frac{n^2}{2\left(1+\pi^2\right)}$ the reason is that Bohr's postulate $mvR_n = n\hbar$.

Used to deduce the permitted radii for hydrogen atom as given above is not accurate.

2) The circumference of the path it not circular and it can be enclosed within two circles with different radii.

Another example is carbon atom where the atomic number $Z = 6$.

We calculate $\tilde{\gamma}_{2j}$ for carbon atom as the same steps followed in the hydrogen atom

$$S_{aj} = \frac{\tilde{\gamma}_{0j}}{\left(2-\tilde{\gamma}_{0j}\right)\pi^2} = 6 \to \tilde{\gamma}_{0j} = 6\pi^2\left(2-\tilde{\gamma}_{0j}\right) = 12\pi^2 - 6\pi^2\tilde{\gamma}_{0j} \to \tilde{\gamma}_{0j} = \frac{12\pi^2}{1+6\pi^2}$$

$$\tilde{\gamma}_{2j} = 2 - \frac{12\pi^2}{1+6\pi^2} = \frac{2+12\pi^2-12\pi^2}{1+6\pi^2} = \frac{2}{1+6\pi^2}$$

Substitute with the value of $\tilde{\gamma}_{2j}$ for the carbon atom into relation (71.4) we get

$$R_{n_j} = \frac{n_j}{2}\lambda_j = \frac{\tilde{\gamma}_{2j}\pi h^2\kappa}{Zm_j e_j^2} n_j^2 = \frac{2\pi h^2\kappa}{Zm_j e_j^2}\frac{n_j^2}{\left(1+6\pi^2\right)} = \frac{1}{2\left(1+6\pi^2\right)}\frac{4\pi\hbar^2\kappa}{Zm_j e_j^2} n_j^2, \quad j = 1,\cdots,N$$

The general formula which determine the permitted radii for any element in the periodic table is given as follows

$$Rl_{n_j} = \frac{1}{2\left(1+l\pi^2\right)}\frac{4\pi\hbar^2\kappa}{Zm_j e_j^2} n_j^2, \quad j = 1,\cdots,N \tag{71.6}$$

where Rl_{n_j} are the permitted radii for the element with atomic number *l*.

Bohr's method is valid for hydrogen atom only and Schrodinger's method focus mainly on solving Schrodinger's equation in order to determine the wave function.

Inverse Square Law of Electromagnetic Waves Intensity

Inverse Square Law of Electromagnetic Waves Intensity describes how the intensity of the electromagnetic waves decreases with the square of the distance from the wave source and it is given by the following relation [5]

$$I_j = \frac{P}{4\pi R_j^2}, \quad j = 1,\cdots,N \tag{78}$$

where I_j is defined as the intensity of the wave at the point *j*.

P is the power of the source of the electromagnetic wave at the origin.

R_j is the distance between the power source and the point *j*.

I_j represents the amount of energy per time delivered by the wave at the point

j and the base units in the International System (SI) units for Intensity at the point j is the same (SI) units of the power, so we write

$$I_j = \frac{\mathrm{d}V_j}{\mathrm{d}t}, \quad j = 1, \cdots, N \tag{79}$$

The differentiation of the potential energy V_j with respect time given by the Equation (16.3) results

$$I_j = \frac{\mathrm{d}V_j}{\mathrm{d}t} = \frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2} \frac{m_j}{R_j^2} R_j', \quad j = 1, \cdots, N \tag{80}$$

Comparing Equation (80) with Equation (78) we get

$$P = \frac{1}{4\pi} \frac{s_j^2}{\phi_j^2} \phi_j' m_j \tilde{v}_j R_j', \quad j = 1, \cdots, N \tag{81}$$

The displacement s_j of particle j in terms of polar coordinates is given by the relation $s_j = R_j \phi_j$. *i.e.*

$$s_j^2 = R_j^2 \phi_j^2, \quad j = 1, \cdots, N$$

Substitute from the above relation into the term $\frac{\phi_j' \tilde{v}_j s_j^2}{\phi_j^2}$ in Equation (81) and write

$$P = \frac{1}{4\pi} \phi_j' m_j \tilde{v}_j R_j^2 R_j', \quad j = 1, \cdots, N \tag{81.1}$$

From Part(I) [1] the momentum p_j for a moving particle j is connected to a wave length λ_j by the relation

$$p_j = m_j \tilde{v}_j = \frac{h}{\lambda_j}, \quad j = 1, \cdots, N \tag{68}$$

where h is Planck's constant which is appropriate to describe the motion of tiny particles.

Consider the following equality

$$2R_j = n_j \lambda_j \to 4R_j^2 = n_j^2 \lambda_j^2 \to R_j^2 = \frac{n_j^2}{4} \lambda_j^2, \quad j = 1, \cdots, N \tag{69}$$

The speed of the electromagnetic wave (a light ray or a laser beam) is R_j'. *i.e.* $R_j' = \frac{1}{2} n_j' \lambda$.

The frequency of the electromagnetic wave is ν_j is related to the angular frequency ϕ_j' with relation $\phi_j' = 2\pi \nu_j$.

Now we Substitute from relation (68) and the equality (69) into Equation (81.1) we get

$$\begin{aligned} P &= \frac{1}{4\pi} 2\pi \nu_j m_j \tilde{v}_j R_j^2 R_j' = \frac{1}{2} \nu_j \left(\frac{h}{\lambda_j}\right) \left(\frac{n_j^2}{4} \lambda_j^2\right) \left(\frac{1}{2} n_j' \lambda\right) \\ &= \frac{1}{16} \nu_j \lambda_j^2 h n_j^2 n_j', \quad j = 1, \cdots, N \end{aligned} \tag{82}$$

$$P = \frac{1}{16} c\lambda_j h n_j^2 n_j' \to \frac{16}{c\lambda_j h} P \mathrm{d}t_j = n_j^2 \mathrm{d}n_j, \quad j = 1, \cdots, N \tag{82.1}$$

Integrating both sides of Equation (82.1) we get

$$\frac{16}{c\lambda_j h} P t_j = \frac{1}{3} n_j^3 + \frac{1}{3} n_{0j}^3, \quad j = 1, \cdots, N \tag{83}$$

We use the relation $c = \frac{R_j}{t_j}$ in Equation (83) and rewrite

$$\frac{48}{c\lambda_j h} P t_j = n_j^3 + n_{0j}^3 = \frac{48P}{R_j \lambda_j h} t_j^2, \quad j = 1, \cdots, N \tag{83.1}$$

From relation (69) we have

$$R_j^3 = \frac{n_j^3}{8} \lambda_j^3 \to n_j^3 = \frac{R_j^3}{\lambda_j^3}, \quad j = 1, \cdots, N \tag{69.1}$$

Substitute from relation (69.1) into Equation (83.1) we get

$$\frac{48P}{R_j \lambda_j h} t_j^2 = \frac{R_j^3}{\lambda_j^3} + n_{0j}^3, \quad j = 1, \cdots, N \tag{83.2}$$

$$\frac{48P}{h} t_j^2 = \frac{1}{\lambda_j^2} R_j^4 + n_{0j}^3 \lambda_j R_j, \quad j = 1, \cdots, N \tag{84}$$

In order to get Equation (83.2) in terms of the intensity of the electromagnetic wave at a point j I_j multiply both sides of Equation (83.2) with $\frac{1}{4\pi R_j}$ we get

$$\frac{48P}{4\pi R_j^2 \lambda_j h} t_j^2 = \frac{1}{4\pi} \frac{R_j^2}{\lambda_j^3} + \frac{1}{4\pi R_j} n_{0j}^3 = \frac{48}{\lambda_j h} t_j^2 I_j, \quad j = 1, \cdots, N$$

$$\frac{48}{\lambda_j h} t_j^2 I_j = \frac{1}{4\pi} \frac{R_j^2}{\lambda_j^3} + \frac{1}{4\pi R_j} n_{0j}^3, \quad j = 1, \cdots, N \tag{85}$$

n_{0j} is the energy level inside the atom of the substance of the source which emitted the wave, now by substitution of $t_j^2 = \left(\frac{R_j}{c}\right)^2$ in Equation (85) we get

$$\frac{48}{\lambda_j h} \frac{R_j^2}{c^2} I_j = \frac{1}{4\pi} \frac{R_j^2}{\lambda_j^3} + \frac{1}{4\pi R_j} n_{0j}^3, \quad j = 1, \cdots, N \tag{85.1}$$

Rearranging Equation (85.1) and rewrite we get

$$\frac{48}{\lambda_j h} \frac{R_j^2}{c^2} I_j = \frac{1}{4\pi} \frac{R_j^2}{\lambda_j^3} + \frac{1}{4\pi R_j} n_{0j}^3$$

$$\to n_{0j}^3 = 4\pi R_j \left(\frac{48}{\lambda_j h} \frac{R_j^2}{c^2} I_j - \frac{1}{4\pi} \frac{R_j^2}{\lambda_j^3} \right), \quad j = 1, \cdots, N$$

$$n_{0j}^3 = R_j^3 \left(\frac{192}{\lambda_j h} \frac{\pi}{c^2} I_j - \frac{1}{\lambda_j^3} \right) = \frac{R_j^3}{\lambda_j^3} \left(\frac{192\pi}{h} \left(\frac{\lambda_j^2}{c^2} \right) I_j - 1 \right), \quad j = 1, \cdots, N \tag{85.2}$$

Now we substitute into Equation (85.2) with $\frac{1}{\nu_j^2} = \frac{\lambda_j^2}{c^2}$ we get

$$n_{0j}^3 = \frac{R_j^3}{\lambda_j^3}\left(\frac{192\pi}{\nu_j^2 h} I_j - 1\right) \to n_0 = \frac{R_j}{\lambda_j}\left(\frac{192\pi}{\nu_j^2 h} I_j - 1\right)^{\frac{1}{3}}, \quad j = 1, \cdots, N \tag{86}$$

In order to get n_0 as function of the power P substitute into Equation (86) from Equation (78).

Equations (84)-(86) have numerous usage and applications, see Section 3.

This section presented the rotational motions of objects and particles. For more information on the remaining motions, please refer to Section 3.

3. Discussion

The derivation of the energy levels used the classical probability $\frac{1}{2}$ the general format of the Spectra Positioning Ratio (SPR) will be given in future parts.

Equations (84)-(86) have numerous usage and applications in all branches of theoretical sciences, telecommunications, navigation, aviation, technology, biomedical equipments, sound waves, earthquake waves, and engineering as well as gravity waves mentioned above These equations can be adapted for use according to the nature of the application being deal with, with great flexibility to switch between its different variables and constants matching the available data and the required outcome.

This article discussed the rotational motions of objects and particles. The remaining motions are as follows:

1) Rotational motion with spinning

2) Translational motion

3) Translational motion with spinning

These motions will be studied in subsequent parts

Based on the relationships derived in this research, it appears that electromagnetic waves are trapped within matter in the form of photons, just like the particles that make up matter, and that these photons are released or stretched outside the matter as soon as the appropriate conditions are met.

4. Conclusions

In this section, I present a brief overview of the results obtained in this research and assessment for some of its implications. The results obtained in this article are derived from two sources. The first source is the derivation operations already performed in this part "Part (II)", which are entirely based on the results obtained in Part(I). The second source is the equations and mathematical formulas already derived in Part (I).

The mathematical derivation of the general formula for complementary energy contributes to proving that complementary energy is potential energy and that it is a wave. This is necessary so that every conclusion drawn from it is based on a strong and solid foundation. The derivation of the general formula for complementary energy led to the determination of the condition under which the particle's associated wave transforms into a radiation wave that propagates outside the matter, as well as to the arrival of a mathematical definition for adhesion forces. Adhesion force is considered a type of dissipative force because its value depends on the speed, but its effect on the motion of the particle differs from other types of dissipative forces because its direction differs from the rest of the dissipative forces. The direction of the adhesion force is perpendicular to the direction of motion, while the direction of the rest of the dissipative forces is opposite to the direction of motion. The forces acting can be expressed in terms of the adhesion force and another dissipative force in the form of two components perpendicular to each other.

Physicists frequently and extensively use the potential energy of a linear oscillator in explaining many physical phenomena. Thanks to the general formula for potential energy that was deduced in Part(I), the potential energy of a linear oscillator was derived, which explains why it is useful in explaining physical phenomena.

Several results emerged after deducing the relationship between electromagnetic waves and particles, as follows:

1) The possibility of benefiting from the legacy of field theories and continuum media. Although this is considered an advantage, it weakens the strength of mathematical derivation due to the inability to deal with scales at the micro and macro scales. This was the problem facing physicists in the 1920s, and the inability to use field theories to explain non-continuum discreet phenomena.

2) Finding the relationship between the electromagnetic wave parameters and the particle parameters has led to the discovery of new relationships that were previously unknown. Firstly, a relationship stating that not every moving charged mass produces a magnetic field. This relationship defines the values of the charged mass that, when moving, generates a magnetic field. These values have been linked to the concept of effective mass, thus providing a deeper understanding of this concept. Secondly, a relationship was deduced linking mass to wavelength. From this relationship, the particle's radius and the photon's kinetic energy, which is related to the mass, were calculated. This can be used to calculate the total radiation energy at a specific point by multiplying the kinetic energy of a single photon by the number of photons, the value of which can be obtained from the relationship between the number of photons and the distance from the radiation source.

3) The study we are conducting examines the motion of a group of particles (N) within an adiabatic boundary—that is, an environment that neither absorbs nor emits energy. Therefore, everything observed is within the space in which the particles move. The studied electromagnetic waves are stored within the material in equilibrium with the rest of the particle system. These waves are emitted when the material is excited by an external energy source to maintain its equilibrium. They then return to their original location within the material when the external source

ceases to excite the material, restoring the material to its initial equilibrium state before excitation. In other words, the process of electromagnetic radiation is a reversible process that occurs between two equilibrium states. Further research can delve deeper into the mathematical aspects of how matter stores electromagnetic waves.

The triangle method, a geometric approach to determining the constant energy surface, was presented. This method illustrated both the orbital and cloud models of the atom, and explained the concepts of energy levels, quantum numbers, and bandwidth geometrically. Furthermore, a proposed model combining the orbital and cloud models, the Taurus model, was presented. Compared to the analytical method for determining the constant energy surfaces, the efficiency of the geometric method exceeds the analytical method for determining the constant energy surface, as the analytical method is restricted to solving elaborate analytical equations. However, the geometric method used in this research is flexible and unleashes the imagination of researchers to discover and innovate new shapes for the constant energy surfaces of various materials.

There are three laws for grand potential energies: Newton's law of universal gravitation, Coulomb's law, and the inverse square law of the intensity of electromagnetic radiation. Because of the general formula for potential energy, which was deduced in the first part, it was possible to deduce a mathematical relationship to calculate the wavelength of the gravity wave associated with each of the planets of the solar system in terms of the universal gravitational constant G, the angular velocity, and a constant coefficient hG corresponding to Planck's constant h, which is used to explain the motion of waves associated with tiny particles. Farther research is required to determine the value of coefficient hG. A mathematical formulation of quantum gravity, which studies the motion of tiny particles in gravitational field, has also been presented. The study of gravitational waves is believed to be the exclusive domain of the general theory of relativity.

Regarding Coulomb's law of charge attraction, the general formula for the energy levels of the atom of different elements was obtained through mathematical deduction. This formula was compared with Bohr's and Schrödinger's formulas for explaining the hydrogen atom, and a perfect match was found with both formulas. The ratio that determines the element was discovered and named the Spectra Positioning Ratio (SRP). Further research is required to determine this ratio.

The third law of grand potentials that was investigated is the inverse square law of the intensity of the electromagnetic wave energy. It does not determine the value of the potential energy at a point in space, but rather the value of the energy that reaches that point from the energy source per unit of time. Three mathematical relationships were derived that link several variables and constants that determine the wave properties such as wavelength, number of photons, wave intensity, and distance from the radiation source. More details are in the discussion. Farther mathematical investigations are required in order to connect wave parameters the magnetic and electric field to the quantum parameters.

The final conclusion concerns proving a mathematical relationship that estimates the length of the perimeter of conic sections, and in particular the perimeter of an ellipse. This relationship is considered a unique mathematical achievement, as numerous attempts to discover such a relationship since the 17th century have failed. The result of all these attempts was the establishment of a branch of mathematics concerned with elliptical functions and elliptical integration, which has proven its importance in all applied fields.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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