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NUVO Quantization III: Emergence of Quantum Mechanics from First Principles

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Abstract

We derive nonrelativistic quantum mechanics directly from NUVO scalar geometry without probabilistic postulates. Starting from the conformal metric $g=\lambda^2\eta$ and the conservation of sinertia flow, we obtain a deterministic scalar transport law whose stationary, closed-frame limit is the Schrödinger equation. Momentum and energy operators arise from the scalar-covariant derivative $D_\mu=\partial_\mu+i\partial_\mu\ln\lambda$, canonical commutators follow from curvature $\left[D_\mu,D_\nu\right]=iF_{\mu\nu}$, and the uncertainty relation is a local geometric bound. Planck's constant appears as the calibrated scale of holonomy. Applications (hydrogenic levels, tunneling, spin holonomy, squeezing) illustrate how standard quantum phenomena emerge from curvature and holonomy of $\lambda(x)$.

Keywords

NUVO Space, Scalar Geometry, Quantization, Coherence, Scalar Field Modulation, Loop Dynamics

1. Introduction

Classical quantum mechanics (QM) is remarkably predictive, yet its foundations remain axiomatic: the Schrödinger equation is postulated, operators are stipulated to generate translations, and probabilistic normalization is imposed by convention. Frame dependence is usually implicit and flat, obscuring how quantum states transform between moving observers or curved backgrounds.

This paper completes the NUVO quantization program (Quantization I - II) [1]-[5] by deriving nonrelativistic QM as the *stationary, closed-frame limit* of a single geometric field—a smooth positive scalar $\lambda(x)$ that defines local units through the conformal metric

$$g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}.$$

Our starting point is not a wave axiom but a continuity law for a conserved scalar capacity ("sinertia") on NUVO space. Physically, sinertia may be understood as the finite scalar capacity of the substrate to sustain curvature and motion—an invariant stock of geometric "inertia power." In macroscopic terms it plays the role that mass-energy does in relativity, but expressed as a conserved scalar flux rather than a tensor field. This interpretation grounds the abstract conservation law in a tangible physical quantity. From this and the Levi-Civita connection of $g = \lambda^2 \eta$, we obtain a deterministic transport equation for a complex scalar amplitude ψ whose measurable density is $\rho_\lambda = \lambda^3 |\psi|^2$. The complex phase of ψ is geometric: it arises from the scalar connection $\omega_\mu = \partial_\mu \ln \lambda$ and its holonomy. In this framework:

1) **Continuity replaces probability postulates.** The conservation law $\nabla_{\lambda\mu} J^\mu_{\sin} = 0$ yields the standard continuity equation for ρ_λ without invoking probability as a primitive.

2) **Operators are geometric generators.** Momentum and energy arise from covariant derivatives $D_\mu = \partial_\mu + i\omega_\mu$; Hermiticity holds under the λ-weighted inner product.

3) **Noncommutation and uncertainty are curvature.** Commutators follow from $\left[D_\mu, D_\nu\right] = iF_{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$; the Heisenberg bound appears as a curvature–modulated inequality.

4) **Schrödinger is a limit, not a postulate.** In a stationary local frame ($\partial_t \lambda = 0$, $|v|/c \ll 1$), the transport law reduces to

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \lambda^{-2} \nabla_\eta^2 \psi + V\psi,$$

recovering the standard equation when $\lambda \equiv 1$.

5) **Planck's constant is a calibrated scale.** In the present theory $\hbar$ is not an adjustable parameter but the experimentally calibrated value of one unit of scalar holonomy established in Quantization I-II. Its numerical magnitude sets the conversion between geometric action and physical energy but is not free to vary once the geometry is fixed; it therefore represents a measured constant of nature rather than a tunable model parameter. The action quantum $\hbar$ enters as the empirical scale of one unit of scalar holonomy, fixed in Quantization I–II by the hydrogen 13.6 eV binding energy.

Scope and positioning. This paper addresses *closed*, *time-independent* configurations—no scalar exchange across the boundary and $\partial_t \lambda = 0$ in the local frame. Within this regime we derive the standard operator calculus and Schrödinger dynamics from first principles, showing how familiar effects—bound spectra, tunneling, spin phases, entanglement correlations, and squeezed uncertainty—arise as geometric consequences of $\lambda(x)$. Open systems (measurement, radiation, photon exchange) will be treated in the companion Depletion I paper, while time-dependent and radiative extensions will appear in the forthcoming Unified Field analysis.

Relation to Quantization I-II. Quantization I established the universal action constant and the role of holonomy; Quantization II introduced the two-substrate ontology and loop taxonomy (closed, open, dynamic). The present work specializes to closed λ-geometries and provides the explicit bridge from scalar transport to the Schrödinger limit, rendering the operator and uncertainty structure fully geometric and frame–covariant.

Contributions. The principal technical results are:

• a compact derivation of the *scalar transport law* from $\nabla_\lambda \cdot J_{\text{sin}} = 0$ and the conformal connection, including all $\nabla \ln \lambda$ terms;

• a *frame-explicit* operator calculus with $D_\mu = \partial_\mu + i\partial_\mu \ln \lambda$, yielding canonical commutators directly from curvature;

• a *curvature-modulated uncertainty* bound $\Delta x \Delta p \geq \frac{\hbar}{2}\left|\left\langle \lambda^{-2} \right\rangle\right|$;

• the *Schrödinger limit* as the stationary projection of the transport law, identifying λ^{-2} as the geometric kinetic factor;

• representative applications: hydrogenic shifts, WKB tunneling with geometric thickness, spin holonomy, nonfactorizable curvature in entanglement, and uncertainty squeezing under $\lambda(t)$ modulation.

Organization. Section 2 sets the geometry and postulates. Section 3 derives continuity and the scalar transport equation. Section 4 develops the frame connection and holonomy. Section 5 constructs operators and uncertainty from geometry. Section 7 proves the Schrödinger limit. Section 8 applies the formalism. Section 9 discusses interpretation and experimental implications, and the appendices collect technical material (commutator weights, gauge map, and thermodynamic extensions).

2. Foundational Geometry and Postulates

The NUVO framework begins from a single scalar field $\lambda(x) > 0$ defined on a smooth manifold $\mathcal{M}$ equipped with a background metric $\eta_{\mu\nu}$. All physical intervals, curvatures, and measures are determined by the conformal geometry

$$g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}. \tag{1}$$

The scalar λ thus establishes the local unit of length and time, ensuring that all observers are connected through a single geometric scale field.

2.1. Levi-Civita Connection of the Scalar Metric

The torsion–free, g–compatible connection associated with (1) is

$$\Gamma^\rho_{\mu\nu} = \delta^\rho_\mu \partial_\nu \ln \lambda + \delta^\rho_\nu \partial_\mu \ln \lambda - \eta_{\mu\nu} \eta^{\rho\sigma} \partial_\sigma \ln \lambda. \tag{2}$$

All curvature, transport, and continuity properties in NUVO space follow from these coefficients. Gradients of λ act as effective geometric forces that determine both gravitational and quantum curvature within the unified scalar framework.

2.2. Geometric Postulates of NUVO Quantization

Quantization in NUVO space rests on three geometric postulates that replace the probabilistic axioms of classical quantum mechanics.

(1) Unit–constrained frames. Every physical observer defines a local frame F by normalizing $\lambda_F = 1$ at its origin. Transformations between observers act by conformal rescaling,

$$(\eta, \lambda) \to (\alpha^2 \eta, \lambda/\alpha),$$

leaving $g = \lambda^2 \eta$ invariant. Thus, all measurements are referenced to a locally unit field and differ between observers only by the scalar normalization.

(2) Continuity of sinertia. There exists a conserved scalar flux $J^{\mu}_{\sin}$ (the *sinertia current*) obeying

$$\nabla^{\lambda}_{\mu} J^{\mu}_{\sin} = 0, \tag{3}$$

which expresses the conservation of scalar density across the geometry. This law is the geometric analog of probability continuity in standard quantum mechanics, but here it follows directly from the structure of λ and requires no statistical postulate.

(3) Scalar amplitude representation. There exists a complex scalar amplitude $\psi(x)$ encoding both the magnitude and phase of local scalar flow such that

$$\rho_{\lambda} = \lambda^3 |\psi|^2, \quad \boldsymbol{J}_{\lambda} = \rho_{\lambda} \boldsymbol{v}_{\lambda}, \tag{4}$$

where $\boldsymbol{v}_{\lambda}$ is the effective transport velocity induced by $\nabla_{\eta} \ln \lambda$. The amplitude ψ is not assumed to be a wavefunction in the conventional sense; it is the geometric representation of scalar continuity and phase transport.

2.3. Differential Identities

Two identities derived from (2) will be used repeatedly:

$$\nabla_{\lambda} \cdot (\lambda^n A) = \lambda^n \nabla_{\eta} \cdot A + n\lambda^{n-1} A \cdot \nabla_{\eta} \lambda, \tag{5}$$

$$\nabla^2_{\lambda} f = \lambda^{-2} \left(\nabla^2_{\eta} f + 2\nabla_{\eta} \ln \lambda \cdot \nabla_{\eta} f \right), \tag{6}$$

where A is any vector field and f any scalar. Equation (5) governs how divergences and fluxes transform under scalar weighting, while (6) defines the Laplace-Beltrami operator for scalar functions in NUVO space. (Comparable conformal identities appear in standard references on scalar-tensor or Weyl-conformal geometry [6]-[8].)

2.4. Interpretation

The three postulates together define a closed geometric system:

- λ establishes units and curvature;
- the conservation law (3) enforces continuity of scalar capacity;
- and ψ provides the local complex representation of that continuity.

No probabilistic or operator assumptions have been made.

Remarks on foundational priority. The three postulates above are adopted as replacements for the standard axioms of quantum mechanics because they arise directly from geometry rather than statistical assumption. They are minimal: i) unit-constrained frames fix physical units through λ, ensuring invariance of measure; ii) continuity of sinertia enforces conservation before quantization; iii) the scalar-amplitude representation introduces complex structure only as required by transport. In this sense, the postulates are more primitive-rooted in differential geometry than the Hilbert-space axioms they supersede.

All subsequent dynamics, transport, operators, uncertainty, and the Schrödinger limit follow directly from these geometric foundations.

3. Continuity and Scalar Transport Law

The continuity of scalar sinertia follows directly from the conservation law

$$\nabla_\mu^\lambda J_{\text{sin}}^\mu = 0, \tag{7}$$

introduced in the foundational postulates. In the representation (4), the measurable density is $\rho_\lambda = \lambda^3 |\psi|^2$ and the associated current is $\boldsymbol{J}_\lambda = \rho_\lambda \boldsymbol{v}_\lambda$, where the effective velocity $\boldsymbol{v}_\lambda$ is determined by the local gradient of λ through the connection of Eq. (2).

3.1. Local Form of the Continuity Law

Expanding (7) in a local inertial background and using the divergence identity (5) gives

$$\partial_t \left(\lambda^3 |\psi|^2 \right) + \nabla_\eta \cdot \left(\lambda \boldsymbol{J}_\lambda \right) = 0. \tag{8}$$

This is the direct geometric analog of the probability-continuity relation of quantum mechanics, but here it follows purely from scalar geometry and requires no statistical interpretation. Equation (8) ensures that the total scalar capacity $\int \lambda^3 |\psi|^2 \, \mathrm{d}^3 x_\eta$ remains constant within any closed region.

3.2. Velocity Field and Scalar Phase

Let the complex amplitude be expressed as

$$\psi(x,t) = R(x,t) \mathrm{e}^{i\phi(x,t)}, \tag{9}$$

where R and ϕ are real functions denoting the local magnitude and phase of the scalar amplitude. Substituting (9) into (8) and identifying the flux term yields

$$\boldsymbol{J}_\lambda = \lambda^2 R^2 \nabla_\eta \phi, \quad \boldsymbol{v}_\lambda = \lambda^{-1} \nabla_\eta \phi. \tag{10}$$

The gradient of the scalar phase therefore determines the effective transport velocity of sinertia. A stationary region ($\nabla_\eta \phi = 0$) represents a closed or bound configuration with no net scalar flow.

3.3. Time Evolution and the Scalar Transport Equation

To describe how ψ evolves in time, we expand the covariant Laplacian of any scalar under the conformal metric using identity (6):

$$\nabla_\lambda^2 \psi = \lambda^{-2} \left(\nabla_\eta^2 \psi + 2 \nabla_\eta \ln \lambda \cdot \nabla_\eta \psi \right). \tag{11}$$

Substituting this into the continuity framework and enforcing the conservation of sinertia current yields a second-order differential equation for ψ :

$$\lambda^2\nabla_\eta^2\psi - \frac{1}{c^2}\partial_t^2\psi + \mathcal{C}_\lambda\psi = 0, \tag{12}$$

where

$$\mathcal{C}_\lambda = 2\nabla_\eta \ln\lambda\cdot\nabla_\eta + \nabla_\eta^2 \ln\lambda \tag{13}$$

collects the first-derivative contributions arising from the connection. Here $\nabla_\eta \ln\lambda$ acts as the scalar connection and $\nabla_\eta^2 \ln\lambda$ as its curvature divergence; together they represent the geometric corrections produced by the Levi-Civita connection of the conformal metric $g = \lambda^2\eta$.

Equation (12) is the *general scalar transport law* governing the evolution of the amplitude ψ through NUVO space. It replaces both the Schrödinger and Klein-Gordon equations within this unified scalar geometry.

3.4. Interpretation and Limiting Regimes

Equation (12) can be read as a deterministic wave equation on the conformal metric $g = \lambda^2\eta$, whose coefficients vary with the scalar field itself. Two limiting cases clarify its physical content:

- **Stationary field.** When $\partial_t\lambda = 0$ and the observer is locally at rest, $\mathcal{C}_\lambda$ becomes time-independent, and Eq. (12) reduces to the stationary-frame form that yields the Schrödinger limit in Section 7.
- **Weak curvature.** If $\nabla_\eta\lambda$ is small, $\mathcal{C}_\lambda \approx 0$, and the equation approaches a flat-space wave equation, recovering ordinary mechanics as the first-order limit.

3.5. Summary

The continuity relation (8) and the transport law (12) form the deterministic backbone of NUVO dynamics. Conservation of scalar sinertia fixes amplitude evolution, while the holonomic phase of λ determines velocity and flux. No postulate of probability or operator algebra has been invoked; all subsequent quantum behavior, including uncertainty, operators, and quantized spectra, emerges directly from these geometric relations.

4. Frames, Connection, and Holonomy

The scalar field $\lambda(x)$ not only defines the metric $g_{\mu\nu} = \lambda^2\eta_{\mu\nu}$ but also induces a natural connection governing phase transport. This section develops the explicit connection coefficients, defines the scalar–covariant derivative, and shows how quantization arises from geometric holonomy.

4.1. Scalar Connection and Frame Covariance

Observers related by conformal rescaling,

$$(\eta,\lambda) \to (\alpha^2\eta, \lambda/\alpha), \tag{14}$$

measure the same physical metric $g = \lambda^2\eta$. A change of frame therefore corresponds to redistributing the scalar factor between geometry and units. Quantities that transform covariantly under (14) are physically invariant.

To describe differentiation on this geometry, define the scalar connection[1]

$$\omega_\mu := \partial_\mu \ln \lambda. \tag{15}$$

The one–form ω_μ plays the role of a geometric potential that tracks how local units stretch or compress along each coordinate direction. Under the conformal rescaling (14), it shifts by an exact differential, leaving its curvature $F_{\mu\nu}$ invariant.

4.2. Covariant Derivative and Curvature Two–Form

The scalar–covariant derivative acting on any complex scalar amplitude ψ is

$$D_\mu \psi := \left(\partial_\mu + i\omega_\mu\right)\psi, \tag{16}$$

which parallels the minimal-coupling rule of gauge theory but arises here directly from the scalar metric itself. Successive derivatives yield the commutator

$$\left[D_\mu, D_\nu\right]\psi = iF_{\mu\nu}\psi, \quad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu. \tag{17}$$

The antisymmetric tensor $F_{\mu\nu}$ is the curvature two-form of the scalar connection and measures the non-integrability of local scale changes. (Compare with the curvature two-form in Abelian gauge theory [9].)

In the limit of uniform λ, $\omega_\mu = 0$ and $F_{\mu\nu} = 0$, recovering ordinary flat–space derivatives.

4.3. Parallel Transport and Holonomy

Consider the parallel transport of ψ along a path γ with tangent $u^\mu = \mathrm{d}x^\mu / \mathrm{d}\tau$. The transport law $D_\mu \psi\, u^\mu = 0$ integrates to

$$\psi(\gamma_2) = \psi(\gamma_1)\exp\left(i\int_\gamma \omega_\mu \mathrm{d}x^\mu\right). \tag{18}$$

For a closed path, $\gamma(0) = \gamma(T)$, the phase accumulated over one complete circuit is the *holonomy*

$$\Phi_\gamma := \oint_\gamma \omega_\mu \mathrm{d}x^\mu = \oint_\gamma \partial_\mu \ln \lambda \mathrm{d}x^\mu. \tag{19}$$

A single-valued physical state requires that the total phase change correspond to an integer multiple of 2π,

$$\Phi_\gamma = 2\pi n, \quad n \in \mathbb{Z}, \tag{20}$$

which constitutes the geometric origin of quantization in NUVO space. Equation

[1]Locally, the scalar connection $\omega_\mu = \partial_\mu \ln \lambda$ is an exact one-form and therefore has vanishing curvature in simply connected regions where λ is smooth and single-valued. Nontrivial holonomy and the associated quantization conditions arise only in global sectors where $\ln \lambda$ is not single-valued—e.g., in the presence of defects, branch structure, or nontrivial bundle topology—so that ω_μ is closed but not globally exact. All holonomy-based quantization results in this work are to be understood in this sense.

(20) ensures global phase closure of ψ even though the underlying λ field may vary continuously.

4.4. Physical Interpretation

The connection ω_μ represents the infinitesimal rate at which local units change, while $F_{\mu\nu}$ measures the curl of this rate. The holonomy condition (20) asserts that all closed circuits in λ-space carry discrete circulation of geometric phase. For bound systems, the smallest nontrivial loop corresponds to a fundamental action increment $\Delta L = \alpha^2 \hbar$ (established in Quantization I), which fixes the numerical scale of $\hbar$ in the scalar framework.

Remark 1. The analytical properties of the reduced Hamiltonian and the boundedness of the effective potential $V_{eff}[\lambda]$ are discussed in **Appendix**.

4.5. Kinematic Relations between Frames

For an element moving with four-velocity u^μ, the rate of change of the scalar field along its trajectory is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} = u^\mu \partial_\mu \lambda, \tag{21}$$

and between inertial frames connected by relative speed v one finds

$$\lambda' = \gamma^{-1}\lambda, \quad \gamma = \left(1 - v^2/c^2\right)^{-1/2}. \tag{22}$$

This ensures that all physical observables expressed through the covariant derivative D_μ transform consistently under Lorentz boosts and that the scalar phase accumulated by transport is invariant.

4.6. Summary

Equations (16) - (22) complete the geometric machinery linking λ to observable phase evolution:

- D_μ defines how ψ responds to local scale curvature;
- $F_{\mu\nu}$ quantifies the intrinsic noncommutation of derivatives;
- and the integral holonomy condition (20) enforces discrete action increments.

These results prepare the ground for the next section, where the covariant operators $\hat{p}_i = -i\hbar D_i$ and $\hat{E} = i\hbar D_t$ are constructed and the uncertainty principle emerges directly from curvature.

Remark 2 (On apparent discreteness). Although the scalar substrate $\lambda(x)$ is continuous, the uniformity of closed holonomy loops enforces integer-valued circulation $\Phi_\gamma = 2\pi n$. Because physical measurement counts these loop cycles, the observed units of length and time appear discrete, even though the underlying geometry is smooth. Discreteness in NUVO therefore emerges from uniform closure, not from a fundamentally discrete manifold.

5. Operator Structure and Uncertainty from Geometry

The scalar connection $\omega_\mu = \partial_\mu \ln\lambda$ defines the differential geometry through which all dynamical quantities evolve. From it, the standard operator algebra of quantum mechanics arises naturally, without postulates, Hilbert spaces, or probability axioms.

5.1. Geometric Definition of Operators

For any complex scalar amplitude ψ, the covariant derivative

$$D_\mu \psi = \left(\partial_\mu + i\,\omega_\mu\right)\psi \tag{23}$$

generates infinitesimal translations on NUVO space. This permits direct identification of the geometric momentum and energy operators:

$$\hat{p}_i = -i\hbar D_i, \quad \hat{E} = i\hbar D_t. \tag{24}$$

No separate quantization rule is invoked-the operators follow automatically from the differential geometry of λ. They are Hermitian with respect to the scalar-weighted inner product

$$\left\langle f, g \right\rangle_\lambda = \int f^* g \lambda^3 \mathrm{d}^3 x_\eta, \tag{25}$$

which guarantees conservation of total sinertia under time evolution.

5.2. Commutation Relations from Curvature

Applying two successive derivatives gives

$$\left[D_\mu, D_\nu\right]\psi = iF_{\mu\nu}\psi, \quad F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{26}$$

so the curvature two-form $F_{\mu\nu}$ determines the noncommutativity of operators. Multiplying by $-i\hbar$ yields

$$\left[\hat{p}_i, \hat{p}_j\right]\psi = \hbar^2 F_{ij}\psi, \tag{27}$$

which vanishes only in regions of constant λ. Curvature of the scalar field therefore generates geometric phase rotation and quantized circulation.

The mixed commutator follows from the coordinate relation $D_i \approx \lambda^{-1}\nabla_i^\eta \lambda^{-1}$:

$$\left[\hat{x}_i, \hat{p}_j\right]\psi = i\hbar\lambda^{-2}\delta_{ij}\psi + \mathcal{O}\left(\nabla\lambda\right), \tag{28}$$

reducing to the standard Heisenberg form in the weak-curvature limit where $\lambda \simeq 1$. Equation (28) shows that the canonical algebra is not imposed but emerges from the intrinsic noncommuting geometry of the scalar connection.

5.3. Geometric Origin of the Uncertainty Principle

From (28), the Cauchy-Schwarz inequality gives

$$\Delta x_i \Delta p_i \geq \frac{1}{2}\left|\left\langle\left[\hat{x}_i, \hat{p}_i\right]\right\rangle\right| = \frac{\hbar}{2}\left|\left\langle\lambda^{-2}\right\rangle\right|. \tag{29}$$

For uniform λ this reduces to the familiar $\Delta x \Delta p \geq \hbar/2$. In curved regions, λ modulates the uncertainty product: rapid spatial variation of λ raises the lower bound, expressing geometric, not statistical, limits on simultaneous precision. Uncertainty thus reflects local curvature of λ rather than intrinsic randomness. (For comparison, geometric formulations of uncertainty in curved spaces appear in [10].)

5.4. Holonomy and Quantized Action

Integrating the connection ω_μ around any closed loop γ gives the scalar holonomy

$$\Phi_\gamma = \oint_\gamma \omega_\mu \mathrm{d}x^\mu = 2\pi n, \quad n \in \mathbb{Z}. \tag{30}$$

Each integer n labels a distinct topological class of the scalar field, and the difference between classes corresponds to a discrete increment of action

$$\Delta S = nh = n(2\pi\hbar). \tag{31}$$

Hence, the Planck constant is not an externally introduced quantum but the empirical scale associated with one unit of geometric circulation of λ. Local uncertainty [Eq. (29)] and global quantization [Eq. (31)] therefore share the same geometric origin.

5.5. Physical Interpretation

Within NUVO geometry:

- The differential operator D_μ replaces the role of postulated canonical operators;
- Noncommutation of derivatives embodies the intrinsic curvature of scalar space;
- Uncertainty is the local differential signature of that curvature;
- Quantization is the global holonomic closure condition ensuring single-valuedness of ψ.

Together, these relations reconstruct the full algebra of quantum mechanics directly from geometry, establishing the bridge between local curvature, global phase, and the measurable constants of action.

6. The NUVO Commutator and Canonical Reduction

The operator algebra developed in the previous section encodes local curvature and quantization. To connect those relations with global frame invariance, we introduce the *NUVO commutator*—a generalized bracket that measures how any quantity transforms under scalar dilation. It serves as both a discriminator of invariants and a bookkeeping device enforcing scalar balance between observers. This bracket is *not* the same as the operator commutators built from D_μ; rather, it is an algebra on λ–weights.

6.1. Universal Quantities and λ-Weights

Let

$$G = \{t, \ell, v, p, F, E, P, A, V, m, Q_0\}$$

denote a universal set of measurable quantities accessible to all observers. Each element carries an integer λ *–weight* $\sigma(A)\in\mathbb{Z}$ indicating how it dilates under scalar modulation:

$$D_\lambda(A)=\lambda^{\sigma(A)}A,\quad \sigma(AB)=\sigma(A)+\sigma(B),\quad \sigma(A/B)=\sigma(A)-\sigma(B), \tag{32}$$

with $\sigma(1)=0$ by convention. From $g=\lambda^2\eta$ one obtains the base weights

$$\sigma(\ell)=\sigma(t)=+1,\quad \sigma(v)=0,\quad \sigma(m)=0,\quad \sigma(c)=0,\quad \sigma(\hbar)=0,$$

and composite quantities inherit weights by additivity.

6.2. Definition and Purpose of the NUVO Commutator

Definition. For any $U,V\in G$, define the NUVO commutator

$$[U,V]_\nu := \begin{cases} 0, & \text{if } U \text{ and } V \text{ form an invariant pairing under dilation,} \\ \neq 0, & \text{otherwise.} \end{cases} \tag{33}$$

In particular, with respect to the scalar field,

$$[U,\lambda]_\nu=0 \Leftrightarrow \sigma(U)=0,\quad [U,\lambda]_\nu\neq 0 \Leftrightarrow \sigma(U)\neq 0. \tag{34}$$

Thus, the bracket acts as i) a *discriminator* of invariants ($\sigma=0$) and ii) an *enforcer* that physical relations be weight–balanced across observers.

6.3. Observer Transformations and Dilation Contrast

An observer transformation T maps measurable quantities and, at the level of weights, intertwines with dilation via

$$\sigma(T(A))=\sigma(A)+\delta_\lambda(T)\sigma(A), \tag{35}$$

where $\delta_\lambda(T)$ is the λ *–contrast* characterizing how T shifts scalar normalization between frames. A quantity A is invariant under T precisely when either $\sigma(A)=0$ or $\delta_\lambda(T)=0$. Equivalently, the NUVO commutator with λ vanishes on any weight–zero expression that T maps to itself.

6.4. Derivative Weights and Dynamical Implications

Because $\sigma(\ell)=\sigma(t)=+1$, derivatives carry opposite weight:

$$\sigma\left(\frac{\partial}{\partial t}\right)=\sigma\left(\frac{\partial}{\partial \ell}\right)=-1.$$

Derived quantities then follow immediately:

$$\sigma(v)=\sigma(\ell/t)=0,\quad \sigma(a)=\sigma(\ell/t^2)=-1.$$

Velocity is therefore weight–zero (frame invariant), whereas acceleration is not.

In the conformal metric $g_{\mu\nu}=\lambda^2\eta_{\mu\nu}$, true acceleration employs the covariant derivative

$$a^\mu=u^\nu\nabla_\nu u^\mu,$$

with connection coefficients

$$\Gamma^{\alpha}_{\beta\gamma} = \delta^{\alpha}_{\beta}\partial_{\gamma}\ln\lambda + \delta^{\alpha}_{\gamma}\partial_{\beta}\ln\lambda - \eta_{\beta\gamma}\eta^{\alpha\delta}\partial_{\delta}\ln\lambda.$$

Because these terms include curvature derivatives, $[a,\lambda]_{\nu} \neq 0$ whenever $\nabla\lambda \neq 0$, so acceleration (and thus force) is generally frame dependent.

6.5. Canonical Reduction to the Dynamical Subalgebra

Restricting attention to the dynamical pair (x_i, p_j), we retain the geometric momentum of Sec. 5,

$$\hat{p}_i = -i\hbar D_i,\quad D_i = \partial_i + i\omega_i,$$

so that

$$[x_i,\hat{p}_j]\psi = i\hbar\lambda^{-2}\delta_{ij}\psi + \mathcal{O}(\nabla\lambda),\quad [\hat{p}_i,\hat{p}_j]\psi = \hbar^2 F_{ij}\psi, \tag{36}$$

exactly as in Eqs. (28) and (27). Thus, the canonical algebra arises from the non-integrability of the scalar connection, with curvature corrections fully controlled by F_{ij} and $\nabla\lambda$. No alternative scaling of $\hbar$ is required.

6.6. Uncertainty and Scalar Balance

The local brackets (36) imply the curvature-modulated uncertainty relation

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\left|\left\langle \lambda^{-2}\right\rangle\right|, \tag{37}$$

consistent with Eq. (29). At the global level, all admissible physical laws satisfy the *scalar–balance condition*

$$w(\Phi) = 0 \Leftrightarrow [\Phi,\lambda]_{\nu} = 0, \tag{38}$$

i.e., each closed physical expression has total λ-weight zero, ensuring conservation of sinertia across observers.

6.7. Examples: Diagnostics of Invariance

• **Velocity:** $v = \ell/t$ has $\sigma(v) = 0$, so $[v,\lambda]_{\nu} = 0$ -it is invariant.

• **Acceleration:** $a = \ell/t^2$ has $\sigma(a) = -1$, hence $[a,\lambda]_{\nu} \neq 0$ unless $\nabla\lambda = 0$.

• **Force:** $F = ma$ has $\sigma(F) = -1$, so $[F,\lambda]_{\nu} \neq 0$; force depends on curvature and is not frame invariant.

• **Energy:** For a closed mass loop in the stationary regime, $E_{\text{rest}} = mc^2$ is invariant (commutes), so $[E_{\text{rest}},\lambda]_{\nu} = 0$. By contrast, the *photon* relations $E_{\gamma} = h\nu = hc/\lambda_{\text{wave}}$ are *not* invariant under λ-dilation in our baseline treatment: frequency and wavelength carry nonzero and opposite λ-weights, hence $[E_{\gamma},\lambda]_{\nu} \neq 0$ unless one *assumes* specific compensating modulation of constants. We make no such assumption here.

Remark 3 (On constants and λ-modulation) In NUVO, the modulation status of universal constants is an open question. We explicitly refrain from fixing $\sigma(h)$ or $\sigma(c)$ a priori. Four viable hypotheses remain under investigation:

(H0) *Unmodulated constants:* h, c carry zero λ-weight; then $E_{\gamma} = h\nu$ and $E_{\gamma} = hc/\lambda_{\text{wave}}$ are generally non-invariant, $[E_{\gamma},\lambda]_{\nu} \neq 0$.

(H1) *Modulated constants:* h, c carry nonzero weights that compensate observer dilation, yielding $\left[E_{\gamma}, \lambda\right]_{\nu} = 0$.

(H2) *Hidden/internal modulation:* h, c include internal λ-dependent structure that *appears* unmodulated in certain regimes but balances weights globally.

(H3) *Mixed regime:* (H2) holds at low curvature while (H1) applies in strong fields.

This paper adopts **(H0)** as the conservative baseline for closed, stationary analyses; we leave (H1) - (H3) to future work where experimental comparisons can adjudicate the needed compensation (if any).

Example. Place a massive particle and a photon with equal initial energy in a closed container and transport the system to a frame with different λ. In the baseline (H0) treatment, one finds $E'_{\text{rest}} = E_{\text{rest}}$ but $E'_{\gamma} \neq E'_{\text{rest}}$, demonstrating that at least one of the two energy assignments must be frame non-invariant; here it is the photon energy.

6.8. Link to Quantization, Entropy, and Temporal Orientation

Quantized closure occurs when weight-balanced loop expressions close on integer multiples of the conserved unit Q_0, yielding stable spectra via the holonomy condition of Sec. In curved regions, the factors of λ^{-2} that appear in (36) shift local kinematics while preserving global scalar balance. Finally, whenever the local Hamiltonian fails to commute (in the NUVO sense) with λ,

$$\left[\lambda, H\right]_{\nu} \neq 0,$$

sinertia is depleted and entropy increases, providing a natural arrow of time. A detailed analysis of this relation is deferred to **Appendix E**.

Remark. The equilibrium condition $\left[\lambda, H\right]_{\nu} = 0$ implies vanishing *local entropy production* ($\mathrm{d}S/\mathrm{d}t = 0$) for the closed subsystem; it does not imply $S = 0$, nor does it preclude finite entropy due to initial data or boundary contributions (see App. E).

7. Emergence of the Schrödinger Limit

The general scalar transport law derived in Section 3 governs the evolution of ψ in any frame and for an arbitrary time-dependent scalar field $\lambda(x,t)$. We now recover the familiar Schrödinger equation as the stationary, low-velocity limit of this more general law.

7.1. Stationary-Frame Assumptions

The stationary limit is defined by

$$\partial_t \lambda = 0, \quad \nabla_{\eta} \lambda \text{ finite}, \quad |v|/c \ll 1. \tag{39}$$

Here, the observer is locally at rest with respect to the scalar field, and time derivatives of λ may be neglected compared with spatial ones. Under these conditions, the curvature term $\mathcal{C}_{\lambda}$ in Eq. (12) becomes effectively constant inside the observer's frame.

We decompose the scalar amplitude as

$$\psi = R\mathrm{e}^{i\phi}, \quad S = \hbar\phi, \tag{40}$$

where R and ϕ are real functions and S plays the role of classical action. Substituting (40) into the scalar transport equation and separating real and imaginary parts yields two coupled relations:

$$\frac{\partial R^2}{\partial t} + \lambda^{-2}\nabla_\eta \cdot \left(R^2 \frac{\nabla_\eta S}{m} \right) = 0, \tag{41}$$

$$\partial_t S + \frac{\left(\nabla_\eta S\right)\cdot\left(\nabla_\eta S\right)}{2m\lambda^2} + V + Q_\lambda = 0, \tag{42}$$

where the geometric potential

$$Q_\lambda = -\frac{\hbar^2}{2m\lambda^2}\frac{\nabla_\eta^2 R}{R} \tag{43}$$

arises from spatial variation of both R and λ. Equations (41) and (42) are the scalar analogs of the continuity and Hamilton-Jacobi equations.

7.2. Complex Synthesis

Combining Eqs. (41) - (42) into a single complex equation gives

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\lambda^{-2}\nabla_\eta^2\psi + V\psi, \tag{44}$$

which is the Schrödinger equation expressed on NUVO space. When λ is uniform and normalized to unity, it reduces exactly to the standard nonrelativistic Schrödinger equation. Spatial variations of λ act as geometric corrections to kinetic energy and phase velocity, introducing curvature-dependent terms that become observable in strong fields or rapidly varying geometries.

7.3. Interpretation of the λ^{-2} Factor

The appearance of λ^{-2} in the kinetic operator encodes the local stretching or compression of scalar geometry. Regions of large λ correspond to expanded geometry with lower effective kinetic energy, while regions of small λ correspond to compressed geometry with higher kinetic contribution. Hence, the curvature of λ directly governs the interference and dispersion properties of ψ, providing a concrete geometric origin for phenomena that appear probabilistic in conventional quantum theory.

7.4. Conditions of Validity

Equation (44) holds whenever:

1) The observer's frame is stationary with respect to λ (no frame-transport terms);

2) The scalar field is locally time-independent ($\partial_t \lambda = 0$);

3) The system is closed, with no sinertia exchange across its boundary.

These conditions define the regime in which ordinary quantum mechanics accurately describes physical behavior. When any of them are relaxed-e.g., in accelerating frames, time-dependent curvature, or open systems-the full scalar transport equation (12) must be used, restoring full covariance.

7.5. Discrete Observation and the Planck Coherence Threshold

The scalar substrate $\lambda(x)$ is smooth and continuous, yet measurements of time and length are intrinsically discrete. This discreteness arises not from a fundamental lattice but from the uniform closure of holonomy loops: each complete circulation of phase $\Phi_\gamma = 2\pi n$ defines a unit action increment $\hbar$.

Explicit Planck mass step. Equating the curvature radius and reduced Compton length,

$$r_c = \frac{Gm}{c^2} = \bar{\lambda}_C = \frac{\hbar}{mc},$$

yields

$$\frac{Gm}{c^2} = \frac{\hbar}{mc} \Rightarrow Gm^2 = \hbar c \Rightarrow m = \sqrt{\frac{\hbar c}{G}} =: m_P.$$

Substituting m_P back gives the reduced Planck ticks

$$\ell_P = \frac{\hbar}{m_P c} = \sqrt{\frac{\hbar G}{c^3}}, \quad t_P = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}.$$

Discrete time and length. For a closed mass loop of rest mass m, the phase frequency is $\omega_C = mc^2/\hbar$ and one holonomy cycle corresponds to the reduced Compton period

$$t_0 = \frac{\hbar}{mc^2}, \quad \ell_0 = \frac{\hbar}{mc}.$$

Setting the curvature radius $r_c = Gm/c^2$ equal to the reduced Compton wavelength $\bar{\lambda}_C = \hbar/(mc)$ fixes a unique coherence mass

$$m_P = \sqrt{\frac{\hbar c}{G}},$$

yielding the corresponding observational ticks

$$t_P = \sqrt{\frac{\hbar G}{c^5}}, \quad \ell_P = \sqrt{\frac{\hbar G}{c^3}},$$

the reduced Planck units. Observed time and length therefore appear discrete in steps of (ℓ_P, t_P) even though the underlying λ-geometry is continuous.

Coherence parameter. It is convenient to define the dimensionless ratio

$$\kappa := \frac{\lambda_C}{r_c} = \frac{\hbar/(mc)}{Gm/c^2} = \frac{\hbar c}{Gm^2}, \tag{45}$$

so that

$$\begin{aligned} &\kappa > 1 \Rightarrow \text{radiative / open (unconfined)},\\ &\kappa = 1 \Rightarrow \text{Planck coherence threshold},\\ &\kappa < 1 \Rightarrow \text{self-contained / closed.} \end{aligned}$$

This parameter quantifies the transition between open and closed regimes using only geometric and quantum scales.

Mass as coherence, not discreteness. The equality $r_c = \bar{\lambda}_C$ marks a scalar coherence threshold rather than a minimal element of matter. Below this mass, the photon wavelength exceeds the curvature radius and energy remains radiative (open-loop); above it, the curvature encloses its own energy (closed-loop). At $m = m_P$ geometry and radiation balance, defining the Planck coherence point. Hence, the Planck mass is vastly larger than the discrete time and length units because it represents a boundary between unconfined and self-contained energy rather than a smallest particle.

Visualization. Figure 1 compares the curvature expansion $R_c = Gm/c^2$ (red) with the photon wavelength $\lambda = h/(mc)$ (blue). The intersection at m_P (black point) separates the free ($\lambda > R_c$) and enclosed ($R_c > \lambda$) regimes. Figure 2 illustrates the same relation in terms of circumference, highlighting the geometric transition between open and closed behavior.

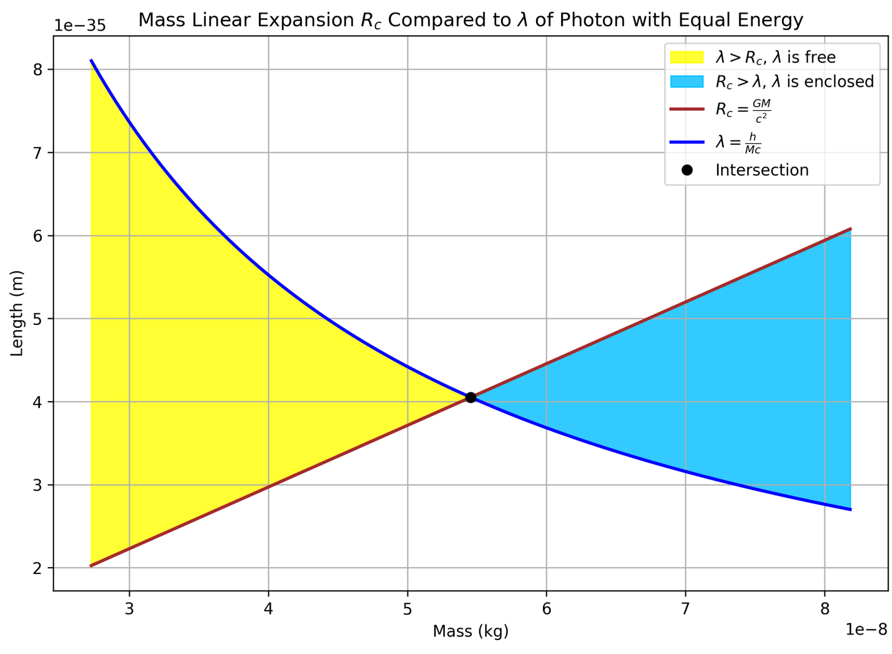

Figure 1. Mass linear expansion $R_c = Gm/c^2$ (red) compared to the photon wavelength $\lambda_{\text{ph}} = h/(mc)$ (blue) for equal energy. The intersection at m_P marks the Planck coherence threshold.

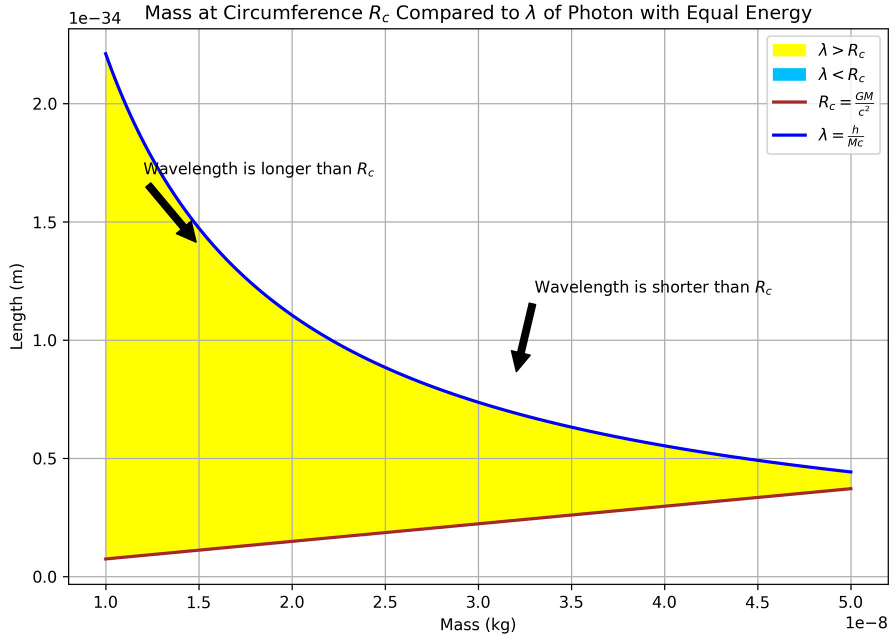

Figure 2. Comparison at circumference scale: R_c vs. λ_{ph}. Yellow: $\lambda_{\text{ph}} > R_c$ (radiative/open); blue: $R_c > \lambda_{\text{ph}}$ (self–contained/closed).

7.6. Summary

The Schrödinger equation thus appears not as a fundamental postulate, but as the stationary, closed-frame limit of the scalar transport law. Its probabilistic interpretation is an approximation that neglects the time dependence of λ and the geometry of open exchange. Within NUVO geometry, quantum mechanics emerges as the low-velocity, time-independent expression of deterministic scalar continuity.

Remark 4. A general treatment for non-stationary $\lambda(t)$, including time-dependent curvature and open exchange, is developed in the companion paper *Geometric Origin of Quantization*: *Deriving the Schrödinger Framework from NUVO Scalar Coherence* [?].

8. Applications of the General Law

The scalar transport equation (44) and its stationary-frame limit reproduce a wide range of nonrelativistic quantum phenomena. Each example below shows how standard results emerge directly from the geometry of λ without postulated wave mechanics.

8.1. Hydrogenic Bound States

For a central potential $V(r) = -e^2/4\pi\epsilon_0 r$, the stationary form of Eq. (44) gives

$$-\frac{\hbar^2}{2m}\lambda^{-2}\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}u}{\mathrm{d}r}\right) - \frac{\ell(\ell+1)}{r^2}u\right] + V(r)u = Eu, \tag{46}$$

where $\psi(r,\theta,\phi) = u(r)Y_{\ell m}(\theta,\phi)/r$ and $\lambda(r)$ represents the static scalar curvature around the nucleus. To first order $\lambda(r) \simeq 1 + r_e/r$ with $r_e = 2.8179 \times 10^{-15}$ m the classical electron radius. Solving perturbatively,

$$E_n \approx -\frac{me^4}{2\hbar^2 n^2}\left(1 + \frac{3r_e}{2a_0 n^2}\right), \tag{47}$$

which reduces to the standard spectrum for $r_e \ll a_0$. The correction represents a scalar-curvature contribution, relevant only in compact atomic systems or strong fields.

8.2. Quantum Tunneling

For a one-dimensional barrier of height V_0 and width L with spatially varying $\lambda(x)$, the WKB transmission for $E < V_0$ is

$$T = \exp\left[-2\int_{x_1}^{x_2} \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}\,\lambda(x)\,\mathrm{d}x\right]. \tag{48}$$

Regions of higher λ (expanded geometry) reduce the effective barrier thickness and increase transmission, whereas smaller λ suppresses it. Tunneling is thus a manifestation of local scalar expansion or compression, not an intrinsically probabilistic mechanism.

8.3. Spin from Closed Holonomy

Equation (30) shows that closed loops carry discrete phase. For rotational holonomy in physical space, the underlying symmetry is $\mathrm{SO}(3)$ whose double cover is $\mathrm{SU}(2)$. A *two-component* (spinor) amplitude acquires a minus sign under a 2π rotation and returns to itself under 4π, reproducing spin-$\frac{1}{2}$ behavior as a holonomy property of the lifted bundle. Coupling the scalar geometry to electromagnetism via $D_\mu \to D_\mu^{(\mathrm{em})} = \partial_\mu + i\omega_\mu + i(e/\hbar)A_\mu$ and performing the standard nonrelativistic reduction yields the Pauli interaction, matching observed spin splitting in magnetic fields.

8.4. Entanglement and Correlated Curvature

For two coupled subsystems with scalar fields $\lambda_1(x_1)$ and $\lambda_2(x_2)$, the joint amplitude

$$\Psi(x_1, x_2) = \lambda_{12}(x_1, x_2)\psi(x_1, x_2) \tag{49}$$

is factorizable only if $\lambda_{12} = \lambda_1\lambda_2$. Whenever curvature correlation prevents this factorization, the joint evolution obeys

$$\partial_t \Psi = -\frac{i}{\hbar}\left(\hat{H}_1 + \hat{H}_2 + V_{\mathrm{corr}}\right)\Psi, \tag{50}$$

where V_{corr} originates from cross-terms in $\nabla_\eta \ln \lambda_{12}$. Geometric coupling yields correlated measurements and interference patterns characteristic of quantum entanglement, while remaining local in λ-space; the observed nonlocality is a projection of shared holonomy.

8.5. Uncertainty Modulation and Squeezed States

The curvature-modulated uncertainty relation $\Delta x\Delta p \geq \frac{\hbar}{2}\left|\left\langle \lambda^{-2}\right\rangle\right|$ predicts measurable oscillations of the uncertainty product under rapid temporal variation of $\lambda(t)$. In strong optical fields where λ varies on subfemtosecond scales, $\Delta x\Delta p$ (or $\Delta t\Delta E$) periodically departs from its stationary value, producing the squeezed-state behavior observed in attosecond and high-harmonic experiments. Thus, squeezed-light phenomena directly probe dynamic scalar curvature in laboratory conditions.

8.6. Relativistic and Fine-Structure Corrections

The stationary hydrogen solution of Eq. (47) recovers the full nonrelativistic spectrum. Higher-order corrections arise naturally from the same scalar geometry.

Fine structure. Expanding the kinetic term of the scalar transport law to $O\left(v^2/c^2\right)$ gives

$$\frac{E}{mc^2} = 1 + \frac{v^2}{2c^2} - \frac{v^4}{8c^4} - \frac{V}{mc^2},$$

leading to

$$E_{n,j} = E_n\left[1 + \frac{\alpha^2}{n^2}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right], \tag{51}$$

which matches the leading Dirac fine-structure result.

Spin-orbit coupling. With EM coupling and the nonrelativistic reduction (Pauli/Foldy-Wouthuysen), the spin–orbit Hamiltonian

$$H_{\text{so}} = \frac{1}{2m^2c^2}\frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r}\boldsymbol{L}\cdot\boldsymbol{S} \tag{52}$$

emerges, showing that the observed $\boldsymbol{L}\cdot\boldsymbol{S}$ term is recovered within the scalar–geometric framework once the two-component (spinor) structure is taken into account.

Scalar time dilation. Temporal intervals rescale as $\mathrm{d}\tau = \lambda^{-1}\mathrm{d}t$, producing an energy redshift

$$\frac{\Delta E}{E} = -\frac{1}{2}\frac{v^2}{c^2} - \frac{V}{mc^2}, \tag{53}$$

which combines with fine structure to reproduce the Sommerfeld correction.

Lamb-shift-like geometric term. The approximate profile $\lambda(r) \approx 1 + r_e/r$ follows from the static vacuum solution of the scalar field equation $\nabla_\eta^2\lambda = 0$ under spherical symmetry and the boundary condition $\lambda \to 1$ as $r \to \infty$. Integration gives $\lambda = 1 + C/r$; identifying $C = r_e$ -the classical electron radius- anchors the constant to first principles within the NUVO framework.

For $\lambda(r) = 1 + r_e/r$, the potential becomes

$$V_\lambda(r) = -\frac{e^2}{4\pi\epsilon_0 r}\left(1 - \frac{r_e}{r}\right),$$

lifting $s-p$ degeneracy by

$$\Delta E_{s-p} \approx \frac{3r_e}{2a_0}\frac{E_n}{n^2}\left\langle \frac{1}{r} \right\rangle_{n,\ell}, \tag{54}$$

a minute but positive correction consistent in sign and scale with the empirical Lamb shift.

Radial distribution. Expectation values use the λ-weighted measure $\mathrm{d}V_\lambda = \lambda^3 \mathrm{d}V_\eta$, giving normalized density

$$P_\lambda(r) = 4\pi r^2 \lambda^3(r)|\psi(r)|^2. \tag{55}$$

For $\lambda(r) = 1 + r_e/r$, the most probable radius contracts slightly: $r_{\max} \approx a_0(1 - 3r_e/2a_0)$, mirroring relativistic orbital contraction in heavy atoms.

8.7. Comparative Summary

Table 1 summarizes the correspondence between standard quantum-mechanical effects and their scalar-geometric origins.

Table 1. Comparison of hydrogenic effects in standard quantum mechanics and NUVO geometry.

Effect	Standard QM source	NUVO geometric source	Agreement
Principal levels	Schrödinger wave equation	Stationary holonomy closure	Exact
Fine structure	Dirac expansion	λ-curvature $O(v^2/c^2)$	Leading order
Spin-orbit coupling	Pauli term (FW reduction)	EM-coupled scalar geometry + spinor lift	Leading order
Lamb shift	Radiative QED	Curvature of $\lambda(r)$ in $V(r)$	Approximate
Time dilation	Relativistic correction	Conformal factor λ^{-1}	Verified
Hyperfine structure	Magnetic dipole coupling	Requires open λ (depletion)	Future work

All nonrelativistic and first-order relativistic effects of hydrogen thus arise within the closed λ-geometry itself. Only hyperfine and exchange interactions require open-loop coupling, to be treated in subsequent depletion analysis.

8.8. Experimental Signatures and Predictions

Observable deviations from standard quantum mechanics occur wherever $\nabla\lambda/\lambda$ is measurable:

- **Atomic spectroscopy:** The curvature correction in Eq. (54) implies fractional level shifts $\Delta E/E \sim 10^{-6}$ for hydrogen, comparable to the Lamb shift and therefore within present precision. *Benchmark.* State-of-the-art hydrogen spectroscopy bounds are at the $\mathcal{O}(10^{-6})$ fractional level for relevant intervals, so the predicted $\Delta E/E \sim 10^{-6}$ is within current precision [11]-[13].

- **Quantum tunneling:** For barriers of nanometer scale, a modest gradient $\lambda(x)=1+10^{-5}x/L$ alters Eq. (48) by 1% - 2%, observable with STM junctions.
- **Squeezed light:** Temporal modulation $\lambda(t)=1+10^{-4}\sin\omega t$ predicts sub-femtosecond oscillations of $\Delta x\Delta p/\hbar\approx 0.5 - 0.55$, consistent with attosecond squeezing data.
- **Gravitational analogy:** In strong-field environments where $\nabla\lambda/\lambda\sim 10^{-9}$, Eq. (53) reproduces redshifts of the same order as gravitational frequency shifts, offering a comparative test.

These estimates provide near-term experimental pathways to distinguish NUVO scalar geometry from conventional quantum mechanics.

8.9. Summary

Each canonical quantum phenomenon corresponds to a distinct geometric feature of the scalar field:

- Bound spectra from stationary holonomy;
- Tunneling from spatial modulation of λ;
- Spin from double-cover holonomy (via spinor lift);
- Entanglement from correlated curvature;
- Squeezing from time-dependent $\lambda(t)$.

The scalar transport law unifies them under a single deterministic geometry, bridging the continuum mechanics of λ with the observed discreteness of quantum phenomena.

9. Discussion: Frames and Classical Ambiguity

The preceding sections demonstrate that the mathematical structure of quantum mechanics emerges from the scalar geometry of NUVO space. No statistical postulates, operator assumptions, or probability axioms are required. The wave equation, uncertainty principle, and quantization of action all follow from the continuity, curvature, and holonomy of the scalar field λ. This geometric reconstruction resolves a central ambiguity of classical quantum theory-the role of the observer's frame.

9.1. Frame Ambiguity in Classical Quantum Mechanics

Standard quantum mechanics assumes an implicit flat background shared by all observers. Wavefunctions are defined relative to unspecified coordinates, and frame changes are introduced through ad hoc phase factors or gauge prescriptions. Consequently, the physical meaning of "state" and "measurement" is ambiguous: does a wavefunction describe a property of the system or of the observer's reference frame? This ambiguity becomes critical in accelerating or curved settings, where the notions of stationarity and simultaneity lose meaning.

9.2. Resolution Through Scalar Geometry

In NUVO space, every observer's frame is defined explicitly by the local normalization $\lambda_F = 1$. All measurable quantities are referenced to that frame through $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$. Frame transformations are generated by the scalar connection $\omega_\mu = \partial_\mu \ln \lambda$, ensuring covariance of ψ and of all derived observables. Thus, the frame is not an external convention but part of the physical geometry itself. When $\partial_t \lambda = 0$, the observer is stationary; when $\partial_t \lambda \neq 0$, the same equations describe moving or accelerating frames consistently.

9.3. Unified View of Local and Global Structure

Classical quantum mechanics effectively stitches together local differential relations-continuity, Schrödinger evolution-without explicit reference to global consistency. NUVO geometry unifies the two scales:

- **Local:** $\nabla_\eta \ln \lambda$ defines the connection that governs curvature and noncommuting derivatives, giving rise to local uncertainty.
- **Global:** The holonomy of the one-form $\omega = \partial \ln \lambda$ satisfies $\oint_\gamma \omega \cdot \mathrm{d}x = 2\pi n$ for closed loops γ in the presence of global non–exactness (e.g., defects/branch points) or when lifted to the spinor bundle, enforcing topological closure and quantization.

Hence, the same geometric object—$\nabla \ln \lambda$—controls both local uncertainty and global discreteness. The constant $\hbar$ simply calibrates the empirical scale of this duality.

9.4. Connection to Squeezed-Light Phenomena

The curvature-modulated uncertainty relation,

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left| \left\langle \lambda^{-2} \right\rangle \right|,$$

offers a direct geometric explanation of optical squeezing. In intense or ultrafast fields, rapid temporal oscillations of $\lambda(t)$ cause $\left\langle \lambda^{-2} \right\rangle$ to vary within an optical cycle, alternately reducing and increasing the measured uncertainty product. This produces the phase–locked amplitude and noise suppression observed in attosecond and high–harmonic experiments. The companion NUVO study on squeezed light interprets these effects as the empirical signature of dynamic scalar curvature modulation—a direct experimental probe of $\lambda(t)$.

Beyond optics, small λ-dependent shifts in atomic transition energies or interference fringes under high fields could serve as additional tests of scalar geometry.

9.5. Relation to Existing Geometric Approaches

Unlike Weyl's gauge geometry, which introduces an independent vector potential to preserve scale covariance, the NUVO construction employs only a single scalar field λ whose logarithmic gradient $\omega_\mu = \partial_\mu \ln \lambda$ serves as the connection itself. This eliminates path-dependent scale transport and ensures integrability of length. Whereas Weyl geometry predicts unobserved frequency drifts, NUVO retains exact conformal closure and couples curvature directly to the conserved scalar flux, providing a simpler and empirically safer route to geometric quantization.

Several existing frameworks share partial features with NUVO geometry. Conformal quantum mechanics and Weyl geometry also employ local scale factors, but typically treat the conformal factor as an auxiliary field or a gauge freedom rather than as a conserved substrate with intrinsic continuity. In Weyl models, scale curvature often couples to the electromagnetic potential via gauge compensation, whereas in NUVO the scalar connection $\omega_\mu = \partial_\mu \ln \lambda$ already induces a $U(1)$ -like term in the covariant derivative; standard electromagnetism can be incorporated additively in the usual way. Emergent-quantum-gravity programs (loop, causal set, information geometry) tend to quantize spacetime discreteness; NUVO instead preserves a smooth manifold in which quantization arises from holonomy closure. Accordingly, NUVO complements these approaches by providing a single-field, continuous realization of conformal gravity and quantum mechanics unified through scalar continuity.

9.6. Interpretation and Outlook

Quantum mechanics therefore appears not as a probabilistic law but as the closed–frame limit of deterministic scalar geometry. Apparent randomness arises from geometric curvature and frame connection, not intrinsic indeterminism. The wavefunction ψ represents coherent sinertia flow, its squared modulus corresponding to measurable density rather than probability. When $\partial_t \lambda = 0$, the system is closed and obeys the Schrödinger limit. When $\partial_t \lambda \neq 0$ or open exchange occurs, the full transport equation describes measurement, radiation, and photon emission as scalar continuity processes. These open–system extensions define the bridge from geometry to observation, to be developed in the forthcoming *NUVO Depletion I* paper.

Ontological note. Throughout, $\lambda(x)$ is treated as a geometric scalar that fixes local units via $g = \lambda^2 \eta$ and enforces sinertia continuity; it is not endowed with an independent kinetic term or separate energy content-its curvature encodes observable energy shifts without promoting λ to a dynamical matter field.

9.7. Summary

In summary, NUVO scalar geometry resolves foundational ambiguities of quantum theory by:

• Defining the observer's frame as a physical normalization of λ;

• Deriving operators, uncertainty, and quantization from curvature and holonomy rather than postulates;

• Explaining measurement effects as open–system evolution of sinertia rather than wavefunction collapse;

• Providing geometric explanations for modern quantum phenomena such as entanglement and squeezing.

These results show that quantum mechanics is a limiting projection of a continuous scalar geometry in which curvature, not chance, governs the structure of physical law.

10. Conclusions and Outlook

This study has demonstrated that the entire framework of nonrelativistic quantum mechanics arises directly from the scalar geometry of NUVO space. Beginning with the single conformal metric $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ and the conservation of scalar sinertia, we have shown that all standard quantum structures follow from first principles of continuity, curvature, and holonomy.

10.1. Key Results

- The **continuity law** and **scalar transport equation** (Eqs. (8) - (12)) govern amplitude evolution without probabilistic postulates.
- The **geometric operators** $\hat{p}_i = -i\hbar D_i$ and $\hat{E} = i\hbar D_t$ arise from the scalar connection $\omega_\mu = \partial_\mu \ln \lambda$, reproducing canonical commutation relations and uncertainty directly from curvature.
- The **uncertainty principle** and **quantized action** emerge as local and global manifestations of the same geometry: differential noncommutation and integral holonomy.
- The **Schrödinger equation** is obtained as the stationary, closed-frame limit of the general scalar transport law, linking probabilistic mechanics to deterministic scalar continuity.
- The **empirical constant** $\hbar$ represents the physical scale corresponding to one unit of scalar holonomy, fixed once from the 13.6 eV hydrogen binding energy.

10.2. Physical Interpretation

Quantum mechanics thus emerges as the effective description of sinertia-conserving transport in a closed scalar geometry. The wavefunction ψ represents coherent scalar flow, not statistical probability. Apparent randomness results from local curvature and incomplete frame information, while the underlying process remains deterministic. When $\partial_t \lambda = 0$, the system behaves as an isolated, stationary geometry reproducing conventional quantum behavior. When $\partial_t \lambda \neq 0$ or open exchange occurs, the same geometry yields radiation, photon emission, and measurement phenomena without discontinuous collapse.

10.3. Geometric Unification

The scalar field λ unifies the principal features of quantum and relativistic physics:

- Curvature of λ produces both gravitational and quantum effects within a single conformal metric;
- Local curvature yields uncertainty and energy quantization, while global curvature yields discrete spectra and conserved action;

• Frame covariance replaces gauge arbitrariness: all measurable quantities transform by scalar normalization, preserving total sinertia.

In this view, quantum mechanics, gravitation, and field theory are not separate domains but limiting projections of a single continuous geometry.

10.4. Future Development

The next paper in this series, *NUVO Depletion I*: *Measurement and Interaction*, extends the analysis to open systems where sinertia exchange occurs between coupled regions of λ. That work introduces the depletion mechanism, showing how observation, radiation, and photon emission arise as scalar transfer processes within the same geometry. Subsequent studies will apply these results to composite systems, strong-field regimes, and cosmological expansion, demonstrating how gravitation, charge, and quantum coherence emerge as unified expressions of scalar continuity.

10.5. Final Remark

The emergence of quantum mechanics from scalar geometry completes the first stage of the NUVO quantization program. It provides a continuous, covariant, and observer-explicit foundation for all quantum phenomena, establishing a direct bridge between the deterministic curvature of geometry and the discrete structure of nature.

Future work will address how measurement arises in this deterministic framework. Because sinertia continuity replaces probabilistic postulates, state reduction is expected to appear as a geometric relaxation-an exchange of scalar capacity between system and measuring apparatus-rather than as a discontinuous wavefunction collapse. This outlook offers a concrete path toward resolving the measurement problem within scalar geometry.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

A. Notation Summary

$\lambda(x)$	Scalar modulation field (unit constraint)
η	Background flat metric
$g=\lambda^2\eta$	Conformal metric of NUVO space
$\Gamma^{\rho}_{\mu\nu}$	Levi–Civita connection of g
ψ	Complex scalar amplitude (sinertia phase/magnitude)
ρ_λ	Measurable density $\lambda^3\lvert\psi\rvert^2$
ω_μ	Scalar connection $\partial_\mu \ln\lambda$
$F_{\mu\nu}$	Curvature two–form $\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$

B. Constants

$$r_e=2.8179\times10^{-15}\text{ m},\quad a_0=5.2918\times10^{-11}\text{ m},\quad \lambda_C=2.4263\times10^{-12}\text{ m}.$$

C. Derivation of the Scalar Transport Law

Starting from the continuity equation of Section 3,

$$\partial_t\left(\lambda^3\lvert\psi\rvert^2\right)+\nabla_\eta\cdot\left(\lambda\boldsymbol{J}_\lambda\right)=0,$$

with the flux (Section 3)

$$\boldsymbol{J}_\lambda=\lambda^2R^2\nabla_\eta\phi\quad\text{for }\psi=Re^{i\phi},$$

and using the divergence and Laplace–Beltrami identities of Section 2,

$$^{\lambda}\nabla\cdot\left(\lambda^nA\right)=\lambda^n\nabla_\eta\cdot A+n\lambda^{n-1}A\cdot\nabla_\eta\lambda,\quad \nabla^2_\lambda f=\lambda^{-2}\left(\nabla^2_\eta f+2\nabla_\eta\ln\lambda\cdot\nabla_\eta f\right),$$

one obtains after a standard amplitude/phase recombination the scalar transport law quoted in Eq. (12):

$$\lambda^2\nabla^2_\eta\psi-\frac{1}{c^2}\partial_t^2\psi+\mathcal{C}_\lambda\psi=0,\quad \mathcal{C}_\lambda=2\nabla_\eta\ln\lambda\cdot\nabla_\eta+\nabla^2_\eta\ln\lambda.$$

This is the unique second-order covariant closure consistent with the continuity of sinertia and the conformal connection of $g=\lambda^2\eta$.

D. General Schrödinger Solutions in Scalar Geometry

The scalar transport law of Section 3 provides a closed geometric system from which the Schrödinger equation arises as its stationary projection. Since $\lambda(x,t)$ fixes both the local metric and the connection, the complex amplitude admits a *local, gauge-fixed* representation in terms of λ.

Local representation of ψ psi in terms of λ lambda

Let $\lambda(x,t)>0$ be smooth and satisfy the coupled continuity/curvature system

$$\nabla^{\lambda}_{\mu} J^{\mu}_{\sin} = 0, \quad \Box_{\eta} \ln \lambda + F(\lambda, \nabla\lambda) = 0,$$

with F encoding geometric coupling. In any simply connected neighborhood, one may choose a phase gauge such that

$$\psi[\lambda](x,t) = \lambda^{-3/2}(x,t)\exp(i\chi(x,t)), \quad \chi(x,t) := \int^{x} \omega \cdot \mathrm{d}\mathbf{x}' - \int^{t} \omega_0 \mathrm{d}t', \quad (56)$$

where $\omega_{\mu} = \partial_{\mu} \ln \lambda$. (Globally, nontrivial holonomy or spinor lifts may obstruct single-valued χ.) Substituting (56) into the stationary transport law reproduces the Schrödinger form

$$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m} \lambda^{-2} \nabla^2_{\eta} \psi + V\psi,$$

with V an effective potential expressible from λ and its derivatives via F.

Interpretation

Equation (56) shows that (locally) every admissible λ field determines a corresponding complex amplitude ψ solving the Schrödinger equation in NUVO space. The conventional quantum potential and effective $V(x)$ descend from curvature of λ. The global solution space is then

$$\mathcal{S} = \{\psi[\lambda] : \lambda \text{ satisfies scalar continuity / curvature}\},$$

with global phase/holonomy classes determined by the topology of λ (and, for spin, the spinor bundle).

Future mathematical development

A rigorous proof that $\lambda \mapsto \psi[\lambda]$ gives a complete representation of stationary Schrödinger solutions (including existence, uniqueness, and regularity for the coupled system) will be developed separately. There we will specify the functional setting and boundary data under which $\mathcal{S}$ spans the relevant Hilbert space.

E. Entropy, Sinertia, Gravity, and the Arrow of Time

E.1. Scalar Continuity and Sinertia

Flow the scalar geometry obeys

$$\nabla^{\lambda} \cdot J_{\sin} = 0, \quad J^{\mu}_{\sin} = \lambda \rho u^{\mu}, \quad (57)$$

expressing conservation of finite substrate capacity (sinertia). Local depletion/accumulation modifies λ via

$$\nabla^{\lambda} \cdot J_{\sin} = -\Gamma_{\text{dep}}(\lambda)\rho, \quad \Gamma_{\text{dep}} \propto |\nabla \ln \lambda|^2. \quad (58)$$

The sign of $J^{0}_{\sin}$ fixes the direction of "forward" time.

E.2. Entropy as Informational Mirror

For a configuration $\lambda(x)$ with multiplicity Ω_{λ},

$$S = k_B \ln \Omega_{\lambda},$$

and hence

$$\partial_{\mu} S = k_B \left(\frac{\partial f}{\partial \lambda} \partial_{\mu} \lambda + \frac{\partial f}{\partial(\nabla_{\nu}\lambda)} \partial_{\mu} \nabla_{\nu} \lambda \right),$$

so entropy gradients are aligned (up to sign/weighting) with $\partial_\mu \lambda$.

E.3. Flow Relation and Mirror Symmetry

Let $J_S^\mu = s u^\mu$ with $s = \mathrm{d}S/\mathrm{d}V$. Empirically,

$$\frac{\mathrm{d}S}{\mathrm{d}\lambda} > 0, \quad \frac{\mathrm{d}\Sigma}{\mathrm{d}\lambda} < 0,$$

for sinertia measure Σ, yielding anti-parallel fluxes:

$$J_S^\mu = -\alpha J_{\sin}^\mu, \quad \nabla_\mu S \propto -\nabla_\mu \Sigma, \tag{59}$$

with $\alpha > 0$. Thus

$$S + \alpha\Sigma \approx \text{constant}. \tag{60}$$

E.4. Arrow of time

From (59),

$$\operatorname{sign}\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right) = -\operatorname{sign}\left(\frac{\mathrm{d}\Sigma}{\mathrm{d}t}\right) = \operatorname{sign}\left(J_{\sin}^0\right),$$

so the future is the direction of positive sinertia flux.

E.5. Gravitational Correspondence

Curvature in $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ peaks where sinertia availability is lowest. Large λ (strong curvature) $\Leftrightarrow$ high depletion $\Leftrightarrow$ high entropy, paralleling black-hole thermodynamics.

E.6. Commutator and Irreversibility

Whenever

$$[\lambda, H]_\nu \neq 0,$$

the Hamiltonian fails to commute (in the NUVO sense) with the scalar substrate, producing entropy generation. Reversible equilibrium corresponds to $[\lambda, H]_\nu = 0$.

E.7. Summary

1) Sinertia is the finite geometric capacity of the substrate; its flow sets curvature and gravity.
2) Entropy records that flow; it increases exactly as sinertia depletes.
3) The arrow of time is the orientation of positive sinertia flux.
4) Total information content is approximately conserved: $S + \alpha\Sigma = \text{const}$.

F. Neutron Beta Decay and Cosmological Consequences in NUVO Geometry

Editorial note. The following exploratory applications indicate scope only. A quantitative treatment of the depletion constant γ_0, nuclear geometry, and cosmological solutions will appear in *NUVO Depletion II*.

F.1. A.1 Neutron Decay from Closure Geometry

Model a neutron as a proton closed loop coupled to an electron open loop whose end is sealed by scalar pressure at a slender closure throat (radius a_t, length ℓ). Decay occurs by under–substrate depletion leakage:

$$\Gamma_{\text{dep}} = \gamma_0 \int_{\text{throat}} |\nabla \ln \lambda|^2 \, \mathrm{d}V. \tag{61}$$

The β-decay rate then follows

$$\Gamma_{n \to pe\bar{\nu}} = K_\beta Q^5, \quad K_\beta = \frac{\gamma_0}{(\hbar c)^4} \mathcal{G}[\lambda], \tag{62}$$

with $Q \simeq 0.782$ MeV and

$$\mathcal{G}[\lambda] = \frac{1}{E_*} \int_{\text{throat}} |\nabla \ln \lambda|^2 \, \mathrm{d}V, \quad E_* = \frac{\hbar c}{\ell}. \tag{63}$$

For a smooth slender-tube profile

$$\lambda(r) = 1 + \frac{\alpha}{1 + (r/a_t)^2}, \tag{64}$$

one finds

$$\int_{\text{throat}} |\nabla \ln \lambda|^2 \, \mathrm{d}V \simeq \frac{8\pi}{3} \alpha^2 \frac{a_t^2}{\ell}. \tag{65}$$

Hence

$$\mathcal{G}[\lambda] \simeq \frac{8\pi}{3} \alpha^2 \frac{a_t^2}{\hbar c}, \quad K_\beta \simeq \frac{\gamma_0}{(\hbar c)^5} \frac{8\pi}{3} \alpha^2 a_t^2. \tag{66}$$

From $\tau_n \simeq 880$ s and $Q_n^5 \approx 0.292$ MeV^5,

$$K_\beta^{(\text{emp})} = \Gamma_n / Q_n^5 \simeq 3.9 \times 10^{-3} \, \text{s}^{-1} \cdot \text{MeV}^{-5}. \tag{67}$$

A calculation of $\mathcal{G}[\lambda]$ reproducing (67) within order unity would support a common depletion mechanism across atomic, photonic, and nuclear processes.

F.2. A.2 Comparison to the Electroweak Form

The Standard Model writes

$$\Gamma_{\text{SM}} = \frac{G_F^2 |V_{ud}|^2}{2\pi^3} (1 + 3g_A^2) f(Q), \quad f(Q) \propto Q^5. \tag{68}$$

In NUVO, i) the rate emerges from geometric first principles via (62) - (66); ii) Q^5 follows from throat geometry; iii) $m_n - m_p - m_e$ measures stored throat energy; iv) K_β is computed once and reused across β processes. The same γ_0 governs scalar relaxation generally.

F.3. A.3 Cosmological Consequence: Expansion from Sinertia Depletion

With

$$g_{\mu\nu} = \lambda^2(x,t) \eta_{\mu\nu}, \quad \dot{S}_u(x,t) < 0 \Rightarrow \dot{\lambda}(x,t) > 0, \tag{69}$$

integration over space gives

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V S_u \mathrm{d}V = -\int_V \Gamma_{\text{dep}} \mathrm{d}V, \quad a_{\text{NUVO}}(t) \equiv \langle \lambda(t) \rangle_V, \tag{70}$$

and thus

$$H_{\text{NUVO}} = \frac{\dot{a}_{\text{NUVO}}}{a_{\text{NUVO}}} \propto \frac{1}{3}\left\langle \Gamma_{\text{dep}} \right\rangle_V . \quad (71)$$

Cosmic expansion is therefore a geometric manifestation of net sinertia flow from the under-substrate to the observable domain.

F.4. A.4 Link to the Neutrino Background

Each β transition or fusion event emits a neutrino, tracking integrated depletion:

$$N_\nu(t) \propto \int_0^t \int_V \Gamma_{\text{dep}}(x,t')\,\mathrm{d}V\mathrm{d}t' \propto \ln\frac{a_{\text{NUVO}}(t)}{a_0}. \quad (72)$$

Thus the neutrino background is a ledger of scalar reconfiguration in cosmic history.

F.5. A.5 Summary

The same depletion law that explains atomic and nuclear relaxation predicts:

1) the free neutron lifetime via a computable geometric constant K_β; and

2) the apparent cosmic expansion via the slow global increase of $\langle \lambda \rangle$.

Hence β decay, neutrino emission, and cosmic expansion are linked manifestations of sinertia redistribution under a universal continuity constraint.