Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(2), 785-803
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Vaccination Decisions in Triadic Interactions: A Coupled ODE Approach

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How to cite this paper: Ibragimov, R. and Ntiamoah, D. (2026) Vaccination Decisions in Triadic Interactions: A Coupled ODE Approach. *Journal of Applied Mathematics and Physics*, **14**, 785-803.
https://doi.org/10.4236/jamp.2026.142041

Received: October 10, 2025
Accepted: February 11, 2026
Published: February 14, 2026

Abstract

This paper presents a mathematical model of vaccination decision-making in small social groups using triadic game theory. It employs coupled ordinary differential equations to illustrate the evolution of vaccination intensities among three individuals, influenced by interaction coefficients that reflect their interconnected decisions. The analysis explores scenarios in which one individual's vaccination stance remains fixed. When the first individual maintains a constant stance, the system exhibits exponential growth or decay. Conversely, if the second individual is fixed, the system displays oscillatory behavior with declining vaccination rates. Additionally, the model is extended to include two coupled triads involving five individuals, revealing emergent behaviors that are not present in isolated groups. The findings emphasize differences in group stability based on which member serves as the decision anchor. This research establishes a mathematical foundation for understanding collective behavior in small social clusters and suggests potential avenues for future studies on vaccination decision-making in families, friend groups, and workplaces.

Keywords

Triadic Game Theory, Vaccination Decision-Making, Coupled Ordinary Differential Equations, Emergent Behavior, Decision Anchor

1. Introduction

The vaccination dilemma, first described by Fine and Clarkson [1] and later studied by Geoffard and Philipson [2], represents a fundamental tension between individual rational self-interest and societal welfare in public health decision-making. Classical game-theoretic analyses predict that populations guided exclusively by individual rational choices will reach Nash equilibria with vaccination coverage below the societal optimum, creating a persistent gap between what is individually rational and collectively beneficial.

Building upon this foundation, contemporary research has investigated how social interactions and behavioral factors influence vaccination decisions beyond pure economic calculations. The work of Xin *et al.* [3] addressed a key limitation in social learning models: while imitation-based approaches using standard Fermi switching functions typically produce vaccination rates below Nash equilibrium levels, introducing greater flexibility in how individuals respond to others' strategies can dramatically improve outcomes. Specifically, their "open-mindedness" parameter captures the degree to which decision-makers remain receptive to alternative strategies, with higher values enabling social learning to push vaccination coverage toward socially optimal levels rather than the suboptimal equilibria predicted by traditional models.

Recent advances in vaccination game theory have further highlighted the limitations of traditional approaches when applied to complex social structures. Perisic and Bauch [4] demonstrated through imitation dynamics that vaccination decisions in structured populations deviate significantly from well-mixed population predictions, while Bauch *et al.* [5] showed that individual vaccination behavior can create clustered susceptibility patterns that persist even when overall coverage appears adequate. Expanding on these structural insights, Kumar *et al.* [6] incorporated adaptive social connections, revealing that individual vaccination behavior combined with dynamic network topology creates complex threshold dynamics not captured by static models.

These structural considerations have been complemented by insights from behavioral economics into the psychological drivers of vaccination decision-making. Oraby *et al.* [7] found that observational social learning significantly affects vaccination uptake, with individuals' decisions influenced by both local and global information about disease prevalence and vaccine safety. This finding aligns with work by Chang *et al.* [8], who demonstrated that social norms vary significantly depending on which reference groups individuals consider as "others," suggesting that vaccination campaigns must account for heterogeneous social influence patterns. Wu *et al.* [9] further extended this behavioral perspective, showing that psychological factors such as risk perception and trust in health authorities create non-linear responses to public health interventions that cannot be captured by traditional epidemiological models.

The recognition that both social structure and behavioral factors matter has led to increasingly sophisticated network-based approaches. Granell *et al.* [10] introduced multiplex network models where disease transmission and opinion formation occur on separate but interconnected layers, building on earlier work by Salathe and Bonhoeffer [11] who demonstrated that opinion clusters on vaccination can persist and spread through social networks independently of disease dynamics. More recently, Bellingeri *et al.* [12] showed that vaccination strategies must account for the underlying network of social interactions, with targeted approaches significantly outperforming random vaccination in structured populations. Chen *et al.* [13] extended this work to show that multiplex social network targeting can overcome vaccine hesitancy more effectively than single-layer approaches.

However, despite these advances, most existing models focus either on pairwise interactions within larger populations or on aggregate population-level dynamics. Reluga [14] noted that cooperation dynamics in vaccination games reveal emergent behaviors in small groups that are not captured by dyadic interaction models, while Cardillo *et al.* [15] emphasized that multi-agent approaches typically focus on spatial rather than social clustering effects. As Manfredi and d'Onofrio [16] argued, the complexity of real-world vaccination decision-making requires models that can capture simultaneous multi-way interactions within social clusters, particularly during emerging infectious disease outbreaks where small social groups coordinate their responses to public health recommendations.

While these findings represent significant progress in understanding how social learning can enhance vaccination outcomes, they primarily consider pairwise interactions within larger populations. Real-world vaccination decisions, however, often occur within small social clusters where multiple individuals simultaneously influence each other's choices. Family units, close friend groups, and workplace teams frequently engage in collective decision-making processes that cannot be fully captured by models focusing on dyadic interactions or large population dynamics (Teslya *et al.* [17]).

To address this gap, we introduce a triad-player game framework that explicitly models the complex interdependencies among three decision-makers during a pre-outbreak vaccination campaign. Our model recognizes that each individual's vaccination decision is simultaneously influenced by their interactions with two other players, creating a system of coupled decision dynamics that exhibits rich behavioral patterns not captured by previous approaches.

The triad structure is particularly relevant for understanding vaccination behavior during emerging infectious disease outbreaks, such as COVID-19, where small social groups often coordinate their responses to public health recommendations. In such contexts, the decision-making process involves not only individual risk assessment but also social influence patterns that evolve dynamically as group members observe and respond to each other's evolving preferences.

Our mathematical framework employs a system of coupled ordinary differential equations to model the temporal evolution of vaccination decisions within the triad. This approach allows us to analyze how small perturbations in individual commitment levels propagate through the group, revealing fundamental stability properties of different decision-making scenarios. We introduce decision-making interaction coefficients γ_{ij} that capture the willingness of individuals i and j to coordinate their vaccination choices, with positive coefficients indicating mutual reinforcement of pro-vaccination decisions and negative coefficients representing mutual reinforcement of vaccine hesitancy.

By analyzing different "forcing" scenarios, where one member of the triad maintains a strong initial position while the other two members' decisions evolve through their interactions, we explore how stable individual preferences and dynamic peer influence combine to shape collective vaccination outcomes. This approach allows us to investigate questions such as: What mathematical dynamics emerge when individuals within triads maintain different levels of commitment to their vaccination decisions? How do these dynamics differ between isolated triads and coupled triad systems? And what are the stability properties of different forcing scenarios in small group decision-making?

The remainder of this paper develops the mathematical model, analyzes the dynamics under different forcing conditions, and discusses the theoretical implications for understanding vaccination decision-making in small social groups.

2. Mathematical Model

This article aims to develop a model that accommodates an arbitrarily large number of individuals participating in a vaccination campaign. The individuals are organized into interacting groups of three, referred to as trios or *triads*, as illustrated schematically in **Figure 1**. Within each triad, the members are designated as 0, 1 and 2. Each individual has an intensity, denoted as A_i (where $i = 0,1,2$), that signifies their likelihood of getting vaccinated or not. In this paper, we relate intensity to a measure of maximum displacement from equilibrium, which shares the same dimensions as other forms of wave amplitude. Consequently, the dimensional representation of intensity is designated as $[L]$, indicating that it measures distance.

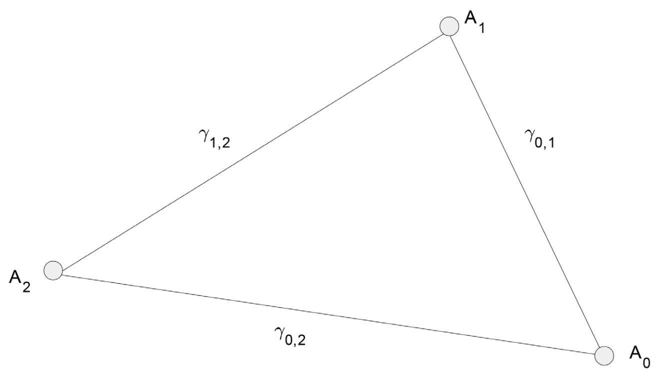

Figure 1. Schematic illustrating the intensities A_i $(i = 0,1,2)$ of willingness to vaccinate distributed among three individuals i $(i = 0,1,2)$ forming a triad. During a pre-outbreak vaccination campaign, each individual i makes an independent decision based on various factors.

The decision-making interaction coefficients $\gamma_{0,1}, \gamma_{0,2}$, and $\gamma_{1,2}$ for vaccination against flu-like infections indicate the intensity of the vaccination effort. For instance, the coefficient $\gamma_{0,1}$ represents the combined intensity of individuals 0 and 1 participating in the vaccination campaign. Thus, the interaction coefficients are defined as follows:

$$\gamma_{0,1} = A_0(0)A_1(0),\ \gamma_{0,2} = A_0(0)A_2(0),\ \gamma_{1,2} = A_1(0)A_2(0). \tag{2.1}$$

In this model, it is assumed that during a vaccination campaign that precedes the actual outbreak of the disease, each focal individual i $(i = 0,1,2)$ makes an independent decision based on various factors. Furthermore, assuming that one or two members of the triad are willing or unwilling to participate in the vaccinate campaign, the coefficients γ_{ij} adhere to the *triad rule*:

$$\gamma_{0,1} + \gamma_{0,2} + \gamma_{1,2} = 0. \tag{2.2}$$

The rule (2.2) reflects the interdependence of the individuals' decisions regarding vaccination. For instance, the coefficient $\gamma_{0,1}$ signifies the overall intensity of the individuals 0 and 1, meaning that the interaction coefficients reflect the intensity of these individuals' decisions to vaccinate or not. A positive coefficient $\gamma_{ij} > 0$ indicates a favorable initial decision-making process of individuals i and j, *i.e.* $A_i(0)A_j(0) > 0$. However, over time $t > 0$, their decision may be influenced by another individual k ($k \neq i$, $k \neq j$). Consequently, the pair (i, j) may either maintain their initial choice or switch to a negative decision (not to vaccinate). Conversely, a negative decision-making coefficients $\gamma_{ij} < 0$ implies that the initial decision-making of both i and j is unfavorable, indicating that both have opted not to vaccinate, $A_i(0)A_j(0) < 0$. In other words, the value A_i represents the intensity of each individual i, while the interaction coefficient γ_{ij} indicates the overall intensity between two individuals, i and j. As will be discussed later, there is a direct relationship between the individual intensity A_i for each person and γ_{ij} for all three individuals in a triad.

Since each person's intensity (e.g. 0), can influence another person's intensity, for example, 2, we can visualize this influence as a vector extending from 0 to 2. From a geometric perspective, we can relate the influences among three individuals in a triad to the triad rule (2.2). This rule states that the sum of the vectors $\boldsymbol{A}_0\boldsymbol{A}_1 + \boldsymbol{A}_1\boldsymbol{A}_2 + \boldsymbol{A}_2\boldsymbol{A}_0 = 0$. This indicates that the wave vectors are in equilibrium, meaning that the triad forms a "closed system", as shown in **Figure 1**. In other words, the intensity does not flow out of the triad, nor does it come in from an external source.

We will analyze a specific scenario where $\gamma_{0,1} < 0$, $\gamma_{0,2} < 0$, and $\gamma_{0,2} > 0$. This situation depicts the case in which the pairs of individuals $(0,1)$ and $(0,2)$ are initially unwilling to participate in vaccination campaign initially. However, the initial negative decision is further influenced by the interaction between the pair of individuals $(1,2)$. Consequently, the choice of initial intensities $A_i(0)$ depends on the numerical values of the coefficients γ_{ij}. In reality, these coefficients can be selected based on real observations. However, since our focus is on qualitative analysis rather than the quantitative evaluation of numerical data, we will use some numerical values that enhance the descriptiveness of the simulation. This approach will help us gain a deeper understanding of the underlying reasons behind the phenomena. Therefore, the simulations will utilize the following numerical choices for the interaction coefficients:

Table 1. Values of parameters for a single triad.

$\gamma_{1,2}$	$\gamma_{0,1}$	$\gamma_{0,2}$
13.09e−08	−8.0e−8	−5.0e−08

In this paper, the numerical values presented in **Table 1** have been selected randomly to gain insight into the qualitative behavior of vaccination intensities. However, we plan to extend the findings of this paper to a larger number of triads, as discussed in the final section. The numerical values of the interaction coefficients will be linked to observational data from [18], where the authors analyzed vaccination strategies for a large population using statistical methods. This will be explored in our future studies.

The coupled system of ODEs that describe a time behavior of a decision on participating or nonparticipating in the vaccination campaign:

$$\frac{\mathrm{d}A_0}{\mathrm{d}t} = \gamma_{1,2} A_1 A_2, \quad A_0(0) = \alpha_0, \tag{2.3}$$

$$\frac{\mathrm{d}A_1}{\mathrm{d}t} = \gamma_{0,2} A_0 A_2, \quad A_1(0) = \alpha_1, \tag{2.4}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}t} = \gamma_{0,1} A_0 A_1, \quad A_2(0) = \alpha_2, \tag{2.5}$$

where $\alpha_i \quad (i = 0,1,2)$ are given data that are related to the interaction coefficients γ_{ij} via (2.1).

3. Constant Intensities

In this section we will examine situations in which one of the participants does not change his or her willingness to vaccinate. The primary focus is to understand how this choice affects the overall behavior of the participants within the triad. Since the interaction coefficients γ_{ij} follow the triad rule (2.2), the signs of the coefficients are not all the same. As a result, the overall behavior depends on the group depends on which specific individual is unwilling to change their decision regarding vaccination. We will first investigate the scenario in which individual 0 is not willing to adjust his or her willingness to vaccinate. Then, we will consider the case in which individual 1 is resistant to changing his or her willingness to vaccinate.

3.1. Case 1: Constant Intensity A_0

We begin by considering the scenario where the vaccination intensity A_0 of an individual 0 remains constant. The primary question we aim to explore is how a small perturbation added to A_0 will influence the overall behavior of the participants within the triad $\{A_0, A_1, A_2\}$. Following this, we will analyze the implications of this perturbation and we let

$$A_0 = A_* + \delta \hat{A}_0, \quad A_1 = \delta \hat{A}_1, \quad A_2 = \delta \hat{A}_2 \tag{3.1}$$

in which $\delta \ll 1$ is a small parameter, $A_* = \text{const.}$, and $\hat{A}_i \ll A_*$, where $i = 0,1,2$.

Then the initial conditions in (2.3) - (2.5) are changed as

$$\hat{A}_1(0)=\frac{\alpha_1}{\delta},\ \hat{A}_2(0)=\frac{\alpha_2}{\delta},\ \hat{A}_0(0)=0 \text{ and } A_*=\alpha_0. \tag{3.2}$$

In this case, within the order $0(\delta)$, Equations (2.3) - (2.5) are written as the following decoupled system:

$$\frac{\mathrm{d}\hat{A}_1}{\mathrm{d}t^2}=\gamma_{0,2}A_*\hat{A}_2, \tag{3.3}$$

$$\frac{\mathrm{d}\hat{A}_2}{\mathrm{d}t^2}=\gamma_{0,1}A_*\hat{A}_1. \tag{3.4}$$

The Equations (3.3) - (3.4) can be decoupled as

$$\frac{\mathrm{d}^2\hat{A}_1}{\mathrm{d}t^2}=\gamma_{0,1}\gamma_{0,2}A_*^2\hat{A}_1, \tag{3.5}$$

$$\frac{\mathrm{d}^2\hat{A}_2}{\mathrm{d}t^2}=\gamma_{0,1}\gamma_{0,2}A_*^2\hat{A}_2. \tag{3.6}$$

The explicit general solution of the system (3.5) - (3.6) is

$$\hat{A}_1=c_1\exp\left(A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right)+c_2\exp\left(-A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right), \tag{3.7}$$

$$\hat{A}_2=d_1\exp\left(A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right)+d_2\exp\left(-A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right). \tag{3.8}$$

In this case, $\gamma_{0,1}\gamma_{0,2}>0$, and the constants c_1,c_2,d_1, and d_2 can be expressed in terms of the initial data α_1 and α_2 as

$$c_{1,2}=\frac{1}{2\delta}\left(\alpha_1\pm\alpha_2\sqrt{\frac{\gamma_{0,2}}{\gamma_{0,1}}}\right), \tag{3.9}$$

$$d_{1,2}=\frac{1}{2\delta}\left(\alpha_2\pm\alpha_1\sqrt{\frac{\gamma_{0,1}}{\gamma_{0,2}}}\right). \tag{3.10}$$

According (2.1), we set

$$\alpha_1=\frac{\gamma_{0,1}}{\alpha_0},\alpha_1=\frac{\gamma_{1,2}}{\gamma_{0,1}},$$

in which

$$\alpha_0=\sqrt{\frac{\gamma_{0,1}\gamma_{0,2}}{\gamma_{1,2}}}$$

Figure 2 illustrates the time behavior of the vaccination intensities A_1 and A_2 for the individuals 1 and 2 affected by constant intensity A_0. We observe that the intensity A_1 is exponentially increasing, while the intensity A_2 is decreasing. This behavior suggests instability in the vaccination decisions of the triad. Specifically, the willingness of the individual 2 to vaccinate is increasing over time, whereas the intensity of the individual 1 is tending towards negative values. This indicates that, despite an initial positive decision to vaccinate, individual 1 will likely become unwilling to vaccinate as time progresses. Although the **Figure 2** shows the vaccination intensity for each individual within the triad, it does not reflect the overall intensity of all three individuals. To address this, we define the total vaccination intensity of the triad consisting of individuals $\{0,1,2\}$ as a product $A_0A_1A_2$. Figure 3 presents the time behavior of this total intensity. The data reveals that the total intensity $A_0A_1A_2$ of the triad is decreasing when A_0 is constant, indicating a general trend against vaccination intentions.

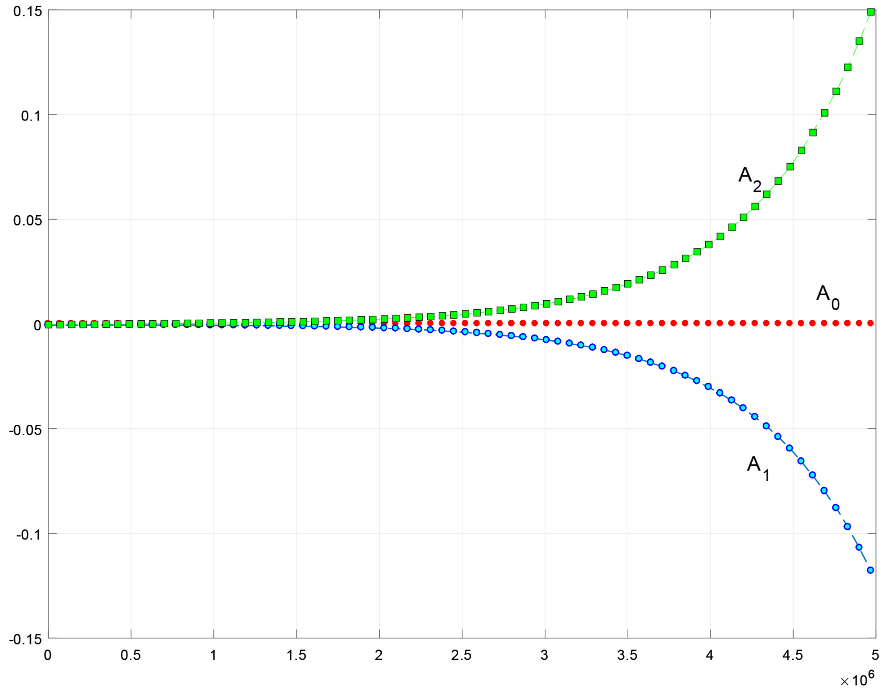

Figure 2. Case 1. Time behavior of the vaccination intensities A_1 and A_2 for the individuals 1 and 2, given that the intensity A_0 is constant.

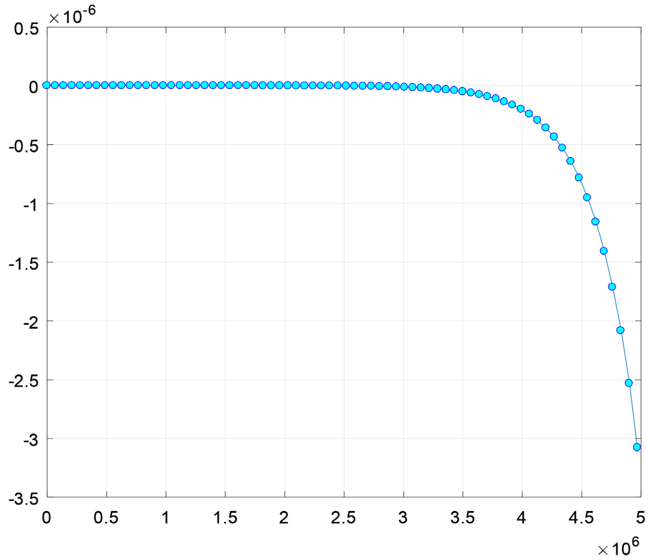

Figure 3. Time behavior of the total intensity $A_0A_1A_2$ of the triad of the individuals $\{0,1,2\}$ is decreasing when A_0 is constant, indicating that the overall intent is not to vaccinate.

3.2. Case 2: Constant Intensity A_1

Now, let's delve into a scenario in which the vaccination intensity A_1 of individual 1 remains constant. Similarly to the case 1, our goal is to investigate how even a minor perturbation to A_1 can significantly influence the overall behavior of the participants within the triad (A_0, A_1, A_2). Following this analysis, we will examine the profound implications of this perturbation, highlighting its potential impact on the dynamics of the group. We then let

$$A_0 = \delta \hat{A}_0,\ \ A_1 = A_* + \delta \hat{A}_1,\ \ A_2 = \delta \hat{A}_2 \tag{3.11}$$

with $\hat{A}_i \ll A_*$, where $i = 0,1,2$. Then the initial conditions in (2.3) - (2.5) are then modified as follows:

$$\hat{A}_0(0) = \frac{\alpha_0}{\delta},\ \ \hat{A}_1(0) = 0,\ \ \hat{A}_2(0) = \frac{\alpha_2}{\delta} \text{ and } A_* = \alpha_1. \tag{3.12}$$

In this case, within the order $0(\delta)$, Equations (2.3) - (2.5) can be expressed as:

$$\frac{\mathrm{d}\hat{A}_0}{\mathrm{d}t} = \gamma_{1,2} A_* \hat{A}_2, \tag{3.13}$$

$$\frac{\mathrm{d}\hat{A}_2}{\mathrm{d}t} = \gamma_{0,1} A_* \hat{A}_0. \tag{3.14}$$

The Equations (3.3) - (3.4) can be decoupled, meaning they can be treated independently as

$$\frac{\mathrm{d}^2\hat{A}_0}{\mathrm{d}t^2} = \gamma_{0,1}\gamma_{1,2} A_*^2 \hat{A}_0, \tag{3.15}$$

$$\frac{\mathrm{d}^2\hat{A}_2}{\mathrm{d}t^2} = \gamma_{0,1}\gamma_{1,2} A_*^2 \hat{A}_2. \tag{3.16}$$

The solution of the system (3.5) - (3.6) is written as

$$\hat{A}_0 = c_1 \exp\left(A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right) + c_2 \exp\left(-A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right), \tag{3.17}$$

$$\hat{A}_2 = d_1 \exp\left(A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right) + d_2 \exp\left(-A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right), \tag{3.18}$$

in which the constants c_1, c_2, d_1, and d_2 can be expressed in terms of α_0, α_2 and δ as follows:

$$c_{1,2} = \frac{\alpha_0 \sqrt{\gamma_{0,1}\gamma_{1,2}} \pm \alpha_2 \gamma_{1,2}}{2\delta \sqrt{\gamma_{0,1}\gamma_{1,2}}},\ \ d_{1,2} = \frac{\alpha_2 \sqrt{\gamma_{0,1}\gamma_{1,2}} \pm \alpha_0 \gamma_{0,1}}{2\delta \sqrt{\gamma_{0,1}\gamma_{1,2}}}. \tag{3.19}$$

Now, since $\gamma_{0,1}\gamma_{0,2} < 0$, the solution of the system (3.5) - (3.6) is written as

$$\hat{A}_0 = k_1 \cos\left(\operatorname{Im}\left\{A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right\}\right) + k_2 \sin\left(\operatorname{Im}\left\{A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right\}\right), \tag{3.20}$$

$$\hat{A}_2 = m_1 \cos\left(\operatorname{Im}\left\{A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right\}\right) + m_2 \sin\left(\operatorname{Im}\left\{A_* \sqrt{\gamma_{0,1}\gamma_{1,2}}\, t\right\}\right), \tag{3.21}$$

in which the constants $k_{1,2}$ and $m_{1,2}$ can also be expressed in terms of α_0, α_2 and δ as

$$k_1 = \frac{\alpha_0}{\delta},\ \ k_2 = \frac{i\alpha_0}{\delta}\sqrt{\frac{\gamma_{1,2}}{\gamma_{0,1}}},\ \ m_1 = \frac{\alpha_2}{\delta},\ \ m_2 = \frac{i\alpha_0}{\delta}\sqrt{\frac{\gamma_{0,1}}{\gamma_{1,2}}} \tag{3.22}$$

As shown in Figure 4, both intensities A_0 and A_2 exhibit oscillatory behavior. This suggests stability in the vaccination decisions of the group. Specifically, when the individual 1 is not willing to change his or her decision about vaccination, it appears that the decision causes individual 0 and 2 to feel uncertain about participation in the vaccination campaing over time. However, since the amplitudes of oscillations for the individuals 0 and 2 are not identical, this creates uncertainty regarding the overall intensity of the group's vaccination over time.

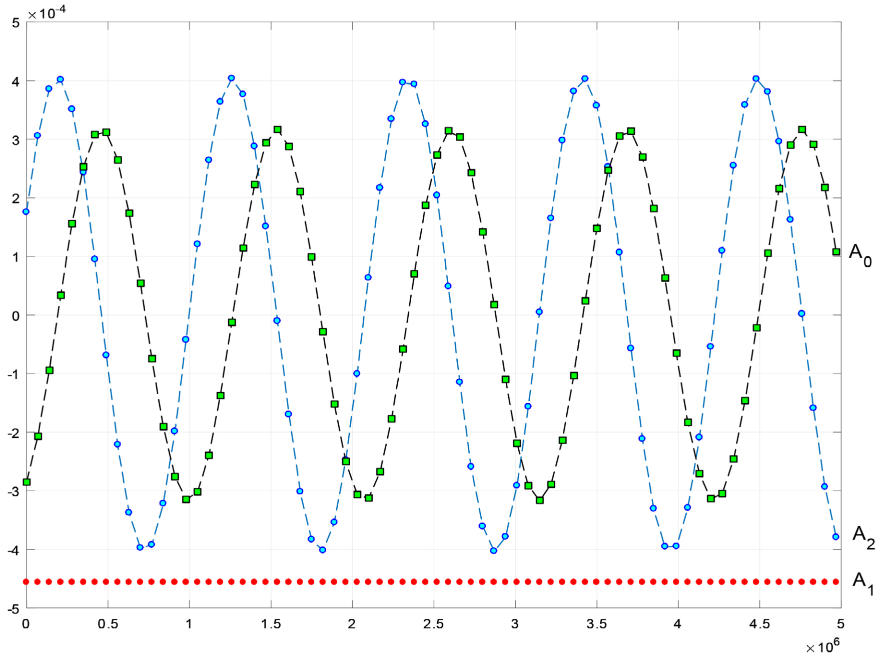

Figure 4. Case 2. Time behavior of the vaccination intensities A_0 and A_2 for the individuals 0 and 2, given that the intensity A_1 is constant.

To address this uncertainty, we next plot the total vaccination intensity $A_0A_1A_2$ of the triad. Figure 5 illustrates the temporal behavior of the total intensity of the triad $\{A_0, A_1, A_2\}$. This figure also shows oscillatory behavior but with a general negative trend, indicating a decreasing intention to vaccinate among the group.

4. Two Triads of Five Individuals

In the provisos section, we studied the case of three individuals, labeled 0, 1, and 2, forming one triad. In this scenario, all three participants can equally influence each other's vaccination decisions. This can be represented with the following information chains: $[0 \leftrightarrow 1]$, $[0 \leftrightarrow 2]$, and $[2 \leftrightarrow 1]$.

Now, let's consider a more complex situation involving five individuals – 0, 1, 2, 3, and 4 - who form two coupled triads: $(0,1,2)$ and $(0,3,4)$, as illustrated in Figure 6. In this case, individual 0 has a direct influence on all other participants. However, the remaining participants can only influence the vaccination decisions of those individuals within their respective triads. This can be illustrated using the following information chains:

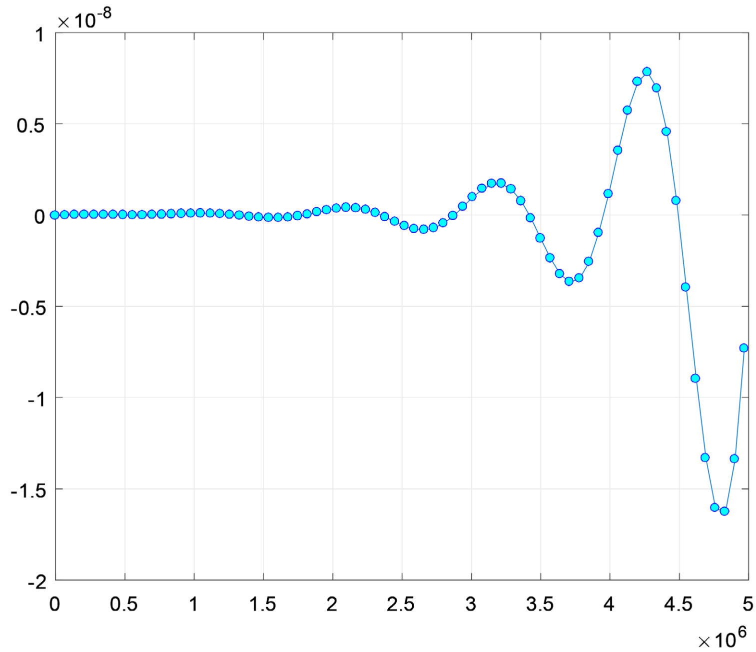

Figure 5. Time behavior of the total intensity $A_0A_1A_2$ of the triad $\{0,1,2\}$ oscillates when A_0 is constant. Since the overall trend is decreasing, it suggests that the overall intent is not to vaccinate.

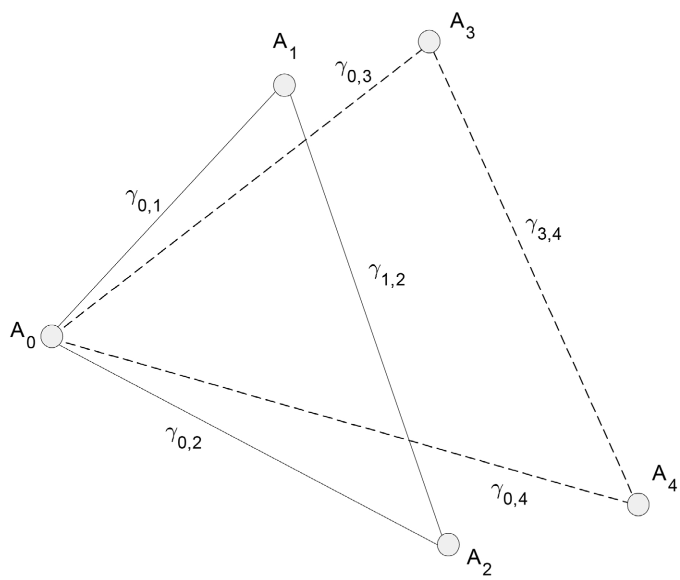

Figure 6. Schematic illustrating the intensities A_i $(i = 0,1,2,3,4)$ of willingness to vaccinate distributed among three individuals i $(i = 0,1,2,3,4)$ forming two coupled triads.

$$[0 \leftrightarrow 1],\ [0 \leftrightarrow 2],\ [0 \leftrightarrow 3],\ [0 \leftrightarrow 4] \tag{4.1}$$

$$[2 \leftrightarrow 1],\ [3 \leftrightarrow 4] \tag{4.2}$$

In this section, we will examine a situation in which individual 0 remains steadfast in their willingness to vaccinate. The primary focus will be to understand how this decision impacts the overall behavior of the participants within the two coupled triads, given that the interaction coefficients γ_{ij} follow the triad rule (2.2), meaning the signs of the coefficients are not all the same.

Case 3: Constant Intensity A_0

We expand upon the previous scenario involving three individuals forming a single triad to consider the case of five participants, who now form two separate triads, as illustrated in **Figure 6**. We assume the following conditions for the first triad $\{A_0, A_1, A_2\}$: $\gamma_{1,2} > 0$, $\gamma_{0,2} < 0$, and $\gamma_{0,1} > 0$. For the second triad $\{A_0, A_3, A_4\}$, the conditions are $\gamma_{3,4} > 0$, $\gamma_{02} < 0$, and $\gamma_{0,3} < 0$, as detailed in **Table 2**.

Table 2. Values of parameters for two triads.

$\gamma_{1,2}$	$\gamma_{0,1}$	$\gamma_{0,2}$	$\gamma_{0,3}$	$\gamma_{0,4}$	$\gamma_{3,4}$
13.09e−08	−8.0e−8	−5.0e−08	−7.0e−8	−9.0e−8	16.0e−8

As in the previous section, according to the **Table 2**, the coefficients γ_{ij} satisfy the *triad rule*: $\gamma_{0,1} + \gamma_{0,2} + \gamma_{1,2} = 0$, and $\gamma_{0,3} + \gamma_{0,4} + \gamma_{3,4} = 0$. Additionally, similarly to the single triad case, (2.1), the decision-making interaction coefficients $\gamma_{i,j}$ for vaccination against flu-like infections are related to the initial intensities A_i by the following relations:

$$\gamma_{0,1} + \gamma_{0,3} = A_0(0)A_1(0) + A_0(0)A_3(0), \tag{4.3}$$

$$\gamma_{0,2} + \gamma_{0,4} = A_0(0)A_2(0) + A_0(0)A_4(0), \tag{4.4}$$

$$\gamma_{1,2} = A_1(0)A_2(0),\ \gamma_{3,4} = A_3(0)A_4(0) \tag{4.5}$$

A time behavior of a decision on participating or nonparticipating in the vaccination campaign is presented by the following coupled system of ODEs:

$$\frac{\mathrm{d}A_0}{\mathrm{d}t} = \gamma_{1,2}A_1A_2 + \gamma_{3,4}A_3A_4,\ A_0(0) = \alpha_0 \tag{4.7}$$

$$\frac{\mathrm{d}A_1}{\mathrm{d}t} = \gamma_{0,2}A_0A_2,\ A_1(0) = \alpha_1 \tag{4.7}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}t} = \gamma_{0,1}A_0A_1,\ A_2(0) = \alpha_2 \tag{4.8}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}t} = \gamma_{0,4}A_0A_4,\ A_3(0) = \alpha_3 \tag{4.9}$$

$$\frac{\mathrm{d}A_4}{\mathrm{d}t} = \gamma_{0,3}A_0A_3,\ A_4(0) = \alpha_4 \tag{4.10}$$

According to (4.3) - (4.5), the initial amplitudes α_j $(j = 0,4)$ are expressed in terms of γ_{ij} as

$$\alpha_1 = \alpha_0\Gamma_1,\ \alpha_2 = \frac{\gamma_{1,2}}{\alpha_0\Gamma_1},\ \alpha_3 = \alpha_0\Gamma_2,\ \alpha_4 = \frac{\gamma_{3,4}}{\alpha_0\Gamma_2}, \tag{4.11}$$

in which

$$\alpha_0 = \sqrt{\frac{\gamma_{0,1} + \gamma_{0,3}}{\Gamma_1 + \Gamma_2}}, \tag{4.12}$$

where we denote

$$\Gamma_1 = \frac{\gamma_{1,2}\left(\gamma_{3,4} - \gamma_{1,2}\right)}{\gamma_{0,1} + \gamma_{0,3} - \gamma_{1,2}\left(\gamma_{0,2} + \gamma_{0,4}\right)},\ \Gamma_2 = \frac{\gamma_{3,4}\left(\gamma_{1,2} - \gamma_{3,4}\right)}{\gamma_{0,1} + \gamma_{0,3} - \gamma_{3,4}\left(\gamma_{0,2} + \gamma_{0,4}\right)}. \tag{4.13}$$

We consider the scenario in which the initial vaccination intensity A_0 of an individual 0 remains constant. Our primary aim is to explore how a small perturbation added to A_0 will influence the overall behavior of the participants within two coupled triads $\{A_0, A_1, A_2\}$ $\{A_0, A_3, A_4\}$. To analyze the implication of this perturbation, we let

$$A_0 = A_* + \delta \hat{A}_0,\ A_1 = \delta \hat{A}_1,\ A_2 = \delta \hat{A}_2,\ A_3 = \delta \hat{A}_3,\ A_4 = \delta \hat{A}_4, \tag{4.14}$$

where $\hat{A}_i \ll A_*$ for $i = 0,1,2,3,4$. As a result, the initial conditions specified in Equations (4.6) - (4.10) are modified to:

$$\hat{A}_1(0) = \frac{\alpha_1}{\delta},\ \hat{A}_2(0) = \frac{\alpha_2}{\delta},\ \hat{A}_3(0) = \frac{\alpha_3}{\delta},\ \hat{A}_4(0) = \frac{\alpha_4}{\delta},\ \hat{A}_0(0) = 0 \text{ and } A_* = \alpha_0.$$

In this case, up to the order $0(\delta)$, the Equations (4.6) - (4.10) reduce to the following decoupled system:

$$\frac{\mathrm{d}\hat{A}_1}{\mathrm{d}t^2} = \gamma_{0,2}\gamma_{0,1}A_*^2\hat{A}_1, \tag{4.15}$$

$$\frac{\mathrm{d}\hat{A}_2}{\mathrm{d}t^2} = \gamma_{0,1}\gamma_{0,2}A_*^2\hat{A}_2, \tag{4.16}$$

$$\frac{\mathrm{d}\hat{A}_3}{\mathrm{d}t^2} = \gamma_{0,4}\gamma_{0,3}A_*^2\hat{A}_3 \tag{4.17}$$

$$\frac{\mathrm{d}\hat{A}_4}{\mathrm{d}t^2} = \gamma_{0,3}\gamma_{0,4}A_*^2\hat{A}_4 \tag{4.18}$$

The solution of the system (4.15) - (4.18) is written as

$$\hat{A}_1 = c_{11}\exp\left(A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right) + c_{12}\exp\left(-A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right), \tag{4.19}$$

$$\hat{A}_2 = c_{21}\exp\left(A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right) + c_{22}\exp\left(-A_*\sqrt{\gamma_{0,1}\gamma_{0,2}t}\right), \tag{4.20}$$

$$\hat{A}_3 = c_{31}\exp\left(A_*\sqrt{\gamma_{0,3}\gamma_{0,4}t}\right) + c_{32}\exp\left(-A_*\sqrt{\gamma_{0,3}\gamma_{0,4}t}\right), \tag{4.21}$$

$$\hat{A}_4 = c_{41}\exp\left(A_*\sqrt{\gamma_{0,3}\gamma_{0,3}t}\right) + c_{42}\exp\left(-A_*\sqrt{\gamma_{0,3}\gamma_{0,4}t}\right), \tag{4.22}$$

where the constants c_{ij} can be expressed in terms of α_i and δ as follows:

$$c_{11,12} = \frac{1}{2\delta}\left(\alpha_1 \pm \alpha_2\sqrt{\frac{\gamma_{0,2}}{\gamma_{0,1}}}\right),\ c_{21,22} = \frac{1}{2\delta}\left(\alpha_2 \pm \alpha_1\sqrt{\frac{\gamma_{0,1}}{\gamma_{0,2}}}\right). \tag{4.23}$$

$$c_{31,32} = \frac{1}{2\delta}\left(\alpha_3 \pm \alpha_4\sqrt{\frac{\gamma_{0,4}}{\gamma_{0,3}}}\right),\ c_{41,42} = \frac{1}{2\delta}\left(\alpha_4 \pm \alpha_3\sqrt{\frac{\gamma_{0,3}}{\gamma_{0,4}}}\right). \tag{4.24}$$

In our case, since $\gamma_{0,1}\gamma_{0,2} > 0$ and $\gamma_{0,3}\gamma_{0,4} > 0$, the perturbation (4.14) added to A_0 will produce instability, as shown in **Figure 7**.

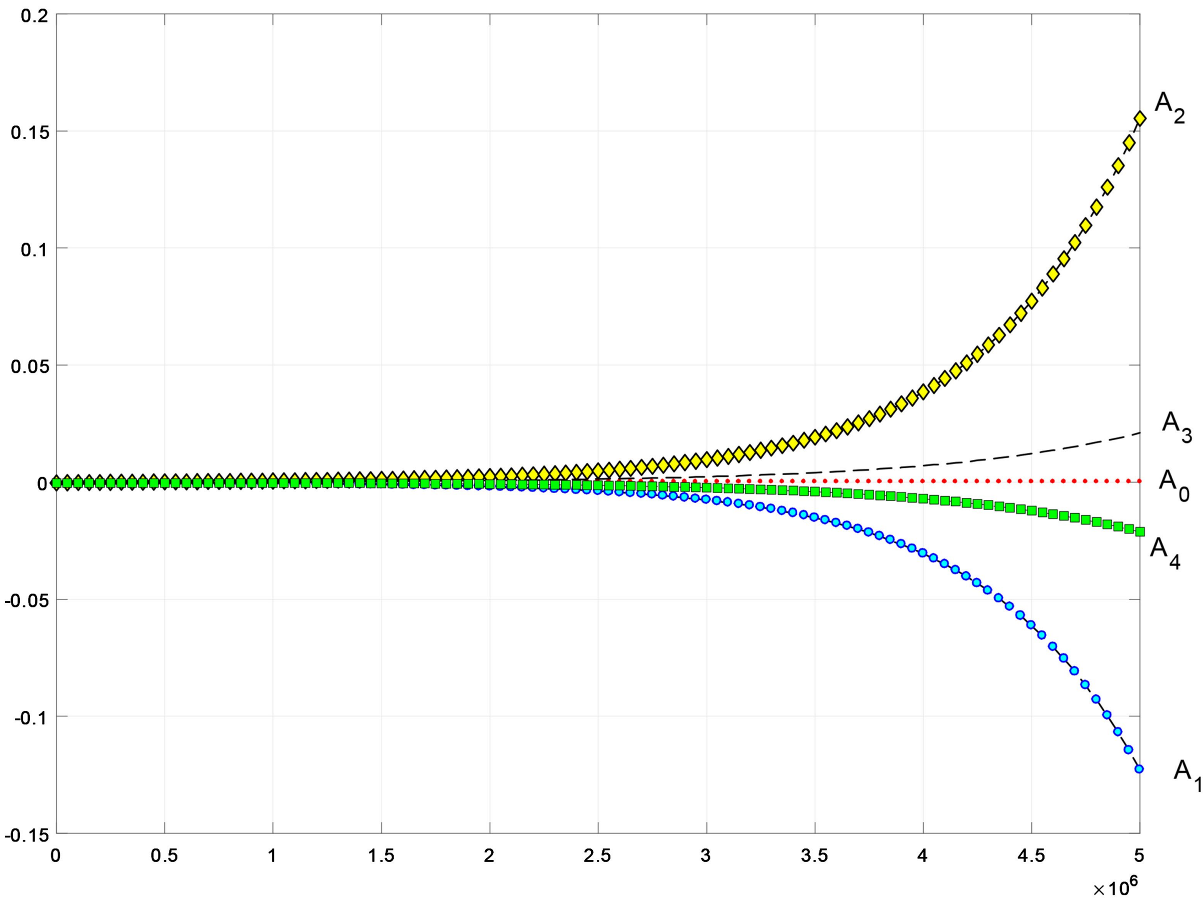

Figure 7. Case 3. Time behavior of the vaccination intensities A_1, A_2, A_3, and A_4 for the individuals 1, 2, 3, and 4 given that the intensity A_0 is constant.

Figure 7 illustrates the time behavior of the vaccination intensities A_1, A_2, A_3, and A_4 for the individuals 1, 2, 3, and 4 affected by constant intensity A_0. We observe that the intensities A_2 and A_3 are increasing exponentially, while the intensities A_1 and A_4 are decreasing. This behavior suggests instability in the vaccination decisions of the coupled triads of the individuals $\{0,1,2\}$ and $\{0,3,4\}$. Specifically, the willingness of the individuals 2 and 3 to vaccinate is increasing over time, whereas the intensities of the individuals 1 and 4 are tending towards negative values. This indicates that, despite an initial positive decision to vaccinate, individuals 2 and 3 are likely to become more willing to vaccinate, while individuals 1 and 4 are likely to become less willing as time progresses. Although **Figure 7** shows the vaccination intensity for each individual within the two coupled triads, it does not reflect the overall intensity of all five individuals. To address this, we plot the total vaccination intensity of two triads consisting of all five individuals $\{0,1,2\}$ and $\{0,3,4\}$ as a product $A_0A_1A_2A_3A_4$. **Figure 8** presents the time behavior of this total intensity. Unlike the Cae 1.1 for a single triad shown in **Figure 3**, the data reveals that the total intensity $A_0A_1A_2A_3A_4$ of the two triads is increasing when A_0 is constant, indicating a general trend toward higher vaccination intentions.

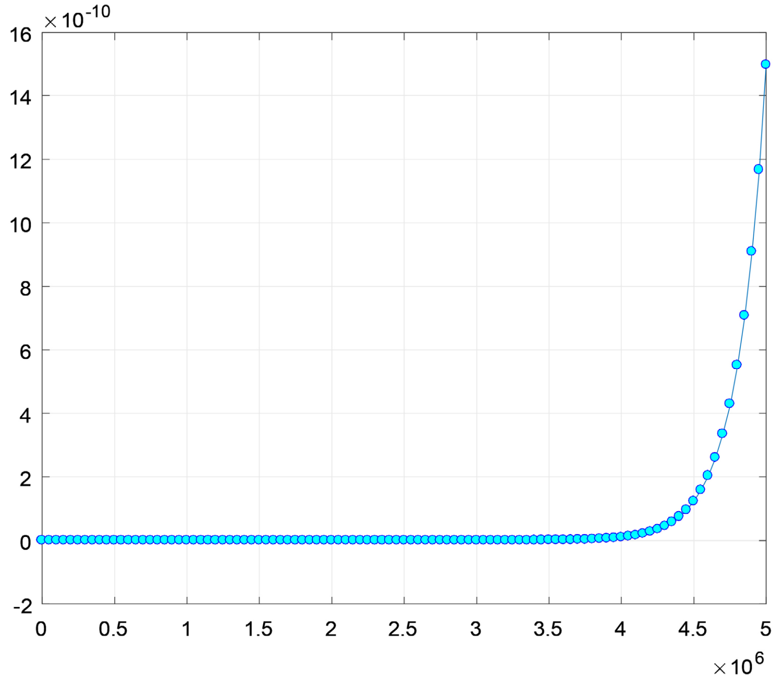

Figure 8. Time behavior of the total intensity $A_0A_1A_2A_3A_4$ of two coupled triads of the individuals $\{0,1,2\}$ and $\{0,2,4\}$ is increasing when the intensity A_0 is constant, indicating that the overall intent is not to vaccinate.

5. Concluding Remarks

This paper presents a novel mathematical framework for modeling vaccination decisions in small social groups through triadic interactions. Our coupled differential equation system reveals complex dynamics that cannot be captured by traditional pairwise interaction models or large population approaches. The introduction of the triad rule $\gamma_{0,1}+\gamma_{0,2}+\gamma_{1,2}=0$ creates realistic constraint structures that reflect the interdependent nature of group decision-making processes. The interaction coefficients γ_{ij} serve as free parameters that can be calibrated based on empirical observations of social influence patterns within specific populations or cultural contexts.

Our analysis demonstrates that the stability and convergence properties of vaccination decisions depend critically on which individual maintains a constant position within the single triad. When individual A_0 remains constant, the system exhibits exponential growth or decay behavior, while constant A_1 leads to oscillatory dynamics with an overall declining trend. These findings suggest that the position and influence of key decision-makers within small social groups can fundamentally alter collective vaccination outcomes.

The extension of coupled triads to include five individuals reveals emergent behaviors that are not present in single triads. Notably, the total vaccination intensity in two coupled triads exhibits different trends compared to single triads under similar forcing conditions. This suggests that network effects become significant even at small scales. For example, when the initial intensity A_1 remains constant, as illustrated in **Figure 6**, individuals "3" and "4" cannot receive information directly from individuals "1" or "2". Instead, they obtain information from "1" and "2" through the transient individual "0".

We then let

$$A_0 = \delta \hat{A}_0,\ A_1 = A_* + \delta \hat{A}_1,\ A_2 = \delta \hat{A}_2,\ A_3 = \delta \hat{A}_3,\ A_4 = \delta \hat{A}_4 \tag{5.1}$$

In this case, within the order $0^2(\delta)$, Equations (4.6) - (4.10) are written as the following decoupled system:

$$\frac{\mathrm{d}A_0}{\mathrm{d}t} = \gamma_{1,2} A_* \hat{A}_2 + \delta\left(\gamma_{1,2} \hat{A}_1 \hat{A}_2 + \gamma_{3,4} \hat{A}_3 \hat{A}_4\right), \tag{5.2}$$

$$\frac{\mathrm{d}A_1}{\mathrm{d}t} = \gamma_{0,2} \delta \hat{A}_0 \hat{A}_2,\ A_1(0) = \alpha_1 \tag{5.3}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}t} = \gamma_{0,1} A_* \hat{A}_0 + \gamma_{0,1} \delta \hat{A}_0 \hat{A}_1,\ A_2(0) = \alpha_2 \tag{5.4}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}t} = \gamma_{0,4} \delta \hat{A}_0 \hat{A}_4,\ A_3(0) = \alpha_3 \tag{5.5}$$

$$\frac{\mathrm{d}A_4}{\mathrm{d}t} = \gamma_{0,3} \delta \hat{A}_0 \hat{A}_3,\ A_4(0) = \alpha_4 \tag{5.6}$$

To the first-order approximation, the two-triad model (5.2) - (5.6) model can be written as

$$\frac{\mathrm{d}^2 \hat{A}_0}{\mathrm{d}t^2} = \gamma_{0,1} \gamma_{1,2} A_*^2 \hat{A}_0, \tag{5.7}$$

$$\frac{\mathrm{d}^2 \hat{A}_2}{\mathrm{d}t^2} = \gamma_{0,1} \gamma_{1,2} A_*^2 \hat{A}_2. \tag{5.8}$$

The solution to the system (5.7) - (5.8) has been previously derived using the formulas (3.17) - (3.18). Given that $\gamma_{0,1}\gamma_{1,2} > 0$, the intensities A_0 and A_2 exhibit exponential growth, which corresponds to case 1. However, the behavior of the intensities A_3 and A_4 can be understood through a higher-order approximation. To accomplish this, we can express the intensities as a series.

$$A_k = \sum_{i=1}^{\infty} \varepsilon^i A_k^{(i)}, \tag{5.9}$$

where ε is another small parameter. In general, $\varepsilon \neq \delta$. However, without loss of generality, it can be assumed that $\varepsilon = \delta$. The analysis of this scenario will be presented in upcoming studies, where we will develop an algorithmic approach to include a large number of triad, as shown schematically in **Figure 9**.

Future research directions include developing algorithmic approaches for modeling arbitrarily large numbers of triads. While our framework is mathematically tractable for small systems, scaling to populations involving thousands or millions of triads would require sophisticated computational methods and automated parameter selection strategies. The challenge lies in developing principled approaches for choosing the interaction coefficients γ_{ij} automatically based on available demographic, social network, or behavioral data.

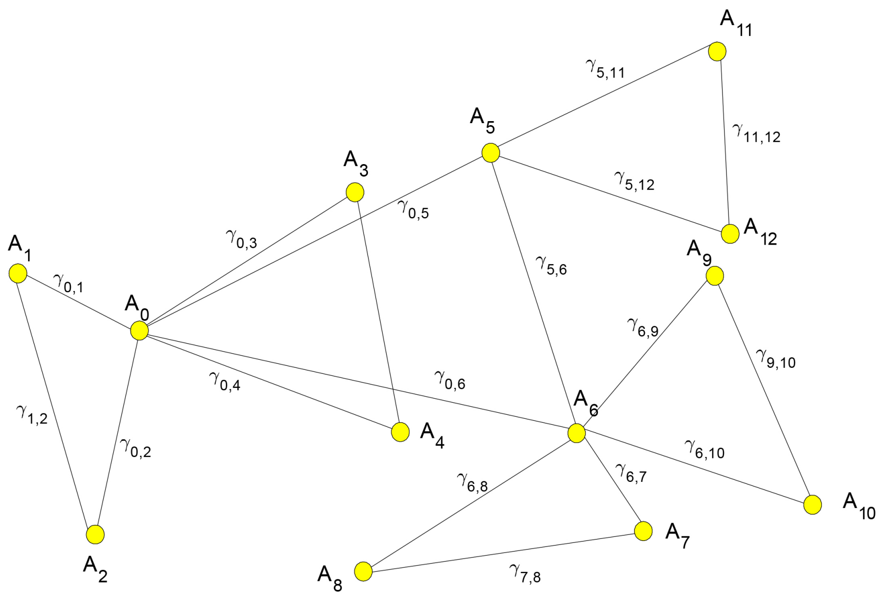

Figure 9. Multiple triad case.

The model provides insights into how stable individual preferences and dynamic peer influence combine to shape collective vaccination outcomes in small social clusters such as families, friend groups, and workplace teams. While our analysis is primarily theoretical, these findings suggest potential directions for public health research, particularly the importance of understanding complex interdependencies among group members rather than treating individuals as independent decision-makers. Future empirical work could explore how to identify and understand key individuals whose positions can anchor group dynamics.

From a theoretical perspective, our findings highlight fundamental differences in group stability depending on which member serves as the decision anchor. The mathematical framework developed here provides a foundation for understanding how vaccination decisions might propagate through interconnected social clusters, though empirical validation would be necessary to confirm these theoretical predictions in real-world settings.

In this paper, we have discovered that the outcomes of a single triad, whether exponential or oscillatory, are relatively straightforward to comprehend if the signs of each interaction coefficient are known. However, when it comes to two triads, the situation becomes much more unpredictable. The outcomes strongly depend on which intensity is maintained constant, as demonstrated in the analysis of two triads involving five individuals.

From a mathematical perspective, a key objective of our future research involving multiple triads is to establish a stability criterion that will enhance observational analyses of vaccination intensity among interacting individuals forming an arbitrarily large number of triads. To our knowledge, this type of analysis has not been conducted previously.

Limitations of our current approach include the assumption of fixed group membership, focus on pre-outbreak scenarios, and the use of specific parameter values for illustration. Future extensions could incorporate dynamic group formation, post-outbreak feedback effects, and systematic parameter sensitivity analysis to better understand the robustness of our findings.

The mathematical framework developed here provides a foundation for understanding how vaccination decisions emerge from the complex interplay of individual preferences and social influence within small, tightly connected groups, which represents a critical gap in existing epidemiological and behavioral models.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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