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Cosmic Entropy Prediction with Extremely High Precision in $R_h = ct$ Cosmology

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Abstract

We present how the Bekenstein-Hawking entropy of a growing black hole variant of $R_h = ct$ cosmology model can be re-written as a function of the Cosmic Microwave Background (CMB) radiation temperature or Hubble parameter, rather than the Hubble radius, as first pointed out by Tatum and Seshavatharam [1]. We then show how our CMB temperature formulae lead to much higher precision in the estimated entropy of the Hubble radius universe, since the CMB temperature can be measured with great precision. We also briefly discuss how the Schwarzschild metric can be re-written as a function of the Bekenstein-Hawking entropy, and how the entropy of the universe can be directly linked to recent estimates of the number of quantum operations in the universe since its beginning.

Keywords

Bekenstein-Hawking Entropy, Black Hole Entropy, Hubble Sphere, CMB Temperature, Cosmological Constant Problem, Hubble Tension, Holographic Universe, Quantum Cosmology

1. Black Hole $R_h = ct$ Cosmology Model Entropy

In this paper, we will mainly focus on $R_h = ct$ cosmology, which covers a group of cosmology models actively discussed as an alternative to the Λ-CDM model; see, for example, [2]-[7]. Melia [8] [9] has recently compared many different kinds of observation with respect to the Λ-CDM and $R_h = ct$ models, and concludes that "$R_h = ct$ *has accounted for the data at least as well as the standard model, and often much better*". Nevertheless, it remains to be determined by the cosmology community which model will ultimately prevail.

There are multiple types of cosmological models following the $R_h = ct$ principle, namely, linear growth of the universal radius at the speed of light. In this paper, the type of $R_h = ct$ model of interest is growing black hole $R_h = ct$ cosmology, within which black hole entropy can be explored.

As early as 1972, Pathria [10] pointed out that the Hubble sphere has mathematical properties similar to those of a black hole. See, for example, [6] [11]-[15]. Herein, our focus will be on a Schwarzschild black hole universe model following a linear $R_h = ct$ expansion. Accordingly, our model entropy follows the Bekenstein-Hawking black hole entropy formula [16]-[18].

The Bekenstein-Hawking entropy is given by:

$$S_{BH} = \frac{A}{l_p^2} = \frac{4\pi R_s^2}{l_p^2} \tag{1}$$

In a critical Friedmann [19] universe, the mass is equal to $M_c = \frac{c^2 R_H}{2G}$. If we solve this for R_H, we get $R_H = \frac{2GM_c}{c^2}$. In other words, the Hubble radius and the Schwarzschild radius are identical in a critical Friedmann universe. If our universe is also following a linear $R_h = ct$ expansion, and is a growing Schwarzschild black hole, then its entropy can presumably be treated as:

$$S_{BH} = \frac{A}{l_p^2} = \frac{4\pi R_H^2}{l_p^2} \tag{2}$$

As early as 2015, Tatum *et al.* [20] suggested the following formula for the Cosmic Microwave Background (CMB) radiation temperature consistent with a growing black hole $R_h = ct$ model and the critical Friedmann universe:

$$T_{cmb} = \frac{\hbar c}{k_b 4\pi \sqrt{R_h 2 l_p}} \tag{3}$$

wherein k_b is the Boltzmann constant, $\hbar$ is the reduced Planck constant (the Dirac constant), and $R_h = \frac{c}{H_0}$. Haug and Wojnow [21] [22] have demonstrated that this formula can be derived from the Stefan-Boltzmann law. Furthermore, Haug and Tatum [23] have shown that the same formula can be derived using a geometric mean approach, and Haug [24] has also demonstrated that it can be derived from the quantization of light bending.

If one solves Formula (3) for H_0, this gives:

$$H_0 = T_{cmb}^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \tag{4}$$

This means that we can rewrite the Bekenstein-Hawking entropy as:

$$S_{BH} = \frac{A}{l_p^2} = \frac{4\pi R_H^2}{l_p^2} = \frac{1}{T_{cmb}^4} \frac{\hbar^4 c^4}{256\pi^3 k_b^4 l_p^4} \tag{5}$$

And, since we know that the Planck [25] time is given by $t_p = \sqrt{\frac{G\hbar}{c^5}} = \frac{l_p}{c}$, this centropy can also be written as:

$$S_{BH}=\frac{A}{l_p^2}=\frac{4\pi R_H^2}{l_p^2}=\frac{1}{T_{cmb}^4}\frac{\hbar^4}{256\pi^3k_b^4t_p^4} \tag{6}$$

Be aware that the Planck time can be found independently of first finding G; see [26] [27]. However, we can also re-write this in a form containing G; in which case, we then have:

$$S_{BH}=\frac{A}{l_p^2}=\frac{4\pi R_H^2}{l_p^2}=\frac{1}{T_{cmb}^4}\frac{\hbar^2c^{10}}{256\pi^3k_b^4G^2} \tag{7}$$

The above formula expressing the Bekenstein-Hawking entropy as a function of the CMB temperature was first presented by Tatum and Seshavatharam in 2018 [1]. In the current paper, we will demonstrate how such a temperature formula leads to an incredibly low STD for the predicted Hubble sphere entropy.

Table 1. This table shows cosmic entropy estimates using our new calculation method applied to several different CMB temperature studies. It gives extremely high precisions, due to relying upon very precise CMB measurements. We have already taken into account uncertainty in the Planck length.

CMB Study	Temperature Measurement	High-Precision Method for S_{BH}
2023: Dhal *et al.* [28]	2.725007 ± 0.000024 K	$S_{BH}=9.2057\pm0.0007\times10^{122}$
2021: Noterdaeme *et al.* [29]	2.725 ± 0.002 K	$S_{BH}=9.2058\pm0.0027\times10^{122}$
2009: Fixsen [30]	2.72548 ± 0.00057 K	$S_{BH}=9.1993\pm0.0081\times10^{122}$

Table 2. This table calculates the Bekenstein-Hawking entropy from the traditional formula that depends on knowing the radius of the black hole, in this case that of the Hubble sphere. The Hubble radius is given by $R_H=\frac{c}{H_0}$. This gives much higher uncertainty in the predicted Hubble sphere entropy than in the new method described in **Table 1**. The reason for this is that there is much higher uncertainty in measured H_0 values than in measured CMB values.

H_0 Study	H_0 Estimate	Standard Method Estimate for S_{BH}
2023: Murakami *et al.* [31]	73.01 ± 0.85 km/s/Mpc	$S_{BH}=7.72\pm0.17\times10^{122}$
2022: Riess *et al.* [32]	73.04 ± 1.04 km/s/Mpc	$S_{BH}=7.72\pm0.22\times10^{122}$
2020: Alves *et al.* [33]	67.4 ± 0.5 km/s/Mpc	$S_{BH}=9.06\pm0.13\times10^{122}$
2023: Balkenhol *et al.* [34]	68.3 ± 1.5 km/s/Mpc	$S_{BH}=8.82^{+0.38}_{-0.40}\times10^{122}$

This new way to express the Schwarzschild black hole entropy is more than just a change of the elements in which it is expressed; there are also important practical implications for cosmology, since the CMB temperature has been measured much more precisely than the Hubble constant. For example, Dhal *et al.* [28] report a CMB temperature of 2.725007 ± 0.000024 K. This leads to a Hubble sphere $R_h=ct$ black hole entropy of $S_{BH}=9.2057\pm0.0007\times10^{122}$. We even account for the uncertainty in the Planck length, which is needed to calculate the Bekenstein-Hawking entropy, using the NIST CODATA value of $l_p = 1.616255 \pm 0.000018 \times 10^{-35}$ m. In Table 1 and Table 2, we have expressed the entropy as the number of entropic states since the beginning of the universe. Often, entropy is expressed in $J \cdot K^{-1}$. To do this, one simply needs to multiply the expressions in our formula by the Boltzmann constant k_b. Since 2019, the Boltzmann constant has been defined exactly as 1.380649×10^{-23} $J \cdot K^{-1}$. This means that converting from expressing entropy as the number of entropic states in the universe since the beginning of time to entropy in $J \cdot K^{-1}$ does not add any uncertainty to our estimates.

Table 1 shows Bekenstein-Hawking entropies estimated using CMB temperature measured in recent studies [28]-[30]. Table 2 shows Bekenstein-Hawking entropies estimated using H_0 values from recent studies [31]-[34]. We clearly see that our new CMB entropy method is much more precise in comparison to the Hubble constant entropy method. In addition, there is what may be referred to as an entropy tension between different H_0 studies, somewhat similar to the well-known Hubble tension. However, this is outside the scope of our present paper. See also [35].

Haug [36] has recently demonstrated that the number of quantum operations since the Planck epoch in a critical Friedmann universe following linear $R_h = ct$ black hole cosmology is given by:

$$\#ops \approx \frac{S_{BH}}{8\pi} \tag{8}$$

This means that, using the Dhal CMB temperature study, for example, formula (8) would imply that the number of such operations is $3.6628 \pm 0.0003 \times 10^{121}$. The magnitude of this number is quite interesting because of its remarkable similarity to that of the well-known cosmological constant problem.

2. The Schwarzschild Metric for a Hubble Sphere Black Hole Written in Entropy Form

The Schwarzschild [37] metric is normally given by:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{9}$$

Since we have:

$$S_{BH} = \frac{A}{l_p^2} = \frac{4\pi r_s^2}{l_p^2} = \frac{4\pi 4G^2M^2}{c^4 l_p^2} \tag{10}$$

we can now solve this for GM and get: $GM = \frac{c^2 l_p}{4}\sqrt{\frac{S_{BH}}{\pi}}$. This means that, for a black hole, the Schwarzschild metric can be re-written as a function of the black hole Bekenstein-Hawking entropy. We then get:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2 = -\left(1 - \frac{l_p}{2r}\sqrt{\frac{S_{BH}}{\pi}}\right)c^2 dt^2 + \left(1 - \frac{l_p}{2r}\sqrt{\frac{S_{BH}}{\pi}}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{11}$$

This is of great interest, since it shows that the black hole metric can now be expressed in terms of the Planck length and Bekenstein-Hawking entropy. Eddington [38] was the first to suggest that the Planck scale would likely play an important role in a future quantum gravity theory, see also [22].

3. The Critical Friedmann Equation

The critical Friedmann equation is given by:

$$H_0^2 = \frac{8\pi G\rho}{3} \tag{12}$$

This can now be re-written to include the Bekenstein-Hawking entropy, the Planck length and the speed of light according to:

$$H_0^2 = \frac{8\pi G\rho}{3}$$

$$H_0^2 = \frac{c^6}{4G^2M^2}$$

$$H_0^2 = \frac{4c^2\pi}{l_p^2 S_{BH}}$$

$$H_0 = \frac{2c}{l_p}\sqrt{\frac{\pi}{S_{BH}}} \tag{13}$$

See also [39] and [40] for more background, including parallels to this, such as our new thermodynamic Friedmann equation. In particular, [39] shows how our $R_h = ct$ black hole universe model can be fully incorporated into the standard FRW metric, thus avoiding horizon-scale causal contradictions.

There is much yet to be learned about how the thermodynamic concept of entropy might apply to the observable universe and to black holes in general. This subject becomes especially relevant with respect to growing black hole models of cosmology. What effect cosmic entropy might have on the phenomena associated with gravity needs to be further explored. The holographic principle, when applied to the universe as a finite global object, is a related subject of great interest, although beyond the scope of the present brief communication.

4. Conclusion

Due to recent theoretical progress in understanding the direct mathematical relationship between the CMB temperature and the Hubble constant, we can now also estimate the Hubble sphere entropy directly from the CMB temperature. Since the CMB temperature is measured much more precisely than the Hubble constant, this allows for a much more accurate and precise estimate of the entropy of the Hubble sphere black hole $R_h = ct$ universe than previously presented. There are remarkable similarities between the magnitude of the cosmic entropy calculated in the present paper and the magnitude of the cosmological constant problem [40]. This poses interesting questions for continuing theoretical investigations, including those that apply the cosmological holographic principle.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Tatum, E.T. and Seshavatharam, U.V.S. (2018) Clues to the Fundamental Nature of Gravity, Dark Energy and Dark Matter. *Journal of Modern Physics*, **9**, 1469-1483. https://doi.org/10.4236/jmp.2018.98091

[2] John, M.V. (2019) $R_h = ct$ and the Eternal Coasting Cosmological Model. *Monthly Notices of the Royal Astronomical Society: Letters*, **484**, L35-L37. https://doi.org/10.1093/mnrasl/sly243

[3] John, M.V. and Joseph, K.B. (2000) Generalized Chen-Wu Type Cosmological Model. *Physical Review D*, **61**, Article 87304. https://doi.org/10.1103/physrevd.61.087304

[4] John, M.V. and Narlikar, J.V. (2002) Comparison of Cosmological Models Using Bayesian Theory. *Physical Review D*, **65**, Article 43506. https://doi.org/10.1103/physrevd.65.043506

[5] Melia, F. (2013) The $R_h = ct$ Universe without Inflation. *Astronomy & Astrophysics*, **553**, A76. https://doi.org/10.1051/0004-6361/201220447

[6] Melia, F. and Shevchuk, A.S.H. (2012) The $R_h = ct$ Universe. *Monthly Notices of the Royal Astronomical Society*, **419**, 2579-2586. https://doi.org/10.1111/j.1365-2966.2011.19906.x

[7] Tatum, E.T. and Seshavatharam, U.V.S. (2018) How a Realistic Linear $R_h = ct$ Model of Cosmology Could Present the Illusion of Late Cosmic Acceleration. *Journal of Modern Physics*, **9**, 1397-1403. https://doi.org/10.4236/jmp.2018.97084

[8] Melia, F. (2023) A Resolution of the Monopole Problem in the $R_h = ct$ Universe. *Physics of the Dark Universe*, **42**, Article 101329. https://doi.org/10.1016/j.dark.2023.101329

[9] Melia, F. (2024) Strong Observational Support for the $R_h = ct$ Timeline in the Early Universe. *Physics of the Dark Universe*, **46**, Article ID: 101587. https://doi.org/10.1016/j.dark.2024.101587

[10] Pathria, R.K. (1972) The Universe as a Black Hole. *Nature*, **240**, 298-299. https://doi.org/10.1038/240298a0

[11] Stuckey, W.M. (1994) The Observable Universe Inside a Black Hole. *American Journal of Physics*, **62**, 788-795. https://doi.org/10.1119/1.17460

[12] Zhang, T.X. and Frederick, C. (2014) Acceleration of Black Hole Universe. *Astrophysics and Space Science*, **349**, 567-573. https://doi.org/10.1007/s10509-013-1644-6

[13] Popławski, N. (2016) Universe in a Black Hole in Einstein-Cartan Gravity. *The Astrophysical Journal*, **832**, Article 96. https://doi.org/10.3847/0004-637x/832/2/96

[14] Popławski, N. (2021) Gravitational Collapse of a Fluid with Torsion into a Universe in a Black Hole. *Journal of Experimental and Theoretical Physics*, **132**, 374-380. https://doi.org/10.1134/s1063776121030092

[15] Gaztanaga, E. (2022) The Black Hole Universe, Part II. *Symmetry*, **14**, Article 1984. https://doi.org/10.3390/sym14101984

[16] Bekenstein, J.D. (1972) Black Holes and the Second Law. *Lettere al Nuovo Cimento Series 2*, **4**, 737-740. https://doi.org/10.1007/bf02757029

[17] Bekenstein, J.D. (1973) Black Holes and Entropy. *Physical Review D*, **7**, 2333-2346. https://doi.org/10.1103/physrevd.7.2333

[18] Hawking, S.W. (1976) Black Holes and Thermodynamics. *Physical Review D*, **13**, 191-197. https://doi.org/10.1103/physrevd.13.191

[19] Friedman, A. (1922) Über die krüng des raumes. *Zeitschrift für Physik*, **10**, 377-386. https://doi.org/10.1007/bf01332580

[20] Tatum, E.T., Seshavatharam, U.V.S. and Lakshminarayana, S. (2015) The Basics of Flat Space Cosmology. *International Journal of Astronomy and Astrophysics*, **5**, 116-124. https://doi.org/10.4236/ijaa.2015.52015

[21] Haug, E.G. and Wojnow, S. (2024) How to Predict the Temperature of the CMB Directly Using the Hubble Parameter and the Planck Scale Using the Stefan-Boltzmann Law. *Journal of Applied Mathematics and Physics*, **12**, 3552-3566. https://doi.org/10.4236/jamp.2024.1210211

[22] Haug, E.G. (2024) CMB, Hawking, Planck, and Hubble Scale Relations Consistent with Recent Quantization of General Relativity Theory. *International Journal of Theoretical Physics*, **63**, Article No. 57. https://doi.org/10.1007/s10773-024-05570-6

[23] Haug, E.G. and Tatum, E.T. (2024) The Hawking Hubble Temperature as the Minimum Temperature, the Planck Temperature as the Maximum Temperature, and the CMB Temperature as Their Geometric Mean Temperature. *Journal of Applied Mathematics and Physics*, **12**, 3328-3348. https://doi.org/10.4236/jamp.2024.1210198

[24] Haug, E.G. (2023) Planck Quantized Bending of Light Leads to the CMB Temperature. Hal Archive. https://hal.science/hal-04357053

[25] Planck, M. (1899) Natuerliche Masseinheiten. Der Königliche Preussischen Akademie Der Wissenschaften. https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up

[26] Haug, E.G. (2024) God Time = Planck Time: Finally Detected! And Its Relation to Hubble Time. *Open Journal of Microphysics*, **14**, 40-66. https://doi.org/10.4236/ojm.2024.142004

[27] Haug, E.G. (2020) Finding the Planck Length Multiplied by the Speed of Light without Any Knowledge of G, c, or $\hbar$, Using a Newton Force Spring. *Journal of Physics Communications*, **4**, Article ID: 075001. https://doi.org/10.1088/2399-6528/ab9dd7

[28] Dhal, S., Singh, S., Konar, K. and Paul, R.K. (2023) Calculation of Cosmic Microwave Background Radiation Parameters Using COBE/FIRAS Dataset. *Experimental Astronomy*, **56**, 715-726. https://doi.org/10.1007/s10686-023-09904-w

[29] Noterdaeme, P., Petitjean, P., Srianand, R., Ledoux, C. and López, S. (2011) The Evolution of the Cosmic Microwave Background Temperature. *Astronomy & Astrophysics*, **526**, L7. https://doi.org/10.1051/0004-6361/201016140

[30] Fixsen, D.J. (2009) The Temperature of the Cosmic Microwave Background. *The Astrophysical Journal*, **707**, 916-920. https://doi.org/10.1088/0004-637x/707/2/916

[31] Murakami, Y.S., Riess, A.G., Stahl, B.E., D'Arcy Kenworthy, W., Pluck, D.A., Macoretta, A., *et al.* (2023) Leveraging SN Ia Spectroscopic Similarity to Improve the Measurement of H_0. *Journal of Cosmology and Astroparticle Physics*, **2023**, Article 046. https://doi.org/10.1088/1475-7516/2023/11/046

[32] Riess, A.G., Yuan, W., Macri, L.M., Scolnic, D., Brout, D., Casertano, S., *et al.* (2022) A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s^{-1} Mpc^{-1} Uncertainty from the Hubble Space Telescope and the SH0ES Team. *The Astrophysical Journal Letters*, **934**, L7. https://doi.org/10.3847/2041-8213/ac5c5b

[33] Alves, J., Forveille, T., Pentericci, L. and Shore, S. (2020) *Planck* 2018 Results. *Astronomy & Astrophysics*, **641**, E1. https://doi.org/10.1051/0004-6361/202039265

[34] Balkenhol, L., *et al.* (2023) Measurement of the CMB Temperature Power Spectrum

and Constraints on Cosmology from the SPT-3G 2018 TT, TE, and EE Dataset. *Physical Review D*, **108**, Article ID: 0023510.

[35] Haug, E.G. and Tatum, E.T. (2025) Solving the Hubble Tension Using the Pantheon-PlusSH0ES Supernova Database. *Journal of Applied Mathematics and Physics*, **13**, 593-622. https://doi.org/10.4236/jamp.2025.132033

[36] Haug, E.G. (2024) The Planck Computer Is the Quantum Gravity Computer: We Live Inside a Gigantic Computer, the Hubble Sphere Computer? *Quantum Reports*, **6**, 482-492. https://doi.org/10.3390/quantum6030032

[37] Schwarzschild, K. (1916) Über das Gravitationsfeld einer Kugel aus inkompressibler Flussigkeit nach der EINSTEINschen Theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik, 424. https://de.wikisource.org/wiki/Gravitationsfeld_einer_Kugel_aus_inkompressibler_Fl%C3%BCssigkeit

[38] Eddington, A.S. (1918) Report on the Relativity Theory of Gravitation. Fleetway Press.

[39] Haug, E.G. and Tatum, E.T. (2025) Friedmann Type Equations in Thermodynamic Form Lead to Much Tighter Constraints on the Critical Density of the Universe. *Discover Space*, **129**, Article No. 6. https://doi.org/10.1007/s11038-025-09566-y

[40] Tatum, E.T. and Seshavatharam, U.V.S. (2018) Flat Space Cosmology as a Model of Light Speed Cosmic Expansion—Implications for the Vacuum Energy Density. *Journal of Modern Physics*, **9**, 2008-2020. https://doi.org/10.4236/jmp.2018.910126