

Closed Form Solution to the Hubble Tension Based on $R_{H_t} = ct$ Cosmology for Generalized Cosmological Redshift Scaling of the Form: $z = \left(R_{H_0} / R_{H_t} \right)^x - 1$ Tested against the Full Distance Ladder of Observed SN Ia Redshift

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Abstract

Haug and Tatum have recently outlined a possible path to solving the Hubble tension within $R_{H_t} = ct$ cosmology models using a trial-and-error algorithm for redshift scaling, specifically $z = \left(R_{H_0} / R_{H_t} \right) - 1$ and $z = \left(R_{H_0} / R_{H_t} \right)^{\frac{1}{2}} - 1$. Their algorithm demonstrates that one can start with the measured CMB temperature and a rough estimate of H_0 . Based on this approach, they nearly perfectly match the entire distance ladder of observed supernovae by identifying a single value for H_0 . This work replaces a previous numerical approach with a formal, closed-form mathematical solution. Furthermore, we will prove that this solution is valid for a much more general case of any cosmological redshift scaling consistent with: $z = \left(R_{H_0} / R_{H_t} \right)^x - 1$. Haug and Tatum have only considered the most common assumptions of $x=1$ and $x=\frac{1}{2}$. Our solution involves simply solving an equation to determine the correct value of H_0 . This is possible because an exact mathematical relation between H_0 and the CMB temperature has recently been established, in combination with the linearity in an $R_{H_t} = ct$ model. We also demonstrate that a thermodynamic form of the Friedmann equation is consistent with a wide range of redshift scalings, namely: $z = \left(R_{H_0} / R_{H_t} \right)^x - 1$.

Keywords

Hubble Tension Close Form, Hubble Constant, Cosmological Redshift, CMB Temperature

1. $R_{H_t} = ct$ Type Cosmological Models

A series of interesting papers have been published in recent years [1]-[3] discussing the Hubble tension. Here, we take a closer look at a proposed solution to the Hubble tension within the Haug-Tatum $R_{H_t} = ct$ cosmological model. The Haug-Tatum model [4] [5] is unique in that it provides an exact mathematical relation between the CMB temperature, the Hubble constant, and the cosmological redshift. The Haug-Tatum cosmological model has evolved over time through multiple stages. It is consistent with the $R_{H_t} = ct$ principle, which describes a universe expanding at the speed of light without accelerated expansion. Several $R_{H_t} = ct$ types of cosmological models exist, and they continue to be actively discussed in the recent literature; see, for example, [6]-[9]. Melia [10] has recently shown that $R_{H_t} = ct$ cosmology appears to be more consistent with observations from the James Webb Space Telescope than the Λ CDM model. The question of which cosmological model best accounts for different observed properties of the universe will undoubtedly remain an ongoing discussion in the years to come. This paper provides additional evidence in support of $R_{H_t} = ct$ cosmology, as it appears that a closed-form mathematical solution can resolve the Hubble tension within this framework.

Standard cosmology is not able to predict the current CMB temperature, T_0 , despite it being one of the best-determined cosmological parameters, measured with extremely high precision. This limitation, for example, has been clearly pointed out in the review article by Narlikar and Padmanabhan [11]: “*The present theory is, however, unable to predict the value of T at $t = t_0$. It is therefore a free parameter in SC (Standard Cosmology)*”. Furthermore, they suggest that if one could link T_0 to other physical processes in the universe, this would: “*clearly mark an improvement over the standard interpretation*”.

In recent years, we have developed a new model based on Einstein’s general relativity theory that not only predicts T_0 with remarkable precision but also mathematically links T_0 to parameters such as H_0 . This advancement even appears to resolve the Hubble tension, a result we will demonstrate both mathematically and experimentally in this paper.

In 2015, Tatum *et al.* [12] heuristically presented the following formula for the Cosmic Microwave Background (CMB) temperature, which was later formally derived based on the Stefan-Boltzmann law [13] [14] by Haug and Wojnow [15] [16]:

$$T_{CMB,t} = \frac{\hbar c^3}{k_b 8\pi G \sqrt{M_c m_p}} = \frac{\hbar c}{k_b 4\pi \sqrt{R_{H_0} 2l_p}} \quad (1)$$

where k_b is the Boltzmann constant and $m_p = \sqrt{\frac{\hbar c}{G}}$ is the Planck mass, $l_p = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length [17] [18], and $R_{H_0} = \frac{c}{H_0}$ is the Hubble radius and $M_c = \frac{c^3}{2GH_0}$ is the mass (equivalent) of the critical Friedmann [19] universe. The Stefan-Boltzmann law was developed basically for black bodies. The CMB temperature has been described as an almost perfect black body, see, for example, Muller *et al.* [20] state that:

“Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057k$ ”.

Equation (1) has also recently been derived using a geometric mean approach; see [21]. Haug [22] has demonstrated that the CMB temperature can simply be written as:

$$T_{cmb,t} = \sqrt{T_{max} T_{min,t}} \quad (2)$$

where $T_{max} = \frac{\hbar c}{8\pi l_p}$ is the maximum possible Hawking [23] temperature, in other words, the temperature for a Planck mass black hole, and $T_{min,t} = \frac{\hbar c}{4\pi R_{H_t}}$ is the

Hawking temperature of the Hubble sphere, which is the minimum Hawking temperature. The geometric mean approach is consistent with the Hubble sphere operating as a Carnot [24] engine; see Haug [25] and also [26]-[33].

Additionally, Haug and Tatum [4] have demonstrated that to be consistent with the observed relation $T_t = T_0 (1+z)$, see [34]-[36], the predicted redshift seems like it must be given by:

$$z = \sqrt{\frac{R_{H_0}}{R_{H_t}}} - 1 \quad (3)$$

However, they also show that one can have the more common $z = \frac{R_{H_0}}{R_{H_t}} - 1$

scaling, but that this leads to $T_t = T_0 (1+z)^{\frac{1}{2}}$, which does not seem to be supported by observational studies. However, we must be careful here, as even in observational studies, there are often assumptions, or hidden assumptions, that need to be carefully revisited before prematurely drawing conclusions. In this paper, we will demonstrate that in a more general model with cosmological redshift scaling of the form:

$$z = \left(\frac{R_{H_0}}{R_{H_t}} \right)^x - 1 \quad (4)$$

where x is what we can call the scaling factor, it is still possible to resolve the Hubble tension. The x is decided by assumption based on observations and logic, for example if one decide the cosmological scaling should be $z = \frac{R_{H_0}}{R_{H_t}} - 1$

(a scaling used in multiple $R_{H_t} = ct$ models such as the Melia model) one simply set $x=1$ or if one want $z = \sqrt{\frac{R_{H_0}}{R_{H_t}}} - 1$ scaling one set $x = \frac{1}{2}$, but one can also set x to any other value and still one will by solving an equation we soon will look at get the one and the same value for H_0 that seems to solves the Hubble tension. Technically, one could even make x time dependent $x(t)$.

It is important to be aware that we only claim to solve the Hubble tension inside a class of $R_{H_t} = ct$ cosmological models this way and not at all inside the Λ -CDM model. The $R_{H_t} = ct$ model of Haug and Tatum has many advantages over the Λ -CDM model, some of them listed in the recent paper [37].

For example, the Melia $R_{H_t} = ct$ model has a cosmological redshift corresponding to $x=1$ in our suggested general redshift scaling formula. Melia has however no equation for the relation between the CMB temperature now and H_0 . Haug and Tatum model B in [4] given above corresponds then to $x = \frac{1}{2}$, the Haug and Tatum model A [4] that has the same redshift scaling as the Melia model corresponds to $x=1$, but this model is still different than the Melia model, as we in this have a tight mathematical relation between CMB temperature and H_0 that Melia does not have in his model.

For the general redshift scaling Equation (4) to be consistent with the CMB temperature formula derived from the Stefan-Boltzmann law, we get the following relation for the CMB temperature now and in past cosmological epochs:

$$z = \left(\frac{R_{H_0}}{R_{H_t}} \right)^x - 1 \tag{5}$$

$$z = \left(\frac{T_t^2}{T_0^2} \right)^x - 1 \tag{6}$$

$$(z + 1)^{\frac{1}{2x}} = \frac{T_t}{T_0} \tag{7}$$

$$T_t = T_0 (1 + z)^{\frac{1}{2x}} \tag{8}$$

Observations seem to favor a $x \approx \frac{1}{2}$, even if the exact value of x not yet is experimentally fully settled. We will not in this paper strongly conclude on the optimal scaling factor x . The important point in this paper is that the Hubble tension itself seems to be solved for any scaling factor x in the closed-form solution we will soon present.

Haug and Tatum demonstrate that the predicted redshift in one of their two models must satisfy:

$$z_{pre} = \sqrt{\frac{R_{H_0}}{R_{H_t}}} - 1 = \sqrt{\frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0 (1 + z_{obs,i}) k_b 4\pi} \right)^2 \frac{1}{2l_p}}} - 1. \tag{9}$$

Be aware that they do similar for $z_{pre} = \frac{R_{H_0}}{R_{H_t}} - 1$ redshift scaling.

They then use a smart trial-and-error algorithm, such as the Newton-Raphson method or the bisection method, to find the value of H_0 that minimizes the sum of the prediction errors $\sum_{i=1}^n \frac{z_{pre,i} - z_{obs,i}}{z_{obs,i}}$. They demonstrate that this approach leads to a single H_0 value that perfectly matches the model with the full observed distance ladder, something that seems to solve the Hubble tension.

However, here we simply solve Equation (9) for H_0 , which yields:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \frac{(1 + z_{obs,i})^2}{(1 + z_{pre,i})^2} \tag{10}$$

In the case where the predicted redshift $z_{pre,i}$ is exactly equal to the observed redshift $z_{obs,i}$, we must have $\frac{(1 + z_{obs,i})^2}{(1 + z_{pre,i})^2} = 1$. Substituting $\frac{(1 + z_{obs,i})^2}{(1 + z_{pre,i})^2} = 1$ back into Equation (10) gives:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} = T_0^2 \mathfrak{U} \tag{11}$$

The last part, the Latin upsilon: $\mathfrak{U} = \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} = \frac{k_b^2 32\pi^2 \sqrt{G}}{\hbar^{3/2} c^{5/2}}$, is a composite constant made up of well-known constants (which we [38] [39] have coined \mathfrak{U}). This is the same formula as given by [38], but here we have for the first time just demonstrated that this formula is strictly valid only when the predicted redshift exactly matches the observed redshift, or as we soon will see we can use Equation (13) to match the full distance ladder of observed supernova redshifts by simply finding this one H_0 value directly from the current measured CMB temperature.

This means that we only need to know T_0 and this Hubble constant to closely match all observed cosmological redshifts. The reason we say “close to perfect” rather than “perfect” is due to small measurement errors in both the measured CMB temperature and in G , and that is the only uncertainty in this method. The Boltzmann constant, the speed of light, and the reduced Planck constant have no uncertainty, as they have been exactly defined since the 2019 NIST CODATA standard.

We can generalize this to any redshift scaling assumption $z_{pre} = \left(\frac{R_{H_t}}{R_{H_0}}\right)^x - 1$ inside $R_{H_t} = ct$ cosmology, as long as we assume Equation (1), which has been derived from the Stefan-Boltzmann law, is correct, we then get:

$$z_{pre} = \left(\frac{R_{H_0}}{R_{H_t}}\right)^x - 1 = \left(\frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0 (1 + z_{obs,i})^{2x} k_b 4\pi} \right)^2 \frac{1}{2l_p}} \right)^x - 1. \tag{12}$$

Solved for H_0 we get:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \frac{(1 + z_{obs,i})^{\frac{1}{x}}}{(1 + z_{pre,i})^{\frac{1}{x}}} \tag{13}$$

when we have (or want) perfect prediction of redshift, we must have $z_{pre,i} = z_{obs,i}$

and then we end up with $\frac{(1 + z_{obs,i})^{\frac{1}{x}}}{(1 + z_{pre,i})^{\frac{1}{x}}} = 1$ and therefore we must have:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \tag{14}$$

That is, for any scaling factor, x one gets exactly the same H_0 dependent on only the CMB temperature measured now, the Boltzmann constant, the Planck length, the speed of light and the Planck constant. The only variable is the CMB temperature that has been measured very precisely.

2. Distance, Hubble Constant and Redshift

If we solve the general redshift formula for R_{H_t} , we get:

$$z = \left(\frac{R_{H_0}}{R_{H_t}} \right)^x - 1$$

$$R_{H_t} = \frac{c}{H_0 (1+z)^{\frac{1}{x}}}. \tag{15}$$

This means the predicted distance to the observed redshift must be:

$$R_{H_0} - R_{H_t} = R_{H_0} - \frac{c}{H_0 (1+z)^{\frac{1}{x}}}$$

$$R_{H_0} - R_{H_t} = \frac{c}{H_0} - \frac{c}{H_0 (1+z)^{\frac{1}{x}}}$$

$$R_{H_0} - R_{H_t} = \frac{c}{H_0} \left(1 - \frac{1}{(1+z)^{\frac{1}{x}}} \right) \tag{16}$$

further if we solve this for H_0 , we get:

$$H_0 = \frac{c}{d} \left(1 - \frac{1}{(1+z)^{\frac{1}{x}}} \right) \tag{17}$$

Here, $d = R_{H_0} - R_{H_t}$ will be the estimated distance to the object emitting the photons. For very low redshift, we have $z \ll 1$ and we can use the first term of the Taylor expansion to get:

$$d \approx \frac{cz}{xH_0} \tag{18}$$

and now solved for z , we get:

$$z \approx \frac{xdH_0}{c} \quad (19)$$

In the case $x = 1$, this is identical to the standard Λ -CDM cosmological redshift prediction formula approximation when used for $z \ll 1$. Haug and Tatum examine both the special case of $x = 1$, where one obtains the standard distance formula when $z \ll 1$, and a model corresponding to $x = \frac{1}{2}$, which predicts twice the distance of Λ -CDM for low z . However, as we will soon demonstrate, any value of x can be used in the redshift scaling and still resolve the Hubble tension. The choice of x therefore depends on other observations beyond predictions of the Hubble constant versus redshift. It is influenced by factors such as determining the optimal β in $T_t = T_0(1+z)^{(1-\beta)} = T_0(1+z)^{\frac{1}{2x}}$ in comparison with observed data; see, for example, [40], which suggests that β should be close to 0. However, it is crucial to carefully examine the assumptions and methods used in any observational study to arrive at its results.

Nevertheless, this is not the primary focus of this paper. The main discussion, as we will see in the next section, is that within the $R_{H_t} = ct$ cosmology model presented here, any choice of x can be used while still allowing us to match all observed SN Ia redshifts with a single H_0 value. Most values of x , and likely all except one, should be ruled out based on other types of observations, such as the observed $T_t = T_0(1+z)^{(1-\beta)}$ scaling.

In the Λ -CDM model, at least three different distances are considered for a given cosmological redshift: the comoving distance, the angular diameter distance, and the luminosity distance. These three distances differ from each other in the Λ -CDM model, which is fully consistent within the model and necessarily accounts for phenomena such as accelerated expansion. In $R_{H_t} = ct$ cosmological models, however, there is no accelerated expansion, and in the Haug-Tatum cosmological model ($x = \frac{1}{2}$), the comoving, luminosity, and angular diameter distances are identical, see [41] for in detailed discussion on this point. We believe this is not a coincidence. Only the redshift scaling $x = \frac{1}{2}$ is consistent with $T_t = T_0(1+z)$, and it is the only redshift scaling where the three distances—comoving, luminosity distance, and angular diameter distance—are identical. For any other x , the three distances are not the same.

Importantly, $x = \frac{1}{2}$ is also consistent with the well-known Etherington equation [42], which is based on purely geometrical principles linked to general relativity. Both the Λ -CDM model and the $R_{H_t} = ct$ model used here are consistent with the Etherington equation: $D_L = (1+z)^2 D_A$, where D_L is the luminosity distance and D_A is the angular diameter distance. In the next section, we will see how our model can match one of the largest databases of supernovae of type SN Ia while simultaneously predicting their distances. We suspect that the Λ -CDM model has become overly complex due to its three different distances for each ob-

served z_i . In $R_{H_i} = ct$ cosmology, things appear to be much simpler, and we are even able to match all the SN Ia with a single H_0 parameter value, as we will explore next. The distance to cosmological redshifts is not the main topic of this article.

3. Predictions Relative to the Observations Using the Full Distance Ladder from the PantheonPlus Compilation

Here, we will see if our model can match all the observed cosmological redshifts by simply determining the H_0 constant from Equations (13) and (17). However, to demonstrate the superiority of Equations (13) and (17), we will first instead use the predicted value for H_0 by for example Riess [43] of $H_0 = 73.04 \pm 1.04$ km/s/Mpc. We plot the Riess value, accounting for 2 Standard Deviations (STDs), and from this, we get **Figure 1**. The blue line represents the predicted redshift from $H_0 = 73.04$ km/s/Mpc, while the green lines represent the 2 STD confidence interval, *i.e.*, $\pm 2 \times 1.04$ km/s/Mpc. We can see that even the 95% confidence interval falls outside the observations, meaning that any H_0 value within this interval does not come close to matching the observed redshifts in our cosmological model.

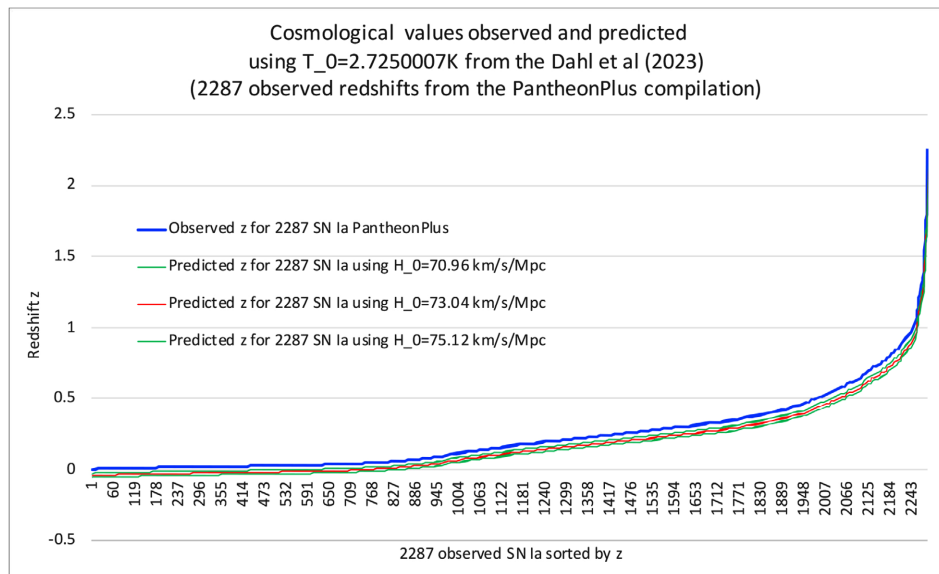


Figure 1. This figure shows observed redshift values from 2287 type Ia supernovae from PantheonPlusSH0ES, sorted by redshift (blue line). Based on the measured CMB temperature by Dhal *et al.* (2023) [44] of 2.725007 K, the blue line represents our predictions based on $H_0 = 73.04$ km/s/Mpc, and the green lines represent the 2 STD confidence interval $\pm 2 \times 1.04$ km/s/Mpc. We find that the Riess H_0 value cannot match the observed redshifts in this $R_{H_i} = ct$ model.

Figure 2 demonstrates the results we get when we instead calculate H_0 based on Equation (11) when using the Dhal *et al.* [44] measured CMB value of $T_0 = 2.725007 \pm 0.000024$ K. According to our theory, this should provide a perfect match between the observed and predicted values, and as we can see, the observed and predicted values lie on top of each other. The confidence interval is

now so narrow that even if we plotted it, it would appear to overlap with the observed values. The predicted $H_0 = 66.8712 \pm 0.0019$ km/s/Mpc when using this measured CMB temperature.

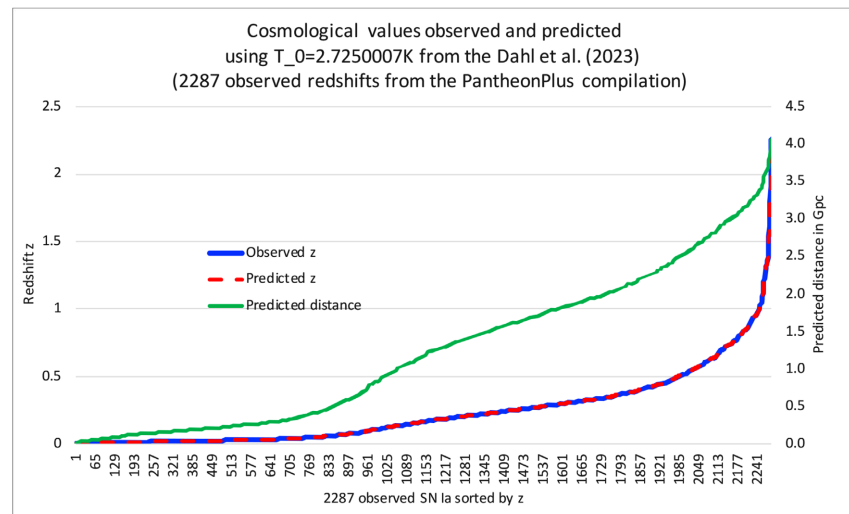


Figure 2. This figure shows observed redshift values from 2287 type Ia supernovae, sorted by redshift (blue line). Based upon the measured CMB temperature by Dhal *et al.* (2023) [44] of 2.725007 K, the red line represents our predictions based on $H_0 = 66.8712$ km/s/Mpc, which we extracted from the data using Equation (11).

Figure 3 demonstrates the results we get when we calculate H_0 based on Equation (14) when the measured CMB value of Fixsen [45]: $T_0 = 2.72548 \pm 0.00057$ K, this leads to a basically perfect match between predicted and observed SN Ia redshifts with a predicted $H_0 = 66.8943 \pm 0.0287$ km/s/Mpc.

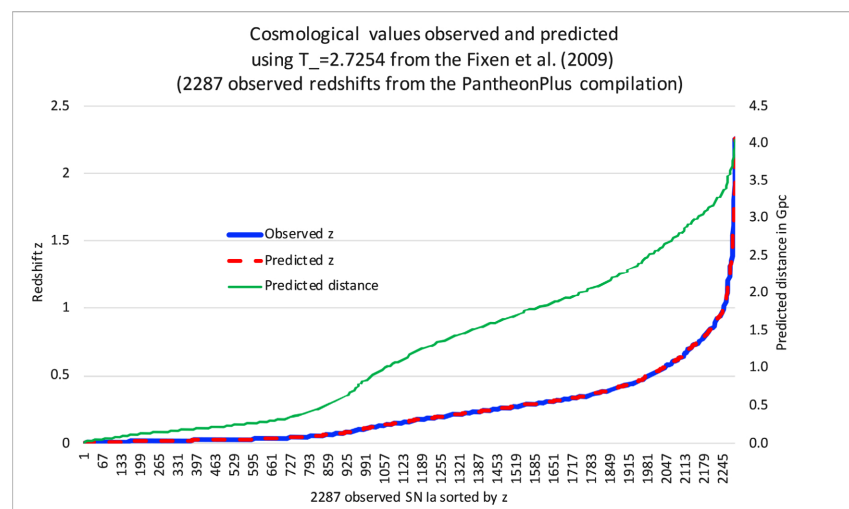


Figure 3. This figure shows observed redshift values from 2287 type Ia supernovae, sorted by redshift (blue line). Based upon the measured CMB temperature by Fixsen (2009) [45] of 2.72548 K, the red line represents our predictions based on $H_0 = 66.8943$ km/s/Mpc calculated from Equation (11).

It is important to understand that the results in this section are independent of the value selected for the scaling factor x in $z = \left(\frac{R_{H_0}}{R_{H_t}}\right)^x - 1$ in our $R_{H_t} = ct$ cosmology.

4. The New Thermodynamic Friedmann Equation Consistent with the General Redshift Scaling of the Form

$$z = \left(\frac{R_{H_0}}{R_{H_t}}\right)^x - 1$$

Haug and Tatum [46] have recently demonstrated that the critical Friedmann [19] equation:

$$H_0^2 = \frac{8\pi G\rho}{3} \tag{20}$$

can be rewritten in thermodynamical form:

$$T_0^4 = \frac{\hbar^4 c^2 G\rho}{384k_b^4 \pi^3 l_p^2} = \frac{\hbar^3 c^5 \rho}{384k_b^4 \pi^3} \tag{21}$$

Keep in mind that when doing the calculations that $G = \frac{l_p^2 c^3}{\hbar}$, something we get by simply solving the Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$, see also [47].

Here, we will generalize this to

$$T_0^4 = \frac{\hbar^3 c^5 \rho}{384k_b^4 \pi^3} \frac{(1+z_{pre,i})^{\frac{2}{x}}}{(1+z_{obs,i})^{\frac{2}{x}}} \tag{22}$$

and when $z_{pre,i} = z_{obs,i}$ this will simply reduce to (21). We have simply replaced

H_0 with $H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \frac{(1+z_{obs,i})^{\frac{1}{x}}}{(1+z_{pre,i})^{\frac{1}{x}}}$. However, when $z_{pre,i} = z_{obs,i}$ we end up

getting Equation (21), which demonstrates that the thermodynamic form of the Haug and Tatum equation is very general and robust, it is valid for a wide range of redshift scaling choices (the choice of x) inside $R_{H_t} = ct$ cosmology.

From the sections above, it is clear that this thermodynamic Friedmann equation is valid and identical for any x scaling factors in $z = \left(\frac{R_{H_0}}{R_{H_t}}\right)^x - 1$, as they all lead to the exactly same H_0 when $z_{pre} \rightarrow z_{obs}$.

More importantly, the thermodynamic Friedmann equation, when carefully studied in relation to our empirical and theoretical work, clearly seems to present a solution to the Hubble tension. However, further discussions and testing by many other researchers are needed before a consensus can be reached. We hope the research community is open-minded enough to carefully consider this possibility and

not simply ignore it due to biases based on the current consensus model, where the Hubble tension has yet to be solved.

5. The Preferred Value for x Based on Other Observations

Even if we can essentially perfectly match all SN Ia in our model for “any” value of x with the same single value of $H_0 = 66.8712 \pm 0.0019$ km/s/Mpc, there are other important aspects that a good cosmological model must also fit. Only when we have $x = \frac{1}{2}$ in our model that is fully consistent with the observationally confirmed relation $T_i = T_0(1+z)$. At the moment, it seems that $x = \frac{1}{2}$ is favored. However, this should be investigated further, as many other aspects of the observed cosmos must also fit the model.

We also have to remember that this solution is only valid within the $R_{H_i} = ct$ principle. More research also needs to be done to compare $R_{H_i} = ct$ cosmology with the Λ -CDM model and other alternatives. We have already compared a series of properties in the two models, where the $R_{H_i} = ct$ appears to outperform the Λ -CDM model; see Haug and Tatum [37].

6. Conclusions

Haug and Tatum have outlined a way to solve the Hubble tension inside $R_{H_i} = ct$ cosmology based on new exact relations between the CMB temperature, the Hubble constant and redshift, they however use a numerical search algorithm to do so

and have only considered $z = \sqrt{\frac{R_{H_0}}{R_{H_i}}} - 1$ and $z = \frac{R_{H_0}}{R_{H_i}} - 1$ cosmological redshift

scaling. Even if their method is intuitive and powerful, we here demonstrate that one can simply solve one of their equations and further based on logic, get to the one single H_0 value that makes their model match all observed SN Ia. In other words, this leads to a closed-form mathematical solution of the Hubble tension inside $R_{H_i} = ct$ cosmology. We get a $H_0 = 66.8712 \pm 0.0019$ km/s/Mpc when relying on the very precise. Dhal *et al.* [44] measured CMB value matching, leading to matching all the observed SN Ia redshifts across the full distance ladder in the PantheonPlusSH0ES compilation. This is the same value Haug and Tatum got from their numerical search algorithm solution when solving the Hubble tension. It is basically the same solution; one is using numerical search algorithm while the latter uses closed-form solution. The closed form solution is naturally more elegant as no numerical search routine with many calculations are needed to find the H_0 that matches all the supernovas. Further, the solution in this paper is generalized for

$z = \left(\frac{R_{H_0}}{R_{H_i}}\right)^x - 1$ cosmological scaling, while Haug and Tatum have only consid-

ered the case equivalent to $x=1$ and $x = \frac{1}{2}$.

So, it looks like we have a path to solving the Hubble tension in $R_{H_i} = ct$ cos-

mology, but this does not solve the Hubble tension inside Λ -CDM cosmology. Further investigation between $R_{H_i} = ct$ cosmology and Λ -CDM cosmology is therefore warranted.

Data Availability Statements

The supernova PantheonPlusSH0ES database that we have used can be found here: https://github.com/PantheonPlusSH0ES/DataRelease/blob/main/Pantheon%2B%20Data/1_DATA/all_redshifts_PVs.csv.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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