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About the Role of Dark Energy in the Formation of Bound States of Matter in the Early Universe

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Abstract

The JWST observations challenge current ideas about the mechanism of galaxy formation in the early Universe, and lead us to the need to unbiasedly revise our understanding of the nature and role of Dark Matter and Dark Energy in the evolution of the early Universe. This paper is devoted to new aspects revealed in light of the obtained observations. We considered a simple model which shows the influence of Dark Energy on the formation of bound states with local binary interaction potential in the form of the rectangular well. The Schrödinger equation was generalized for the curved space-time with anti de Sitter metric. It found spectrum of bound states and their wave functions with taking into account the parameter of Dark Energy given in terms of Λ values. It was directly shown that the sensitivity of the energy spectrum of bound states to the change of Λ parameter. The obtained results are presented in tables and figures.

Keywords

Anti de Sitter Space, Schrödinger Equation, Dark Matter, Bound States, The Wave Functions, Eigenstates, Curved Spacetime, Local Interaction Potential

1. Introduction

In this paper, we consider the influence of Dark Energy (DE) on the formation of bound matter structures in the early Universe. We generalize the Schrödinger equation in the case of curved Einstein space and find its solutions taking into account the potential $U_{\Lambda\pm} = \pm\frac{\hbar^2}{6m}|\Lambda|$ induced by the curvature caused by the non-zero value of the parameter $\Lambda \neq 0$. The structure of the paper is as follows:

in addition to the Abstract, where we briefly outline the main goal of the paper, there follows a short Introduction in which we describe the structure of our paper. Then there is Section 2, in which we trace how and why the concept of DE arose. We included this material because we believe it is important to understand how and why this concept arose in cosmology in order to show that this was not an arbitrary choice of researchers, but followed from the need to understand the observed dynamics of the Universe discovered in the works [1].

Next, in Section 3, we formulate the equations of quantum mechanics in curved Einstein space and for a model problem where the local interaction potential is determined by a rectangular well and the contribution of the potential induced by the space-time curvature, find an analytical solution to the eigenvalue problems, and find a transcendental equation for determining the spectrum of bound states. Further in this section, we study numerically the spectrum of bound states and the corresponding wave functions. The main results are presented in the form of tables and graphs. In the Final Section 4 of the article is presented a discussion and conclusion section, where we summarize the results and outline the prospects for further research.

Acknowledgments and a list of references follow at the end of the paper.

2. Brief Consideration of the Formation of the Concept of Dark Energy

The observational results obtained by the JWST telescope cannot be explained within the framework of existing cosmological concepts of the mechanisms of formation of galactic structures. The need for our understanding of the role of Dark Energy (DE) and Dark Matter (DM) in the evolutionary processes in the early Universe is particularly acute. Therefore, it is interesting to recall what prompted the introduction of Dark Energy and Dark Matter into the conceptual basis of modern cosmology, the nature of which, at the moment, has no generally accepted rational explanation. The history of the birth of DE, as a necessary physical reality, begins in 1917, when Einstein, based on the then existing ideas about the stationarity of the Universe, introduced into his gravitational field equations the quantity, which should have led to the existence of stationary solutions of the field equations. Later, in 1922-1927, when Friedmann obtained dynamic solutions of Einstein's equations, and in 1929 Hubble discovered the expansion of the Universe by observing the redshifts of the emission lines of distant galaxies, Einstein abandoned the need to include the parameter in the equations of the gravitational field. However, already in the 60s, a contradiction arose between the observations of the intensity of emission from quasars and quasags and the values of their redshifts at [2], which again led to the need to introduce the value into cosmological models for the simplest explanation of these data [3]. The selectivity of the absorption of emission lines from quasars and quasags could not be explained by theories assuming that the main absorption of emission occurs in the region of local formation of the quasar, as was assumed to explain these observations in the work of

[4]. In this regard, it is interesting to trace how our understanding of the laws operating in the Universe has changed with the expansion of technical capabilities and the receipt of new data. Thus, in connection with the improvement of the methods of observing type Ia supernova explosions, surprising results were obtained, which showed that the Universe is not simply expanding, but accelerating during expansion [1]. These observations contradicted the usual ideas, when the expansion of the Universe slows down under the action of its own gravitational field. The energy that contributes to this accelerated expansion began to be called Dark Energy and this concept again found its new birth. The physical nature of this energy, at the moment, has not received a generally accepted explanation and is the subject of heated debate. Nevertheless, the formal introduction of the term with into the Einstein equations allows us to obtain dynamic solutions, in the spirit of Friedmann's, which allow us to describe this accelerated expansion. The introduction leads to the fact that the Einstein equations can be written in the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Here we use the conventional notations for the Ricci tensor, scalar curvature, metric tensor, energy-momentum tensor, speed of light and gravitational constant [5]. We represent the energy-momentum tensor as:

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{\Lambda \cdot c^4}{8\pi G} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad T_{\mu\nu} = \begin{pmatrix} p & 0 & 0 & 0 \\ 0 & -\rho \cdot c^2 & 0 & 0 \\ 0 & 0 & -\rho \cdot c^2 & 0 \\ 0 & 0 & 0 & -\rho \cdot c^2 \end{pmatrix}$$

Taking this into account, Einstein's equation can be written in the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \tilde{T}_{\mu\nu}$$

In this representation, the DE is interpreted as a medium with uniform matter density $\rho_\Lambda = \frac{\Lambda \cdot c^2}{8\pi G}$ and pressure $P_\Lambda = -\varepsilon_\Lambda = \rho_\Lambda \cdot c^2$. This is one of the existing representations for the DE. However, another form of representation of Einstein's equations is also possible, with $\Lambda \neq 0$, when: $\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Then the change in the left side of the equation is due to the geometric nature of the quantity Λ. Indeed, gravity, in the Einstein interpretation, is nothing more than the curvature of the 4-dimensional space-time manifold under the action of matter. However, it is easy to understand that such a curvature can also be provided by topological anomalies of this manifold, without the presence of matter as a source of deformation of space and time. This leads to a change in the geometric properties of space-time, determined by the modified metric tensor $\tilde{g}_{\mu\nu}$. In this case, the nature of the geodesic trajectories of the bodies' motion will be determined by the geometry of space, as shown in **Figure 1**, where in the case of the (a) existence of stable states is impossible, in contrast to the space described by the case of (b).

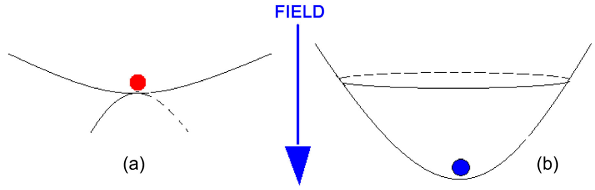

Figure 1. Example of a saddle point with Gaussian curvature. $R_S = \frac{R_1 R_2}{R_2 \pm R_2}$ [1].

These simple examples show that bound states can be determined not only by the local nature of the interaction, but also by the geometry of the space-time in which the physical system is formed. Moreover, we can strengthen this assertion by assuming that the geometry of the space in which the physical system is formed actually sets the system of connections that largely determine the nature of the motion. Ignoring this leads us to the need, when describing the observed phenomena, to introduce fictitious fields that do not correspond to reality. Let us now explain why we touched on this point in such detail.

JWST observations have found galaxy-like structures at redshifts, at which, according to current ideas, such structures cannot form during this time. At this stage of understanding these results, the reason for such a discrepancy between the structures observed by JWST and our ideas about the mechanisms of their formation is not clear. The redshift z, determined for distant sources emitting at the moment t of an electromagnetic wave with a length $\lambda(t)$ and received by an observer at the moment of t_0 a wave with a wavelength λ_0, is calculated as: $z = \frac{\lambda_0(t_0)}{\lambda(t)} - 1 = \frac{a_0(t_0)}{a(t)} - 1$, where $a(t)$ is the scale factor determining the dynamics of the expansion of the Universe.

Since the time since the emission of distant sources is determined by measuring the Hubble constant $H(t) = a_{,t}/a$, where we have introduced the notation $a_{,t} = \frac{\partial a}{\partial t}$, it depends on the nature of the equation of state of the medium at different moments of the formation of the early Universe. At present, the upper limit of the value Λ is estimated as $|\Lambda| < 10^{-56}\ \text{cm}^{-2}$, which indicates a negligibly small local influence of this parameter on the formation of bound states on scales comparable to the sizes of atoms and molecules. However, due to the fact that $[\Lambda] = \text{cm}^{-2}$, it is reasonable to assume that at the early stages of the development of the Universe this value could have been of great importance. Then the question arises about the influence of this parameter on the formation of bound states in the early Universe.

Earlier, in [6], we showed that introducing the term $\Lambda g_{\mu\nu}$ into the Einstein equations leads to the fact that at the local level, due to the curvature of space-time, an additional field $U_{\Lambda\pm} = \pm \frac{\hbar^2}{6m}|\Lambda|$ is generated. Let us consider what the value of the parameter Λ should be and at what red shifts in order to influence

[1]Here $D = 2$ is the spatial dimension.

the formation of the spectrum of hydrogen atoms. To do this, we assume that the energy of the Coulomb interaction in the hydrogen atom should be of the order of the effective $U_{\Lambda\pm}$: $U_{Coulamb} = -k\frac{e^2}{r} \to |U_{Coulamb}| = k\frac{e^2}{r} \sim |U_{\Lambda\pm}| = \frac{\hbar^2}{6m}|\Lambda|$, from which we get:

$$|\Lambda| = k\frac{6k\cdot m_e\cdot e^2}{\hbar^2\cdot r_H} \approx \frac{6}{137}\left(\frac{0.511}{200\times 10^{-13}}\right)\frac{1}{0.529\times 10^{-8}}\,\text{cm}^{-2} \approx 2.1\times 10^{17}\,\text{cm}^{-2}$$

Assuming that $\Lambda = \frac{C_\Lambda}{l^2}$, where for estimates we take $C_\Lambda \sim 1$, and can find the radius of the Universe at which the influence of DE on the formation of the energy spectrum of the hydrogen atom H begins to affect and then the radius of the Universe, at this moment, is $l = 1/\sqrt{\Lambda} \approx 0.21\times 10^{-8}\,\text{cm}$. Since, here we have that $l \sim r_H$, and by this moment, the environment in which atoms are formed is too hot for bound states of atoms to form. However, these conclusions may well change if we assume that the estimate $C_\Lambda \sim 1$ for the constant $\Lambda = \frac{C_\Lambda}{l^2}$, was underestimated by us. Calculating the value of C_Λ, is a separate problem. Let us now consider the influence of the parameter Λ on the formation of deuteron nuclei D, the content of which is most sensitive to the parameters of the cosmological model, since in stars, during the thermonuclear synthesis of heavy elements, it almost completely burns out. The binding energy of the deuteron is $\varepsilon_D \approx 2.2\,\text{MeV}$, and the mass is $M_D \approx 1875.6\,\text{MeV}$. For the assessment, we assume that: $\varepsilon_D \sim |U_{\Lambda\pm}| = \frac{\hbar^2}{6M_D}|\Lambda|$

$$\to |\Lambda| = 6\varepsilon_D\left(\frac{M_D}{\hbar^2}\right) \approx \frac{6\times 2.2\times 1875.6}{4000}\times 10^{26}\,\text{cm}^{-2} \approx 6.2\times 10^{26}\,\text{cm}^{-2}.$$

From the last relation we obtain that $l \approx 0.4\times 10^{-13}\,\text{cm}$, that is, that at these Λ values an acceptable value is achieved, capable of influencing the spectrum of the deuteron. After these assessments, in the next section, we will move on to a model problem where the influence of the DE is superimposed on the local interaction, which generates additional interaction at the local level, effectively playing the role of a third force, introducing properties into the system that cannot be explained within the framework of only binary interaction, without a third force.

3. On the Role of DE in the Formation of Bound States of Matter in Einstein Spaces

In this section we examine in more detail and refine the results of the work [6], where we consider the solution of the Schrödinger equation in Einstein space[2], for

[2]The Einstein equation in this case can be written as $R_{\mu\nu} = \pm\Lambda g_{\mu\nu}$ and in our approach we can consider both of these cases, but we start with a negative value of $\Lambda < 0$, which generates an additional interaction to the local potential, defined by the expression: $U_{\Lambda\pm} = \pm\frac{\hbar^2}{6m}|\Lambda|$, as introduced in Section 2. We show that in principle, for any sign of Λ, there will be a contribution to the formation of the spectrum of the bound state and the number of bound states will either increase or decrease.

which the energy-momentum tensor is defined by the relation (1):

$$T_{\mu\nu} = \pm\frac{\Lambda\cdot c^4}{8\pi G}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \tag{1}$$

Note that in this case, there is no baryonic or dark matter and DE dominates. Then, taking into account (1), the space-time metric is written as [6]:

$$\mathrm{d}s^2 = \left(1\pm\frac{|\Lambda|}{3}r^2\right)\cdot c^2\mathrm{d}t^2 - \left(1\pm\frac{|\Lambda|}{3}r^2\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\vartheta^2 - r^2\sin^2\vartheta\cdot\mathrm{d}\phi^2 \tag{2}$$

In this case[3], the Schrödinger equation, in curved Einstein space, looks like:

$$g^{ij}\left[\Psi_{,ij} - \Gamma^k_{ij}\Psi_{,k}\right] + \frac{2m}{\hbar^2}\left(E - U(r)\right)\Psi = 0 \tag{3}$$

where i, j, k notations take the values 1, 2, 3 and $x^\mu = (ct, \vec{r})$, $x_\mu = (ct, -\vec{r})$. Taking into account the metric (2) and the introduction of the notation $\Psi = \frac{f}{r}$, Equation (3) can be written in the following form [6]:

$$\left(1 - \frac{1}{3}\Lambda r^2\right)\cdot f_{,rr} - \frac{1}{3}\Lambda r f_{,r} + \left[\frac{2m}{\hbar^2}\left(U(r) - E\right) - \frac{1}{3}\Lambda\right]f = 0 \tag{4}$$

Let us introduce the model potential of local interaction in its simplest form:

$$U(r) = \begin{cases}-|U_0|, & r\in[0,a]\\ 0, & r\notin[0,a]\end{cases} \tag{5}$$

Then Equation (4) can be represented as[4]:

$$\left(1 - \frac{1}{3}\Lambda r^2\right)\cdot f_{1,rr} - \frac{1}{3}\Lambda r f_{1,r} - \frac{2m}{\hbar^2}\left[-E - |U_0| - \frac{\hbar^2|\Lambda|}{6m}\right]f_1 = 0,\quad r\in[0,a] \tag{6}$$

$$\left(1 - \frac{1}{3}\Lambda r^2\right)\cdot f_{2,rr} - \frac{1}{3}\Lambda r f_{2,r} - \frac{2m}{\hbar^2}\left[-E - \frac{\hbar^2|\Lambda|}{6m}\right]f_2 = 0,\quad r\notin[0,a] \tag{7}$$

By introducing the notation $\alpha^2 = \frac{1}{3}|\Lambda|$, for $\Lambda < 0$, Equations (6)-(7) are written as:

$$\left(1 + \alpha^2 r^2\right)\cdot f_{1,rr} + \alpha^2 r\cdot f_{1,r} + \frac{2m}{\hbar^2}\left[E + |U_0| + \frac{\hbar^2\alpha^2}{2m}\right]f_1 = 0,\quad r\in[0,a] \tag{6.1}$$

$$\left(1 + \alpha^2 r^2\right)\cdot f_{2,rr} + \alpha^2 r\cdot f_{2,r} + \frac{2m}{\hbar^2}\left[E + \frac{\hbar^2\alpha^2}{2m}\right]f_2 = 0,\quad r\notin[0,a] \tag{7.1}$$

Using a new variable ξ defined as:

$$\alpha\cdot r = sh(\alpha\cdot\xi) \to \xi = \frac{1}{\alpha}\ln\left[\alpha\cdot r + \sqrt{1 + \alpha^2\cdot r^2}\right] \tag{8}$$

[3]For detailed calculations see [6].

[4]For bound states the following condition must be satisfied: $-|U_0| - \frac{\hbar^2\alpha^2}{2m} \le E \le -\frac{\hbar^2\alpha^2}{2m}$.

Equations (6.1) - (7.1) take the form:

$$f_{1,\xi\xi}+\frac{2m}{\hbar^2}\left(E+|U_0|+\frac{\hbar^2\alpha^2}{2m}\right)f_1=0\,,\quad \xi\in[0,\xi_a] \tag{9}$$

$$f_{2,\xi\xi}+\frac{2m}{\hbar^2}\left(E+\frac{\hbar^2\alpha^2}{2m}\right)f_2=0\,,\quad \xi\notin[0,\xi_a]\,, \tag{10}$$

here $\xi_a=\frac{1}{\alpha}\ln\left[\alpha\cdot a+\sqrt{1+\alpha^2\cdot a^2}\right]$. In the final form, introducing the notations:

$$-\lambda_2^2=\frac{2m}{\hbar^2}\left(E+\frac{\hbar^2\alpha^2}{2m}\right)\le 0 \quad\text{and}\quad \lambda_1^2=-\lambda_2^2+\frac{2m}{\hbar^2}|U_0|\,, \tag{11}$$

we bring the equations to the following form:

$$\begin{cases} f_{1,\xi\xi}+\lambda_1^2 f_1=0, & \xi\in[0,\xi_a] \\ f_{2,\xi\xi}-\lambda_2^2 f_2=0, & \xi\notin[0,\xi_a] \end{cases} \tag{12}$$

Note that caution should be exercised when solving the system of Equations (12), since the energy of the bound states E will be considered in two different forms: E_- and E_+, which are determined by the following relations:

$$-|U_0|-\frac{\hbar^2\alpha^2}{2m}\le E_-\le-\frac{\hbar^2\alpha^2}{2m}$$

$$-\frac{\hbar^2\alpha^2}{2m}\le E_+\le 0$$

It follows that for bound states E_- and E_+ the values of λ_0^2 and λ_1^2 should be considered separately. Bound state with energy E_-, we will call deeply bound state, by the reason, that it is more localized compared with E_+ which we call weakly bound state. The binding energy is determined by the values of both the local potential, determined by relation (5), and the value of the additional potential $U_\Lambda=\frac{\hbar^2\alpha^2}{2m}$ generated by the DE, determined by the parameter Λ [5]. Bound states, with binding energy E_+, we will call weakly bound states. Their energy is determined by the value of the potential well barrier and additional contribution from DE, given by $U_\Lambda=\frac{\hbar^2\alpha^2}{2m}$.

$$\begin{cases} f_1=C_1\sin(\lambda_1\xi)+C_2\cos(\lambda_1\xi), & \xi\in[0,\xi_a] \\ f_2=C_3e^{-|\lambda_2|\xi}+C_4e^{|\lambda_2|\xi}, & \xi\notin[0,\xi_a] \end{cases}, \tag{13}$$

With taking into account boundary conditions, we can write:

$$\begin{cases} f_1=C_1\sin(\lambda_1\xi), & \xi\in[0,\xi_a] \\ f_2=C_3e^{-|\lambda_2|\xi}, & \xi\notin[0,\xi_a] \end{cases} \tag{14}$$

[5] Λ value determines effective pressure which take the form: $P_{eff}=p-\frac{\Lambda\cdot c^4}{8\pi G}$ and in the case $\Lambda>0$ it will give negative contribution to the pressure p and leads to the repulsion. For $\Lambda<0$ effective pressure $P_{eff}=p+\frac{|\Lambda|\cdot c^4}{8\pi G}$ and contribution to the energy density in the right hand side of the Einstein equation is positive.

The energy spectrum is obtained by stitching together the external and internal solutions, from which:

$$\begin{cases} f_1(\xi_a) = f_2(\xi_a), \\ f_{1,\xi}\big|_{\xi=\xi_a} = f_{2,\xi}\big|_{\xi=\xi_a}, \end{cases} \tag{15}$$

$$\begin{cases} C_1 \sin(\lambda_1 \xi) = C_3 e^{-|\lambda_2|\xi}, & \xi = \xi_a \\ -C_1 \lambda_1 \cos(\lambda_1 \xi) = -|\lambda_2| C_3 e^{-|\lambda_2|\xi}, & \xi = \xi_a \end{cases} \tag{16}$$

From where:

$$tg(\lambda_1 \cdot \xi_a) = \frac{\lambda_2}{\lambda_1} \to \cos^2(\lambda_1 \cdot \xi_a) = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2} \tag{17}$$

Simplification of the condition for the existence of bound states is reduced to the relationship:

$$\cos(\lambda_1 \cdot \xi_a) = \pm\sqrt{1 + \frac{2m \cdot E + (\hbar\alpha)^2}{2m \cdot |U_0|}} \tag{18}$$

From the solution of the transcendental Equation (18), we can find out eigenstates for the given input values which are presented in Figure 2:

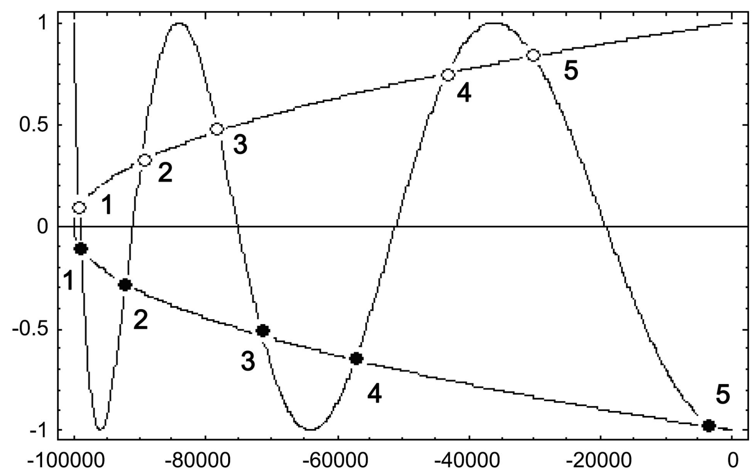

Figure 2. Graphical solution of Equation (18) at the given parameters: $a = 10.0\ \text{fm}$, $mc^2 \approx 0.5\ \text{MeV}$, $\Lambda = -0.001\ \text{cm}^{-2}$, $\hbar c \approx 200\ \text{MeV} \cdot \text{fm}$, $U_0 = -1.0 \times 10^5\ \text{MeV}$.

Exact results of the numerical calculations are given in Table 1.

Table 1. Results of calculations for the $\frac{E_i}{|U_0|}$ for the set of parameters to Figure 2.

№ of state	1	2	3	4	5
○, $\frac{E_i}{\|U_0\|}$	0.991314	0.897464	0.780680	0.432342	0.301118
●, $\frac{E_i}{\|U_0\|}$	0.988750	0.920875	0.713489	0.572519	0.020300

Here it is interesting to investigate sensitivity of the bound states energy spectrum to the change of the parameters: a, Λ and U_0, but first of all from the parameter Λ. For it, let's calculate the difference in the bound states energy spectrum at the change of the Λ. Result of calculation is presented in **Figure 3**.

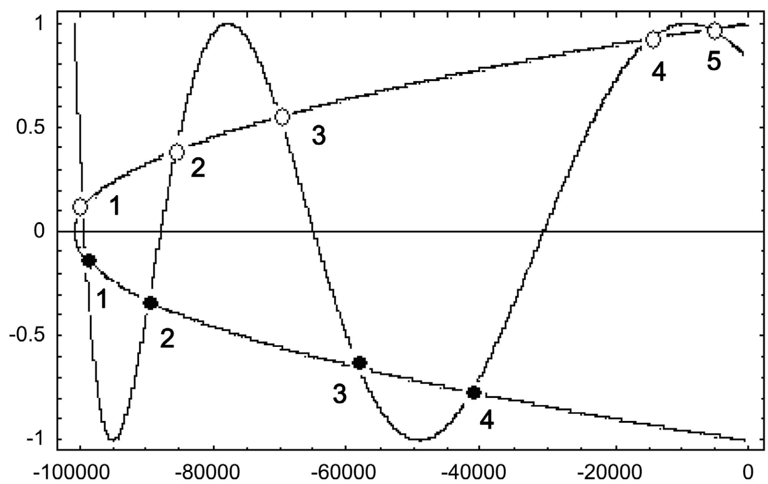

Figure 3. Graphical solution of Equation (18) with the same parameter as in **Figure 2**, only with exchanged values for the parameter $\Lambda = -0.05\ \text{cm}^{-2}$.

Table 2 presents numerical values for the bound state spectrum with $\Lambda = -0.05\ \text{cm}^{-2}$:

Table 2. Results of calculations for the $\frac{E_i}{|U_0|}$ for the set of parameters to **Figure 3**.

№ of state	1	2	3	4	5
○, $\frac{E_i}{\lvert U_0\rvert}$	0.994331	0.855337	0.700772	0.147282	0.0584696
●, $\frac{E_i}{\lvert U_0\rvert}$	0.9889920	0.895919	0.581965	0.413943	-

The wave functions of the eigenstates are calculated as follows:

$$\Psi_i(r) = \begin{cases} \dfrac{C_1 \sin\left(\lambda_{1(i)}\xi(r)\right)}{r}, & r \in [0,a] \\ \dfrac{C_3 e^{-|\lambda_{2(i)}|\xi(r)}}{r}, & r \notin [a,\infty] \end{cases} \tag{19}$$

where, taking into account (11), we can rewrite it in the following form:

$$\lambda_{2(i)} = \sqrt{\frac{2m}{\hbar^2}\left|E_i + \frac{\hbar^2\alpha^2}{2m}\right|} \quad \text{and} \quad \lambda_{2(i)} = \sqrt{\frac{2m}{\hbar^2}\left|E_i + |U_0|\right| + \alpha^2}.$$

Let us now consider the expression for the eigenwave function $\Psi_i(r)$ corresponding to the energy state E_i in general form, and then consider their behavior for the values of $i = 1,2,3,4,5$ respectively. We will use the Heaviside function $\eta(r)$:

$$\Psi_i(r) = \frac{1}{r}\Big[\eta\big[r(a-r)\big]C_1\sin\big(\lambda_{1(i)}\xi(r)\big) + \eta\big[r(a-r)\big]C_3\mathrm{e}^{-\lambda_{2(i)}\xi(r)}\Big], \tag{20}$$

which, taking into account the boundary condition (16), can be represented as:

$$\begin{aligned}\Psi_i(r) = \frac{C_1}{r}\Big\{&\eta\big[r(a-r)\big]\sin\big(\lambda_{1(i)}\xi(r)\big)\\ &+\eta\big[r(a-r)\big]\sin\big(\lambda_{1(i)}\xi_a\big)\mathrm{e}^{-\lambda_{2(i)}(\xi(r)-\xi_a)}\Big\}\end{aligned} \tag{21}$$

$$\xi(r) = \frac{1}{\alpha}\ln\Big[\alpha\cdot r + \sqrt{1+\alpha^2\cdot r^2}\Big],\quad \xi_a = \xi(r=a) = \frac{1}{\alpha}\ln\Big[\alpha\cdot a + \sqrt{1+\alpha^2\cdot a^2}\Big].$$

The family of wave functions that describe the formation of the i-th bound states of particles with mass m can be written in the form:

$$\begin{aligned}\Psi_{i(e)}(r) = \frac{C_\Sigma}{r}\Big\{&\eta\big[r(a-r)\big]\sin\big(\lambda_{1(i)}\xi(r)\big)\\ &+\eta\big[r(a-r)\big]\cos\big(\lambda_{2(i)}\xi_a\big)\mathrm{e}^{\lambda_{2(i)}(\xi_a-\xi(r))}\Big\}\end{aligned} \tag{22}$$

4. Discussion and Conclusions

In our article, we investigated the influence of the space-time curvature given by the DE, parameterized by value Λ. This work was mainly motivated by the JWST observations, from which it was possible to see the formation of galaxies at a very early time after the origin of the Universe. In our previous article, we investigated formation of the bound states for the matter under the influence of the DE which we described by the parameter Λ. In this article, we investigate it in more detail and using a simple model as an example, we show numerically how changing the Λ parameter changes the spectrum of eigenstates of the coupled system. For us it was interesting to show that DE can code simpler bound systems and form their physical properties. It means that information of the DE is saved in the structure of the energy spectrum of bound systems formed under the influence of the DE. Further we plan to generalize obtained results on the high dimensional spacetime using their metric in the given analytic form.

Some additional remarks we want to add due to the choice of the input parameters a, U_0 and Λ values. Namely, an influence of their values on the formation of the bound states was a subject of our investigation. In fact, the matter forms a potential well, which we define by its depth U_0 and width a and this local potential is induced by matter and does not depend directly from the time. From the time depends the contribution that come from the change of the $\Lambda \sim l(t)^{-2}$. In figures and tables, we showed how a change of the DE parameter Λ leads to the change of the bound states' energy spectrum.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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