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Out of Equilibrium Extended Electrodynamics, Dynamic Thomson Voltages and Helical Thermal Waves on Rotating Conductors Exposed to Chopped Laser Beam

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Abstract

We propose a new out of equilibrium thermodynamic approach to Schiff Electrodynamics aimed to generalize in rotating frames a recent reformulation of extended Ahranov-Bohm electrodynamic model. In accordance with these theories, we introduce a gauge breaking scalar field S linearly dependent on a thermal field T generated by a chopped laser beam, showing that under particular hypotheses it satisfies the hyperbolic telegraph equation. Exploiting then a particular gauge generalized condition suggested recently, we deduce that T and S are proportional to a thermoelectric scalar field which satisfies a Klein-Gordon equation, suggesting it can be interpreted as a dynamic Thomson voltage induced by rotation. We then illustrate briefly a more general theory of anisotropic heat diffusion on rotating conductors exposed to a chopped polarized laser beam formulated and developed in the Ph.D. thesis of the author. We show the existence of new helicoidal thermal waves that satisfy a telegraphist dissipative equation, whose isothermal wavefronts are quantized. We give a simple stationary estimate of a new dynamic Thomson effect induced by the chopped laser beam on the rotating conductors which is similar to a rotational Tolman effect. Finally, it is briefly outlined the relevance of the new anisotropic wavelike heat diffusion model proposed for paving the way to a new dynamic approach to thermal management and to future implementation of tunable thermal emissivity on thermal metamaterials bypassing conventional Kirchoff reciprocity law.

Keywords

Generalized Schiff Rotational Electrodynamics, Out of Equilibrium Extended Electrodynamic Theory, Polarized Heat Vortex Beams, Dynamic Thomson

Voltage, Quantized Helical Massive Thermal Waves, Rotational Tolman Cooling Effect

1. Introduction

Out of Equilibrium Rotational Extended Electrodynamics

In this work we will discuss in the first paragraph how to generalize the extended Ahranov-Bohm electrodynamics theory [1]-[3], called Extended Electrodynamic Theory in rotating frames taking in account the Schiff theory [4]-[7] and the out of equilibrium thermal effect induced by a chopped laser beam incident on a rotating conductive disk.

In fact, this well known generalization of Maxwell theory although it is actively investigated recently in the literature was never generalized, as far as we know, considering the coupling between dynamic thermal effects and induced electromagnetic fields. On the contrary, in our model, that we will illustrate in the second paragraph, it is assumed that the scalar field S of the Extended Electromagnetic Theory is proportional to the thermal field induced by a chopped laser polarized beam. We will show, assuming a new anisotropic wavelike heat diffusion model due to a generalized Righi-Leduc effect caused by a temperature dependent Barnett field B(T), that on the surface of a rotating disk heat propagates in helicoidal patterns transporting angular momentum [8].

Now we will start to illustrate briefly the Schiff theory of electrodynamics in rotating frames outlining its anomalous properties compared with the conventional Maxwell theory, that will be exploited to introduce gauge breaking scalar thermoelectric fields.

In fact, although the study of electromagnetic fields in a rotating system was already implicit in Biot-Savat's law of 1820 and had attracted Maxwell's attention in 1873, the success of Einstein's special theory of relativity directed research in the first decades of the last century towards the study of symmetries with respect to Lorentz's group of Maxwell's electromagnetic theory, perhaps also because in Einstein's revolutionary work of 1905 entitled "On the electrodynamics of bodies in motion", only uniform rectilinear motions of the observer and not rotational are analyzed.

Consequently, the study of electromagnetic waves in rotating systems, such as the Earth despite the light coming to us from the Sun, which is a rotating star, was not deepened, as far as we know, until 1939, when the physicist Schiff, inspired by Einstein's theory of general relativity, did not propose a first model [4]. In particular, in this work, the scientist introduced a tensor-based method to describe the laws of transformations of an electromagnetic field from an inertial to a rotating frame that inspired a new work by Irvine two decades later, in 1964 [5], where he introduced a different method based on orthogonal tetrads.

Although both theories are mathematically correct, the transformation laws are different, and this dichotomy inspired a debate that has developed, which still lasts today, on the correct formulation of a rotational electrodynamic theory implementing a covariant extension of Maxwell's relativistic theory [6] [7]. We note that, curiously, the original issue on electric field on rotational frames was already investigated by Faraday himself and inspired him to create his famous homopolar motor which showed that a metallic disk rotating parallel to an external magnetic field develops a voltage difference at its ends, that has been rediscovered in 1915 by Stewart and Tolman [5].

We will now briefly describe the generalization of Maxwell's theory in a rotating system formulated by Schiff and then reworked by Irvine, focusing the analysis on some aspects concerning out of equilibrium thermodynamics that we will use for the model we will expose in the next paragraph.

The Schiff-Irivine model is based on the following generalization of the first and the fourth of Maxwell's equations

$$div\boldsymbol{E}' = div\left(\boldsymbol{v}\times\boldsymbol{B}'\right) = div\left(\boldsymbol{\Omega}\times\boldsymbol{r}\right)\times\boldsymbol{B}' = 2\boldsymbol{\Omega}\cdot\boldsymbol{B}'\left(T\right) \tag{1}$$

$$div\boldsymbol{B}'\left(T\right) = -\frac{div\left(\boldsymbol{v}\times\boldsymbol{E}'\right)}{c^2} = -\frac{div\left(\boldsymbol{\Omega}\times\boldsymbol{r}\right)\times\boldsymbol{E}'}{c^2} \tag{2}$$

with $\boldsymbol{\Omega}$ the angular velocity of the rotating metal and $\boldsymbol{B}'\left(T\right)$ the effective thermal Barnett magnetic field associated to rotation [8].

These equations imply that rotation induces effective sources of electric and magnetic charge proportional to the angular velocity Ω, making a neutral disk an effective polarized metamaterial differently from what is predicted from Maxwell's theory in vacuum in inertial frames.

This prediction, although paradoxical for Maxwell's theory, can be interpreted as a byproduct of Lorentz invariance symmetry breaking in rotating systems, in a similar way to what has recently been discussed [6] on reworkings of the extended Ahranov-Bohm electrodynamic theory. In such a new model, the out-of-equilibrium electrodynamic state of a body is described by introducing a scalar magnetic field S, which will be shown to be proportional to a thermal field T, that it is defined, in accordance with Extended Electrodynamics theory [1]-[3], by breaking the Lorentz gauge condition by the following equation

$$S = \frac{\partial\varphi}{c^2\partial t} + div\boldsymbol{A} = \alpha\left(T - T_0\right) \tag{3}$$

where φ and $\boldsymbol{A}$ are respectively the scalar and vectorial electromagnetic field, α a dimensional constant, different for each metal, T the thermal field and T_0 the vacuum equilibrium temperature associated to cosmic microwave background.

We start from the conventional equations of Extended Electrodynamic Theory [1]-[3], which generalizes Maxwell's equations, taking in account the thermal field T induced by the Joule effect on the rotating metal

$$div\boldsymbol{E}' = -\frac{\partial S}{\partial t} = -\frac{\partial S}{\partial T}\frac{\partial T}{\partial t} = \tau\frac{\partial\rho}{\epsilon\partial t}; \tag{4}$$

$$rot\boldsymbol{B}' = \frac{\partial\boldsymbol{E}'}{c^2\partial t} + \nabla S \tag{5}$$

where the scalar magnetic field S satisfies the conventional non homogeneous wave equation [1], given by

$$\Delta S = -\mu\left(\frac{\partial\rho}{\partial t} + div\boldsymbol{J}\right) \tag{6}$$

with μ the medium magnetic permeability, while ρ the electric charge density and $\boldsymbol{J}$ the electric charge density vector are induced by rotation.

If we assume that the second member of (6) is proportional to the time derivative solidary to the rotating disk, it is possible to deduce that either S that the thermal field T satisfies the same telegraph wave equation,

$$\text{a)}\quad \Delta S = \tau\frac{\partial S}{c^2\partial t} = \frac{\partial S}{D\partial t}\,;\ \text{b)}\quad \Delta T = \tau\frac{\partial T}{c^2\partial t} = \frac{\partial T}{D\partial t} \tag{7}$$

We note that this approach, assuming a coupling between electromagnetic fields and thermal fields induced by Joule effect, once it is known the local electron relaxation time τ and the local thermal diffusivity D of the metal implements a general hyperbolic wavelike model of heat diffusion in accordance with a recent interesting model published recently by Markus and Gambar [9].

In fact it is possible to deduce the same telegraph Equation (6) either for S than for T, assuming that the charge density is time dependent and rewriting the second member of (4) by

$$div\boldsymbol{E}' = \tau\frac{\partial\rho}{\epsilon\partial t} \tag{8}$$

which together with the (4) gives after a simple integration in the time variable

$$S = -\frac{\epsilon}{\tau}\rho \tag{9}$$

In fact assuming that the charge density ρ and therefore S are scalar field associated to thermal vacuum polarization and proportional to the time delayed variation of the vacuum temperature associated to the cosmic microwave background

$$S \propto \Delta T(t+\tau) \tag{10}$$

and introducing a new dynamic Thomson field defined by

$$\nabla\varphi = -\boldsymbol{E}' = \alpha_{TH}(T)\nabla T = -\frac{\boldsymbol{J}}{\sigma} \tag{11}$$

with $\alpha_{TH}(T)$ the Thomson coefficient [8], and σ the electric conductivity, it is easy to deduce the telegraph equation for the vacuum scalar field S and the thermal field T (6a) and 6b) deduced previously using conventional Extended Electrodynamic Theory.

We note that, as discussed recently in the literature [1], this magnetic scalar wave, called in the literature a gauge breaking wave, although in inertial frames it does not carry energy, has the anomalous property of being associated to a negative energy density

$$u = -\frac{S^2}{2\mu_0} \tag{12}$$

with μ_0 the vacuum magnetic permeability.

We remark that since the scalar field S of (6), describing negative energy waves, are associated, in our approach, to out of equilibrium thermal electrodynamic processes, it can be assumed, as we did by (7a), to be explicitly dependent on the helicoidal thermal waves T investigated recently by the author [10], that will be described in detail in the following paragraph.

The new gauge breaking scalar field S defined in Equations (5) and (6) allows in our out of equilibrium generalization of Extended Electrodynamic Theory by exploiting (11) to introduce a generalized dynamic Thomson voltage associated to the out of equilibrium rotational electric field of (1) and (4).

It can be rewritten the new thermal dependent rotational electric field making it explicitly dependent on the thermal field T and the angular velocity Ω of the rotating disk by

$$\boldsymbol{E}'(T) = -\nabla\varphi(T,\Omega) \tag{13}$$

by identifying the Thomson voltage ΔV with the gauge breaking scalar field φ

$$\Delta V(T,\Omega) = \varphi(T,\Omega) \tag{14}$$

It can be shown [8] that the rotation induced electromotive force of (11) tends to contrast the Joule heating effect by the rotation induced enhanced Thomson cooling effect associated to thermal hysteresis loops similar to a recently published dynamic model on thermal management exploiting time dependent thermal gradients [11].

In our model a similar dynamic thermal hysteresis process, whenever confirmed experimentally in the future, could pave the way to a new approach to rotation induced control of thermal diffusion on thermal metamaterials which could find important technological applications.

We will mow show that the new Thomson thermal electromotive force of (11) satisfy Klein-Gordon equation and, by imposing the Zel'dovich [8], will satisfy the telegraph equation too, as the thermal field T.

In fact by imposing the following gauge breaking condition associated with the non-conservation of the charge on the rotating conductors [2].

$$div\boldsymbol{A} + \frac{\partial\varphi}{\partial tc^2} = -\frac{4M(T)\pi}{h}\varphi(T) \tag{15}$$

with

$$\frac{\mathrm{d}\varphi}{\mathrm{d}T} = \alpha_{TH}(T) \tag{16}$$

with α_{TH} the dynamic Thomson voltage induced by rotation and the chopped laser beam.

It is possible to show that either the new temperature dependent Thomson voltage $\varphi(T,\Omega)$ as the conventional field φ of Extended Electrodynamic Theory satisfies a relativistic Klein-Gordon equation [2].

$$\frac{\mathrm{d}^2\varphi(T)}{\mathrm{d}t^2} - c^2\nabla^2\varphi(T) = -c^2\mu^2(T)\varphi(T) \tag{17}$$

with $\frac{1}{\mu(T)} = \frac{h}{2\pi M(T)c}$ an effective thermal Compton wavelength whose inverse is proportional to the wave number k, and M the effective thermal inertia of heat waves propagating on the rotating conductor, in accordance with the Tolman effect [8].

We note that assuming that this equation admits time harmonic solution [8].

$$\varphi \propto \mathrm{e}^{-i\omega' t} \tag{18}$$

with $\omega' = \omega - m\Omega$ due, as we will see in the next paragraph, to the thermal analogue of the Zel'dovich effect, Equation (17) can be reformulated as a telegraphist like equation if it is satisfied the quadratic dispersive law

$$-c^2\mu^2 = -\frac{c^2k^2}{(2\pi)^2} = i\frac{\omega'}{\tau} \tag{19}$$

which implies a light speed with a complex index of refraction making the rotating disk an effective dissipative non linear metamaterial.

We remark, as discussed previously, that the condition (19) is in perfect accordance with a new model published in recent work [11] aimed to Lagrangian formulations of thermal fields coupled by Joule effect to electromagnetic fields, supporting, satisfying the telegraph equation, our wavelike model of heat diffusion that we are going to illustrate in the next paragraph.

In fact these new massive Thomson fields, if it is satisfied (18) are solutions of telegraph equation too and therefore are evidence, we think, of wavelike quantized polarized heat vortex beams induced by the Thomson effect on the rotating disks. We will interpret them as consequences of a generalized Tolman hypothesis on the inertia of heat in rotating frames [8], whose predicted cooling effect will be estimated in the following paragraph.

2. Quantized Helical Thermal Waves and Rotational Tolman Effect on Rotating Disks

The recent achievements on attosecond laser beams and more generally the applications of laser physics on thermal metamaterials based on the photothermal effect inspired some authors [12] [13] to investigate non Fourier wavelike heat diffusion model exploiting the well known Cattaneo-Vernotte model [14] [15]

We will assume the hyperbolic model of heat diffusion based on a generalized non Fourier thermal conduction equation and that the local equation of the energy balance has terms due to the Joule effect, the Thomson effect and the heat emitted per unit of time by radiation, according to Kirchoff's law, *i.e.* that it holds

$$\gamma\frac{\mathrm{d}T}{\mathrm{d}t} + div\boldsymbol{q} + w_{Joule}(T) + w_{TH}(T) + w_{\epsilon}(T) = 0 \tag{20}$$

with the convective time derivative integral to the rotating conductor given by, used in recent works to study rotational superradiance in the acoustic field [10] [16]

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \theta} \cdot \tag{21}$$

with Ω constant angular velocity of the rotating conductor.

We will also assume that the following energy balance condition holds for the nonlinear thermoelectric processes induced on the rotating conductor

$$w_{Joule}(T) + w_{TH}(T) + w_{\epsilon}(T) = 0 \tag{22}$$

with the rate of heat released in the unit of time per emission, the rate of heat dissipated by the Joule effect in the unit of time and the rate of heat seated or absorbed in the unit of time depending on the direction of the electric current, due to the Thomson effect.

We note that since nonlinear thermoelectric phenomena depend on the complex thermal field T, the absorption of heat in the rotating conductor will be accompanied by a complex dielectric constant, the thermal emissivity, will be dependent on this field T and on the angular velocity In fact, as we will see, due to a thermal analogue of the Zel'dovich effect of rotational superradiance [10] [16], the radiation emitted infrared will be modulable by varying the angular velocity of the rotating conductor, implementing the inertial effect an effective doping of the conductor, in a similar way to what has recently been studied on doped metamaterials with tunable emissivity $\epsilon(T,\Omega)$ [17] [18].

From (20) and (21), applying the divergence to the anisotropic heat flux vector defined by (1)

$$\boldsymbol{q}(t+\tau) = \boldsymbol{q}_k + \boldsymbol{q}_\Omega = -k\nabla T + A\boldsymbol{B}'(T) \times \nabla T \tag{23}$$

with k the thermal conductivity, $\boldsymbol{B}'(T)$ the Barnett's effective magnetic field parallel to the angular velocity vector of the body $\boldsymbol{\Omega}$, A the Righi-Leduc average coefficient [5]

$$\boldsymbol{B}'(T) = \frac{b(T)}{A}\boldsymbol{\Omega} \tag{24}$$

with a gyromagnetic unknown function $b(T) = 2e/m(T)$ to be determined experimentally, it can be deduced the hyperbolic telegraphist equation satisfied by the new thermal fields T

$$\gamma\tau \frac{\mathrm{d}^2 T}{\mathrm{d}t^2} + \gamma \frac{\mathrm{d}T}{\mathrm{d}t} - k\nabla^2 T = 0 \tag{25}$$

with γ and k respectively the specific heat and thermal conductivity of the conductor at rest, while τ is the local relaxation time of the rotating conductor.

Rewriting the two-dimensional Laplacian differential operator defined on the surface of the rotating disk

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{r \partial r} + \frac{\partial^2 T}{r^2 \partial \theta^2} \tag{26}$$

we will look for solutions corresponding to the Dirichlet harmonic boundary condition in time [12].

$$T(R, \theta = 0) = T_0 \mathrm{e}^{-i\omega' t} \tag{27}$$

As a result, the new thermal waves will carry medium energy, part of which will be partly emitted in the form of infrared radiation with nonlinear emissivity and a part will propagate by conduction and will be absorbed by the disk, or, based on the Zel'dovich effect, will be transferred to the thermal wave, as we will see, in the case where it has a negative pulsation.

This equation can be rewritten in the form of a nonlinear telegrapher's equation for a thermal field T propagating on the rotating disk surface, using (21) and (22), in accordance with a recent work [19]

$$\frac{\mathrm{d}^2 T}{\mathrm{d}t^2}+\frac{\mathrm{d}T}{\tau \mathrm{d}t}-\frac{k}{\gamma\tau}\nabla^2 T=\frac{\mathrm{d}^2 T}{\mathrm{d}t^2}+\frac{\mathrm{d}T}{\tau \mathrm{d}t}-v_T'^2\nabla^2 T=0 \tag{28}$$

with

$$v_T'=\frac{v_T}{n}=\sqrt{\frac{k}{\gamma\tau}} \tag{29}$$

is the variable phase velocity of the thermal wave dependent on an effective thermal refractive index n, which we will determine below.

We will also assume that the thermal field in the rotating system has a shift due to a rotational Doppler effect identical to that predicted by the Zel'dovich effect originally predicted for electromagnetic waves, and recently verified for both acoustic waves, surface waves and electromagnetic waves [20]

$$\omega'=\omega-m\Omega \tag{30}$$

with m being the integer associated with the angular momentum carried by the thermal field T, so that we have

$$\frac{\mathrm{d}T}{\mathrm{d}t}=-i\omega' T=-i\left(\omega-m\Omega\right)T \tag{31}$$

Before illustrating the new thermal waves predicted by the model, we want to point out that we will not describe the solutions of Equation (19) with general formalisms recently adopted to study the thermal Doppler effect generated by a laser in harmonic motion in interesting recent theoretical works by various authors [21].

In fact, they use the mathematical techniques usually applied in the telegraphist's equations based on the Laplace transformation with respect to time t and on the Fourier transformation with respect to spatial coordinates or on the integral decomposition technique, which, however, are based on the hypothesis, not valid in our model, that the dissipative parameter is constant, despite the fact that in the bibliography the solutions have been extended to the case in which it is variable in the case of the equation of the telegraphist of Nagumo [22].

Conversely, we will use the simplest technique based on the separation of the variables of the field T and assuming the existence of particular solutions given by

$$T\left(r,\theta,t\right)=T_0 \mathrm{e}^{i\left[\beta(r)r+m\theta-\omega' t\right]} \tag{32}$$

with local wave number given $\beta\left(r\right)$

$$\frac{\mathrm{d}\left(Re\beta(r)r\right)}{\mathrm{d}r} = f(r) \tag{33}$$

and m an integer number associated to the angular momentum $L = mh$ of the heat vortex beams due to incident polarized laser beam generating the propagation of the thermal field T on the rotating disk.

It is possible to deduce (1) the algebraic equation,

$$-\tau\left(\omega'^2 + \omega'\right) + k\left(f^2 + \frac{m^2}{r^2} - i\frac{\mathrm{d}f}{\mathrm{d}r} - i\frac{f}{r}\right) = 0 \tag{34}$$

and by imposing that this time of local relaxation is real, we obtain

$$\tau(r) = \frac{k}{\gamma\omega'^2}\left(f^2 + \frac{m^2}{r^2}\right) \tag{35}$$

with f satisfying the following first-order differential equation

$$\frac{\mathrm{d}f}{\mathrm{d}r} + \frac{f}{r} + \frac{\gamma\omega'}{k} = 0 \tag{36}$$

which is supplemented by the explicit formula of f as a function of r

$$f(r) = \frac{Rea}{r} - \frac{\gamma\omega'}{2k}r \tag{37}$$

with an arbitrary dimensionless constant fixed by the initial boundary condition on the thermal field T.

Inserting (37) as the second member of (33) gives the differential equation of the local wavenumber $\beta(r)$

$$\frac{\mathrm{d}\beta(r)r}{\mathrm{d}r} = \frac{a}{r} - \frac{\gamma\omega'}{2k}r \tag{38}$$

whose integration in r in an elementary way and, dividing by r, we obtain for the real part of the local wave number of the new thermal waves

$$Re\beta(r) = \frac{Rea}{r}\ln\frac{r}{R} - \frac{\gamma\omega'}{4k}r = \frac{R\omega'}{V_T ?r}\ln\frac{r}{R} - \frac{\gamma\omega'}{4k}r \tag{39}$$

with R radius of the disk.

By redefining the constant to dimensionless as a function of the pulsation, the radius of the disk R and the phase velocity of the thermal wave of the conductor at rest ω'

$$Rea = \frac{R\omega'}{v_T} \tag{40}$$

its value is obtained by knowing the phase velocity v_T of the conductor thermal wave at rest.

From the telegraphist's equation we derive the spiraling wavefront patterns of the quantized thermal fields, which in polar coordinates are given by

$$\theta(r) = \frac{R\omega'}{mv_T}\ln\frac{r}{R} - \frac{\gamma\omega'}{4\,\mathrm{km}}r^2 \tag{41}$$

since the curves defined by the previous relation are discretized spirals dependent on the azimuthal integer m, being proportional to the pulsation ω' given by the Zel'dovich condition (21).

Note that the particular isothermal curves defined by (41) are not well defined for *zero r*, and on the edge of the rotating disk become a simple parabolic plot with azimuthal angle of sign opposite to that of rotation of the conductor ω'

$$\theta(R) = -\frac{\gamma\omega'}{4\text{ km}}R^2 \tag{42}$$

Equation (41) proves that the wavefront of the isothermal profile are helixs that winds in the opposite direction to that of rotation of the conductor if it is positive, while in the same direction if it is negative, *i.e.* if the above-mentioned rotational superradiance phenomenon occurs due to the Zel'dovich condition.

We believe that such a sign reversal relation at the thermal Zel'dovich effect could be tested in the future in the Thermophotonics Laboratories allowing us to verify the existence of the thermal rotational superradiance effect predicted by our model.

For example, by placing the conducting disk on a rotating platform with angular velocity that satisfies the following inequality

$$\Omega \geq \omega = 100\text{ Hz} \tag{43}$$

with pulsation of the *laser* chopper the change of sign predicted by (32) should be observed by modulating the angular velocity increasing around 100 Hz.

The eventual experimental confirmation of (41) would allow to investigate in general the existence of the new phenomenon of rotational superradiance of thermal fields generated by the laser on rotating conductors, and, we believe, could stimulate the search for a new approach for the control of the thermal emissivity of rotating thermoelectric materials, not based, as studied so far, on the regulation of the doping of metamaterial films, but obtained by exploiting the coupling with the nonlinear dynamic Thomson effect previously discussed

In particular, we believe that the hyperbolic wavelike heat diffusion model is suitable to describe the anisotropic diffusion of heat on very small rotating disks, such as those composed of metamaterials, since they allow to control the heat flux and thus to modulate the rate of heat propagation [14] [15].

For the purpose of illustrating the potential technological applications of the model on thermal management, we can deduce an estimate of the temperature difference at the edge of the R disk between a rotating disk and an identical one at rest, both exposed to the same laser source with chopper.

Introducing the following dimensionless quantity and

$$\epsilon(R,\Omega,\theta=0) = \frac{Re\Delta T(R,\Omega)}{Re\Delta T(R,0)} \tag{44}$$

one can derive, exploiting its dependence on the parameters of the model,

$$\epsilon(R,\Omega,\omega,\theta=0) = \frac{\cos\frac{\gamma\omega'}{4\text{ km}}R^2}{\cos\frac{\gamma\omega}{4\text{ km}}R^2} \tag{45}$$

which, approximating to the second order the cosine function that appears at the second member, gives an estimate of the relative thermal gradients between the two disks given by

$$\epsilon(R,\Omega,\omega) \cong \frac{1-\frac{1}{2}\left(\frac{\gamma\omega'}{4\,\mathrm{km}}R^2\right)^2}{1-\frac{1}{2}\left(\frac{\gamma\omega}{4\,\mathrm{km}}R^2\right)^2} \tag{46}$$

We note that if $\omega = 0$, that is when the laser is switched off, the thermal gradient is non-zero only in the rotating conductor and becomes a quadratic function in the angular velocity, which by putting $\Omega = 100\,\mathrm{Hz}$, $m = 1$, $R = 1\,\mathrm{cm}$, gives the further simple estimate, for example for a ferromagnetic disk,

$$\epsilon(R,\Omega,0) \cong \Delta T(\Omega) \cong 1-\frac{\Omega^2}{2}\left(\frac{\gamma}{4k}\right)^2 R^4 \cong 1-\frac{2.2}{2}\times 10^{-4} \tag{47}$$

For example, if the temperature difference for the disk at rest is 10 K between the focus of the laser and the border R of the disk the relative cooling effect with respect to the rotating disk will be of the order of 1mK, that could be easily verified with modern thermocouples.

We remark that this new rotational Tolman cooling effect could be observed on every rotating metal and metamaterial and whenever confirmed could be exploited to tune dynamically magnetic phase transitions and thermal emission allowing violation of conventional Kirchoff law implementing non Planckian polarized thermal radiation, as recently investigated [23] [24].

3. Conclusions

We illustrated a new anisotropic wavelike model of heat diffusion exposed to a chopped polarized laser beam showing the existence of new quantized helicoidal thermal waves propagating on rotating conductors associated to a temperature dependent effective Barnett magnetic field. We deduced a naïve estimate of a relative average cooling effect induced by rotation which implements a generalized rotational Tolman effect.

We hope that the angular momentum of the new quantized helical thermal fields T predicted by our model can be tested experimentally and, whenever confirmed, will be exploited in the future to implement a new approach to thermal management and thermal harvesting allowing the realization of rotating oscillating thermos genetic batteries.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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