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Wormholes with Low Energy Density

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Abstract

In spite of their speculative nature, traversable wormholes are a topic of interest that started with the Einstein-Rosen bridge in 1935 and became a major research area with the introduction of the Morris-Thorne wormhole in 1988. It also became apparent in time that such wormholes are likely to be compact stellar objects, akin to neutron stars. Although widely discussed, wormholes having a low energy density may therefore not be massive enough to exist on a macroscopic scale. Important examples are wormholes based on a noncommutative geometry background and wormholes supported by the negative energy density sourced by the Casimir effect. The main goal of this paper is to invoke $f(Q)$ modified gravity to provide the extra degrees of freedom to help overcome these obstacles.

Keywords

Low-Energy-Density Wormholes, Noncommutative Geometry, Casimir Effect, $f(Q)$ Gravity

1. Introduction

Wormholes are handles or tunnels in spacetime connecting widely separated regions of our Universe or different universes altogether. While wormholes may be as good a prediction of Einstein's theory as black holes, they are subject to severe restrictions from quantum field theory, calling for the existence of "exotic matter" [1]. This violation is more of a practical than conceptual problem, as illustrated by the Casimir effect [2]: exotic matter can be made in the laboratory. Being a rather small effect, the practical challenge lies in generating and sustaining a negative energy density of sufficient magnitude and volume to support a macroscopic object. In this paper we discuss the more general problem of wormholes having a low energy density by invoking $f(Q)$ gravity, a fairly recent modification of Einstein's theory.

2. Background

2.1. Morris-Thorne Wormholes

In 1988, Morris and Thorne [1] proposed the following static and spherically symmetric line element for a wormhole spacetime, possibly motivated by the original Schwarzschild solution:

$$\mathrm{d}s^2 = -\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-b(r)/r} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \tag{1}$$

using units in which $c = G = 1$. Here $\Phi = \Phi(r)$ is called the *redshift function*, which must be finite everywhere to prevent the occurrence of an event horizon. The function $b = b(r)$ is called the *shape function* since it determines the spatial shape of the wormhole when viewed, for example, in an embedding diagram. The spherical surface $r = r_0$ is called the *throat* of the wormhole. According to Ref. [1], at the throat, $b = b(r)$ must satisfy the following conditions: $b(r_0) = r_0$, $b(r) < r$ for $r > r_0$, and $b'(r_0) < 1$, called the *flare-out condition*. This condition can only be met by violating the null energy condition (NEC), which states that

$$T_{\alpha\beta}k^\alpha k^\beta \geq 0 \tag{2}$$

for all null vectors k^α, where $T_{\alpha\beta}$ is the energy momentum tensor. Matter that violates the NEC is called "exotic" in Ref. [1]. In particular, for the radial outgoing null vector $(1,1,0,0)$, the violation reads $T_{\alpha\beta}k^\alpha k^\beta = \rho + p_r < 0$. Here $T^t{}_t = -\rho(r)$ is the energy density, $T^r{}_r = p_r(r)$ is the radial pressure, and $T^\theta{}_\theta = T^\phi{}_\phi = p_t(r)$ is the lateral (transverse) pressure. Our final requirement is *asymptotic flatness:* $\lim_{r\to\infty}\Phi(r) = 0$ and $\lim_{r\to\infty} b(r)/r = 0$.

For later reference, we now state the Einstein field equations:

$$\rho(r) = \frac{1}{8\pi}\frac{b'}{r^2}, \tag{3}$$

$$p_r(r) = \frac{1}{8\pi}\left[-\frac{b}{r^3} + 2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\right], \tag{4}$$

and

$$p_t(r) = \frac{1}{8\pi}\left(1-\frac{b}{r}\right)\left[\Phi'' - \frac{b'r-b}{2r(r-b)}\Phi' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r-b}{2r^2(r-b)}\right]. \tag{5}$$

2.2. Wormholes with Low Energy Density

As noted in the Abstract, wormholes are likely to be compact stellar objects, but the original formulation of Morris-Thorne wormholes [1] does not call for any such requirement. To appreciate the problem, let us recall that noncommutative geometry, an offshoot of string theory, can in principle support traversable wormholes. This is based on the realization that coordinates may become noncommutative operators on a *D*-brane [3] [4]. A critical feature based on the uncertainty principle is that noncommutativity replaces point-like particles by smeared objects [5]-[7]. The idea is to eliminate the divergences that normally occur in general relativity. As discussed in Ref. [6], this objective can be met by showing that spacetime can be encoded in the commutator $\left[\mathbf{x}^{\mu},\mathbf{x}^{\nu}\right]=i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an antisymmetric matrix that determines the fundamental cell discretization of spacetime in the same sense that Planck's constant $\hbar$ discretizes phase space. According to Refs. [8] [9], a relatively simple way to model the smearing is by means of the so-called Lorentzian distribution of minimal length $\sqrt{\gamma}$ instead of the commonly employed Dirac delta function, *i.e.*, we replace the point-like Dirac delta function by the following smooth distribution: the energy density of a static and spherically symmetric and particle-like gravitational source is given by

$$\rho(r)=\frac{m\sqrt{\gamma}}{\pi^{2}\left(r^{2}+\gamma\right)^{2}}. \tag{6}$$

The usual interpretation is that the gravitational source causes the mass m of a particle to be diffused throughout the region of linear dimension $\sqrt{\gamma}$ due to the uncertainty.

Based on these considerations, it is not immediately obvious how one can determine the size and mass of the wormhole. So we first need to lay the groundwork by following Ref. [6]. Here it is pointed out that it is possible to implement the noncommutative effects in the Einstein field equations $G_{\mu\nu}=\frac{8\pi G}{c^{4}}T_{\mu\nu}$ by modifying only the energy momentum tensor, while leaving the Einstein tensor $G_{\mu\nu}$ intact. It is emphasized in Ref. [6] that the noncommutative-geometry background is an intrinsic property of spacetime rather than some kind of superimposed structure. So this has a direct effect on the mass-energy and momentum distributions. The concomitant determination of the spacetime curvature then explains why the Einstein tensor can be left unchanged. As a consequence, when describing a wormhole, both the length scales and mass can be macroscopic, to be confirmed in Section 3. Moreover, noncommutative-geometry wormholes based on the Casimir effect are discussed in Ref. [10]. Both are examples of wormholes with a low energy density.

2.3. $f(Q)$ Gravity

Attempts to overcome the theoretical and practical problems confronting Morris-Thorne wormholes have relied heavily on various modified gravitational theories. A recently proposed modified theory, called $f(Q)$ gravity, is due to Jimenez, *et al.* [11]. Here Q is the non-metricity scalar from the field of differential geometry. The action for this gravitational theory is

$$S=\int\frac{1}{2}f(Q)\sqrt{-g}\,\mathrm{d}^{4}x+\int\mathcal{L}_{m}\sqrt{-g}\,\mathrm{d}x^{4}, \tag{7}$$

where $f(Q)$ is an arbitrary function of Q, $\mathcal{L}_{m}$ is the Lagrangian density of matter, and g is the determinant of the metric tensor $g_{\mu\nu}$. Even though it is a fairly new theory, numerous applications have already been found; see, for example, Refs. [11]-[20]. This topic will be discussed further in Section 5.

3. Throat Size and Mass

In this section, we will determine both the throat size and mass of the wormhole. So our first task is to obtain the shape function from Equations (3) and (6), previously discussed in Ref. [21].

$$
\begin{aligned}
b(r) &= \int_{r_0}^{r} 8\pi (r')^2 \rho(r')\mathrm{d}r' + r_0 \\
&= \frac{4m}{\pi}\left[\tan^{-1}\frac{r}{\sqrt{\gamma}} - \sqrt{\gamma}\frac{r}{r^2+\gamma} - \tan^{-1}\frac{r_0}{\sqrt{\gamma}} + \sqrt{\gamma}\frac{r_0}{r_0^2+\gamma}\right] + r_0 \qquad (8)\\
&= \frac{4m}{\pi}\frac{1}{r}\left[r\tan^{-1}\frac{r}{\sqrt{\gamma}} - \sqrt{\gamma}\frac{r^2}{r^2+\gamma} - r\tan^{-1}\frac{r_0}{\sqrt{\gamma}} + \sqrt{\gamma}\frac{r_0 r}{r_0^2+\gamma}\right] + r_0.
\end{aligned}
$$

Observe that $\lim_{r\to\infty} b(r)/r = 0$; to ensure asymptotic flatness, we retain the assumption $\lim_{r\to\infty}\Phi(r) = 0$. Here we can simply let $B = b/\sqrt{\gamma}$ be the form of the shape function even though $B(r_0) \neq r_0$. The reason is that B can be rewritten as a function of $r/\sqrt{\gamma}$:

$$
\begin{aligned}
\frac{1}{\sqrt{\gamma}}b(r) &= B\left(\frac{r}{\sqrt{\gamma}}\right) \\
&= \frac{1}{\sqrt{\gamma}}\frac{4m}{\pi}\left(\frac{\sqrt{\gamma}}{r}\right)\left[\frac{r}{\sqrt{\gamma}}\tan^{-1}\frac{r}{\sqrt{\gamma}} - \frac{\left(\frac{r}{\sqrt{\gamma}}\right)^2}{\left(\frac{r}{\sqrt{\gamma}}\right)^2+1} - \frac{r}{\sqrt{\gamma}}\tan^{-1}\frac{r_0}{\sqrt{\gamma}} \right. \qquad (9)\\
&\qquad \left. + \frac{r}{\sqrt{\gamma}}\frac{\frac{r_0}{\sqrt{\gamma}}}{\left(\frac{r_0}{\sqrt{\gamma}}\right)^2+1}\right] + \frac{r_0}{\sqrt{\gamma}}.
\end{aligned}
$$

Observe that

$$B\left(\frac{r_0}{\sqrt{\gamma}}\right) = \frac{r_0}{\sqrt{\gamma}}, \qquad (10)$$

the analogue of $b(r_0) = r_0$. Since B is a function of r, we may consider the line element

$$\mathrm{d}s^2 = -\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{B\left(r/\sqrt{\gamma}\right)}{r/\sqrt{\gamma}}} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \qquad (11)$$

It now becomes apparent that in view of Equation (10), this line element represents a wormhole with throat radius $r_0/\sqrt{\gamma}$, while retaining asymptotic flatness. It is shown in Ref. [21] that the flare-out condition is met. Given that γ is a small constant, it also follows that $r_0/\sqrt{\gamma}$ is macroscopic. Similar comments can be made about wormholes whose energy violation is due to the Casimir effect, whose energy density is given by $-\hbar c\pi^2/720r^4$.

We are now in a position to estimate the mass $m(r)$ of the wormhole. From Equation (6),

$$m(r)=\int_{r_0}^{r}\rho(r')4\pi(r')^2\,\mathrm{d}r'$$
$$=\frac{2m}{\pi}\left[\tan^{-1}\frac{r}{\sqrt{\gamma}}-\frac{r\sqrt{\gamma}}{r^2+\gamma}-\tan^{-1}\frac{r_0}{\sqrt{\gamma}}+\frac{r_0\sqrt{\gamma}}{r_0^2+\gamma}\right]. \tag{12}$$

Since m in Equation (6) represents the mass of a particle, we conclude that the mass $m(r)$ cannot be very large. It has been shown, however, that Morris-Thorne wormholes are actually compact stellar objects [22]. The implication is that noncommutative-geometry inspired and Casimir wormholes are likely to be microscopic after all. This will be discussed further in the next two sections.

4. Inflating Lorentzian Wormholes

Before continuing, let us briefly consider the question of inflating Lorentzian wormholes. It has been suggested that wormholes of the Morris-Thorne type may actually exist on microscopic scales and that a sufficiently far advanced civilization may therefore be able to enlarge such a wormhole to macroscopic size. This possibility was explored in Ref. [23] by assuming that the wormhole is embedded in a flat de Sitter space. Assuming that $\Phi'(r)\equiv 0$, the time-dependent inflationary background is given by

$$\mathrm{d}s^2=-\mathrm{d}t^2+\mathrm{e}^{2\chi t}\left[\frac{\mathrm{d}r^2}{1-b(r)/r}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right)\right], \tag{13}$$

where $\chi=\sqrt{\Lambda/3}$ and Λ is the cosmological constant. Suppose we now have two observers situated on opposite sides of the wormhole throat and separated by an initial proper distance l_0 at $t=0$. If $l(T)$ is the separation at the end of inflation at $t=T$, then, according to Ref. [23],

$$l_0<\frac{\mathrm{e}^{-\chi T}}{\chi}. \tag{14}$$

This yields $l_0<10^{-67}\text{ cm}\ll 10^{-33}\text{ cm}$, the Planck length. Since the Planck length is usually regarded as the smallest distance that makes physical sense, Condition (14) cannot be met. Similarly, an initially Planck-sized wormhole would be enlarged enormously and could even exceed our present cosmological horizon. So inflation alone could not give rise to macroscopic wormholes.

5. Wormholes in $f(Q)$ Gravity

Following the discussion in Ref. [16], the non-metricity scalar Q for line element (1) is given by

$$Q=-\frac{b}{r^2}\left[\frac{rb'-b}{r(r-b)}+\phi'\right] \tag{15}$$

and the field equations are

$$8\pi\rho(r)=\frac{1}{2r^2}\left(1-\frac{b}{r}\right)\left[2rf_{QQ}Q'\frac{b}{r-b}\right.$$
$$\left.+f_Q\left(\frac{b}{r-b}(2+r\phi')+\frac{(2r-b)(rb'-b)}{(r-b)^2}\right)+f\frac{r^3}{r-b}\right], \tag{16}$$

$$8\pi p_r(r)=-\frac{1}{2r^2}\left(1-\frac{b}{r}\right)\left[2rf_{QQ}Q'\frac{b}{r-b}\right.$$
$$\left.+f_Q\left(\frac{b}{r-b}\left(2+\frac{rb'-b}{r-b}+r\phi'\right)-2r\phi'\right)+f\frac{r^3}{r-b}\right], \tag{17}$$

$$8\pi p_t(r)=-\frac{1}{4r}\left(1-\frac{b}{r}\right)\left[-2r\phi' f_{QQ}Q'+f_Q\left(2\phi'\frac{2b-r}{r-b}-r(\phi')^2\right.\right.$$
$$\left.\left.+\frac{rb'-b}{r(r-b)}\left(\frac{2r}{r-b}+r\phi'\right)-2r\phi''\right)+2f\frac{r^2}{r-b}\right], \tag{18}$$

where $f=f(Q)$, $f_Q=\frac{\mathrm{d}f(Q)}{\mathrm{d}Q}$, and $f_{QQ}=\frac{\mathrm{d}^2f(Q)}{\mathrm{d}Q^2}$. To keep the analysis tractable, we will again follow Ref. [16] and assume that $\phi'(r)\equiv 0$, also called the "zero-tidel-force solution".

Our next step is to check the null energy condition given the $f(Q)$-gravity background. Using Equations (16) and (17), we get

$$\rho(r_0)+p_r(r_0)=\frac{1}{8\pi}f_Q\frac{1}{r_0^2}\left[b'(r_0)-1\right]<0 \tag{19}$$

due to the flare-out condition. So the NEC is indeed violated, as long as f_Q is positive.

Since we are primarily interested in qualitative results, our focus is necessarily more narrow. Returning to the non-metricity scalar Q, it was noted after Equation (7) that $f(Q)$ is an arbitrary function of Q. This gives us considerable leeway in choosing $f(Q)$, starting with the highly idealized form $f(Q)=\alpha Q+\beta$ [12], where α and β constants, also discussed in Ref. [21]. This form has one serious drawback however: since $f''(Q)=0$ and $f'(Q)=$ a constant, this produces the Einstein field equations with a cosmological constant [20]. Since $f(Q)$ is arbitrary, we could simply choose $f(Q)=\alpha Q^{1+\epsilon}+\beta$, $0<\epsilon\ll 1$. This form is arbitrarily close to $f(Q)=\alpha Q+\beta$, while avoiding the above drawbacks. In other words, we can view $f(Q)=\alpha Q+\beta$ as a convenient approximation that is sufficient for our purposes, as we will confirm below. (From now on, we assume that α is positive to ensure that f_Q is also positive.)

A preferred approach, proposed in this paper, is to use the form $f(Q)=cQ+d\mathrm{e}^{\alpha Q}$. Here $\alpha>0$ is assumed to be sufficiently small for the form $f(Q)=cQ+d\mathrm{e}^{\alpha Q}$ to become an adequate approximation for the linear form $f(Q)=cQ+d$. This will also allow us to use the simpler form

$$f(Q)=d\mathrm{e}^{\alpha Q}, \tag{20}$$

as can be readily shown: returning to Equation (16) with $\Phi'(r) \equiv 0$, we obtain

$$\begin{aligned} 8\pi\rho(r) &= \frac{1}{2r^3}(r-b)\Bigg[2rf_{QQ}Q'\frac{b}{r-b} \\ &\quad + f_Q\left(\frac{2b}{r-b} + \frac{(2r-b)(b'r-b)}{(r-b)^2}\right) + f\frac{r^3}{r-b}\Bigg] \\ &= \frac{1}{2r^3}\left[2rf_{QQ}Q'b + f_Q\left(2b + \frac{(2r-b)(b'r-b)}{r-b}\right) + fr^3\right], \end{aligned} \tag{21}$$

where $f_Q = d\alpha e^{\alpha Q}$ and $f_{QQ} = d\alpha^2 e^{\alpha Q}$. Observe that near the throat, where $b(r_0) = r_0$, the fractional part of the second term on the right-hand side dominates, making the other terms negligible. As a result,

$$8\pi\rho(r) \approx \frac{1}{2r^3}\left(d\alpha e^{\alpha Q}\right)\frac{(2r-b)(b'r-b)}{r-b}. \tag{22}$$

Solving for $b'(r)$, we get

$$b'(r) = \frac{1}{\alpha}(8\pi)\rho(r)\frac{2r^2}{de^{\alpha Q}}\frac{r-b}{2r-b} + \frac{b}{r} \tag{23}$$

and

$$b(r) = \frac{1}{\alpha}\int_{r_0}^{r}(8\pi)\rho(r')\frac{2(r')^2}{de^{\alpha Q}}\frac{r'-b}{2r'-b}\mathrm{d}r' + \int_{r_0}^{r}\frac{b(r')}{r'}\mathrm{d}r' + r_0. \tag{24}$$

We saw earlier that the energy density $\rho(r)$ can be quite small, based on our discussion of noncommutative-geometry and Casimir wormholes. To see the significance of using a small positive α, let us return to the linear form $f(Q) = \alpha Q + \beta$. It is shown in Ref. [21] that for the noncommutative-geometry case, Equation (6), the linear form $f(Q) = \alpha Q + \beta$, with $\beta = 0$, yields

$$b(r) = \frac{1}{\alpha}\frac{m\sqrt{\gamma}}{\pi^2}\left[\frac{\tan^{-1}\frac{r}{\sqrt{\gamma}}}{2\sqrt{\gamma}} - \frac{r}{2(r^2+\gamma)} - \frac{\tan^{-1}\frac{r_0}{\sqrt{\gamma}}}{2\sqrt{\gamma}} + \frac{r_0}{2(r_0^2+\gamma)}\right] + r_0. \tag{25}$$

Thanks to the free parameter α from $f(Q)$ gravity, the mass of the wormhole, $m(r) = \int_{r_0}^{r}\rho(r')4\pi(r')^2\,\mathrm{d}r' = \frac{1}{2}b(r)$ from Equation (3) can now be macroscopic. The point is that this conclusion is also valid for the general case, Equation (24), due to the assumption that α is sufficiently small. So by invoking $f(Q)$ modified gravity, we have shown that low energy-density wormholes can be macroscopic.

6. Summary

This paper discusses viable models for macroscopic wormholes characterized by a low energy density. Such wormholes may not have a sufficiently large mass to exist on a macroscopic scale. However, Morris-Thorne wormholes are likely to be compact stellar objects, akin to neutron stars, and would normally be quite massive. By invoking $f(Q)$ modified gravity, it is shown that the resulting extra degrees of freedom enable us to overcome these obstacles, thereby allowing certain wormholes to be sufficiently massive despite the low energy densities. Particular attention is paid to two important special cases, wormholes based on a noncommutative geometry background and wormholes whose energy violation is due to the Casimir effect.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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